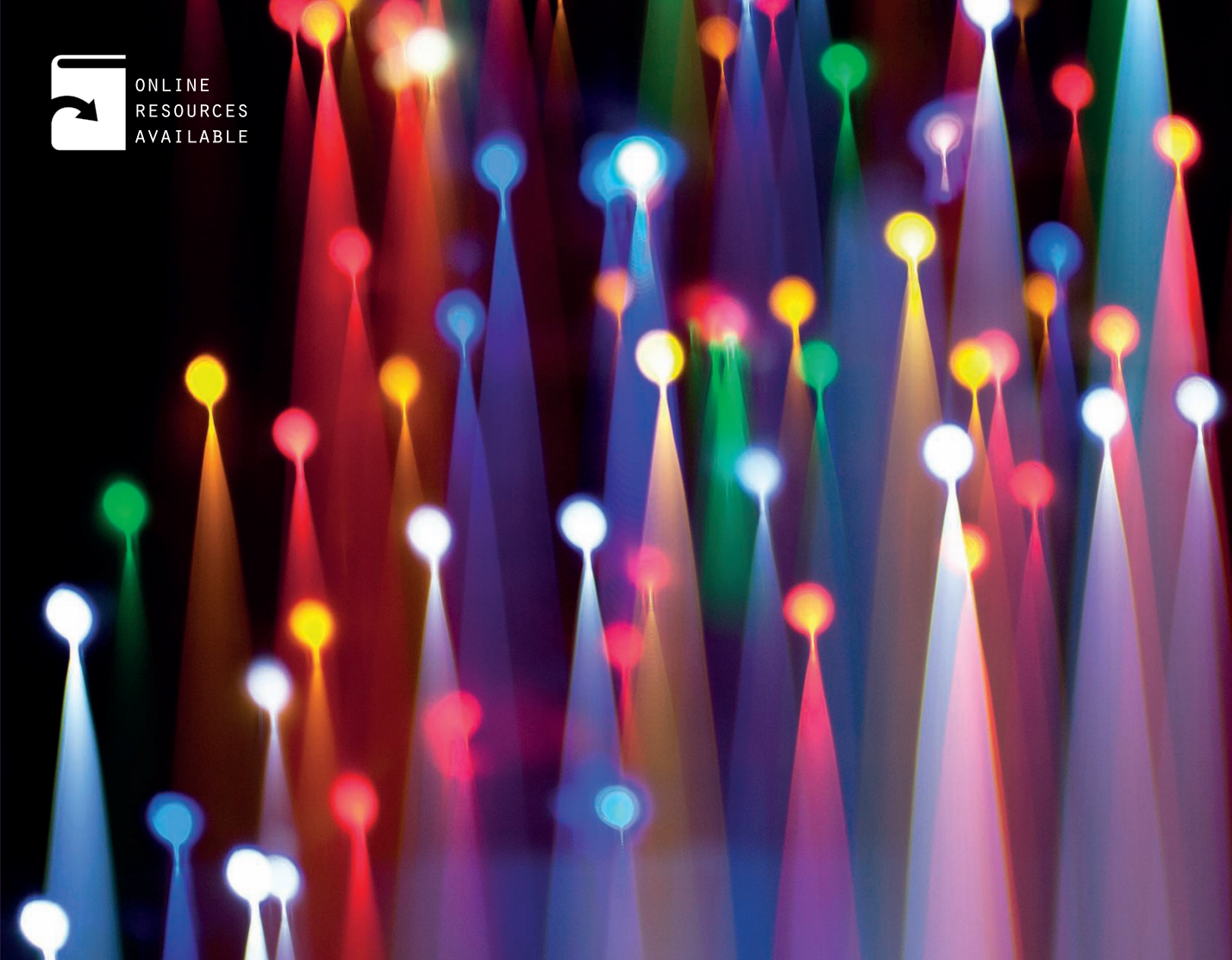




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Preface

This book provides a first course in physics for those studying access or foundation programmes in Physics at university or college, and for non-specialists on degree courses in Biological sciences, Environmental science, Chemistry and Engineering for whom physics is a subsidiary subject in their first year at university. The book is also suitable for trainee science teachers and medical students who need to develop a solid background in physics.

No previous experience of the subject is assumed as the book starts from 'scratch'. Guidance on how the book may be used is provided opposite. The introductory section on 'Units and Measurements' is designed to equip students with essential knowledge and skills in order to gain confidence and make rapid progress at the outset of the course. Each unit commences with a list of objectives and is divided into topics. Each topic introduces mathematical skills as appropriate and ends with questions to ensure the topic is assimilated before progressing to the next. A list of the 73 experiments described in the book is provided on pp. 497–8. Each unit ends with a summary of the key points and a set of revision questions. The book contains over 400 questions with complete solutions as well as a further 115 questions with numerical answers only.

The fourth edition includes all the content in the third edition as well as an additional unit ([unit 30](#)) on rotational dynamics. Practical experiments on rotational dynamics in [unit 30](#) have been added to the website which may be found at www.palgrave.com/foundations/breithaupt. The website contains case studies on applications of physics. Notes to develop the essential mathematical skills featured in the book have been added to the website and a section on calculus has also been added. In addition, the spreadsheet modelling features referred to in the book

can be accessed through the website, which also includes proofs and experiments. Further colour has been introduced in this edition in the text and in the figures.

The units of the book are grouped into parts, with each part being a well-defined area of Physics. The first four parts (Mechanics, Materials, Waves and Electricity) each provide an independent route into the subject, thus providing flexibility for lecturers as well as for students. The part on Fields and the part on Atomic and Nuclear Physics build on topics studied in the first four parts. The final part on Further Physics provides an opportunity for selected in-depth studies appropriate to the needs of each student, either in support of specialist studies in another academic discipline or in preparation for further specialist studies in Physics or another academic discipline.

The final part of the book includes an appendix on the use of spreadsheets in physics, an appendix listing all the experiments described in the book and an appendix on where mathematical skills are introduced in the book. In addition, a summary of equations and a list of useful data are provided. Complete solutions to all revision questions in the book are provided in this part. Key terms, definitions and equations are highlighted in bold in the text and a glossary of key terms is also provided for reference and revision purposes.

The features described above are intended to enable the book to be used by the student in class, in the laboratory, in 'tutorial' workshops and for private study. The book should provide flexible support for teachers of foundation courses in Physics, used as appropriate with students according to their backgrounds and specific needs. Whether you are a student or a teacher, I hope the book serves your needs.

Jim Breithaupt

How to use this book

To the student

- This book is designed to introduce you to Physics and to give you confidence to use the subject or to progress to further studies in the subject.
- The book is designed so that you can work on the subject with the minimum of help.
- Make sure you master the introduction 'Units and Measurements' at the outset.
- Take note of the advice offered at the start of each part.
- If possible, study each unit topic-by-topic in sequence.
- Use the glossary of key definitions when a term from another topic holds up your progress.
- Do the questions at the end of each topic and at the end of each unit. When you complete each question, check your solution against the solution provided in the answers section. Questions with numerical answers only are also provided for further practice.
- Move on to the next topic only when you are satisfied that you have mastered the previous topic.
- Establish where your physics course links to your other courses and develop these links as you progress through your physics course.
- Use the location guide to mathematical skills (see p. 499) if necessary when a difficult mathematical skill appears. The guide will direct you to relevant notes and exercises (with answers) on the website, www.palgrave.com/foundations/breithaupt.
- Keep a folder of your laboratory reports and retain your own solutions for revision purposes.
- Use the website as appropriate. The mathematical exercises are intended to be used when practice is needed in each mathematical skill. The spreadsheet section provides rapid access to software for modelling dynamics and decay processes referred to in the book. Case studies are provided to illustrate applications of basic principles. Some proofs and experiments from the first edition are now only on the website and these are for reference as appropriate.

To the teacher

The website www.palgrave.com/foundations/breithaupt is organised in 8 sections which are listed below. A reference table on page 2 of the website gives a summary of each section unit by unit.

- 1 Data analysis.
- 2 Further experiments and investigations.
- 3 Further proofs.
- 4 Notes and exercises on essential mathematics with answers.
- 5 Spreadsheet models.
- 6 Case studies.
- 7 An introduction to calculus.

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The production of this book has involved a considerable number of people and I wish to thank all who have contributed to the task of converting the manuscript into the book. In particular I would like to thank Frances Arnold, Suzannah Burywood and Bryony Ross of the publishers, and Ann Edmondson (editor) for their particular expertise, advice and courtesy.

I am indebted to my wife and my daughters for their cheerful support and encouragement throughout.

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Safety note

Whilst all the practical activities in this book have been checked for safety implications, no liability is accepted for any loss, damage or injury arising from the use of the information provided. A risk assessment must be done by the teacher before any practical work (including demonstrations) is carried out. All necessary safety information should be supplied to students by the teacher.

Units and Measurements

U-1 Philosophy and practice in science

Experiments play a vital part in the development of all branches of science. Over five hundred years ago, Nicolas Copernicus' theory that the Earth and the planets orbit the Sun was rejected by the Church because it displaced the Earth from the centre of the Universe. Copernicus had no experimental evidence to support his theory, so there was no reason to change the theory established in Ancient Greece. Many years after Copernicus, Galileo used the newly invented telescope to observe the motion of the moons of Jupiter and he realised that the planets orbit the Sun just like the moons of Jupiter orbit Jupiter. Galileo was arrested when he published his evidence for the Copernican view of the Universe but his evidence could not be denied as it was based on observations. See [Fig. U.1](#).

Scientists now believe the Universe originated in a massive explosion, referred to as the Big Bang. The evidence is based on accurate and reliable measurements by astronomers. In the mid-twentieth century, some astronomers theorised that the Universe was in a steady state, i.e. the same now as it has always been. The steady state theory was rejected in favour of the Big Bang Theory when astronomers discovered the presence of low-intensity microwave radiation in all directions in space radiation that originated in the Big Bang which is still travelling through space. The controversy was settled by experimental evidence which supports the Big Bang theory. Nevertheless, experimental evidence in support of a theory is not the same as proof, and just one experiment is all that is needed to disprove a theory. The more experimental tests that are found to support a theory, the stronger that theory becomes.

U-2 The S.I. system of units

Different units for the same quantity can be very misleading. For example, energy supplied by gas is measured in therms, whereas energy supplied by electricity is in kilowatt hours. If gas and electricity suppliers used the same unit, the consumer would be able to compare costs more effectively. Scientists have agreed on a single system of units to avoid unnecessary effort and time converting between different units of the same quantity. This system, the **Système International**, is based on a defined unit for each of five physical quantities, as listed in [Table U.1](#). Units of all other quantities are derived from the S.I. base units.

CONTENTS

- U.1 Philosophy and practice in science
- U.2 The S.I. system of units
- U.3 Symbols and equations in physics
- U.4 Making measurements in physics
- U.5 Uncertainty and accuracy
- Summary
- Revision questions



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Fig. U.1 Galileo, the father of the Scientific Age

Table U.1 S.I. base units

Quantity	Unit
Mass	kilogram (kg)
Length	metre (m)
Time	second (s)
Electric current	ampere (A)
Temperature	kelvin (K)

Note

The cubic centimetre (cm^3) and the gram (g) are used widely outside science and are therefore allowed exceptions to the system of prefixes illustrated here.

Numerical prefixes

These are used to avoid unwieldy numerical values. The most common numerical prefixes are:

nano-	10^{-9}	(n)
micro-	10^{-6}	(μ)
milli-	10^{-3}	(m)
kilo-	10^3	(k)
mega-	10^6	(M)
giga-	10^9	(G)

Numbers and units**Derived units**

The following examples show how the units of all other physical quantities are derived from the base units.

- (a) The unit of area is the square metre, abbreviated to m^2 .
- (b) The unit of volume is the cubic metre, abbreviated to m^3 .
- (c) Density is defined as mass per unit volume. Its unit is the kilogram per cubic metre, abbreviated to kg m^{-3} . The negative sign in a unit means 'per' (e.g. per m^3).
- (d) The unit of speed is the metre per second, abbreviated to m s^{-1} .

Standard form

Numerical values smaller than 0.001 and greater than 1000 are usually expressed in **standard form** for convenience. The numerical value is written as a number between 1 and 10, multiplied by an appropriate power of ten corresponding to how many places the decimal point needs to be moved until the number is between 1 and 10. If the decimal point is moved to the right, the power of 10 will have a negative value. Engineers usually express numerical values in **ternary form** where the power of 10 is a multiple of 3 (i.e. 10^3 , 10^6 , etc.)

Examples

- (a) The mean radius of the Earth is 6 360 000 m. The decimal point after the final zero needs to be moved six places to the left to arrive at 6.36. Therefore, in standard form, the Earth's radius is 6.36×10^6 m.
- (b) The speed of light in free space is 300 000 000 m s^{-1} . This is written as $3.0 \times 10^8 \text{ m s}^{-1}$ in standard form.
- (c) The wavelength of yellow light from a sodium lamp is 0.000 000 59 m or 5.9×10^{-7} m in standard form. Note the negative power of 10 because the decimal point is moved to the right to arrive at 5.9.

Significant figures

If a numerical value is expressed in standard notation, the number of digits in the decimal number is the number of significant figures of the numerical value. So, in Example (c), the wavelength of yellow light is given to 2 significant figures whereas in (a), the radius of the Earth is expressed to 3 significant figures. In Example (b), the speed of light is expressed to 2 significant figures as 3.0×10^8 ; a value of 3×10^8 is expressed to one significant figure only.

Calculators give numerical answers to as many significant figures as there are digits on the display. This can be misleading, since the answer to a calculation should always be to the same number of significant figures as the data supplied. Most data for problem-solving exercises are usually given to no more than 3 significant figures. A numerical answer should therefore be rounded up or rounded down to match the number of significant figures of the data supplied. The general rule is to round up if the last digit is 5 or more; round down otherwise. For example, a value of 6.36 rounded to 2 significant figures would become 6.4, whereas a value of 6.33 would become 6.3.

U.1 QUESTIONS

- 1 Copy and complete the following conversions.
 - a) (i) 500 mm = _____ m, (ii) 3.2 m = _____ cm, (iii) 9560 cm = _____ m.
 - b) (i) 0.45 kg = _____ g, (ii) 1997 g = _____ kg, (iii) 54000 kg = _____ $\times 10$ g.
- 2 Write the following values in standard form to 3 significant figures.
 - a) 150 million kilometres in metres,
 - b) 365 days in seconds,
 - c) 630 nanometres in metres,
 - d) 25.78 micrograms in kilograms,
 - e) 1502 metres in millimetres,
 - f) 1.245 micrometres in metres.

Reminder

Mass:

$$1000 \text{ g} = 1 \text{ kg}$$

Length:

$$1000 \text{ mm} = 100 \text{ cm} \\ = 1 \text{ m}$$

Area:

$$10^6 \text{ mm}^2 = 10^4 \text{ cm}^2 \\ = 1 \text{ m}^2$$

Volume:

$$10^9 \text{ mm}^3 = 10^6 \text{ cm}^3 \\ = 1 \text{ m}^3$$

U-3 Symbols and equations in physics

Physical quantities are usually represented by a standard symbol. For example, time is usually represented by the symbol t . Greek symbols are sometimes used (e.g. the Greek letter ρ (pronounced 'rho') is used for density). The symbol represents a numerical value and its unit.

Equations and formulae in physics are usually presented in symbolic form. For example, the word equation

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

can be written

$$\rho = \frac{m}{V}$$

Note that the S.I. unit of density is kilogram per cubic metre, usually abbreviated as kg m^{-3} .

To use an equation:

- write down the equation and the values of the known quantities
- rearrange the equation so the unknown quantity is the subject of the equation
- substitute the known values into the rearranged equation. In simple equations, include the units
- calculate the unknown quantity and write down its numerical value and unit.

WORKED EXAMPLE

Calculate the mass of a steel block of volume 0.15 m^3 and density 7900 kg m^{-3} .

Solution

$$\rho = \frac{m}{V} \quad V = 0.15 \text{ m}^3 \quad \rho = 7900 \text{ kg m}^{-3}$$

Rearranging the equation gives

$$m = \rho V = 7900 \text{ kg m}^{-3} \times 0.15 \text{ m}^3 \\ = 1185 \text{ kg.}$$

Note

Electronic calculators save time, provided you don't press the wrong button. Calculator layouts and usage may differ from one type to another but the essential principles of operation remain unchanged. A scientific calculator is needed for physics at this level.

WORKED EXAMPLE

The area A of a circle of diameter d is given by the formula $A = \frac{\pi d^2}{4}$.

Calculate the diameter of a circular plate of area 0.10 m^2 .

Solution

Rearrange the equation to give $d^2 = \frac{4A}{\pi}$

Substituting $A = 0.10 \text{ m}^2$ gives $d^2 = \frac{4 \times 0.10}{\pi} = 0.127 \text{ m}^2$

Hence $d = \sqrt{0.127} = 0.36 \text{ m}$ (to 2 sig. figs).

U-4 Making measurements in physics

You will meet a wide range of instruments for making measurements in your physics course. Instruments such as stopwatches, metre rules and top-pan balances are easier to use than others, such as oscilloscopes and Geiger counters. Many instruments, such as frequency meters and electrical multimeters, have digital read-outs instead of scales. However, accurate and reliable measurements from any instrument require practice and care. Here are some hints and tips on how to use the more common instruments.

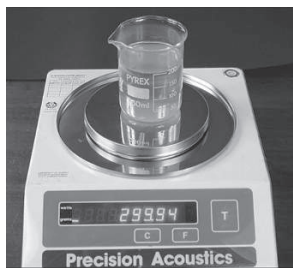


Fig. U.2 A top-pan balance



Fig. U.3 Using vernier calipers



Fig. U.4 Using a micrometer

- The **top-pan balance** is used to measure mass. See Fig. U.2. It should be level and in a draught-free environment. Its read-out can easily be checked using a set of standard masses. Most top-pan balances have more than one range. To avoid overloading a balance, always use the highest range first and then select the most appropriate range for the object under test. A 'tare' button on the balance enables the read-out to be set to zero with a beaker on the pan. If the beaker is removed, filled and then replaced, the read-out gives the mass of its contents directly. The precision of a balance (e.g. to 0.01 g) is given by the least significant figure on the read-out.
- **Vernier calipers** are used to measure the length of an object to within 0.1 mm , for lengths up to about 200 mm . They may be used to measure the external diameter or the internal diameter of a pipe. Fig. 3 shows a vernier scale. Each interval of the slider scale is 0.9 mm . This is so that the position of the zero of the slider scale beyond the nearest millimetre mark on the main scale can be determined by seeing where a mark on the slider scale is aligned with a millimetre mark on the main scale.
- A **micrometer** is used to measure lengths to within 0.01 mm , up to about 50 mm . Fig. U.4 shows a micrometer. Each complete turn of the barrel alters the gap by 0.5 mm exactly. There are usually 50 equal divisions on the barrel scale so that each division corresponds to 0.01 mm . If used to measure the diameter of a wire, several readings should be made at different positions and an average value calculated.
- A **stopwatch** is used to measure time intervals and may be checked against the 'speaking clock' on any phone system. Human error in starting and stopping a stopwatch can be minimised by counting down.

Density tests

1. Regular solids

Measure the mass and dimensions of rectangular and cylindrical blocks of different materials. Calculate the volume of each object using the appropriate formula from Fig. U.5. Use your results to calculate the density of each object, using the formula

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

If possible, use a data book of density values to identify the material of each object. Record your measurements and calculations.

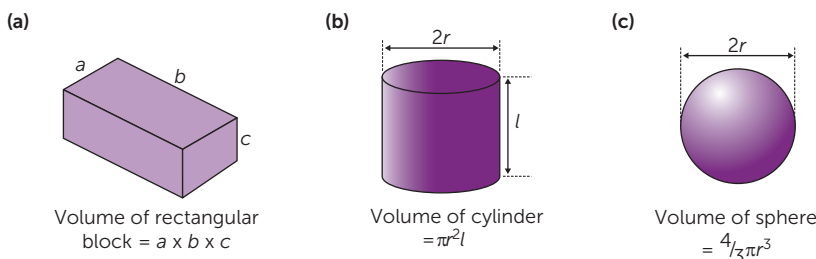


Fig. U.5 Volume formulae

2. Liquids

Measure the mass of a given quantity of liquid using a suitable beaker and a top-pan balance. Also, measure the volume of the liquid using a suitable measuring cylinder, as in Fig. U.6a. Hence calculate the density of the liquid.

3. Irregular objects

Measure the mass of the object. Then suspend the object from a thread and immerse it in water in a measuring cylinder, as in Fig. U.6b. The rise of the water level is equal to the volume of the object. Hence calculate the density of the object.

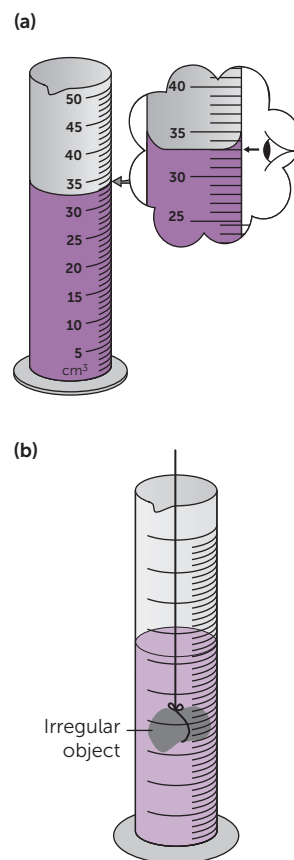


Fig. U.6 Using a measuring cylinder (a) to measure the volume of a liquid, (b) to measure the volume of an irregular object

U.2 QUESTIONS

- A rectangular steel block has dimensions 120 mm $\times$ 80 mm $\times$ 25 mm. Its mass is 1900 g. Calculate
 - its mass in kg,
 - its volume in m^3 ,
 - its density in kg m^{-3} .
- A solid metal cylinder of diameter 32 mm and length 22 mm has a mass of 48 g. Calculate
 - its mass in kg,
 - its volume in m^3 ,
 - its density in kg m^{-3} .
- An empty can of mass 160 g was filled to a depth of 120 mm with water. The total mass of the can and contents was 1.120 kg. The density of water is 1000 kg m^{-3} .
 - Calculate (i) the mass of water in the can, (ii) the volume of water in the can, (iii) the internal diameter of the can.
 - A glass object was then immersed in the can, causing the water level to rise by 5 mm. The mass of the tin and its contents was re-measured and found to be 1.225 kg. Calculate (i) the mass of the glass object in kg, (ii) its volume in m^3 , (iii) its density in kg m^{-3} .

Note

Give your density values in kg m^{-3} . Use the conversion factors on p. 3 to convert from grams to kilograms, etc. Measuring cylinders are calibrated in cm^3 . To calculate a volume in m^3 , divide by the volume in cm^3 by 10^6 . Note that $1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$.

U-5 Uncertainty and accuracy

In the density tests on p. 5, how confident can you be of your results? Statements such as ‘I’m very confident about my results because I was very careful’ carry little weight in science. Measurements need to be checked by repeating the readings for each measurement and using them as outlined below to calculate the following:

- 1 The **mean value**, which is obtained by adding the readings and dividing by the number of readings. For example, suppose a stopwatch is used to time 20 swings of a pendulum five times, giving the following measurements.

25.2 s 24.7 s 25.1 s 24.7 s 24.9 s

The mean value of the timing is therefore

$$\frac{25.2 + 24.7 + 25.1 + 24.7 + 24.9}{5} = 24.9 \text{ s.}$$

Note

The probable error of a measurement is an alternative indicator of the spread of a set of readings.

- 2 The **uncertainty**, which is a measure of the spread of the readings. For example, the timings above are mostly within 0.2 s of the mean value. A reasonable estimate of the uncertainty in the timing is therefore 0.2 s. The mean value of the timing may then be written as 24.9 ± 0.2 s. More precise statistical methods for obtaining the uncertainty of a measurement are used by scientists and engineers. At this level, there is no need to go further than the ‘estimating’ method outlined above.

Repeated readings of a measurement with a sufficiently precise instrument will, in general, give a spread of readings. The readings will vary due to **errors** of measurements which may be human errors (e.g. reaction time when stopping and starting a stopwatch) or errors due to varying conditions. If there is no pattern to the variation in a set of readings, the errors are said to be **random**. If, however, some sort of pattern or trend is evident in successive measurements, the errors are said to be **systematic**.

If repeated readings of a measurement are the same, the instrument could be replaced by a more precise instrument. If this is not possible, the precision of the instrument is a reasonable estimate of the uncertainty of the measurement. For example, suppose the read-out on a top-pan balance shows 35.6 g. The measurement could lie between 35.55 and 35.65 g, as shown in Fig. U.7. The uncertainty in the measurement is therefore 0.05 g. Since there is also a similar uncertainty for the zero reading, the uncertainty in the mass is therefore 0.1 g. The value of the mass is therefore written as 35.6 ± 0.1 g.

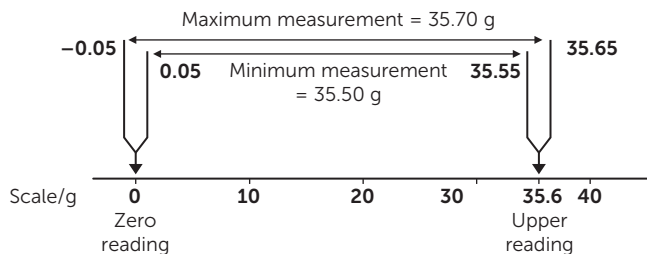


Fig. U.7 Probable errors

The **overall uncertainty** of a quantity calculated from several measurements can be estimated by using the upper and lower limits of each measurement. For example, suppose the width of a rectangular sheet of metal lies between 20.8 and 21.2 mm and its length lies between 26.8 and 27.2 mm. The area of the sheet therefore lies between 557 mm^2 ($= 20.8 \text{ mm} \times 26.8 \text{ mm}$) and 577 mm^2 ($= 21.2 \text{ mm} \times 27.2 \text{ mm}$). The area is therefore $567 \pm 10 \text{ mm}^2$.

WORKED EXAMPLE

A metal cube has dimensions 20 mm x 20 mm x 20 mm, each dimension accurate to 0.2 mm. Its mass is $64.2 \pm 0.2 \text{ g}$. Calculate

- its volume,
- the uncertainty in its volume,
- its density,
- the uncertainty in its density.

Solution

a) Volume $= 20 \times 20 \times 20 \text{ mm}^3$
 $= 8000 \text{ mm}^3 = 8.0 \times 10^{-6} \text{ m}^3$.

b) Maximum volume $= 20.2 \times 20.2 \times 20.2 \text{ mm}^3 = 8240 \text{ mm}^3$.

Minimum volume $= 19.8 \times 19.8 \times 19.8 \text{ mm}^3 = 7760 \text{ mm}^3$.

The uncertainty in the volume is therefore $240 \text{ mm}^3 = 2.4 \times 10^{-7} \text{ m}^3$.

c) Density $= \frac{\text{mass}}{\text{volume}} = \frac{64.2 \times 10^{-3} \text{ kg}}{8.0 \times 10^{-6} \text{ m}^3}$
 $= 8025 \text{ kg m}^{-3}$.

d) The maximum value of the density
 $= \frac{\text{maximum mass}}{\text{minimum volume}} = \frac{64.4 \times 10^{-3} \text{ kg}}{7760 \times 10^{-9} \text{ m}^3}$
 $= 8300 \text{ kg m}^{-3}$.

The minimum value of density

$$= \frac{\text{minimum mass}}{\text{maximum volume}} = \frac{64.0 \times 10^{-3} \text{ kg}}{8240 \times 10^{-9} \text{ m}^3}$$

$$= 7770 \text{ kg m}^{-3}.$$

The value of the density is therefore $8030 \pm 270 \text{ kg m}^{-3}$ to 3 significant figures.

The uncertainty of 270 kg m^{-3} is approximately 3% of the measured density.

The percentage uncertainty or 'error' in the measured density is therefore 3%.

See www.palgrave.com/foundations/breithaupt for more about uncertainties in calculations.

EXERCISE

Revisit the density tests you carried out and estimate the uncertainty in your measurements. Hence determine the percentage uncertainty for each result.

SUMMARY

- The **S.I. system** of scientific units is based on the metre, the kilogram, the second, the ampere (electric current) and the kelvin (temperature). All other units are derived from these five base units.
- **Prefixes** commonly used in science are nano- (n), micro- (μ), milli- (m), kilo- (k), mega- (M) and giga- (G), representing 10^{-9} , 10^{-6} , 10^{-3} , 10^3 , 10^6 and 10^9 respectively.
- **Commonly used instruments** in physics include vernier calipers and the micrometer (lengths), the top-pan balance (mass), the stopwatch (time intervals) and the measuring cylinder (volume).
- The **uncertainty** of a measurement gives the range within which the measurement lies.
- **Area and volume formulae**

The area of a circle of radius $r = \pi r^2$

The surface area of a sphere of radius $r = 4\pi r^2$

The volume of a rectangular block = length $\times$ width $\times$ height

The volume of a cylinder of radius r and height $h = \pi r^2 h$

The volume of a sphere of radius $r = \frac{4}{3}\pi r^3$

REVISION QUESTIONS

1 Which of the following instruments A–E, would you use to measure

- the volume of a liquid,
- the thickness of a coin,
- the mass of a beaker,
- the diameter of a wire?

A vernier calipers B top-pan balance
C measuring cylinder D mm rule E micrometer

2 Fig. U.8 shows a top-pan balance with an empty beaker on. The balance read zero before the beaker was placed on the pan.

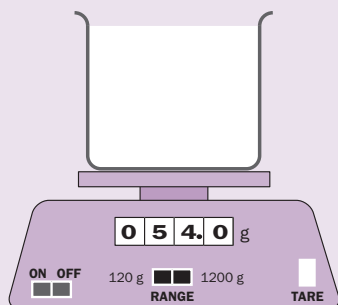


Fig. U.8

- What is the mass of the beaker?
 - How accurate is the read-out?
- The beaker was removed and partly filled with liquid. When the beaker was replaced on the pan, the read-out displayed 157.4 g.
 - Calculate the mass of liquid in the beaker.
 - Write down the uncertainty in the mass of liquid.

c) The liquid was poured into the measuring cylinder shown in Fig. U.9.

- What is the volume of the liquid in the measuring cylinder in Fig. U.9?
 - Estimate the uncertainty in this volume measurement.
- Calculate the density of the liquid.
 - Estimate the percentage uncertainty in this value of density.

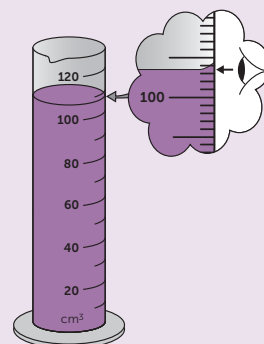


Fig. U.9

3 A rectangular metal plate of thickness 2.5 mm is 100 mm long and 65 mm wide. Its mass is 120 g. Calculate

- the distance around its edge in m,
- the area of one of its flat faces in m^2 ,
- its volume in m^3 ,
- its density in kg m^{-3} .

4 A solid aluminium rod of length 1.2 m has a uniform diameter of 52 mm. The density of aluminium is 2700 kg m^{-3} . Calculate

- its volume in m^3 ,
- its mass in kg.

5

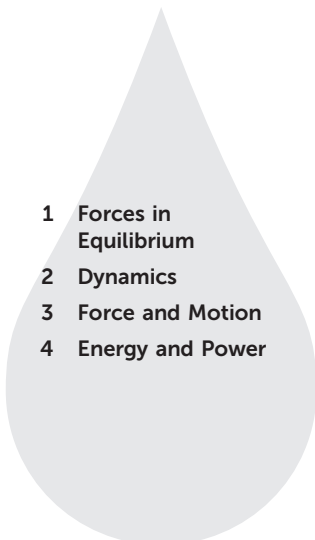
- Calculate the volume of a steel sphere of diameter 24 mm.
- Calculate the diameter of a steel sphere of volume 0.60 cm^3 .

Mechanics

PART

1

Part 1 introduces you to key principles of mechanics, including equilibrium, the link between force and motion and the link between work, energy and power. Experiments are provided to reinforce understanding and to link with applications such as friction and efficiency. The mathematical level in this part is developed from the basic skills developed in the Introduction to a level which should enable you to cope with the mathematical demands in all other parts of this book.

- 
- 1 Forces in Equilibrium**
 - 2 Dynamics**
 - 3 Force and Motion**
 - 4 Energy and Power**

Forces in Equilibrium

UNIT

1

OBJECTIVES

After working through this unit, you should be able to:

- represent a force geometrically or numerically
- combine two or more forces
- resolve a force into perpendicular components
- describe the condition for a point object to be in equilibrium
- draw a free-body diagram of the forces acting on a body
- understand and use the concept of centre of gravity

CONTENTS

- 1 Force as a vector
 - 2 Turning effects
 - 3 Stability
 - 4 Friction
 - 5 Equilibrium conditions
- Summary
Revision questions

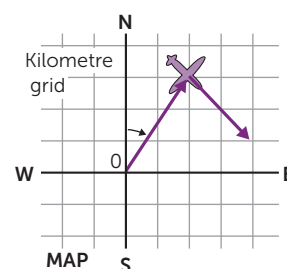
1.1 Force as a vector

Vectors and scalars

An air traffic controller needs to know the position, height, speed and direction of motion of an aircraft in order to monitor its flight path. Each piece of information is important, including the direction of motion. The position and height of an aircraft gives its **displacement**, which is its distance and direction from the air traffic controller. Its speed and direction of motion define its **velocity**. See Fig. 1.1.

Displacement and velocity are examples of **vector** quantities because they have a direction as well as a magnitude. Distance and speed are examples of **scalar** quantities because they have a magnitude only. Any vector may be represented by an arrow of length in proportion to the magnitude of the vector and in the appropriate direction.

- A **vector** is defined as any physical quantity that has a direction. Examples include weight, force, displacement (i.e. distance in a given direction), velocity, acceleration and momentum.
- A **scalar** is any physical quantity that has no direction. Examples include mass, energy and speed.



Velocity = 60 ms^{-1} south-east
Displacement from 0 = 3.5 km
36°E of due North (height not shown)

Fig. 1.1 Vectors and scalars

Force as a vector

Force is measured in units called **newtons** (N). The weight of a mass of 1 kg is approximately 10 N on the surface of the Earth. An object acted on by two forces will be at rest if the two forces are equal and opposite, as in Fig. 1.2a. However, if one of the forces is larger than the other, their combined effect is the difference between the two forces, as in Fig. 1.2b.

Each force in Fig. 1.2 can be represented by a vector (i.e. an arrow of length proportional to the force in the appropriate direction). From now on in this book, a bold symbol will denote a vector. For example, in Fig. 1.2b, $\mathbf{F}_1$ denotes force vector F_1 and $\mathbf{F}_2$ denotes force vector F_2 .

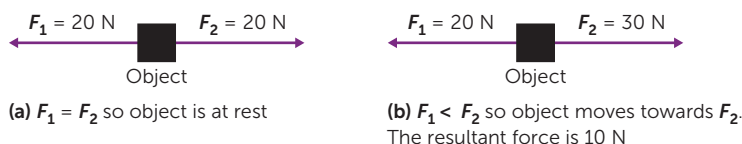


Fig. 1.2 Forces in opposite directions

The parallelogram of forces

Fig. 1.3 shows how to work out the effect of two forces, $\mathbf{A}$ and $\mathbf{B}$, acting on a point object but not along the same line. Each force is represented by a vector, such that the two vectors form adjacent sides of a parallelogram. The combined effect of the two forces, the **resultant**, is the diagonal of the parallelogram between the two vectors. The resultant is therefore given by force vector $\mathbf{B}$ added onto force vector $\mathbf{A}$. Fig. 1.4 shows how to balance out the resultant of $\mathbf{A}$ and $\mathbf{B}$ with an equal and opposite force, $\mathbf{C}$.

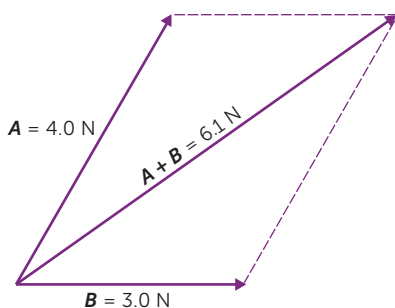


Fig. 1.3 The parallelogram of forces

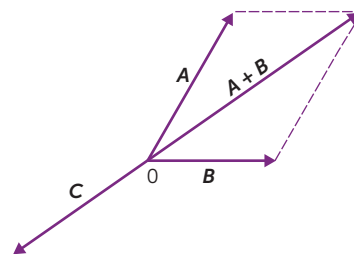


Fig. 1.4 Equilibrium

1.1A QUESTIONS

- A point object is acted on by a force of 5.0 N acting due East and a force of 12.0 N. Use the parallelogram method to determine the magnitude and direction of the resultant force when the 12.0 N force acts
 - due East,
 - due West,
 - due North,
 - 60° North of due East,
 - 60° North of due West.
- A point object is acted on by forces of 6.0 N and 8.0 N. Determine the magnitude of the resultant of these two forces if the angle between them is
 - 90°,
 - 45°.

The condition for equilibrium of a point object

In general, a point object acted on by two or more forces is in equilibrium (i.e. at rest) if the resultant of the forces is zero.

- 1 For equilibrium of a point object acted on only by two forces, A and B , the two forces must be equal and opposite to each other. The resultant of the two forces must be zero.

$$\mathbf{A} + \mathbf{B} = 0$$

- 2 For equilibrium of a point object acted on by three forces, A , B and C , the resultant of any two of the three forces is equal and opposite to the third force. For example, $\mathbf{A} + \mathbf{B} = -\mathbf{C}$. Rearranging this equation gives

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

The three force vectors add together to give a zero resultant if the object is in equilibrium. This can be represented as a triangle, as in Fig. 1.5.

- 3 In general, a point object is in equilibrium if the forces acting on it form a vector diagram which is a closed polygon. This rule is called the **closed polygon rule**.

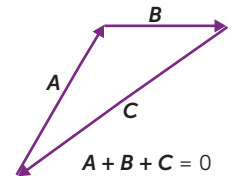


Fig. 1.5 Equilibrium of a point object

Resolving a force into perpendicular components

This is a mathematical technique which enables equilibrium situations to be analysed without scale diagrams. Sketch diagrams are nevertheless useful to visualise situations.

Consider a force, A , acting at point, O , of an x - y coordinate system, as in Fig. 1.6. The magnitude of A is represented by A and the direction of A is at angle θ to the x -axis. This force may be resolved into two perpendicular components.

- 1 $A_x = A \cos \theta$ along the x -axis.
- 2 $A_y = A \sin \theta$ along the y -axis.

Force A may be written in terms of its components in the form

$$\mathbf{A} = A \cos \theta \mathbf{i} + A \sin \theta \mathbf{j}$$

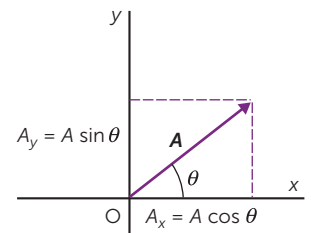
where $\mathbf{i}$ and $\mathbf{j}$ are vectors of unit length along the x -axis and the y -axis respectively.

Using Pythagoras' theorem gives

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

and using the rules of trigonometry gives

$$\tan \theta = \frac{A_y}{A_x}$$



$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$

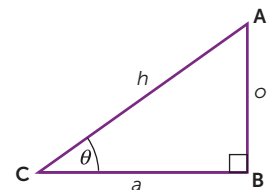
Fig. 1.6 Resolving a force

Notes

- 1 Pythagoras' theorem states that for a right-angled triangle ABC ,
 $AB^2 + BC^2 = AC^2$.
- 2 The following trigonometric functions are defined from this triangle

$$\sin \theta = \frac{AB}{AC} \left(= \frac{o}{h} \right) \quad \cos \theta = \frac{BC}{AC} \left(= \frac{a}{h} \right) \quad \tan \theta = \frac{AB}{BC} \left(= \frac{o}{a} \right)$$

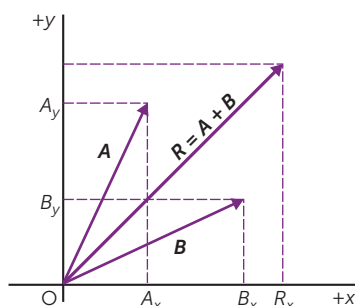
where o is the opposite side and a is the adjacent side to θ , and h is the hypotenuse.



How to calculate the resultant of two or more forces

Consider a point object, O, acted on by two or more forces, A , B , C , etc, as shown in Fig. 1.7. The resultant of these forces can be calculated by following these steps:

- 1 Resolve each force into perpendicular components along the x -axis and the y -axis.
- 2 For each axis, add the components, taking account of $+$ or $-$ directions, to give the component of the resultant along each axis.
- 3 Use Pythagoras' equation to calculate the magnitude of the resultant, and use the trigonometry equations to calculate the direction of the resultant.



$$R = \sqrt{R_x^2 + R_y^2}$$

$$\text{where } R_x = A_x + B_x$$

$$\text{and } R_y = A_y + B_y$$

Fig. 1.7 Using a calculator

1.1 WORKED EXAMPLE

A point object, O, is acted on by three forces, A , B and C , as shown in Fig. 1.8. Calculate (a) the magnitude and (b) the direction of the resultant of these three forces.

Solution

Resolving the three forces into components along the i -axis and the j -axis gives:

$$A = 10 \cos 45^\circ i + 10 \sin 45^\circ j$$

$$B = -8 \sin 30^\circ i + 8 \cos 30^\circ j$$

$$C = -5j$$

Therefore the resultant is

$$R = A + B + C$$

$$= (10 \cos 45^\circ - 8 \sin 30^\circ)i + (10 \sin 45^\circ + 8 \cos 30^\circ - 5)j$$

$$\text{thus } R = 3.1i + 1.0j$$

a) The magnitude of the resultant, $R = \sqrt{(3.1^2 + 1.0^2)} = 9.5 \text{ N}$

b) The angle between the resultant and the x -axis, θ , is given by

$$\tan \theta = \frac{R_y}{R_x} = \frac{1.0}{3.1} = 0.32$$

Hence $\theta = 10^\circ$.

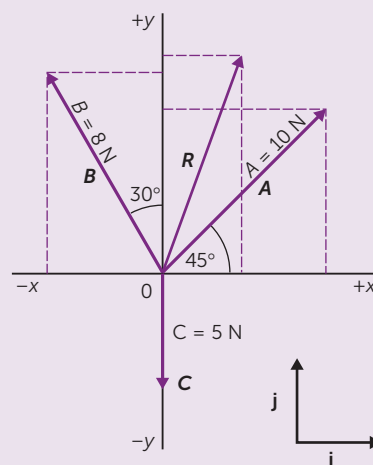


Fig. 1.8

1.1B QUESTIONS

- 1 Calculate the magnitude and direction of the resultant of two forces, **A** and **B**, acting on a point object for each of the arrangements shown in Fig. 1.9.
- 2 For each arrangement shown in Fig. 1.9, state the magnitude and direction of a third force, **C**, acting on **O** which would cancel out the resultant of **A** and **B**.

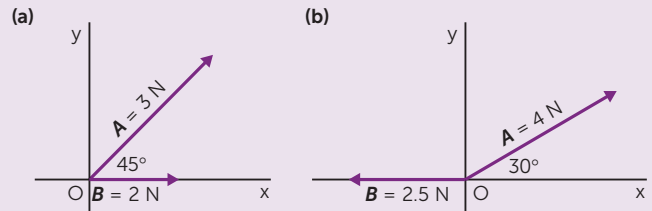


Fig. 1.9

1.2 Turning effects

When you need to use a spanner to unscrew a nut, the longer the handle of the spanner, the easier the task will be. The same applies to a claw hammer if ever you need to remove a nail from a piece of wood. The longer the handle, the greater the leverage about the turning point. See Fig. 1.10.

The **moment of a force about a point** is defined as the magnitude of the force $\times$ the perpendicular distance from its line of action to the point (see Fig. 1.11).

The unit of a moment is the newton metre (Nm). The term **torque** is used to describe the moment of a force about an axis.

For example, if a force of 40 N is applied to a spanner at a perpendicular distance of 0.20 m from the turning point, the moment of the force about that point is equal to 8.0 Nm ($= 40 \text{ N} \times 0.20 \text{ m}$). The same moment could be achieved by applying a force of 80 N at a perpendicular distance of 0.1 m from the pivot ($= 80 \text{ N} \times 0.1 \text{ m}$). Now you can see why a lesser force gives the same leverage if it acts at a greater distance. Prove for yourself that a force of 16 N acting at a perpendicular distance of 0.5 m from the turning point gives the same moment as 40 N at 0.20 m.

The principle of moments

Any object that is not a point object can be turned when one or more forces act on it. For example, a door turns on its hinges when it is given a push. The term **body** is used for any object that is not a point object.

- A body acted on by only one force cannot be in equilibrium and will turn if the force does not act through its centre of gravity.
- A body acted on by two or more forces may or may not turn, depending on the position and direction of the forces acting on it. For example, a balanced see-saw, as shown in Fig. 1.12 carrying a child either side of the pivot will become unbalanced if one of the children climbs off. For balance, a child of weight 250 N would need to be nearer the pivot than a child of weight 300 N. The condition for a balanced see-saw is that the moment of the child on one side about the pivot should equal the moment of the other child about the same pivot. For example, if the 250 N child is 1.20 m from the pivot (moment $= 250 \times 1.20 = 300 \text{ Nm}$), the other child would need to be 1.0 m from the pivot (moment $= 300 \text{ N} \times 1.0 \text{ m}$) for balance.

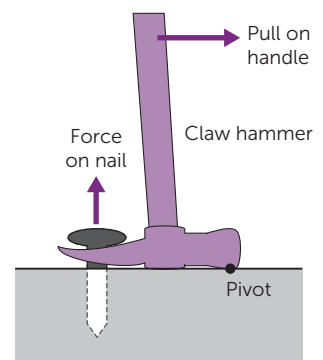
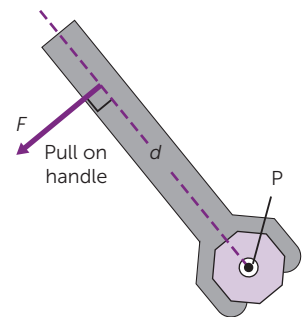


Fig. 1.10 Applying leverage



Moment about $P = Fd$

Fig. 1.11 Moment = force $\times$ perpendicular distance

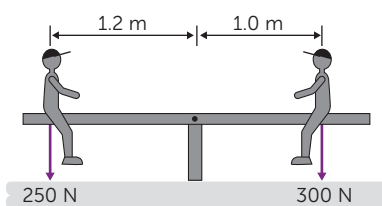


Fig. 1.12 A balanced see-saw

The balanced see-saw is an example of a more general rule, the Principle of Moments, which applies to any body in equilibrium.

The **Principle of Moments** states that for a body in equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments.

1.2 WORKED EXAMPLE

A uniform beam of length 4.0 m pivoted horizontally about its centre supports a 5.0 N weight at a distance of 1.5 m from the centre, and a 6.0 N weight at the other side of the centre. Calculate the distance from the 6.0 N weight to the centre of the beam.

Solution

Let d represent the distance from the centre to the 6.0 N weight.

Applying the principle of moments about the centre gives $6.0d = 5.0 \times 1.5$.

$$\text{Hence } d = \frac{5.0 \times 1.5}{6.0} = 1.25 \text{ m}$$

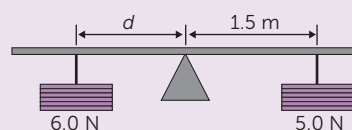


Fig. 1.13

1.2 QUESTIONS

- 1 The uniform beam in Fig. 1.14 is pivoted at its centre and supports three weights, W_1 , W_2 and W_3 , at distances d_1 , d_2 and d_3 as shown.

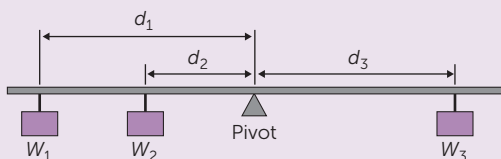


Fig. 1.14

For each set of weights and distances in the table, calculate the value of the missing quantity necessary to maintain the beam in equilibrium.

W_1/N	d_1/m	W_2/N	d_2/m	W_3/N	d_3/m
6.0	2.0	4.0	1.0	8.0	(a)
4.0	(b)	3.0	2.0	8.0	1.5
4.0	1.5	6.0	(c)	4.5	2.0
(d)	1.0	2.0	2.0	4.0	1.5

- 2 A simple beam balance is shown in Fig. 1.15. The scale pan is suspended from one end of a metre rule which is pivoted at its centre. The rule is balanced by adjusting the position of a 2.0 N weight suspended from the rule at the other side of the pivot to the scale pan.

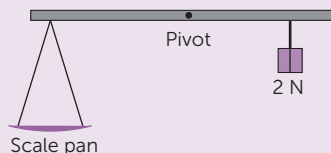


Fig. 1.15

- a) When the scale pan is empty, the 2.0 N weight must be positioned 150 mm from the centre of the rule to keep the rule horizontal. Calculate the weight of the scale pan.
- b) An object, X, of unknown weight is placed on the scale pan and the 2.0 N weight is moved to a distance of 360 mm from the centre to keep the rule horizontal. Calculate the weight of X.

1.3 Stability

Mass and weight

If you want to lose weight, go to the Moon where the force of gravity is one-sixth of the Earth's gravity. Unfortunately, you won't become slimmer because your mass will be unchanged.

- The **mass** of an object is the amount of matter it possesses. The unit of mass is the kilogram (kg), defined by means of a standard kilogram at BIPM (Bureau International des Poids et Mesures), Paris.
- The **weight** of an object is the force of gravity acting on it. The unit of weight is the newton (N).
- The **force of gravity per unit mass**, g , at the surface of the Earth, is 9.8 N kg^{-1} . The value of g changes slightly with position on the Earth's surface, ranging from 9.78 N kg^{-1} at the equator to 9.81 N kg^{-1} at the poles. The cause of this variation is due partly to the shape of the Earth not being exactly spherical and partly to the effect of the Earth's rotation. Strictly, this latter effect is not due to the force of gravity but arises from the circular motion of an object at the equator. The force of gravity on an object on the Moon, at 1.6 N kg^{-1} , is much smaller than that on an object on the Earth because the Moon is smaller in size than the Earth.

For an object of mass m , its weight, W , can be calculated from the equation

$$W = mg$$

Stable and unstable equilibrium

At a bowling alley, have you noticed how easily a pin falls over? Each pin is designed to be top-heavy with a small base. Fig. 1.16 shows a bowling pin in comparison with 'Wobbly Walter', a bottom-heavy child's toy that rights itself when knocked over.

- The child's toy is an example of an object in **stable equilibrium** – it returns to equilibrium when knocked over because its centre of gravity is always over its base.
- The standing bowling pin is an example of an object in **unstable equilibrium** – it moves away from equilibrium when released from a tilted position.
- The bowling pin lying on its side is an example of an object in **neutral equilibrium** – it stays wherever it is moved to, neither returning to nor moving away from where it was.

Centre of gravity

The **centre of gravity of an object** is the point where its weight can be considered to act.

- An object freely suspended from a point and then released will come to rest with its centre of gravity directly beneath the point. Fig. 1.17 shows how to find the centre of gravity of a flat object.
- A tall object tilted too far will topple over if released. This happens if it is tilted beyond the point where its centre of gravity is directly above the point about which it turns, as in Fig. 1.18. If released beyond this position, the moment of the weight about the point of turning causes the object to overbalance.

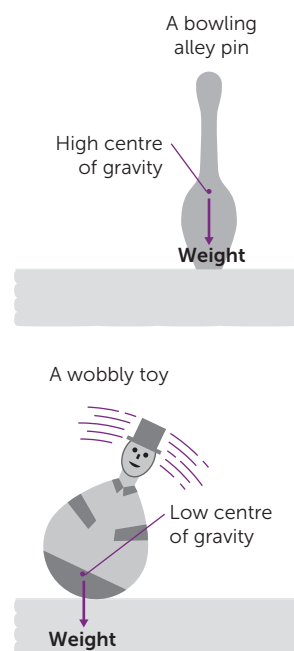


Fig. 1.16 Equilibrium

EXERCISE

Locating the centre of gravity of a flat card

Use a plumbline to mark a vertical line through the pivot on the card as in Fig. 1.17.

Repeat for a different pivot.

The centre of gravity is where the lines intersect.

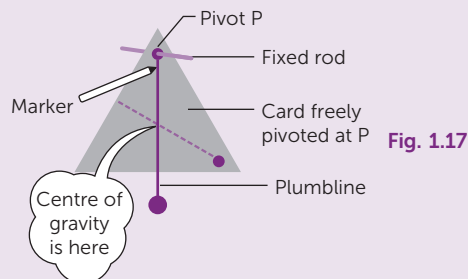


Fig. 1.17

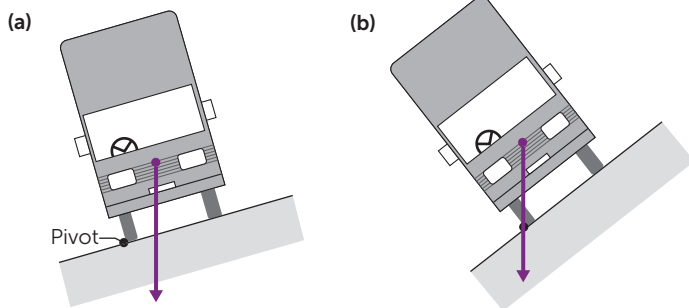


Fig. 1.18 Stability (a) tilting without toppling, (b) on the point of toppling

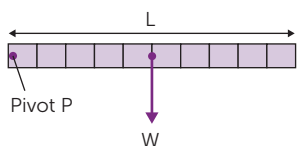


Fig. 1.19 Centre of gravity of a rule

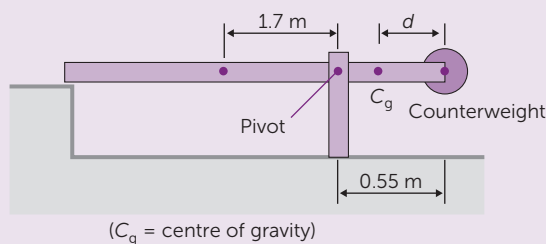
See www.palgrave.com/foundations/breithaupt for a proof that the centre of gravity of a uniform beam is at its midpoint.

- The moment of an object due to its weight, W , about any point = Wd , where d is the perpendicular distance from the point to the vertical line through its centre of gravity. For example, Fig. 1.19 shows a rule pivoted at one end. The rule can be held horizontal by a vertical string attached to the other end. The moment of the rule's weight about the pivot can be proved to be equal to $WL/2$, where L is its length. Therefore its centre of gravity is at a distance $L/2$ from the pivot.
- An object may be balanced by a single vertical force applied at its centre of gravity. For example, it ought to be possible to support a plate on the end of a vertical rod, provided the plate is placed with its centre on the end of the rod. Another example is when someone carries a ladder in a horizontal position; the task is much easier if the ladder is supported at its centre of gravity, because the ladder is balanced at this point.

1.3 WORKED EXAMPLE

A road gate consists of a 4.5 m uniform steel tube of weight 120 N which has a 400 N counterweight fixed to one end, as shown in Fig. 1.20. It is pivoted about a horizontal axis which passes through the beam 0.55 m from the centre of the counterweight.

- Calculate the position of the centre of gravity of the road gate.
- Calculate the magnitude and direction of the force that must be applied to the counterweight to keep the beam horizontal.



(C_g = centre of gravity)

Fig. 1.20

1.3 WORKED EXAMPLE *continued***Solution**

- a) Let d = the distance from the centre of gravity C_g of the road gate to the counterweight. The distance from the centre of gravity of the counterweight to the centre of gravity of the tube = 2.25 m. The moment of the tube about the counterweight = $120 \text{ N} \times 2.25 \text{ m} = 270 \text{ Nm}$ anticlockwise. This is the same as the moment of the whole gate about the counterweight = $(400 + 120)d$ anticlockwise.

Hence $520d = 270 \text{ Nm} \quad \therefore d = 270 \div 520 = 0.52 \text{ m}$.

- b) Because C_g is between the counterweight and the pivot, the road gate will turn clockwise if no additional force is applied to it. Let F = the upward vertical force on the counterweight needed to keep the road gate horizontal.

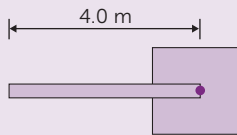
Applying the principle of moments about the pivot, the moment of the road gate about the pivot = $(400 + 120) \times 0.03 = 16 \text{ Nm}$ clockwise.

The moment of force F about the pivot = $0.55 F$ anticlockwise.

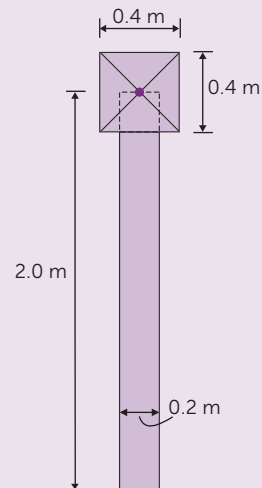
Hence $0.55 F = 16 \quad \therefore F = 16 \div 0.55 = 28 \text{ N}$

1.3 QUESTIONS

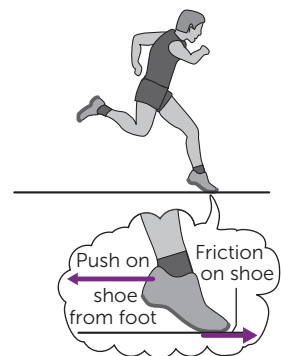
- 1 A placard consists of a uniform pole of length 4.0 m and of weight 30 N which has a square board of weight 20 N fixed to one end, as shown in Fig. 1.21. Calculate the distance from the other end of the pole to the centre of gravity of the placard.

**Fig. 1.21**

- 2 An advertising sign consists of a uniform board as in Fig. 1.22. The total mass of the board is 15 kg.
- Calculate the weight of the rectangular section of the board.
 - Calculate the weight of the square section of the board.
 - Calculate how high above the base the centre of gravity of the whole board is.

**Fig. 1.22****1.4 Friction**

Friction exists between any two solid surfaces that slide over one another. Walking and running would be very difficult without friction (e.g. on ice). See Fig. 1.23. However, friction causes undesirable wear and heating in machines and engines. Friction always acts on a surface in the opposite direction to the movement of the surface. Oil between two solid surfaces reduces the friction because it keeps them apart and reduces the points of direct contact.

**Fig. 1.23** Forces on a running shoe

Measuring friction

The force of friction between two flat surfaces can be measured by:

- measuring the force needed to pull a block across a fixed horizontal surface. One of the surfaces to be tested is fixed to the underside of the block. The other surface is the fixed surface. If the applied force is gradually increased from zero using a spring balance, as in Fig. 1.24, the block does not slide until the applied force reaches a certain value. Once the block starts to move, the applied force becomes slightly less. At the point of sliding, the force needed to make the block move is equal to the maximum possible frictional force between the two surfaces, referred to as the limiting value of the frictional force

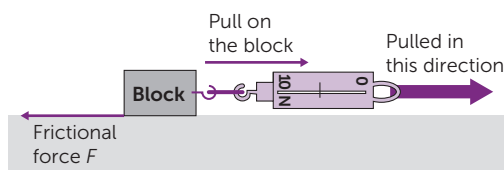


Fig. 1.24 Measuring friction on a horizontal surface

- using an inclined plane as in Fig. 1.25. The angle of the incline is increased until the block on the plane is at the point of sliding down the incline. The limiting value of the frictional force is equal to the component of the block's weight, $W \sin \theta$, acting down the incline.

The coefficient of static friction, μ

The arrangement in Fig. 1.24 may be used to prove that the limiting value of the frictional force, F , is proportional to the weight of the block. Since the weight of the block on a horizontal surface is equal and opposite to the normal reaction, N , of the surface on the block, the frictional force F is proportional to N . In other words, the ratio F/N is a constant for the two surfaces. This ratio is referred to as the coefficient of static friction, μ .

$$\mu = \frac{F}{N}$$

In Fig. 1.25, the normal reaction on the block is equal to $W \cos \theta$, the component of the block's weight normal to the surface. If the plane is made steeper, the block will slip if its component of weight parallel to the slope, $W \sin \theta$, exceeds the frictional force. At the point of sliding, the frictional force is equal to $W \sin \theta$. Therefore the coefficient of static friction is

$$\mu = \frac{F}{N} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta, \text{ since } \frac{\sin \theta}{\cos \theta} = \tan \theta.$$

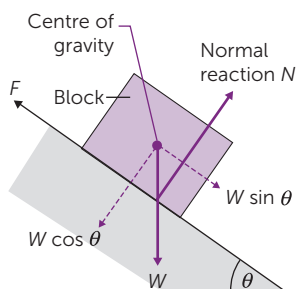


Fig. 1.25 Measuring friction using an inclined plane

1.4 WORKED EXAMPLE

A cupboard of weight 350 N is pushed at steady speed across a horizontal floor, by a force of 150 N applied horizontally near its base, as in Fig. 1.26. Calculate

- the coefficient of static friction between the two surfaces that slide over each other,
- the force needed if the weight of the cupboard is reduced to 200 N by removing some of its contents.

1.4 WORKED EXAMPLE *continued*

Solution

- a) The normal reaction $N = 350$ N since the surfaces are horizontal. The force of friction is 350 N.

$$\text{Hence } \mu = \frac{F}{N} = \frac{150}{350} = 0.43$$

- b) With less weight in the cupboard, the normal reaction is now reduced to 200 N. Hence the force needed to move it is $F = \mu N = 0.43 \times 200 = 86$ N.

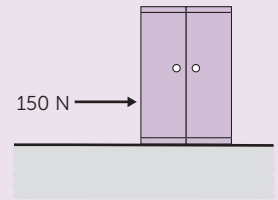


Fig. 1.26

EXERCISE

Investigating friction

Design and carry out an experiment to measure the coefficient of static friction between two surfaces both without and with oil.

Free-body force diagrams

This is a diagram that just shows a body with the forces acting on it, without showing the other bodies that cause these forces. Force diagrams can become very complicated where the forces on more than one body are shown. For example, Fig. 1.27a shows the forces acting on a book and on the table on which it rests. The free-body diagram of the book in Fig. 1.27b is much easier to understand.

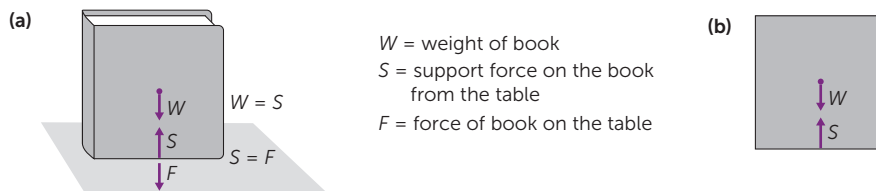


Fig. 1.27 Force diagrams (a) forces on the book and table, (b) free-body force diagram

1.4 QUESTIONS

- A block of weight 12 N rests on an inclined plane, as in Fig. 1.28. The angle of the plane to the horizontal, θ , is increased until the block slips at $\theta = 58^\circ$.

Fig. 1.28

 - Show that the limiting value of the frictional force is 10.2 N.
 - Calculate the normal component of the reaction force on the block due to the slope.
 - Calculate the coefficient of static friction, μ , between the block and the incline.
- A ladder of weight 220 N and of length 8.0 m rests with its lower end on a horizontal concrete surface and its upper end against a smooth wall, as in Fig. 1.29. Calculate the maximum angle between the ladder and the wall at which no slippage can occur, for a coefficient of friction between the ladder and the floor of 0.4. (Hint: take moments about the lower end of the ladder to calculate the normal reaction on the ladder due to the wall.)

Fig. 1.29

1.5 Equilibrium conditions

For any body to be in equilibrium:

- the force vectors must form a closed polygon
- the principle of moments must hold.

If the lines of action of the forces lie in the same plane, the first condition can be established by resolving all the forces into two perpendicular directions in that plane. In each direction, the force components one way should be equal to the force components the other way.

1.5 WORKED EXAMPLE

A uniform shelf of weight 20 N and of width 0.60 m is hinged horizontally to a vertical wall by a small bracket and a metal stay at the centre of the shelf, as shown in Fig. 1.30. The stay makes an angle of 50° with the wall.

- Show that the moment of the shelf about the hinge is 6.0 N m.
- Calculate the tension in the stay.
- Calculate the magnitude and direction of the force of the hinge on the shelf.

Solution

- The centre of gravity of the shelf is 0.30 m from the hinge.
 $\therefore$ the moment of the shelf about the hinge = $20 \text{ N} \times 0.30 \text{ m} = 6.0 \text{ N m}$.
- Fig. 1.31 shows a free-body diagram of the forces acting on the shelf. Since the stay is at 40° to the shelf, the perpendicular distance from the hinge to the stay, $d = 0.60 \sin 40 = 0.39 \text{ m}$.

Taking moments about the hinge:

the total clockwise moment = 6.0 N m (due to the shelf's weight)

the total anticlockwise moment = Td , where T is the tension in the stay.

Hence $Td = 6.0$

$$\therefore T = \frac{6.0}{d} = \frac{6.0}{0.39} = 15.6 \text{ N}.$$

- The three forces acting on the shelf form a closed polygon, as in Fig. 1.32. The magnitude and direction of the force of the hinge on the shelf, R , can therefore be determined by a scale diagram.

Alternatively, the forces can be resolved into vertical and horizontal components, as in Table 1.1. The vertical and horizontal components of R can then be determined, to give its magnitude and direction.

Table 1.1 Vertical and horizontal components of R

	Vertical components	Horizontal components
W	20 N down	0.0
T	$15.6 \cos 50$ up	$15.6 \sin 50$ leftwards
R	R_v (assume up)	R_h (rightwards)

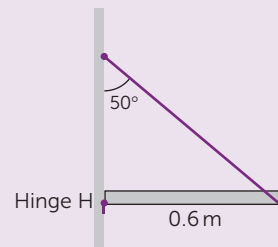


Fig. 1.30

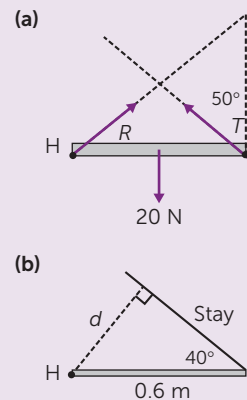


Fig. 1.31

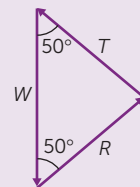


Fig. 1.32

1.5 WORKED EXAMPLE *continued*

From Table 1.1, $R_v = 20 - 15.6 \cos 50 = 10.0$ N upwards

$$R_h = 15.6 \sin 50 = 11.9 \text{ N rightwards}$$

$$\text{Hence } R = \sqrt{(R_v^2 + R_h^2)} = \sqrt{(10.0^2 + 11.9^2)} = 15.6 \text{ N}$$

The angle between the line of action of R and the wall, θ , is given by $\tan \theta = \frac{11.9}{10.0}$. Hence $\theta = 50^\circ$

1.6 WORKED EXAMPLE

A plank of weight 160 N and of length 5.0 m rests horizontally on two pillars. One pillar, X, supports the plank at one end and the other pillar, Y, supports it 1.0 m from the other end.

- Sketch a free-body diagram for this arrangement.
- Calculate the support force from each pillar on the plank.

Solution

- See Fig. 1.33.
- Let S_X and S_Y represent the support forces at X and Y.
 $\therefore S_X + S_Y = W$, where $W = 160$ N.
 Taking moments about X: $S_Y \times 4.0 = W \times 2.5$,
 hence $S_Y = 160 \times \frac{2.5}{4.0} = 100$ N and $S_X = 160 - 100 = 60$ N

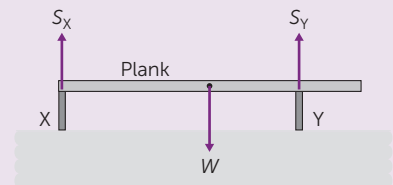


Fig. 1.33

Couples

A couple consists of two equal and opposite forces, acting parallel to each other but not along the same straight line. For a pair of equal and opposite forces, F , at perpendicular distance, d , apart,

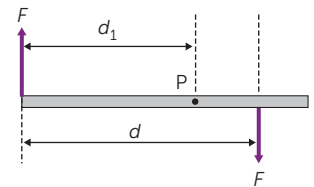
the moment of the couple = Fd about any point

This can be proved by choosing an arbitrary point, P, along a perpendicular line between the two forces at distance d_1 from one of the forces. See Fig. 1.34.

The moment of this force about P = Fd_1 .

The moment of the other force about P = $F(d - d_1)$ in the same sense of rotation.

Therefore, the total moment = $Fd_1 + F(d - d_1) = Fd$ which is independent of the position of P.



Moment of couple = Fd

Fig. 1.34 A couple

Simplifying free-body force diagrams

A single force, F , acting on a body at any point has the same effect as an equal force acting through the centre of gravity and a couple of moment Fd , where d is the perpendicular distance from the centre of gravity to the line of action of the force. Fig. 1.35 shows why this is so. This rule means that all the forces acting on a body can be reduced to a single resultant force acting through the centre of gravity and a resultant couple. It also means that for a body to be in equilibrium, the resultant force on it must be zero and there must be no overall couple.

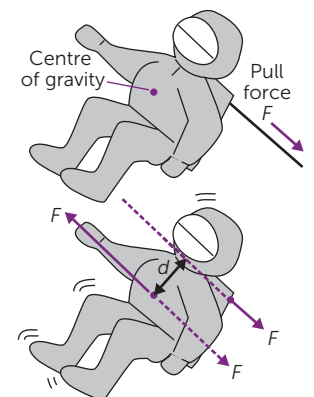


Fig. 1.35 Forces in space

1.7 WORKED EXAMPLE

A caravan of total weight 3200 N is towed at steady speed along a horizontal road. The centre of gravity of the caravan is 1.0 m in front of the wheels at a height of 1.5 m above the ground. The perpendicular distance between the wheel axis and the vertical line through the point of attachment of the tow bar is 3.0 m. The towing vehicle pulls on the caravan with a force of 2.2 kN, as shown in Fig. 1.36.

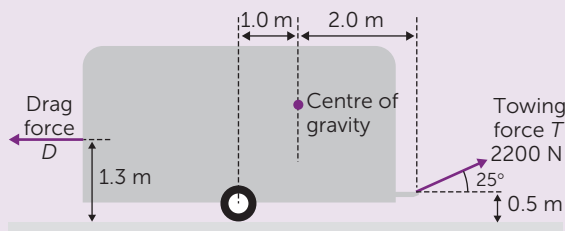


Fig. 1.36 Pulling a caravan

- Sketch the free-body force diagram showing the forces acting on the caravan. Assume the drag force acts horizontally above the road.
- (i) By resolving the forces vertically and horizontally, consider the vertical components and show that the normal reaction of the road on the road wheels is equal to 2.3 kN.
(ii) The coefficient of friction between the road and the tyres is 0.4. Calculate the frictional force on the road wheels and hence calculate the drag force on the caravan.
- Use the principle of moments to show that the line of action of the drag force is 1.3 m above the road.

Solution

a) See Fig. 1.37.

b) (i) Vertically: $R + 2200 \sin 25 = 3200$

$$\text{Hence } R = 3200 - 2200 \sin 25 = 2270 \text{ N}$$

$$(ii) F = \mu R = 0.4 \times 2270 = 908 \text{ N}$$

$$\text{Hence the drag force, } D = 2200 \cos 25 - 908 = 1086 \text{ N.}$$

c) Apply the principle of moments about the point where the tow bar is attached to the caravan, which is 0.5 m above the road and 3.0 m in front of the wheels, as in Fig. 1.37.

$$\text{Total clockwise moment} = 3.0R + 0.5F = 7264 \text{ N.}$$

$$\text{Total anticlockwise moment} = 2.0W + (h - 0.5)D = 5857 + 1086h,$$

where h is the height of the drag force above the road.

$$\text{Hence } 5857 + 1086h = 7264 \quad \therefore h = (7264 - 5857) / 1086 = 1.3 \text{ m.}$$

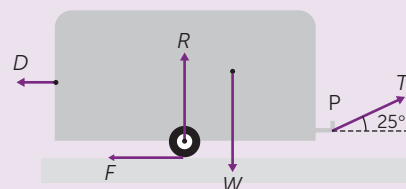


Fig. 1.37 Free-body force diagram

1.5 QUESTIONS

- A uniform horizontal shelf of weight 25 N and length 3.0 m is supported by two brackets fixed at 0.5 m from either end. The shelf supports a can of paint of weight 40 N at 1.0 m from one end, as in Fig. 1.38. Calculate the support force of each bracket on the shelf.

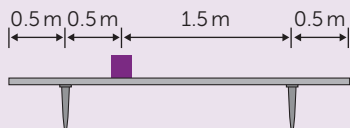


Fig. 1.38

- For the caravan in Worked example 1.7, calculate the magnitude and direction of the towing force if the drag force is reduced by 50% at the same height.



Fig. 1.39

SUMMARY

- The **components of a force, F** , parallel and perpendicular to a line at angle θ , are $F \cos \theta$ parallel to the line and $F \sin \theta$ perpendicular to the line.
- The **moment of a force, F , about a point** $= Fd$, where d is the perpendicular distance from the point to the line of action of the force.
- The **principle of moments** states that for a body in equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point.
- The **coefficient of friction, μ** , between two solid surfaces is the ratio of the frictional force to the normal reaction between the two surfaces.
- **Conditions for equilibrium of a body**
 - 1 The force vectors form a closed polygon.
 - 2 The principle of moments applies.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

1.1 A concrete paving stone of dimensions $50 \text{ mm} \times 500 \text{ mm} \times 800 \text{ mm}$ rests end-on against a smooth wall, as shown in Fig. 1.40.

a) The density of the concrete is 2700 kg m^{-3} . Calculate

- (i) the mass,
- (ii) the weight of the paving stone.

b) The paving stone is at angle θ to the wall.

(i) Sketch a free-body diagram showing the forces acting on the paving stone, including the frictional force and the normal reaction at the point of contact on the floor.

(ii) Why is the normal reaction at the point of contact on the floor equal and opposite to the weight?

(iii) The coefficient of friction between the paving stone and the floor is 0.40. Calculate the frictional force due to the floor on the paving stone.

(iv) Show that the maximum value of θ for no slip is 39° .

1.2 Fig. 1.41 shows a rubbish skip and its contents being raised from a driveway.

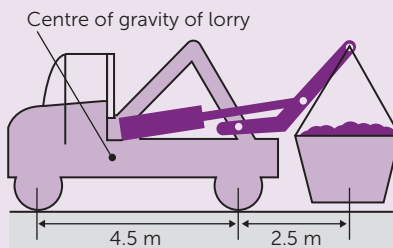


Fig. 1.41

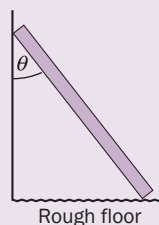


Fig. 1.40

- a) Describe how the distribution of the total weight of the skip and the lorry on the front and rear wheels changes as the skip is lifted off the roadway onto the back of the lorry.
- b) The weight of the lorry without the skip is 120 kN and its centre of gravity is 3.2 m from the back axle. The front axle is 4.5 m from the back axle. For a total weight of 10 kN for the skip and its contents, calculate the reaction of the ground on each pair of wheels just as the lorry raises the skip off the ground. In this position, the centre of gravity of the skip is 2.5 m from the back axle.

1.3 Fig. 1.42 shows a diagram of a model arm in which a 500 mm rule, pivoted at its 10 mm mark, represents the lower arm pivoted at the elbow joint. The spring balance, attached to the rule at its 50 mm mark, represents the biceps muscle.

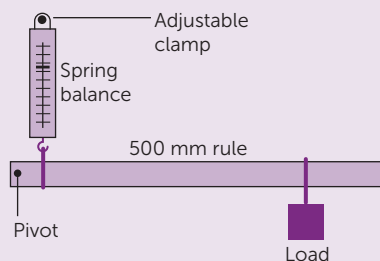


Fig. 1.42

- a) With no load on the rule, the spring balance reads 6 N after being adjusted so it is vertical and the rule is horizontal.
 - (i) Show that the weight of the rule is 1.0 N.
 - (ii) Calculate the force of the pivot on the rule.
- b) A weight of 2.0 N is suspended from the rule at its 410 mm mark. The spring balance is readjusted until it is vertical and the rule is horizontal. Calculate the reading on the spring balance now.

REVISION QUESTIONS

1.4 A uniform beam of length 8.0 m and weight 2400 N rests horizontally on pillars at either end. A pulley hoist of weight 80 N is suspended from the beam at 2.0 m from one end. The hoist is to be used to lift a crate of mass 50 kg, as shown in Fig. 1.43.

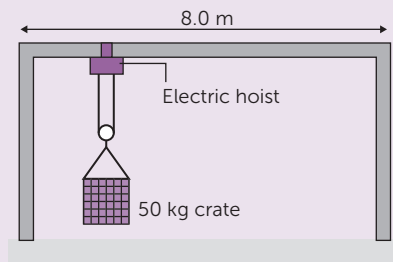


Fig. 1.43

- Calculate the downward pull on the beam due to the hoist when the crate is being raised.
- Calculate the support forces on the beam due to the pillars at either end when the downward force in (a) is exerted on the beam.

1.5 A piano of weight 1.5 kN rests on four small wheels at each corner. The centre of gravity of the piano is midway along its length at a perpendicular distance of 0.25 m from its rear panel and at a height of 0.70 m above the floor, as shown in Fig. 1.44.

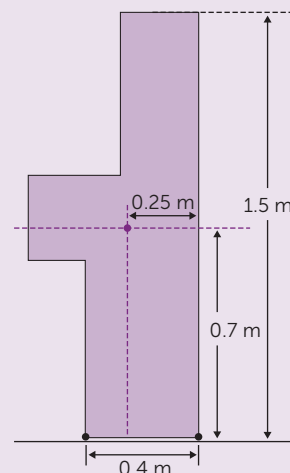


Fig. 1.44

- The base of the piano is 0.40 m wide. Calculate the maximum horizontal force that could be applied at the back edge of the piano, 1.5 m above the floor, to make it tilt about its front wheels.
- A horizontal force of 300 N is necessary to move the piano. Calculate the maximum height at which this force can be applied without tilting the piano about its front wheels.

Dynamics

UNIT

2

OBJECTIVES

After working through this unit, you should be able to:

- state the difference between displacement and distance and between speed and velocity
- plot a displacement versus time graph and use it to find the velocity of an object and the distance it moves
- plot a velocity versus time graph and use it to find the acceleration of an object
- explain what is meant by uniform speed and uniform acceleration
- carry out calculations using the equations for uniform acceleration

CONTENTS

- 2.1 Speed and distance
- 2.2 Speed and velocity
- 2.3 Acceleration
- 2.4 The equations for uniform acceleration
- 2.5 Acceleration due to gravity
- 2.6 Rates of change
- Summary
- Revision questions

2.1 Speed and distance

On the road

On motorways in certain countries, fines are imposed at toll stations on motorists who have exceeded the speed limit. For example, suppose a motorist travels a distance of 150 km between two toll stations in a time of 75 minutes on a motorway with a speed limit of 110 km per hour. The motorist's average speed is 2 km per minute ($= 150 \text{ km}/75 \text{ minutes}$) which is equal to 120 km per hour ($= 2 \text{ km per minute} \times 60 \text{ minutes}$) and, therefore, the motorist exceeded the speed limit. Ignoring speed limits can be costly as well as dangerous.

Conversion factors

The S.I. unit of speed is the metre per second (m s^{-1}). Since 1 kilometre equals 1000 metres and there are 3600 seconds in 1 hour, then a speed of 100 km h^{-1} is equal to 28 m s^{-1} ($= 100 \times 1000 \text{ m}/3600 \text{ s}$). Note that $18 \text{ km h}^{-1} = 5 \text{ m s}^{-1}$ exactly.

Speed limits and speedometer readings are usually stated either in kilometres per hour (km h^{-1}) or in miles per hour (mph). The speed limit on UK motorways is 70 mph which is equal to 112 km h^{-1} , since 1 mile = 1.6 km. Prove for yourself that 112 km h^{-1} is equal to 31 m s^{-1} .



Fig. 2.1 This speed limit sign instructs drivers that their speed should not exceed 60 mph

The speed equation

Speed is defined as distance travelled per unit time. For an object that travels distance, s , in time, t , at constant speed, v ,

$$v = \frac{s}{t}$$

This equation can be rearranged to give

$$s = vt, \text{ or } t = \frac{s}{v}$$

If the speed of an object changes, the equation $v = s/t$ gives the **average speed** over time t .

2.1 WORKED EXAMPLE

A motorist on a motorway journey of length 50 km travels the first 30 km in exactly 20 minutes and completes the rest of the journey at a constant speed of 100 km h^{-1} .

- Calculate the average speed of the motorist in metres per second over the first 30 km of the journey.
- (i) Calculate the time taken, in seconds, for the second part of the journey.
 ii) Sketch a graph to show how the motorist's speed varied with time for the whole journey.
 iii) Calculate the average speed in metres per second for the whole journey.

Solution

a) Average speed = $\frac{\text{distance}}{\text{time}} = \frac{30\,000}{20 \times 60} = 25 \text{ m s}^{-1}$

b) (i) 100 km h^{-1} converts to 28 m s^{-1} as explained earlier.

$\therefore \text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20\,000 \text{ m}}{28 \text{ m s}^{-1}} = 714 \text{ s}$

ii) See Fig. 2.2.

iii) Average speed = $\frac{\text{distance}}{\text{time}} = \frac{50\,000 \text{ m}}{(1200 + 710) \text{ s}} = 26 \text{ m s}^{-1}$

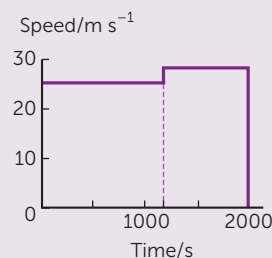


Fig. 2.2

2.1 QUESTIONS

The road map in Fig. 2.3 shows the progress of a motorist from joining a motorway to leaving it 2 hours later. The total distance travelled by the motorist on the motorway was 180 km, including a 20 km section of roadworks which took 30 minutes to pass through.

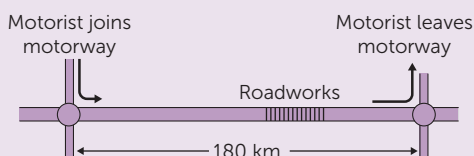


Fig. 2.3 On the road

- The motorist travelled at a steady speed of 110 km h^{-1} for the first hour. How far did the motorist travel in this time?
- The motorist took 30 minutes to travel 20 km through the roadworks. What was the motorist's average speed in km h^{-1} in this section?
- How far was the last section of the journey after the roadworks?
 - How long did this last section take to complete?
 - What was the average speed of the motorist in this final section?
- Sketch a graph to show how the motorist's speed changed with time.
 - What was the average speed for the whole journey in (i) km h^{-1} , (ii) m s^{-1} ?

2.2 Speed and velocity

Vectors and scalars

A vehicle travelling due South on a motorway at a speed of 100 km h^{-1} does not have the same velocity as a vehicle travelling at the same speed due North on the opposite carriageway. Velocity is speed in a given direction and is therefore a vector quantity. The idea of a vector was introduced in Unit 1. A **vector** is a physical quantity which has direction as well as magnitude. Displacement and velocity are examples of vectors. A **scalar** is a physical quantity that has magnitude only. Distance and speed are examples of scalars.

- **Displacement** is distance travelled in a given direction.
- **Velocity** is speed in a given direction.

On the M25

Fig. 2.4 shows the M25, London's orbital motorway, which stretches 196 km around the capital. Suppose a motorist joined it at junction 5 and travelled anticlockwise to junction 25, a total distance of 64 km, taking 40 minutes for the journey.

- 1 Junction 25 is 44 km due North and 15 km due West of junction 5. The motorist's displacement on exit was therefore 44 km due North, 15 km due West from the point of entry. This is a direct distance of 46.4 km at an angle of 19° West of due North. Fig. 2.5 shows how to use Pythagoras' theorem and the rules of trigonometry to work out this distance and angle.

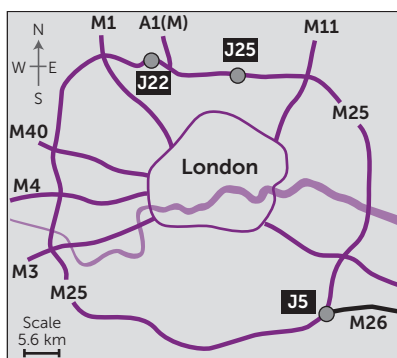


Fig. 2.4 On the M25

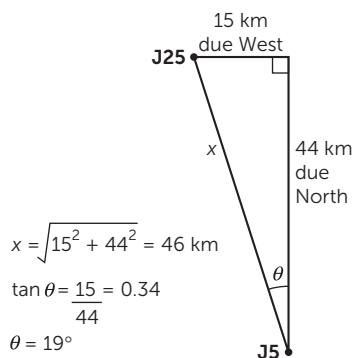


Fig. 2.5 Using trigonometry

- 2 For the whole journey, the average speed

$$= \frac{\text{distance}}{\text{time}} = \frac{64 \text{ km}}{(40 \div 60) \text{ hr}} = 96 \text{ km h}^{-1}$$

- 3 The motorist's direction changed gradually during the journey, initially travelling North at Junction 5 and ending the journey travelling westwards at junction 25. The velocity of the motorist's vehicle therefore changed, even if the speed remained the same throughout.

2.2 QUESTION

- 1 A motorist joins the M25 at junction 22. He travels a distance of 84 km clockwise before leaving the motorway 70 minutes later at junction 5, which is 48 km due South and 30 km due East of junction 22.
- a) Calculate the average speed of the motorist on the motorway (i) in km h^{-1} , (ii) m s^{-1} .
- b) Calculate the displacement from junction 22 to junction 5.
- c) What direction was the motorist travelling on (i) joining the motorway, (ii) leaving the motorway?

2.3 Acceleration

Uniform acceleration along a straight line

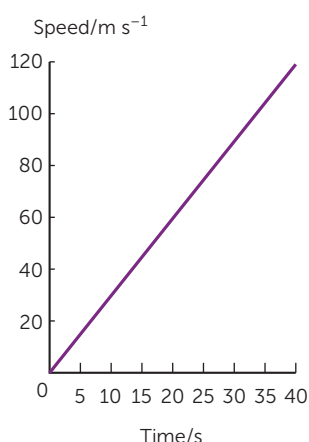


Fig. 2.6 Speed versus time for an aircraft taking off

Consider an aircraft that accelerates from rest on a runway, taking 40 s to reach its take-off speed of 120 m s^{-1} . Fig. 2.6 shows its speed increases 3 m s^{-1} each second during take-off. The graph shows that the speed increases steadily since the gradient of the line is constant. This is an example of **uniform acceleration** which is where the speed changes at a constant rate. The acceleration of the aircraft is 3 m s^{-2} .

Acceleration is defined as change of velocity per second.

The unit of acceleration is the metre per second per second, abbreviated as m s^{-2} or m/s^2 . **Deceleration**, sometimes described as negative acceleration, is where an object slows down.

Consider an object that accelerates uniformly from initial speed, u , to speed, v , in time, t , without a change of direction. Its change of speed is $(v - u)$, therefore its acceleration, a , can be calculated from the equation:

$$a = \frac{(v - u)}{t}$$

This equation may be rearranged as follows to calculate v given values for u , a and t .

$$at = (v - u)$$

hence

$$v = u + at$$

2.2 WORKED EXAMPLE

A car is capable of accelerating from rest to a speed of 20 m s^{-1} in 4.0 s.

- a) Calculate its acceleration.
- b) Calculate its speed after a further 2.0 seconds if the acceleration does not change.

Solution

a) Acceleration, $a = \frac{(20 - 0)}{4.0} = 5.0 \text{ m s}^{-2}$

b) For $a = 5.0 \text{ m s}^{-2}$, $u = 0$, $t = 4.0 + 2.0 = 6.0 \text{ s}$
 $v = u + at = 0 + (5.0 \times 6.0) = 30 \text{ m s}^{-1}$

Using graphs to represent motion

1. Distance versus time graphs

For **constant speed**, the distance travelled increases steadily with time. Fig. 2.7 shows distance versus time graphs on the same axes for two vehicles moving at different speeds. The speed of X is greater than the speed of Y because X travels a greater distance than Y in the same time.

The speed of X or Y can be determined from the **gradient** of the appropriate line. In Fig. 2.7, the height and the base of the gradient triangle for Y represent the distance moved ($= 75 \text{ m}$) and the corresponding time taken ($= 5.0 \text{ s}$). The gradient or slope, the height divided by the base, therefore represents the speed which is 15 m s^{-1} ($= 75 \text{ m}/5 \text{ s}$). Show for yourself that the speed of X is 20 m s^{-1} .

For **changing speed**, the distance travelled per second varies. Fig. 2.8 shows how the distance changes with time for an object moving in a straight line with constant acceleration. The gradient becomes steeper as its speed increases.

To find the speed at any given point, the gradient of the tangent to the curve is measured at that point. The tangent at a point is a straight line that touches the curve without cutting through the curve at that point. Show for yourself that in Fig. 2.8 the speed at point P is 20 m s^{-1} .

The gradient of a distance versus time graph represents the speed.

2. Speed versus time graphs

Consider an object that accelerates uniformly from initial speed, u , to speed v , in time, t , without a change of direction. Its motion is represented by the speed-time graph in Fig. 2.9.

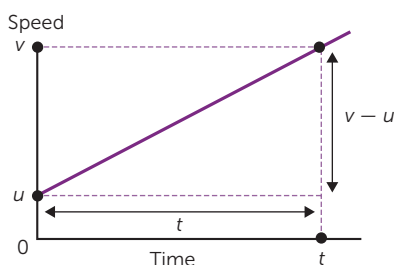
$$\text{Its acceleration, } a = \frac{\text{change of velocity}}{\text{time taken}} = \frac{(v - u)}{t}$$

The gradient of a speed versus time graph represents the acceleration.

In Fig. 2.9, the gradient of the line is $\frac{(v - u)}{t}$ since $(v - u)$, the change of speed, is represented by the height of the gradient triangle and t , the time taken, represents the base.

The **distance travelled**, s = average speed $\times$ time taken $= \frac{(u + v)}{2} t$
since the average speed is $\frac{(u + v)}{2}$.

The area under the line of a speed versus time graph represents the distance travelled.



$$\begin{aligned} \text{Gradient of line} &= \frac{v - u}{t} \\ &= \text{acceleration} \end{aligned}$$

$$\begin{aligned} \text{Area under line} &= \frac{(u + v)t}{2} \\ &= \text{distance moved} \end{aligned}$$

Fig. 2.9
Uniform
acceleration

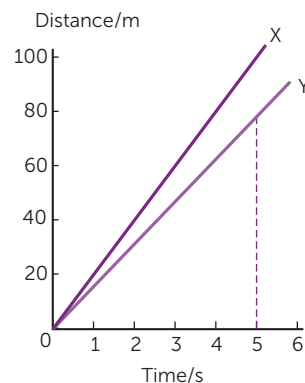


Fig. 2.7 Distance versus time
for constant speed

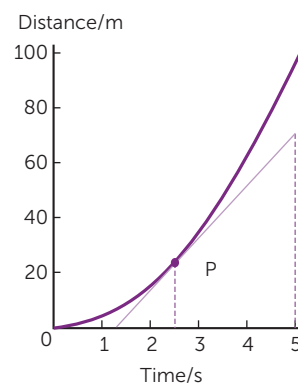


Fig. 2.8 Distance versus time
for changing speed

See www.palgrave.com/foundations/breithaupt for information about investigating motion

In Fig. 2.9, the area under the line is $\frac{(u+v)}{2}t$ because the shape under the line (a trapezium) has the same area as a rectangle of the same width (t) and of height $\frac{(u+v)}{2}$ which represents motion at a constant speed of $\frac{(u+v)}{2}$ for time t .

2.3 WORKED EXAMPLE

A vehicle accelerates from rest to a speed of 15 m s^{-1} in 5.0 s at uniform acceleration.

a) Sketch a graph to show how its speed changes with time.

b) Calculate (i) the acceleration of the vehicle, (ii) the distance travelled from rest in 5 s .

Solution

a) See Fig. 2.10.

b) (i) Acceleration = gradient of speed versus time graph

$$= \frac{(15 - 0)}{5.0}$$

$$= 3.0 \text{ m s}^{-2}$$

(ii) Distance travelled = area under the speed versus time graph from 0 to 5 s

$$= \frac{1}{2} \text{ height} \times \text{base}$$

$$= \frac{1}{2} \times 15 \times 5$$

$$= 37.5 \text{ m}$$

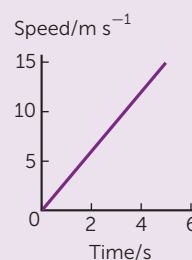


Fig. 2.10

2.3 QUESTIONS

- 1 A vehicle travelling at 30 m s^{-1} leaves a motorway at a slip road and comes to rest 20 s later at traffic lights at the end of the slip road. Fig. 2.11 shows how its speed decreases with time.

- a) Show that the acceleration of the vehicle is -1.5 m s^{-2} .
- b) Calculate the distance travelled along the slip road by the vehicle.

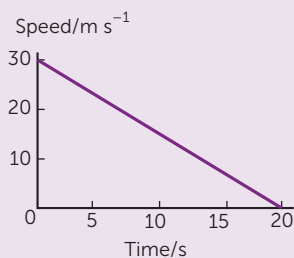


Fig. 2.11

- 2 A train left a station and accelerated uniformly from rest for 30 s to a speed of 12 m s^{-1} . It then travelled at this speed for 100 s before decelerating uniformly to rest in 20 s . Fig. 2.12 shows how its speed changed with time.

- a) Calculate its acceleration and the distance it travelled in the first 30 s .
- b) (i) Explain why its acceleration was zero during the next 100 s .
(ii) Show that the distance it travelled in this time was 1200 m .
- c) Calculate its acceleration and distance travelled in the last 20 s .
- d) Calculate the total distance it travelled and show that its average speed for the whole journey was 10 m s^{-1} .

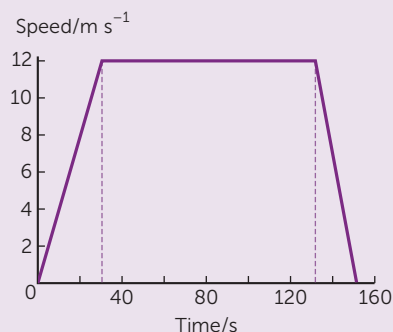


Fig. 2.12

2.4 The equations for uniform acceleration

For an object that accelerates uniformly from initial speed, u , to speed, v , in time, t , without a change of direction, its acceleration, $a = (v - u) \div t$. Rearranging this equation gives $at = (v - u)$ and making v the subject gives the equation in the form

$$v = u + at \quad (1)$$

The distance travelled is

$$s = \frac{(u + v)}{2} t \quad (2)$$

Substituting ' $u + at$ ' for v in equation (2) gives an equation for s without v present, as follows:

$$s = \frac{(u + (u + at))}{2} t = \frac{(2u + at)}{2} t = \frac{(2ut + at^2)}{2} = ut + \frac{1}{2} at^2$$

$$s = ut + \frac{1}{2} at^2 \quad (3)$$

Equations (1) and (2) may be combined to eliminate t as follows:

$$(v - u) = at \quad \text{from equation (1)}$$

$$(v + u) = \frac{2s}{t} \quad \text{from equation (2)}$$

Multiplying the two equations together gives

$$(v - u)(v + u) = at \frac{2s}{t}$$

$$\text{which gives } v^2 + uv - uv - u^2 = 2as$$

$$\text{hence } v^2 - u^2 = 2as$$

$$\text{or } v^2 = u^2 + 2as \quad (4)$$

Using equations

To find the unknown dynamics quantity u , v , a , s or t :

- 1 List the quantities with their values if known.
- 2 Choose the dynamics equation in which all the quantities are known except for one unknown quantity.
- 3 If any quantity is zero, simplify the chosen equation by inserting its value of zero into the equation.
- 4 Rearrange the equation to make the unknown quantity the subject of the equation.
- 5 Substitute the known values into the equation.
- 6 Calculate the unknown quantity, writing down its magnitude and the unit as the answer.

2.4 WORKED EXAMPLE

In a crash test, a car moving at a speed of 20 m s^{-1} is driven automatically at a concrete wall. The impact causes the front part of the car to crumple, shortening the vehicle by a distance of 0.60 m . Calculate

- a) the time taken for the impact,
- b) the deceleration of the car on impact.

Solution

Initial speed, $u = 20 \text{ m s}^{-1}$, Final speed, $v = 0$.

Distance moved during the impact, $s = 0.60 \text{ m}$.

- a) To find the time taken, t , choose the equation which contains s , u , v and t .

$$s = \frac{(u + v)}{2} t = \frac{ut}{2} \quad \text{since } v = 0.$$

$$\text{Rearranging this equation gives } t = \frac{2s}{u} = \frac{2 \times 0.60}{20} = 0.06 \text{ s}$$

- b) To find the deceleration, choose an equation with acceleration, a .

$$a = \frac{(v - u)}{t} = \frac{0 - 20}{0.06} = -330 \text{ m s}^{-2}$$

Notes

- 1 The acceleration is negative which means it slows down.
- 2 The acceleration could have been calculated using $v^2 = u^2 + 2as$. Since $v = 0$, this equation reduces to $0 = u^2 + 2as$. This rearranges as

$$a = -u^2 / 2s$$

$$= -20^2 / (2 \times 0.6)$$

$$= -330 \text{ m s}^{-2}.$$

Application 1

Stopping distances

The shortest distance in which a vehicle can be stopped by its driver without skidding is called the ‘stopping distance’. Table 2.1 shows how the speed of a vehicle affects its stopping distance.

Table 2.1 Stopping distances						
Speed / mph (miles per hour)	20	30	40	50	60	70
Speed / m s^{-1}	9	13.5	18	22.5	27	31
Thinking distance / m	6	9	12	15	18	21
Braking distance / m	6	14	24	38	54	75
Stopping distance / m	12	23	36	53	72	96

The stopping distance is the sum of the ‘thinking distance’ and the ‘braking distance’.

- The **thinking distance** is how far the car travels in the time it takes the driver to react. A ‘normal’ driver has a reaction time of just two-thirds of a second. Using the equation $s = ut + \frac{1}{2}at^2$ with acceleration $a = 0$ and $t = \frac{2}{3}$ s gives the thinking distances for different speeds, as in Table 2.1.
- The **braking distance** is the least distance the car travels without skidding when the brakes are applied. This distance depends on the braking force that is applied. On a flat dry road, skidding occurs if the braking force exceeds two-thirds of the vehicle’s weight. As explained in topic 2.4, the deceleration at this limit is $\frac{2}{3}g$ or 6.7 m s^{-2} . The braking distances in Table 1 are calculated using the equation $v^2 = u^2 + 2as$ with final speed $v = 0$ and acceleration $a = -6.7 \text{ m s}^{-2}$.

Tiredness, poor visibility, drugs and alcohol all increase thinking distances because they increase reaction times. Badly-maintained tyres and brakes and wet road surfaces all increase braking distances as braking forces for no skidding need to be reduced.

2.4 QUESTIONS

- 1 An aircraft takes off from a standstill position on an aircraft carrier with the assistance of a catapult mechanism. It reaches a speed of 85 m s^{-1} in a distance of 210 m from rest. Calculate
 - a) the time taken for the launch,
 - b) the acceleration during take-off.
- 2 A bullet moving at a speed of 115 m s^{-1} hits a tree and embeds itself to a depth of 45 mm. Calculate
 - a) the time taken for the impact,
 - b) the deceleration of the bullet.
- 3 A cricket ball moving at a speed of 28 m s^{-1} is caught with one hand by a cricketer. The cricketer’s hand moves back through a distance of 1.2 m in stopping the ball. Calculate the deceleration of the cricket ball.

2.5 Acceleration due to gravity

Free fall

A freely falling object is acted on by the force of gravity only. Fig. 2.13 represents a video sequence of a sphere falling freely. The video camera recorded the object's position against a vertical scale every tenth of a second after it was released. Table 2.2 gives the distance fallen from its initial position every 0.10 s after release.

Table 2.2 An object in free fall

Time, t , after release/s	0.000	0.100	0.200	0.300	0.400	0.500
Distance fallen, s , after release/m	0.000	0.050	0.200	0.450	0.800	1.250

If the acceleration is constant, the distance fallen, s , in time t , is given by

$$s = ut + \frac{1}{2}at^2$$

Since the object was released from rest, its initial speed, $u = 0$. Hence the above equation reduces to

$$s = \frac{1}{2}at^2$$

A graph of s on the vertical axis against t^2 on the horizontal axis should therefore give a straight line of gradient $\frac{1}{2}a$, as shown in Fig. 2.14. From this graph, it can be concluded that

- 1 the acceleration is constant because the graph is a straight line,
- 2 the acceleration is equal to 9.8 m s^{-2} since the gradient $(= \frac{1}{2}a) = 4.9 \text{ m s}^{-2}$.

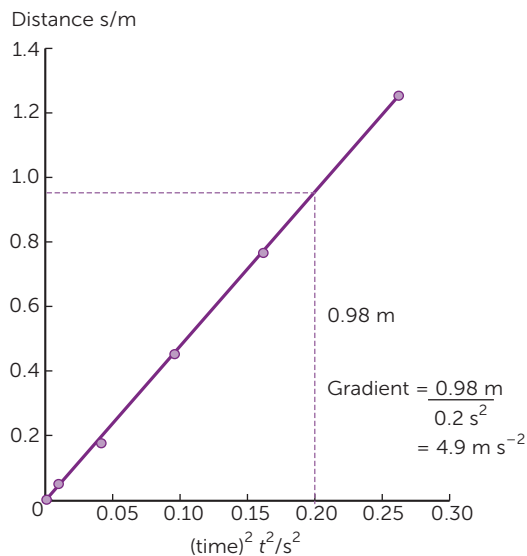


Fig. 2.14 Distance versus time ^{2}

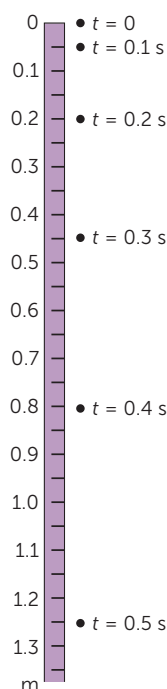


Fig. 2.13 Free fall

Maths workshop: straight line graphs

Fig. 2.15 shows a straight line drawn on a graph. The general equation for this line is $y = mx + c$, where m is the gradient and c is the intercept on the y -axis.

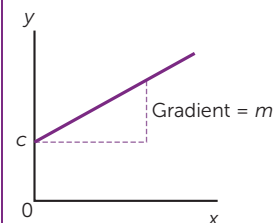


Fig. 2.15 $y = mx + c$

Compare the equation $s = \frac{1}{2}at^2$ with $y = mx + c$. Plotting s on the y -axis and t^2 on the x -axis therefore gives a straight line passing through the origin ($c = 0$) with a gradient of $\frac{1}{2}a$.

The acceleration due to gravity of an object falling freely is constant, equal to 9.8 m s^{-2} .

The symbol g is used to denote this acceleration. The value of g varies slightly over the Earth's surface from 9.81 m s^{-2} at the poles to 9.78 m s^{-2} at the Equator. The value of 9.8 m s^{-2} for g is an average value over the Earth's surface.

EXERCISE

Experiment to measure g using the falling ball method

Use an electronic timer to time a steel ball falling from rest through a measured distance, as shown in Fig. 2.16. Repeat the measurement several times to give an average time. Repeat the procedure for different distances. Plot a graph of distance (on the vertical axis) against time² as Fig. 2.14. Measure the gradient of the graph and determine g .

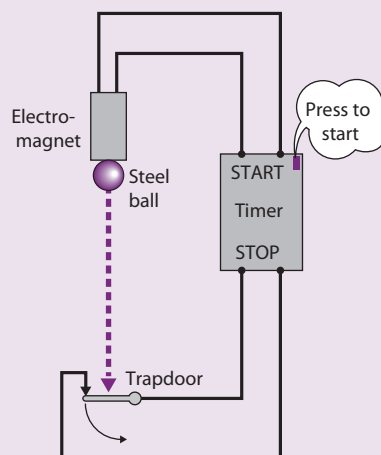


Fig. 2.16 The falling ball method to determine g

2.5 WORKED EXAMPLE

Assume $g = 9.8 \text{ m s}^{-2}$

An object is released from rest at the top of a tower of height 47 m and falls to the ground. Calculate

- the time it takes to fall to the ground,
- its speed just before impact at the ground.

Solution

Initial speed, $u = 0$, distance fallen, $s = 47 \text{ m}$, acceleration, $a = 9.8 \text{ m s}^{-2}$

- a) To find the time taken, t , use

$$s = ut + \frac{1}{2}at^2$$

Substituting $u = 0$ gives

$$s = \frac{1}{2}at^2,$$

$$\therefore t^2 = \frac{2s}{a} = \frac{2 \times 47}{9.8} = 9.6$$

$$\text{Hence } t = \sqrt{9.6} = 3.1 \text{ s.}$$

- b) To find the speed just before impact, v , use

$$v = u + at$$

$$= 0 + (9.8 \times 3.1) = 30 \text{ m s}^{-1}.$$

Up and down

Gravity always acts downwards, regardless of whether the motion of an object is upwards or downwards. For example, an object thrown vertically up into the air slows down, stops momentarily at its highest point then falls back towards the ground. A direction code is needed here to distinguish between the two possible directions, up and down. The usual convention is to assign $+$ values for upward motion and $-$ values for downward motion. Its velocity is initially positive (i.e. upwards) then becomes negative when it moves downwards. However, its acceleration is always negative (i.e. downwards). Fig. 2.17 shows how its **velocity** changes with time. Note that the gradient of the line (i.e. the acceleration) is constant, equal to -9.8 m s^{-2} .

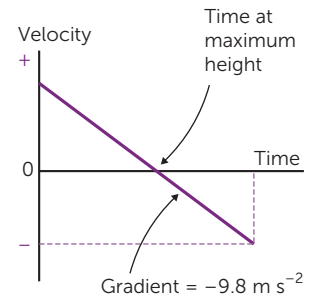


Fig. 2.17 Vertical projection

2.6 WORKED EXAMPLE

Assume $g = 9.8 \text{ m s}^{-2}$

A package is released from a hot air balloon at a certain height above the ground when the balloon is ascending at a steady speed of 4.0 m s^{-1} . The package takes 5.0 s to fall to the ground. Calculate the height of the balloon

- when the package is released,
- when the package hits the ground.

Solution

Use the direction code ' $+$ is up; $-$ is down';
 $u = +4.0 \text{ m s}^{-1}$, $a = 9.8 \text{ m s}^{-2}$, $t = 5.0 \text{ s}$.

- a) To calculate the height of the balloon when the package is released, use

$$s = ut + \frac{1}{2}at^2 = (4.0 \times 5.0) - (0.5 \times 9.8 \times 5.0^2) = -102.5 \text{ m}.$$

The minus sign means the package has moved downwards.

- b) In the 5 seconds the package takes to fall to the ground, the balloon moves up at a steady speed of 4.0 m s^{-1} , gaining a height of 20.0 m ($= 4.0 \text{ m s}^{-1} \times 5.0 \text{ s}$) in this time. The balloon is therefore 122.5 m above the ground when the package hits the ground.

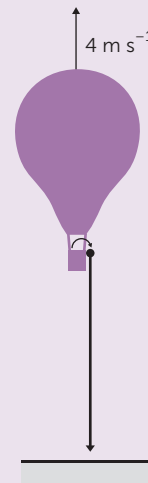


Fig. 2.18

2.5A QUESTIONS

Assume $g = 9.8 \text{ m s}^{-2}$

- A coin released from rest at the top of a well strikes the bottom of the well 1.8 s later. Calculate
 - the depth of the well,
 - the speed of the coin just before impact.
- A package is released from a hot air balloon at a height of 36 m above the ground when the balloon is descending at a steady speed of 4.0 m s^{-1} (Fig. 2.19). Calculate
 - the speed of the package just before impact,
 - the time taken by the package to fall to the ground.

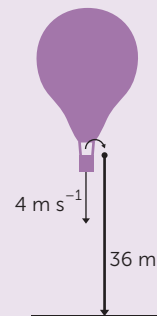


Fig. 2.19

2.5B QUESTIONS

Assume $g = 9.8 \text{ m s}^{-2}$

- 1 A ball is thrown vertically into the air and takes 4.4 s to return to the thrower (Fig. 2.20). Calculate
 - a) the time it takes to reach maximum height,
 - b) its speed of projection,
 - c) its maximum height,
 - d) its speed on returning to the thrower.
- 2 A helicopter is ascending vertically at a steady speed of 15 m s^{-1} when a package is dropped from it at a height of 90 m above the ground (Fig. 2.21). Calculate
 - a) the speed of impact of the package at the ground,
 - b) the time taken for the package to fall to the ground.

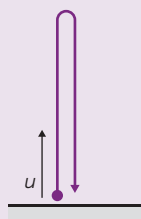


Fig. 2.20

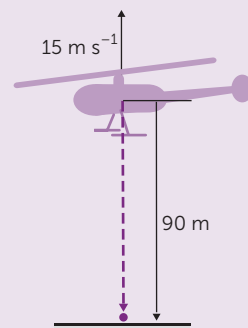


Fig. 2.21

2-6 Rates of change

Velocity

The **velocity** of an object is its rate of change of displacement with time. This may be written in the form

$$v = \frac{ds}{dt}$$

where v is the velocity, s is the displacement and d/dt means 'rate of change'. Fig. 2.22 shows how the displacement of an object released in water changes with time as the object falls. The gradient of any displacement–time curve is the velocity. In Fig. 2.22, the velocity becomes constant due to the resistance of the water. This velocity is referred to as the **terminal velocity** of the ball. To find the velocity at any point on the curve, a tangent to the curve is drawn and the gradient of the tangent is measured.



Fig. 2.22 Displacement versus time for an object released in a fluid

Acceleration

The **acceleration** of an object is its rate of change of velocity. This may be written in the form

$$a = \frac{dv}{dt}$$

where a is the acceleration, v is the velocity and d/dt means 'rate of change'. Fig. 2.23 shows how the velocity of an object released in water increases with time. The gradient of this curve is the acceleration, which can be determined by measuring the gradient of the tangent to the curve. In Fig. 2.23, the acceleration becomes zero as the object reaches its terminal velocity. See Fig. 4.10, p. 58.

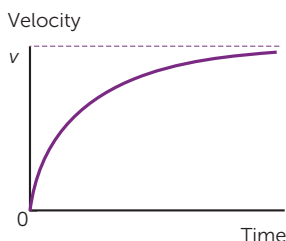


Fig. 2.23 Velocity versus time for an object released in a fluid

A spreadsheet exercise

Let the acceleration of an object at an instant $= a$. In a short time interval, δt , its change of velocity, $\delta v = a\delta t$.

Let v = its velocity at the start of the interval, then its velocity at the end of the interval $= v + \delta v$; this can then be used as the velocity at the start of the next time

Table 2.3 Using a spreadsheet

t	v	$a = g - 0.1 v$	$\delta v = a \delta t$	$v + \delta v$
0.000	0.000	9.800	1.960	1.960
0.200	1.960	9.600	1.920	3.880
0.400	3.880	9.410	1.880	5.760
0.600	5.760	9.220	1.840	7.610
0.800	7.610	9.040	1.810	9.420
1.000	9.420	8.860	1.770	11.190

See www.palgrave.com/foundations/breithaupt to model motion in a straight line using a spreadsheet.

interval. This sequence can be used repeatedly to calculate the velocity of an object if its acceleration is known.

Table 2.3 shows how the velocity of an object released in a fluid changes. As explained in topic 4.2, its acceleration $a = g - bv$, where b is a constant determined by the shape of the object and the resistance to flow of the fluid. In Table 2.3, the value of $b = 0.1$ and $\delta t = 0.2$ seconds.

The results are plotted on a graph shown in Fig. 2.24, in which the calculations are continued to $t = 4.0$ s. The velocity eventually becomes constant because the fluid resistance eventually counteracts the force of gravity.

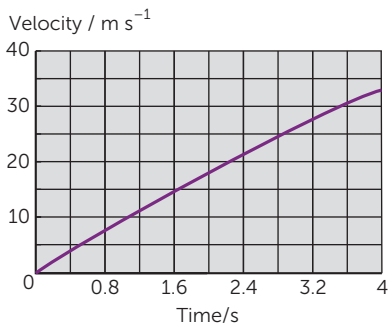


Fig. 2.24 Velocity versus time for an object released in a fluid

The values in Table 2.3 can be calculated using a spreadsheet, as explained on p. 00. The instructions can be used to calculate the motion of any object, given its acceleration in terms of its displacement and velocity. The data from a spreadsheet can be imported into a chart. Fig. 2.24 is a graph of velocity against time imported from a spreadsheet used to calculate the data in Table 2.3

2.6 QUESTIONS

Assume $g = 9.8 \text{ m s}^{-2}$

- 1 Use the dynamics equations to calculate the displacement and velocity of a ball projected vertically upwards with an initial velocity of 24.5 m s^{-1} , every 0.5 s after projection for 5.0 s .
- 2 Plot graphs of the results in Q1 to show how the displacement and velocity vary with time.
- 3 Repeat the procedure using a spreadsheet and display the results in charts to show how the displacement and position vary with time.

SUMMARY

- **Displacement** is distance in a given direction.
- **Speed** is rate of change of distance.
- **Velocity** is rate of change of displacement.
- **Acceleration** is rate of change of velocity.

- **Equations for uniform acceleration:**

$$v = u + at \quad s = \frac{1}{2}(u + v)t \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as$$

- Graphs

- 1 **Displacement versus time:**

gradient = velocity

- 2 **Velocity versus time:**

gradient = acceleration,

area under line = displacement.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

Assume $g = 9.8 \text{ m s}^{-2}$

2.1 A car initially at rest, accelerates at 1.5 m s^{-2} for 20 s, then travels at constant speed for 90 s, then decelerates to rest in a further 40 s. Its direction does not change.

- a) Calculate its maximum speed.
- b) Sketch a graph to show how its speed changes from its initial position to where it stops again.
- c) Calculate its deceleration.
- d) Calculate
 - (i) the distance moved in each of the three parts of its journey,
 - (ii) its average speed for the whole journey.

2.2 A cyclist accelerates uniformly from rest to a speed of 6.5 m s^{-1} in 30 s, then brakes to rest in a distance of 80 m.

- a) Calculate the acceleration and distance moved in the first 30 s.
- b) Calculate the deceleration and how long the brakes are applied before the cyclist stops.
- c) (i) Sketch a speed versus time graph.
(ii) Calculate the average speed of the cyclist.

2.3 A tennis ball is thrown vertically upwards at an initial speed of 15 m s^{-1} .

- a) Calculate its maximum height and the time it takes to reach this height.
- b) (i) Calculate its velocity when it is 5.0 m above the ground moving downwards.
(ii) How long does it take to reach this position?

2.4 A parachutist is descending vertically at a steady speed of 3.2 m s^{-1} when she releases an object from a height of 100 m above the ground.

- a) Calculate the speed of the object just before impact.
- b) How long does it take the object to fall freely to the ground?
- c) How far is the parachutist above the ground when the object strikes the ground?

2.5 A rocket is launched vertically and accelerates uniformly at 8.0 m s^{-2} for 40 s. Its engine then cuts out and it eventually returns to the ground.

- a) Calculate
 - (i) its speed and height when the engine cuts out,
 - (ii) its maximum height,
 - (iii) its velocity just before impact,
 - (iv) its time of flight.
- b) Sketch a velocity versus time graph for the whole flight of the rocket.

Force and Motion

UNIT

3

OBJECTIVES

After working through this unit, you should be able to:

- calculate the motion of a body acted upon by a resultant force
- explain the link between mass and weight
- relate the resultant force on a body to its change of momentum
- explain the thrust of a jet or a rocket
- use the principle of conservation of momentum to analyse collisions and explosions

CONTENTS

- 3.1 Newton's laws of motion
- 3.2 Force and momentum
- 3.3 Conservation of momentum
- Summary
- Revision questions

3.1 Newton's laws of motion

The nature of force

Friction is almost entirely absent on ice. A moving vehicle on an icy road is very difficult to control because there is hardly any grip between its wheels and the road surface. At a bend, a vehicle on ice would continue in a straight line because the icy surface is unable to exert a force on the wheels.

In the absence of any resultant force acting on it, an object remains at rest or continues to move at constant velocity. This statement is known as **Newton's first law of motion**. Essentially, this law defines a force as anything that can change the velocity of an object.

- An ice hockey puck moving on ice keeps moving without change of direction or change of speed because there is negligible frictional force between the puck and the ice surface. See [Fig. 3.1](#). No force means no change of motion.
- When a cupboard is pushed across a floor, a force must be applied to keep the cupboard moving. This is because the hidden force of friction balances out the applied force, so there is no resultant force on the cupboard and it keeps moving at constant velocity. See [Fig. 3.2](#).

Force and acceleration

To investigate the link between force and acceleration, we need to eliminate friction, then time the motion of an object when a constant force is applied to it. The arrangement shown in [Fig. 3.3](#) shows one way this can be achieved. [Table 3.1](#) gives some typical results from such an experiment.



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Fig. 3.1 On ice there is virtually no friction

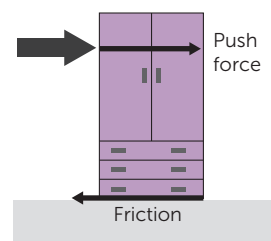


Fig. 3.2 The hidden force of friction

EXERCISE

- 1 A card with friction-free bearings is pulled along a horizontal track by a string which hangs over a pulley and supports a falling weight.
- 2 Different forces are applied using different weights, transferred to or from the cart to keep the total mass constant.
- 3 A pair of light gates is used to time how long the cart takes to travel from rest at the first light gate through a measured distance.
- 4 The acceleration, a , is calculated using $a = 2s \div t^2$ (from $s = ut + \frac{1}{2}at^2$ with $u = 0$) where s is the distance between the light gates and t is the time taken for the cart to travel through this distance.
- 5 Other methods of measuring acceleration may be used. For example, a motion sensor linked to a computer as in Fig. 2.10 can be used to record and display the change of speed with time.

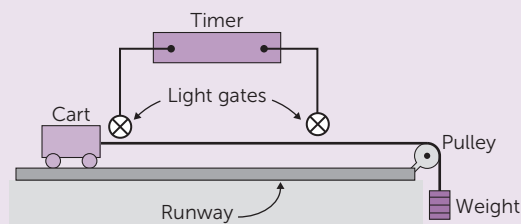


Fig. 3.3 Force and friction

Notes

- 1 The equation $F = ma$ can be rearranged to give

$$a = \frac{F}{m} \text{ to find } a,$$
 given values of F and m ,
 or

$$m = \frac{F}{a} \text{ to find } m,$$
 given values of F and a .
- 2 If several forces act on an object and F is the resultant of these forces, the acceleration, a , of the object is given by

$$a = \frac{F}{m}$$
- 3 If the resultant force F is zero, the acceleration of the object is zero in accordance with Newton's first law.

Table 3.1 Typical results

Force/N	1.0	2.0	3.0	1.0	2.0	3.0
Mass/kg	0.5	0.5	0.5	1.0	1.0	1.0
Acceleration/ m s^{-2}	2.0	4.0	6.0	1.0	2.0	3.0
Mass $\times$ acceleration/ kg m s^{-2}	1.0	2.0	3.0	1.0	2.0	3.0

The results show that the force is proportional to the mass $\times$ acceleration.

Force, $F \propto$ mass, $m \times$ acceleration, a

This may be written as an equation

$$F = ma$$

where the unit of force, the newton (N), is defined as the amount of force that would give a mass of 1 kg an acceleration of 1 m s^{-2} .

The equation $F = ma$ is **Newton's second law** for constant mass. You will meet the general form of Newton's second law of motion in Section 3.2.

3.1 WORKED EXAMPLE

A car of total mass 600 kg accelerates from rest to a speed of 15 m s^{-1} in 12 s. Calculate

- a) its acceleration,
- b) the force needed to produce this acceleration.

Solution

- a) Data to calculate acceleration, a :

Initial speed, $u = 0$, time taken, $t = 12 \text{ s}$, final velocity, $v = 15 \text{ m s}^{-1}$

$$\text{Using } v = u + at \text{ gives } a = \frac{(v - u)}{t} = \frac{(15 - 0)}{12} = 1.25 \text{ m s}^{-2}$$

- b) $F = ma = 600 \text{ kg} \times 1.25 \text{ m s}^{-2} = 750 \text{ N}$

Weight and weightlessness

A freely falling object descends at constant acceleration, g . This acceleration is due to the force of gravity on the object (i.e. its weight). Therefore, using $F = ma$ gives its weight as its mass $\times g$.

Weight, $W = mg$, for an object of mass m

An object in free fall is unsupported, but it is not weightless because the force of gravity continues to act on it. An astronaut 'floating' in an orbiting space vehicle is unsupported but not weightless. The force of gravity (i.e. the astronaut's weight) is necessary to keep the astronaut in orbit, otherwise the astronaut would move off at a tangent.

On the launch pad

For a rocket of mass, m , to take off vertically from its launch pad, as shown in Fig. 3.4, the thrust, T , from its engines must exceed its weight, mg . The resultant force is $T - mg$. Using $F = ma$ therefore gives

$T - mg = ma$, where a is the acceleration of the rocket.

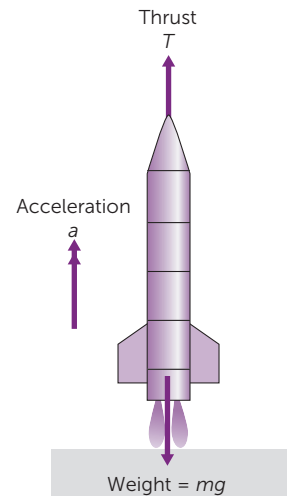


Fig. 3.4 Blast-off

3.2 WORKED EXAMPLE

$$g = 9.8 \text{ m s}^{-2}$$

A rocket of mass 5000 kg is powered by rocket engines capable of developing a total thrust of 69 000 N. Calculate

- the resultant force on lift-off,
- the acceleration at lift-off.

Solution

a) Resultant force = thrust – weight = $69\,000 - (5000 \times 9.8) = 20\,000 \text{ N}$

b) Acceleration = $\frac{\text{resultant force}}{\text{mass}} = \frac{20\,000}{5000} = 4.0 \text{ m s}^{-2}$

Lift problems

When a descending lift stops, the passengers in the lift feel a greater push than normal from the floor of the lift as it slows down. This extra push is necessary to stop the passengers. The lift cable is under extra tension as the lift stops.

Consider a lift of mass, m , supported by a cable as shown in Fig. 3.5.

- When the lift is at rest or moving at constant speed, the tension, T , in the cable is equal and opposite to the weight of the lift, i.e. $T = mg$.
- When the lift is accelerating or decelerating, the tension is not equal to the weight. The difference between the tension and the weight is the resultant force and is therefore equal to the mass $\times$ the acceleration, a , of the lift. In general terms

$$T - mg = ma$$

where upward quantities are assigned $+$ values and downward quantities are assigned $-$ values.

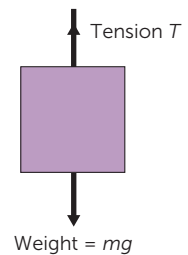


Fig. 3.5 Lift problems

There are four situations to consider.

- (a) Lift accelerating as it moves upwards:
 $T > mg$, since $a > 0$ (as acceleration is directed up, in the same direction as its velocity).
- (b) Lift accelerating as it moves downwards:
 $T < mg$, since $a < 0$ (as acceleration is directed down, in the same direction as the velocity).
- (c) Lift decelerating as it moves upwards:
 $T < mg$, since $a < 0$ (as acceleration is directed down, in the opposite direction to its velocity).
- (d) Lift decelerating as it moves downwards:
 $T > mg$, since $a > 0$ (as acceleration is directed up, in the opposite direction to the velocity).

3.3 WORKED EXAMPLE

$$g = 9.8 \text{ m s}^{-2}$$

A lift of mass 450 kg and a maximum load capacity of 320 kg is supported by a steel cable. Calculate the tension in the cable when

- a) it ascends at a constant speed,
- b) it ascends with an acceleration of 2.0 m s^{-2} ,
- c) it descends and decelerates at 2.0 m s^{-2} ,
- d) it ascends with a deceleration of 2.0 m s^{-2} ,
- e) it descends with an acceleration of 2.0 m s^{-2} .

Solution

a) $a = 0$ $\therefore$ tension in the lift cable, $T = mg = (450 + 320) \times 9.8 = 7550 \text{ N}$

b) $a = +2.0 \text{ m s}^{-2}$ $\therefore T - mg = ma$, where T is the tension
Hence $T = mg + ma = (770 \times 9.8) + (770 \times 2.0) = 9090 \text{ N}$

c) The velocity is downwards and the deceleration is upwards.
 $\therefore a = +2.0 \text{ m s}^{-2}$ (+ for upwards which is the opposite sign to the velocity)
Tension, T , is given by $T - mg = ma$
 $\therefore T = mg + ma = 770 \times 9.8 + 770 \times 2.0 = 9090 \text{ N}$

d) The velocity is upwards and the deceleration is downwards
 $\therefore a = -2.0 \text{ m s}^{-2}$
 $\therefore$ Tension, T , is given by $T - mg = ma$
 $\therefore T = mg + ma = (770 \times 9.8) + (770 \times -2.0) = 6010 \text{ N}$

e) The velocity is downwards and the acceleration is downwards
 $\therefore a = -2.0 \text{ m s}^{-2}$
 $\therefore$ Tension, T , is given by $T - mg = ma$
 $\therefore T = mg + ma = (770 \times 9.8) + (770 \times -2.0) = 6010 \text{ N}$

3.1 QUESTIONS

- A 1200 kg vehicle travelling at a speed of 25 m s^{-1} brakes to a halt in 8.0 s. Calculate
 - its deceleration,
 - the force needed to produce this deceleration.
- A rocket of mass 8000 kg lifts off with an initial acceleration of 6.0 m s^{-2} . Calculate
 - its weight at lift-off,
 - the thrust from its engines.

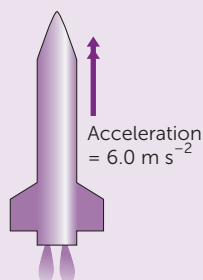


Fig. 3.6

- A lift of total mass 1500 kg is supported by a steel cable. Calculate the cable tension when the lift
 - moves at constant speed,
 - moves upwards with an acceleration of 1.2 m s^{-2} ,
 - slows down with a deceleration of 1.2 m s^{-2} as it moves downwards.

3.2 Force and momentum

A moving body possesses momentum due to its motion. If ever you have collided with someone running at top speed, you will know about momentum!

The momentum of a moving body is defined as its mass $\times$ its velocity.

The unit of momentum is the kilogram metre per second (kg m s^{-1}). Momentum is a vector quantity in the same direction as the velocity of the body. For a body of mass, m , moving at velocity, v , its momentum is given by the equation

$$\text{momentum} = mv$$

Fig. 3.7 shows a body of constant mass acted on by a constant force, F , which increases its speed from u to v in time t without a change of direction. Its acceleration, a , can be written as

$$a = \frac{(v - u)}{t}$$

$$\begin{aligned} \text{hence force, } F = ma &= \frac{m(v - u)}{t} \\ &= \frac{mv - mu}{t} = \frac{\text{change of momentum}}{\text{time taken}} \end{aligned}$$

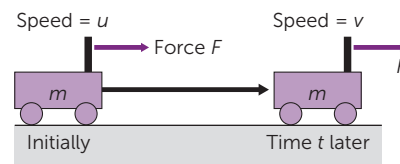


Fig. 3.7 Force and motion

3.4 WORKED EXAMPLE

A railway locomotive of mass 30 000 kg, moving at a speed of 8.5 m s^{-1} , was stopped on a level track in a time of 15.0 s when its brakes were applied.

Calculate

- its change of momentum,
- the braking force.

Solution

- Change of momentum = final momentum – initial momentum
 $= 0 - (30\,000 \times 8.5) = -255\,000 \text{ kg m s}^{-1}$

- Force = $\frac{\text{change of momentum}}{\text{time taken}} = \frac{-255\,000}{15} = -17\,000 \text{ N}$ (– indicates a retarding force)



Fig. 3.8

3.5 WORKED EXAMPLE

A rubber ball of mass 0.12 kg , moving at a speed of 25 m s^{-1} , rebounds perpendicularly from a smooth vertical wall without loss of speed. The time of contact between the wall and the ball was 0.20 ms . Calculate

- the change of momentum of the ball due to the impact,
- the force of impact.

Solution

a) Initial momentum = $0.12 \times 25 = 3.0 \text{ kg m s}^{-1}$

Final momentum = $-0.12 \times 25 = -3.0 \text{ kg m s}^{-1}$

(– for away from the wall; + for towards the wall)

$\therefore$ Change of momentum = final momentum – initial momentum

$$= (-3.0) - (3.0)$$

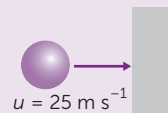
$$= -6.0 \text{ kg m s}^{-1}$$

b) Force of impact = $\frac{\text{change of momentum}}{\text{impact time}}$

$$= \frac{-6.0}{0.20 \times 10^{-3}}$$

$$= -30\,000 \text{ N}$$

(a) Before



$$v = -25 \text{ m s}^{-1}$$



(b) After

Fig. 3.9

Note: The minus sign means that the force on the ball is directed away from the wall.

Newton's second law of motion

The equation

$$\text{force} = \frac{\text{change of momentum}}{\text{time taken}}$$

is an expression of Newton's second law of motion.

The rate of change of momentum of an object is proportional to the resultant force on it.

In more general terms, Newton's second law may be written as

$$F = \frac{d}{dt}(mv)$$

where $\frac{d}{dt}$ means 'rate of change' (see p. 38).

- For constant mass,

$$F = \frac{d}{dt}(mv) = m \frac{dv}{dt} = ma$$

since acceleration, a = rate of change of velocity $\frac{dv}{dt}$

- For a constant rate of change of mass, $\frac{dm}{dt}$ (e.g. a rocket or a jet engine),

$$F = \frac{d}{dt}(mv) = v \frac{dm}{dt}$$

where v is the relative speed at which mass is lost or gained.

3.6 WORKED EXAMPLE

A rocket's engines generate a thrust of 127 kN by expelling hot gas at a speed of 1500 m s^{-1} .

- Calculate the rate at which mass is expelled from the rocket's engines.
- The rocket carries 7500 kg of fuel. Calculate how long it would take to burn this amount of fuel.



Fig. 3.10

Solution

- a) Use $F = v \frac{dm}{dt}$, hence $127\,000 = 1500 \frac{dm}{dt}$
 to give $\frac{dm}{dt} = \frac{127\,000}{1500}$
 $= 85 \text{ kg s}^{-1}$

- b) Since 85 kg of fuel is burned each second, the time taken to burn 7500 kg = $\frac{7500}{85} = 88 \text{ s}$

3.2 QUESTIONS

$$g = 9.8 \text{ m s}^{-2}$$

- A rubber ball of mass 0.120 kg, moving at a speed of 25 m s^{-1} perpendicular to a smooth vertical wall, rebounds from the wall without loss of speed in an impact lasting 0.004 s. Calculate
 - the change of momentum of the ball,
 - the impact force.
- A rocket of mass 2000 kg is launched vertically with an initial acceleration of 6.5 m s^{-2} . Calculate
 - the weight of the rocket on the launch pad,
 - the thrust of its engines.

3.3 Conservation of momentum

An impact puzzle

In an impact between a heavy lorry and a car, which vehicle experiences the greatest impact force? The car suffers most damage so you might think it experiences the greatest force. In fact, the two vehicles experience equal and opposite forces. The impact damage to the car is greater though because it is lighter. Whenever two bodies interact, they exert equal and opposite forces on one another. This is known as **Newton's third law of motion**.

When two bodies interact, they exert equal and opposite forces on one another.

Some further examples of Newton's third law at work are discussed below.

- When a chair is made to slide across a rough floor, the floor exerts a force on the chair in the opposite direction to its motion. See Fig. 3.11. The chair exerts an equal force on the floor in the direction the chair is moving.
- Someone leaning on a wall pushes on the wall with a force equal and opposite to the force of the wall on that person.

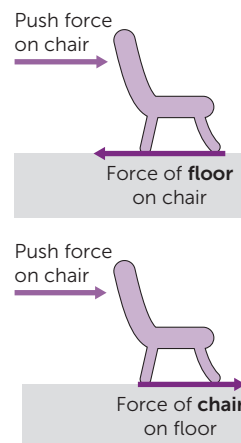


Fig. 3.11 Equal and opposite forces

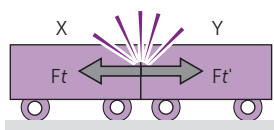


Fig. 3.12 Equal and opposite impulses

Collisions

The **impulse** of a force is defined as the force $\times$ the time for which it acts. From Newton's second law,

$$\text{force} = \frac{\text{change of momentum}}{\text{time taken}}$$

Hence the impulse of a force = force $\times$ time = change of momentum. Therefore, when two bodies X and Y interact, as shown in Fig. 3.12, the impulse of X on Y is equal and opposite to the impulse of Y on X since the force of X on Y is equal and opposite to the force of Y on X. Since impulse equals change of momentum, the change of momentum of X is equal and opposite to the change of momentum of Y when X and Y interact.

In a collision between two bodies, momentum is transferred from one to the other. Therefore, the momentum lost by one body is equal and opposite to the momentum gain of the other body. The total momentum is therefore unchanged, because one object loses the same amount of momentum as the other object gains. This is an example of the principle of conservation of momentum.

The principle of conservation of momentum states that when two or more bodies interact, the total momentum is conserved (i.e. not changed), provided no external force acts on the bodies.

Consider a body of mass, m_1 , moving along a straight line at an initial speed, u_1 , which collides with another body of mass, m_2 , moving along the same line at speed, u_2 as shown in Fig. 3.13. The two bodies move apart after the collision at speeds, v_1 and v_2 , along the same straight line as before.

$$\text{The total initial momentum} = m_1u_1 + m_2u_2$$

$$\text{The total final momentum} = m_1v_1 + m_2v_2$$

Using the principle of conservation of momentum therefore gives

the total final momentum = the total initial momentum

$$(m_1v_1 + m_2v_2) = (m_1u_1 + m_2u_2)$$

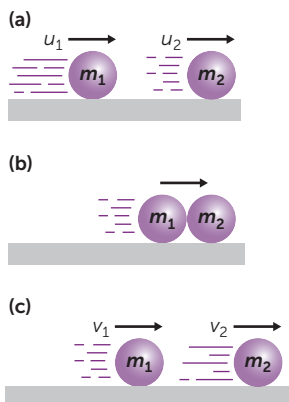


Fig. 3.13 Conservation of momentum (a) before impact, (b) on impact, (c) after impact

Experiment to verify the principle of conservation of momentum

Fig. 3.14 shows a moving dynamics cart, A, on a horizontal track about to collide with a stationary cart, B. The two carts stick together on impact. Use a motion sensor to measure the speed of the dynamics cart before and after impact, on a horizontal runway.

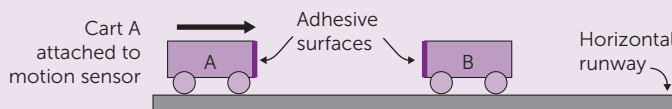


Fig. 3.14 Testing conservation of momentum

The total momentum before impact = mass of A $\times$ speed of A before impact.

The total momentum after impact = (mass of A + mass of B) $\times$ speed of A after impact.

The measurements should demonstrate that the total momentum is unchanged by the impact.

3.7 WORKED EXAMPLE



Fig. 3.15

A vehicle of mass 600 kg, moving at a speed of 30 m s^{-1} , collides with a stationary vehicle of mass 900 kg. The two vehicles lock together on impact. Calculate

- the speed of the two vehicles immediately after the impact,
- the force of the impact, if the duration of the collision was 0.15 s,
- the acceleration of each vehicle during the impact.

Solution

- a) Using the principle of conservation of momentum,

$$(m_1v_1 + m_2v_2) = (m_1u_1 + m_2u_2)$$

$$600V + 900V = (600 \times 30) + (900 \times 0)$$

where V is the speed of the two bodies locked together immediately after the impact.

$$\text{Hence } 1500V = 18\,000$$

$$V = \frac{18\,000}{1500} = 12 \text{ m s}^{-1}$$

- b) To calculate the force on vehicle 1, use

$$\begin{aligned} F &= \frac{m_1v_1 - m_1u_1}{t} \\ &= \frac{(600 \times 12) - (600 \times 30)}{0.15} \\ &= \frac{7200 - 18000}{0.15} = -72\,000 \text{ N} \end{aligned}$$

An equal and opposite force is exerted on vehicle 2.

- c) Acceleration = $\frac{\text{force}}{\text{mass}}$

$$\text{For vehicle 1, acceleration} = \frac{-72\,000 \text{ N}}{600 \text{ kg}} = -120 \text{ m s}^{-2} \text{ (i.e. deceleration)}$$

$$\text{For vehicle 2, acceleration} = \frac{72\,000 \text{ N}}{900 \text{ kg}} = 80 \text{ m s}^{-2}$$

Explosions

In an explosion, bits fly in all directions, each bit taking momentum away. Assuming the object that exploded was initially at rest, the total initial momentum must have been zero. The total final momentum is therefore also zero since the total momentum does not change.

To understand why the total final momentum is zero, consider an explosion where two bodies, X and Y, initially at rest, fly apart as shown in Fig. 3.16. They push each other away with equal and opposite forces for the same length of time. Therefore they exert equal and opposite impulses on each other. Since impulse is

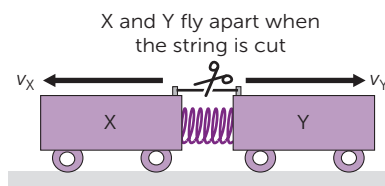


Fig. 3.16 A controlled explosion

change of momentum, they therefore gain equal and opposite amounts of momentum. The gain of momentum of X is equal and opposite to the gain of momentum of Y because they move in opposite directions. The two objects recoil from each other with equal and opposite amounts of momentum.

Momentum is a vector quantity so if one direction is defined as positive and the other direction negative, one object carries away positive momentum and the other object carries away an equal amount of negative momentum.

Momentum of X after explosion = $m_X v_X$

where m_X is the mass of X and v_X is the speed of X immediately after the explosion

Momentum of Y after the explosion = $-m_Y v_Y$ (– for the opposite direction to X)

where m_Y is the mass of Y and v_Y is the speed of Y immediately after the explosion

∴ Total momentum after the explosion = $m_X v_X - m_Y v_Y$

Also, the total initial momentum = 0 since the objects were initially at rest. Therefore, according to the principle of conservation of momentum,

$$m_X v_X - m_Y v_Y = 0$$

$$\therefore m_X v_X = m_Y v_Y$$

See www.palgrave.com/foundations/breithaupt for an experiment to compare masses using the principle of conservation of momentum.

3.8 WORKED EXAMPLE

A student of mass 60 kg, standing on a stationary skateboard of mass 3.0 kg, jumps off the skateboard at a speed of 0.5 m s^{-1} . Calculate the speed of recoil of the skateboard.

Solution

Momentum of student immediately after jumping off the skateboard

$$= 60 \times 0.5 = 30 \text{ kg m s}^{-1}$$

Momentum of skateboard = $3.0v$, where v is the speed of the skateboard.

$$3.0v = 30 \quad \therefore v = 10 \text{ m s}^{-1}$$

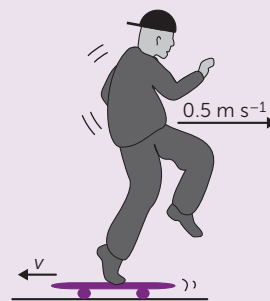


Fig. 3.17

3.3 QUESTIONS

- 1 A vehicle of mass 800 kg, moving at a speed of 25 m s^{-1} , collides with a second vehicle of mass 1600 kg which is initially at rest. The impact reduces the speed of the first vehicle to 5 m s^{-1} without change of direction. Calculate
 - a) the loss of momentum of the first vehicle,
 - b) the speed of the second vehicle immediately after the impact, (c) the force of the impact, if the duration of the impact was 0.2 s.
- 2 An artillery gun of total mass 500 kg is designed to fire a shell of mass 4.0 kg at a speed of 115 m s^{-1} . Calculate the recoil speed of the gun.

SUMMARY

- **Momentum** = mass $\times$ velocity.
- **Newton's laws of motion:**

1st law An object remains at rest or in uniform motion unless acted on by a resultant force.

2nd law The resultant force on an object is proportional to the rate of change of momentum of the object.

3rd law When two objects interact, they exert equal and opposite forces on each other.

- **The principle of conservation of momentum** states that the total momentum of any system of bodies is constant, provided no external resultant force acts on the system.

- **Equation for Newton's second law**

In general, $F = \frac{d}{dt}(mv)$

For constant mass, $F = m \frac{dv}{dt} = ma$

This may be written as $F = m \frac{(v - u)}{t} = \frac{mv - mu}{t}$ for constant acceleration.

For constant rate of change of mass, $F = v \frac{dm}{dt}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

$$g = 9.8 \text{ m s}^{-2}$$

3.1

- A vehicle of total mass 1500 kg accelerates from rest to a speed of 12 m s^{-1} in 10 s. Calculate
 - the acceleration of the vehicle,
 - the force required to cause this acceleration.
- A lift of total mass 850 kg is suspended from four identical steel cables. Calculate the tension in each cable when the lift descends
 - at steady speed,
 - with a deceleration of 0.5 m s^{-2} .
- A rectangular block of mass 12 kg on a rough surface is acted on by a force of 50 N parallel to the surface. The coefficient of friction between the block and the surface is 0.4.

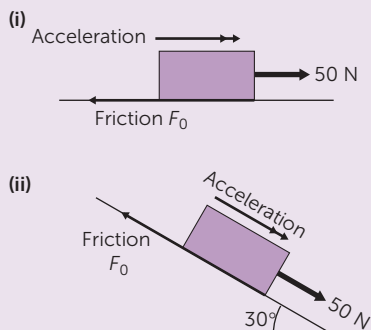


Fig. 3.18

Calculate the acceleration of the block if the surface is

- horizontal,
- inclined at 30° to the horizontal such that the force acts down the slope.

- An air-rifle pellet of mass 0.001 kg , moving at a speed of 100 m s^{-1} , strikes a tree and embeds itself to a depth of 50 mm in the wood. Calculate

- the loss of momentum of the pellet due to the impact,
- the duration of the impact,
- the force of impact.

- A rocket of total mass 1200 kg accelerates directly upwards off the launch pad with a constant thrust from its engines of 16 kN.

- Calculate its weight and show that its acceleration is 3.5 m s^{-2} .
- Calculate the rate of loss of mass of fuel from the rocket if the exhaust gases are ejected at a speed of 1200 m s^{-1} .
- Calculate how long its fuel would last if its initial mass of fuel was 800 kg.

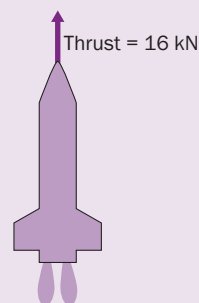


Fig. 3.19

REVISION QUESTIONS

3.4 A railway wagon of mass 1000 kg, moving along a horizontal track at a speed of 3.0 m s^{-1} , collided with a wagon of mass 2000 kg and coupled to it. Calculate the speed of the two wagons immediately after the impact if the second wagon was initially

- a) at rest,
- b) moving at a speed of 2.0 m s^{-1} in the same direction as the 1000 kg wagon was moving,
- c) moving at a speed of 2.0 m s^{-1} in the opposite direction to the direction the 1000 kg wagon was moving before the impact.

3.5 In an experiment to measure the mass of an unknown object, the unknown object is fixed to a dynamics cart of mass 0.50 kg. A spring-loaded cart of mass 0.60 kg is then placed in contact with the first cart on a horizontal track. When the spring is released, the two carts recoil from each other due to the action of the spring. The spring-loaded cart moves away at a speed of 0.35 m s^{-1} and the other cart moves in the opposite direction at a speed of 0.22 m s^{-1} . Calculate the mass of the unknown object.

Energy and Power

UNIT

4

OBJECTIVES

After working through this unit, you should be able to:

- calculate the work done when a force moves its point of application
- describe different forms of energy and analyse energy transformations
- derive and use formulae for kinetic energy and change of potential energy
- define power and carry out power calculations
- relate the power of an engine to force and speed
- explain what is meant by useful energy, waste energy and efficiency when energy changes occur
- carry out efficiency calculations and explain the main causes of inefficiency
- describe sources of useful energy

CONTENTS

- 4.1 Work and energy
- 4.2 Power and energy
- 4.3 Efficiency
- 4.4 Energy resources
- Summary
- Revision questions

4.1 Work and energy

The meaning of work

Working out in a gym usually involves lifting weights or moving about vigorously. A weightlifter who raises a 400 N weight by 1.0 m does twice as much work as someone who raises a 200 N weight by 1.0 m or who raises a 400 N weight by 0.5 m. On an exercise bike, you do work by pedalling to make an axle rotate against friction. The force of friction can be altered by adjusting the 'brake system' pressing on the axle.

Work is done by a force when the force moves its point of application in the direction of the force. For example, work is done by the force of gravity when an object falls. However, if an object is moved horizontally, no work is done by the force of gravity on the object because there is no vertical movement of the object. On an exercise bike, work is done by the force exerted on the pedals but only when the pedals are moving.

The work done by a force is defined as the force $\times$ the distance moved by the point of application of the force in the direction of the force.



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Fig. 4.1 Doing work on a cardio-vascular machine

The unit of work is the **joule (J)**, defined as the work done when a force of one newton moves its point of application through one metre in the direction of the force.

Work done, $W = Fd$

where F is the force and d is the distance moved by the point of application of the force in the direction of the force.

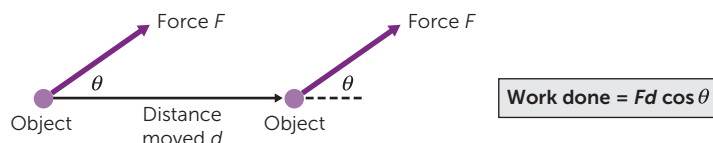


Fig. 4.2 Work and force

Where the direction of movement is at angle θ to the direction of the force as shown in Fig. 4.2, the component of force in the direction of movement is $F \cos \theta$. Hence the work done $= Fd \cos \theta$. Note that if $\theta = 90^\circ$, the work done is zero, since $\cos 90^\circ = 0$ and the force is at right angles to the displacement.

Energy

- **Energy is defined as the capacity to do work.** The unit of energy is the joule. An object can lose or gain energy, either as a result of work done by a force or through heat transfer. For example, if 100 J of work is done by a force acting on an object and 40 J of heat transfer from the object occurs, the net energy gain by the object is 60 J.
- **Energy exists in different forms**, including kinetic energy (i.e. energy of a moving object due to its motion), potential energy (i.e. energy due to position or stored energy), chemical energy, nuclear energy, electrical energy, light and sound.
- **Energy can be transformed from any one form into other forms.** Whenever energy changes from one form into other forms, the total amount of energy is unchanged. This general rule is known as the **principle of conservation of energy**.

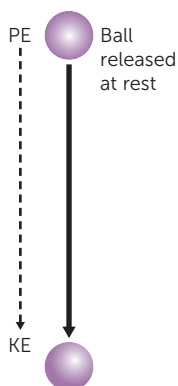


Fig. 4.3 A falling ball

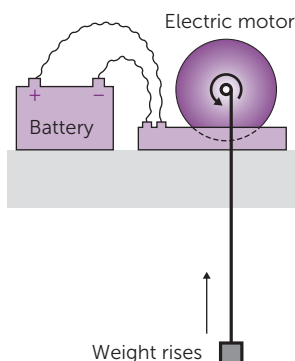


Fig. 4.4 Raising a load

Figs 4.3 and 4.4 show two examples of energy transformations. In Fig. 4.3, the ball loses potential energy as it falls and gains kinetic energy. Because air resistance is negligible, the gain of kinetic energy is equal to the loss of potential energy.

In Fig. 4.4, the weight gains potential energy as it rises as a result of electrical energy from the battery enabling the motor to do work. Not all the electrical energy from the battery is converted to potential energy, as heat is produced in the wires due to their resistance; friction at the bearings of the motor also generates heat.

Gravitational potential energy

To raise a body, a force must be applied which is equal and opposite to its weight. The work done by the applied force is therefore equal to the weight of the body $\times$ its height gain. This is equal to the gain of potential energy of the body. (Note: strictly, potential energy is gained by the body and the Earth as a result of separating them.) Since the weight of an object equals mg , where m is its mass,

the change of potential energy, $\Delta E_p = mgh$

where h is its height gain or loss.

Kinetic energy

For a body of mass, m , moving at speed, v , its kinetic energy E_k is given by

$$E_k = \frac{1}{2}mv^2$$

To prove this equation, consider an object of mass, m , which accelerates from rest to speed, u , in time, t , due to a constant force, F , acting on the object.

1 The distance it moves, $s = \frac{1}{2}(u + v)t = \frac{vt}{2}$

2 The force acting on it, $F = ma = \frac{m(v - u)}{t} = \frac{mv}{t}$

Hence the work done by the force is

$$Fs = \frac{mv}{t} \times \frac{vt}{2} = \frac{1}{2}mv^2$$

which is therefore the kinetic energy of the body.

This equation is not valid at speeds approaching the speed of light, c ($= 300\,000\text{ km s}^{-1}$) and Einstein's famous equation, $E = mc^2$, is used in such circumstances.

4.1 WORKED EXAMPLE

On a roller coaster, a passenger train of total mass 2500 kg descends from rest at the top of a steep incline 75 m high. Calculate

- the loss of potential energy (p.e.) as a result of descending the incline,
- (i) its kinetic energy (k.e.) at the bottom of the incline,
(ii) its speed at the bottom of the incline.

Solution

- Loss of p.e. $= mgh = 2500 \times 9.8 \times 75 = 1.84\text{ MJ}$
- (i) Gain of k.e. = loss of p.e. provided air resistance and friction are negligible. Hence its k.e. at the bottom $= 1.84\text{ MJ}$
(ii) $\frac{1}{2}mv^2 = 1.84 \times 10^6\text{ J}$, hence $\frac{1}{2} \times 2500 \times v^2 = 1.84 \times 10^6$
 $\therefore v^2 = 2 \times 1.84 \times 10^6 \div 2500 = 1470$ Hence $v = 38\text{ m s}^{-1}$

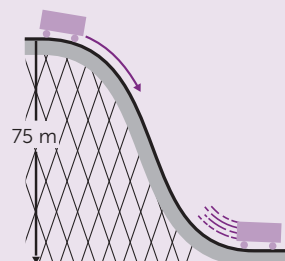


Fig. 4.5

Note The speed is the same, regardless of the mass, since $\frac{1}{2}mv^2 = mgh$ $v = \sqrt{(2gh)}$ which is independent of m .

4.1 QUESTIONS

$$g = 9.8\text{ m s}^{-2}$$

- A ball of mass 0.12 kg is thrown vertically up at a speed of 22 m s^{-1} . Calculate
 - its initial kinetic energy,
 - its kinetic energy at maximum height,
 - its gain of potential energy on reaching maximum height,
 - its maximum height gain.
- A bobsleigh of total mass 150 kg accelerates down a 1500 m incline from rest, descending a total height of 200 m and reaching a speed of 55 m s^{-1} at the foot of the incline. Calculate
 - the loss of potential energy of the bobsleigh,
 - its gain of kinetic energy,
 - the work done against friction and air resistance,
 - the average force resisting the motion of the bobsleigh.

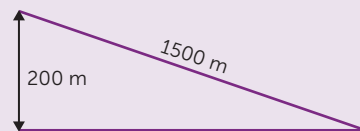


Fig. 4.6

4.2 Power and energy

In an aerobics class, a student of weight 540 N steps repeatedly on and off a box of height 0.5 m. For each step, 270 J ($= 540 \text{ N} \times 0.5 \text{ m}$) of work is done. If the student takes 1.8 s for each complete step, then she does 150 J of work each second ($= 270 \text{ J} \div 1.8 \text{ s}$). This is her power output. In the weightlifting section, another student lifts a weight of 300 N through a height of 0.80 m in a time of 3.0 s. The weightlifter does 240 J ($= 300 \text{ N} \times 0.80 \text{ m}$) of work in this process and has a power output of 80 J s^{-1} .

Power is defined as rate of transfer of energy.

The unit of power is the **watt**, equal to a rate of energy transfer of 1 J s^{-1} . Note that 1 kilowatt = 1000 watts.

For energy transfer, E , in time t ,

$$\text{the average power, } P = \frac{E}{t}$$

4.2 WORKED EXAMPLE

A student of weight 450 N climbed 4.2 m up a vertical rope in 15 s. Calculate
a) the gain of potential energy,
b) the power of the student's muscles.

Solution

$$\begin{aligned} \text{a) Gain of p.e.} &= 450 \text{ N} \times 4.2 \text{ m} \\ &= 1890 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{b) Power} &= \frac{\text{p.e. gain}}{\text{time taken}} \\ &= \frac{1890 \text{ J}}{15 \text{ s}} = 126 \text{ W} \end{aligned}$$

4.3 WORKED EXAMPLE

A vehicle of total weight 20 000 N travels up an incline for 100 s at a steady speed of 8.0 m s^{-1} , gaining a height of 120 m.

- a) Calculate the gain of potential energy of the vehicle.
b) Estimate the output power of its engine.

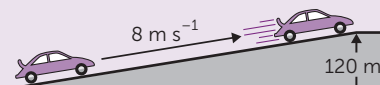


Fig. 4.7

Solution

$$\text{a) Gain of p.e.} = mgh = 20\,000 \times 120 = 2.4 \text{ MJ}$$

b) Assume all the work done by the engine increases the p.e.

$$\begin{aligned} \therefore \text{output power} &= \frac{\text{work done by the engine}}{\text{time taken}} \\ &= \frac{2.4 \text{ MJ}}{100 \text{ s}} = 24\,000 \text{ W} \end{aligned}$$

Engine power and speed

Consider a vehicle of mass, m , initially at rest, driven by an engine that delivers a constant driving force, F , as shown in Fig. 4.8. The initial acceleration of the vehicle is therefore $F \div m$. Because the drag force, D , on it due to air resistance increases with speed, the resultant force becomes less and less as the vehicle speeds up. Its acceleration therefore becomes less and less and it reaches a maximum speed when the drag force is equal and opposite to the driving force. In general terms,

$$\text{acceleration, } a = \frac{F - D}{m}$$

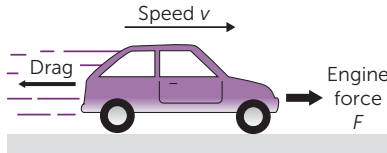


Fig. 4.8 Engine power

- The top speed is reached when $D = F$, corresponding to $a = 0$ in the above equation.
- The initial acceleration $= F \div m$, corresponding to $D = 0$ in the above equation.
- The output power from the engine, $P_E = \text{work done per second} = \text{engine force} \times \text{distance moved per second}$.

$$P_E = Fv$$

- The difference between the work done by the engine and the gain of kinetic energy is the energy wasted due to resistive forces. At top speed, all the work done by the engine is wasted.

Drag factors

The drag force on a moving vehicle depends on the speed of the vehicle, its shape and on the area, A , of the vehicle as seen from the front. A moving vehicle creates turbulence in the surrounding air except at very low speed. Investigations have shown that where turbulence is created, the drag force, D , is given by the equation:

$$D = C_D \times \frac{1}{2} A \rho v^2$$

where v is the vehicle speed, ρ is the fluid density and C_D is the drag coefficient of the vehicle. The term $\frac{1}{2} A \rho v^2$ is the force due to pressure on a flat plate of area, A , moving at speed, v , hence C_D represents the shape of the vehicle. Fig. 4.9 shows how the drag coefficient varies with speed for different vehicles.

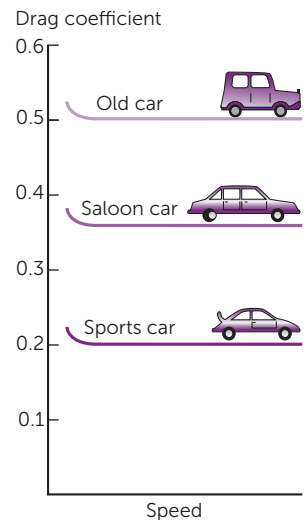


Fig. 4.9 Shape and drag

Viscous drag

Internal friction in a fluid causes a drag force on surfaces which move relative to the fluid. Such friction is due to the viscosity of the fluid, which is a measure of how easily it can flow. When viscous effects are dominant and turbulence is negligible, the drag force is proportional to the speed according to the equation

$$D = kv$$

where k is a constant that depends on the object and on the viscosity of the fluid it moves through. For example, a ball released at rest in water reaches terminal speed

when the drag force becomes equal and opposite to its weight. Its acceleration, a , decreases in accordance with the equation

$$a = \frac{mg - kv}{m} = g - bv \quad \text{where } b = \frac{k}{m}$$

Fig. 4.10 shows how the speed and the acceleration change with time.

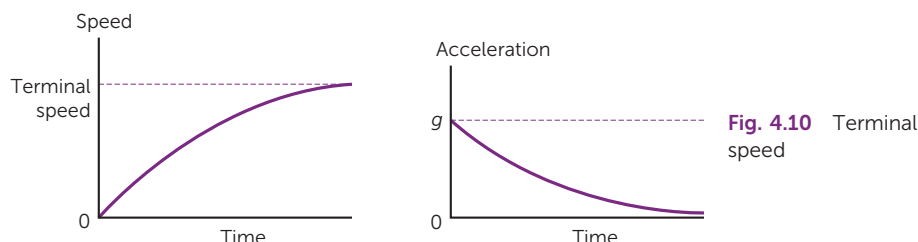


Fig. 4.10 Terminal speed

See www.palgrave.com/foundations/breithaupt to investigate motion using a spreadsheet.

Note

The equation $a = \frac{F - kv}{m}$ can be written $a = \alpha - bv$, where $\alpha = \frac{F}{m}$ and $b = \frac{k}{m}$

The general dynamics spreadsheet can be used to solve this equation numerically. The results may then be displayed as a graph. See p. 494.

4.2 QUESTIONS

$$g = 9.8 \text{ m s}^{-2}$$

- 1 a) A sprinter of mass 70 kg is capable of reaching a speed of 10 m s^{-1} from rest in 3.5 s. Calculate (i) the kinetic energy gain of the sprinter, (ii) the average power output of the sprinter's leg muscles.
- b) In a keep-fit session, a person of weight 55 kg repeatedly steps on and off a box of height 0.40 m at a rate of 20 steps per minute. Calculate (i) the person's gain of potential energy as a result of stepping onto the box, (ii) the average power output of the person's leg muscles.
- 2 A locomotive engine of mass 20 000 kg, which has an output power of 175 kW, pulls carriages of total mass 40 000 kg at a maximum speed of 55 m s^{-1} along a level track. Calculate
 - a) the output force of the engine,
 - b) the total resistive force opposing the motion,
 - c) the average force needed to stop the train in a distance of 1000 m if the engine is switched off and the brakes are applied.

4.3 Efficiency

Useful energy

In Fig. 4.11, an electric motor connected to a battery is used to raise a weight. Chemical energy from the battery is converted to potential energy of the weight. Some of the chemical energy is wasted due to friction in the motor and resistance heating in the connecting wires. See Fig. 4.12. The chemical energy converted to potential energy is referred to as **useful energy**, the energy needed for the task of raising the weight.

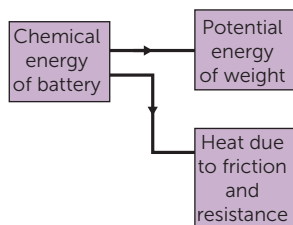


Fig. 4.12 An energy flow diagram

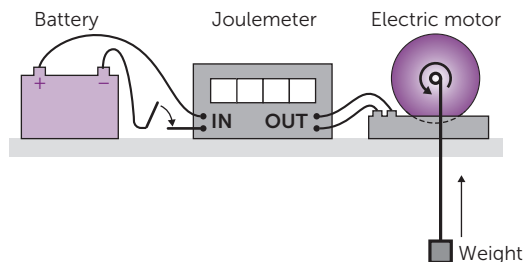


Fig. 4.11 Measuring efficiency

Friction in a machine causes energy to be wasted. The energy supplied to the machine to enable a given task to be carried out must be greater than the useful energy needed because energy is wasted due to friction, increasing the energy of the surroundings.

The **efficiency** of a machine is defined as

$$\frac{\text{the useful power delivered by the machine (i.e. output power)}}{\text{the power supplied to the machine (i.e. input power)}}$$

This is the same as the ratio

$$\frac{\text{the useful energy delivered by the machine}}{\text{the total energy supplied to the machine in the same time}}$$

Efficiency values sometimes are quoted as a percentage (i.e. this fraction $\times 100$).

Experiment to measure the efficiency of an electric winch

Fig. 4.11 shows an electric winch used to raise a weight. The potential energy gained by the weight can be calculated from its mass, m , and its height gain, h . The winch motor is connected to a suitable power supply via a joulemeter. The joulemeter is used to measure the energy supplied to the motor when the motor is used to raise a known weight by a measured height. The efficiency can be calculated by dividing the p.e. gain ($= mgh$) by the energy supplied via the joulemeter.

Sample results

Mass = 0.50 kg, height gain = 1.2 m

Increase of joulemeter reading = 8.2 J

$$\text{Efficiency} = \frac{\text{p.e. gain}}{\text{electrical energy supplied}} = \frac{0.50 \times 9.8 \times 1.2}{8.2} = \frac{5.9}{8.2} = 0.71$$

The problem with energy

Energy tends to spread out and become less useful when it changes from one form into other forms. For example, friction between the moving parts of a machine causes heating and reduces the efficiency of the machine. Energy is wasted due to friction and this energy spreads out, becoming less useful as a result. Even in processes where energy is concentrated (for example, charging a car battery), energy must be used to make the process happen and some of this energy will be wasted (for example, the charging current will heat the circuit wires when a car battery is charged). Another example is when energy is stored in an elastic object. In theory, all the stored energy in a perfectly elastic object can be used usefully. For example, a perfectly elastic ball released above a horizontal surface would repeatedly rebound to the same height. In practice, the ball would lose height after each successive bounce and eventually it would stop bouncing. Part of the reason for this is that such a ball is not perfectly elastic. Its initial potential energy is gradually converted to internal energy of the ball and its surroundings. In general, when energy changes from one form into other forms, the total energy is conserved but it spreads out and becomes less useful. This is why fuel reserves must be used as efficiently as possible.

4.4 WORKED EXAMPLE

A 300 W electric winch takes 15 s to raise a crate of total weight 540 N through a height of 1.6 m. Calculate

- the energy supplied to the winch in this time,
- the potential energy gain of the crate,
- the energy wasted when the crate is raised 1.5 m,
- the efficiency of the electric winch.

Solution

- Energy = power $\times$ time = $300 \times 15 = 4500$ J
- p.e. gain = $540 \times 1.6 = 864$ J
- Energy wasted = $4500 - 864 = 3636$ J
- Efficiency = $\frac{864}{4500} \times 100\% = 19\%$

4.3 QUESTIONS

- A 300 W electric motor attached to a winch is used to raise a load of total weight 320 N through a height of 9.5 m in 25 s. Calculate
 - the electrical energy supplied to the motor in this time,
 - the useful energy supplied by the motor,
 - the energy wasted,
 - the efficiency of the motor.
- A load of 150 N is raised with the aid of a pulley system using an effort of 40 N. The effort must move through a distance of 4 m to raise the load by 1 m. Calculate
 - the p.e. gain of the load when it is raised by 2.5 m,
 - the work done by the effort to raise the load by 2.5 m,
 - the efficiency of the pulley system.

4.4 Energy resources

Fuel supply and demand

Each person in a developed country uses about 5000 J of energy each second on average, corresponding to about 150 000 MJ per year. Fossil fuels such as wood, coal, oil and gas release no more than about 50 MJ per kilogram when burned. Prove for yourself that at least 3000 kg of fuel each year is needed to supply your energy needs, before taking account of inefficiencies and energy needed to extract and transport fuel to where it is needed. With these factors included, you probably need about 10 000 kg of fuel each year to maintain your lifestyle. The world's fuel reserves are finite and unlikely to last more than 200 years or so at the present rate of consumption. In comparison, people in remote areas in third-world countries use far less fuel, mostly wood for cooking. As living standards rise in these countries, energy consumption will rise and fuel reserves will be used more quickly.

Fig. 4.13a shows how the world demand for energy increased between 1940 and 2010. In 1980, world demand exceeded 3×10^{20} J and looks likely to double by 2020. Present reserves of fossil fuels have been estimated as equivalent to about 2.3 million million tonnes of coal, corresponding to about 5.5×10^{20} J or about 90 years of supplies at an energy consumption rate of 6×10^{20} J per year.

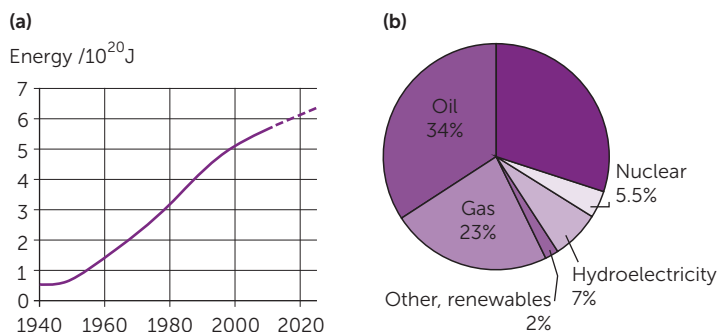


Fig. 4.13 World energy demand

Table 4.1 Fuel reserves in millions of tonnes of oil equivalent (Mtoe)

	World reserves	World use per year
Oil	230 000	5900
Gas	167 000	4100
Coal	1 340 000	5000
Nuclear: Thermal	50 000	560
Fast breeder	250 000	—
Hydroelectricity	—	860
Other renewables	—	280

Table 4.1 shows world fuel usage (2012) and fuel reserves for different types of fuels. Present reserves of oil and gas are unlikely to last much beyond 2060. The fact that coal reserves are likely to last much longer has interesting, long-term implications. The present generation of nuclear power stations uses only the U 235 content of uranium which consists of more than 99% U 238 and less than 1% U 235. At the present rate of use, world uranium reserves are likely to last no more than about 100 years. The fast-breeder nuclear reactor uses plutonium as its fuel. This is an artificial element which is produced from U 238 in thermal reactors. The world's uranium reserves could therefore last many centuries if fast-breeder reactors replaced present thermal reactors.

Fuel and the environment

Fossil fuels

Fossil fuels produce so-called 'greenhouse' gases, such as carbon dioxide, when burned. These gases are thought to be responsible for atmospheric warming in recent decades which could cause the polar ice caps to melt. Low-level countries bordering the oceans would be threatened if the ice caps melted. In addition, burning coal releases sulphur dioxide into the atmosphere which then dissolves in rain droplets to produce acid rain. This can have a severe effect on plant growth far away from the coal-burning sites.

Nuclear fuel

Nuclear fuel releases about a hundred thousand times as much energy per kilogram as fossil fuel. Your annual energy needs would be met by about 100 kg of uranium. In a nuclear reactor, energy is released when nuclei of U 235 are split as a result of neutron bombardment. Each fission (i.e. split) of a U 235 nucleus releases two or three neutrons, which then proceed to split other U 235 nuclei as shown in Fig. 4.14. This chain reaction is controlled in a nuclear reactor by the presence of neutron-absorbing control rods which ensure only one neutron per fission goes on to produce further fission. See p. 376.

The fission products in a fuel rod are extremely radioactive. In addition, the spent fuel rods contain highly radioactive plutonium produced as a result of U 238 nuclei absorbing neutrons. After removal from the reactor, the spent fuel rods are first

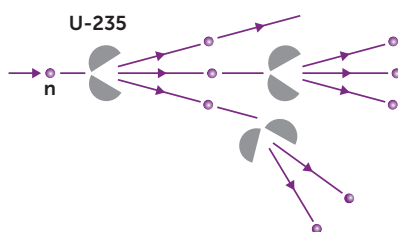


Fig. 4.14 A chain reaction

placed in cooling ponds for several months until they are cool enough to be processed. The unused uranium and plutonium are removed and stored for possible re-use at this stage. The fission products are highly radioactive for many years and are stored in sealed containers in safe, underground conditions.

Renewable energy resources

The Sun radiates energy at a rate of about 4×10^{26} J every second. In one year, the Earth receives about 5×10^{24} J of energy from the Sun. World energy demands would be met if just 0.01% of the energy received by the Earth from the Sun could be used. Solar heating panels and solar cells use sunlight directly. See Fig. 4.15. Solar panels are an example of a renewable energy resource, which is an energy resource that does not use fuel. Other renewable energy resources such as hydroelectric power, wind power and wave power use solar energy indirectly. The Sun heats the atmosphere and the oceans in the equatorial regions, causing circulation currents that generate weather systems responsible for waves and winds.

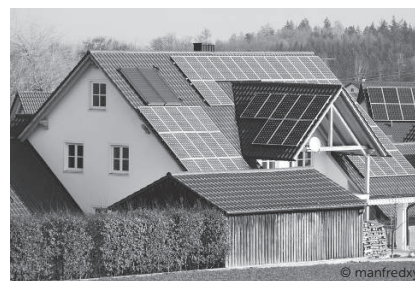
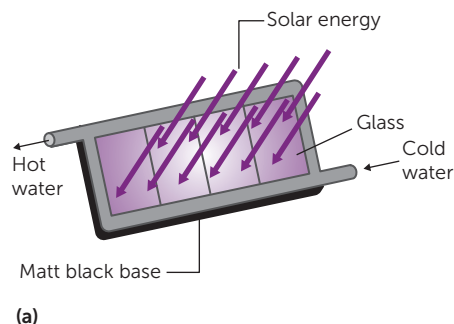


Fig. 4.15 (a) a solar heating panel, (b) panels of solar cells

Solar panels

Above the Earth's atmosphere, a surface of area 1 m^2 , facing the Sun directly, would receive 1400 J every second. Much less energy is received at sea level outside the equatorial regions. Even so, a solar heating panel of area 1 m^2 facing South, in Northern Europe or North America, could absorb 500 J per second from the Sun.

Solar cells in panels are used to provide power to satellites in orbit above the atmosphere. At present, these cells are not very efficient, although they have been developed to the extent that they can provide sufficient power to specially designed vehicles such as 'Solar challenger'.

Hydroelectric power

Hydroelectric power could make a significant contribution to energy supplies in hilly areas. A lot of rainfall is necessary. Prove for yourself that 1 cm of rain (density 1000 kg m^{-3}) over an area of 1 km^2 ($= 10^6 \text{ m}^2$) running downhill by a vertical distance of 500 m would release $5 \times 10^{10} \text{ J}$ of potential energy. At present, hydroelectric power stations worldwide meet no more than about 7% of world energy needs.

Wind power

Wind power, generated by means of wind turbines as shown in Fig. 4.16, makes a small but increasing contribution to national energy needs in some countries. The blades of a wind turbine are forced to rotate by the wind, turning an electricity generator in the wind turbine tower. A wind farm of wind turbines, each capable of producing up to 1 MW of electrical power, could meet the needs of a small town.

Wave power

Wave power is generated by floating devices when waves make one part of the device move relative to the other parts. Scientists reckon that more than 50 kW of power can be obtained per metre length of wavefront in certain coastal areas.

Tidal power

Tidal power is due to the rise and fall of ocean tides. This occurs as a result of the gravitational pull of the Moon on the Earth's oceans. A tidal barrier is designed to trap each high tide, releasing it gradually through turbines that drive electricity generators. Sites such as the Severn Estuary in Britain are particularly suitable, as the tide is heightened by the shape of the estuary. Sea water over an area of 10 km by 10 km, dropping in height by a 5 m drop, would release 12.5 million million joules of potential energy over 6 hours between high tide and low tide. Prove for yourself that this corresponds to an average power output of more than 250 MW.

Geothermal power

Geothermal power is due to heat flow from the Earth's interior to the surface. In some parts of the Earth, this heat flow has raised the temperature of underground rock basins to more than 200°C . Water pumped into these rock basins produces steam, which is then used to drive turbines connected to electricity generators.



Fig. 4.16 Wind turbines on a wind farm. Wind power creates no emissions and is reasonably efficient

See www.palgrave.com/foundations/breithaupt for more about energy resources.

4.4 QUESTIONS

- 1 a) Solar cells at a certain site are capable of generating 500 W of electrical power per square metre. What would be the area of a panel of these cells capable of generating 5 kW of electrical power?
b) A solar heating panel is capable of heating water flowing through it at a rate of 0.012 kg s^{-1} , from 15°C to 30°C . Calculate the rate at which solar energy is absorbed by this panel. The specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$.
- 2 The power, P , generated by a wind turbine, depends on the wind speed, v , in accordance with the equation $P = \frac{1}{2} \rho A v^3$, where A is the area swept out by the blades and ρ is the density of air. Calculate the power from a wind turbine which has blades of length 15 m when the wind speed is
a) 5 m s^{-1} ,
b) 15 m s^{-1} .
The density of air is 1.2 kg m^{-3} .

SUMMARY

- **Work** = force $\times$ distance moved in the direction of the force.
- **Energy** is the capacity of a body to do work.
- **Power** = rate of transfer of energy.
- **The principle of conservation of energy** states that when energy changes from one form to other forms, the total amount of energy is unchanged.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{useful energy delivered}}{\text{total energy supplied}}$$

- **Equations**

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$\text{Change of potential energy} = mgh$$

$$\text{Power, } P = \frac{\text{Energy transferred}}{\text{time taken}}$$

- **Energy resources**

Fuels: fossil fuels (coal, oil, gas), nuclear fuel (uranium 235 and plutonium).

Renewable energy resources: solar heating panels, solar cells, hydro-electricity, wind power, wave power, tidal power, geothermal power.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

4.1 A luggage trolley and luggage cases, total mass 120 kg, is pushed up a ramp of length 40 m which is inclined at an angle of 5° , in 45 seconds.

Trolley to be pushed up slope

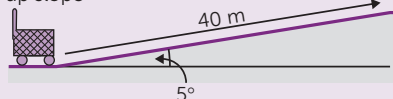


Fig. 4.17

- Calculate
 - the height gain of the trolley,
 - the gain of potential energy of the trolley and the luggage.
- Calculate the minimum force needed to push the trolley up the ramp.
 - Explain why more force is needed in practice.
- Calculate the minimum amount of work done each second.

4.2 A car of total mass 600 kg travelled at a constant speed of 21 m s^{-1} up a steady 1 in 20 incline.

- Calculate
 - the height gain of the car each second,
 - its potential energy gain each second.
- The output power of the car engine when travelling up the incline was 25 kW. Calculate
 - the work done against resistive forces each second,
 - the resistive force.

4.3 A pile driver consisting of an iron block of mass 4000 kg is dropped from rest, through a height of 3.0 m, onto the top end of a vertical steel girder of mass 6000 kg which has its lower end resting on the ground.



Fig. 4.18

- Calculate the speed of the pile driver just before impact with the girder.
- Use the conservation of momentum to calculate the speed of the pile driver and the girder just after impact, assuming the pile driver does not rebound from the girder.
- As a result of the impact, the girder penetrates 1.5 m into the ground. Calculate the force of friction on the girder due to the ground.

4.4 A steel ball of mass 0.25 kg hanging from the end of a thread at rest, is in contact with a steel ball of mass 0.15 kg which also hangs from a thread. The heavier ball is raised through a height of 0.10 m with the thread taut, by drawing it aside. It is then released and it strikes the lighter ball, causing the lighter ball to gain a height of 0.05 m .

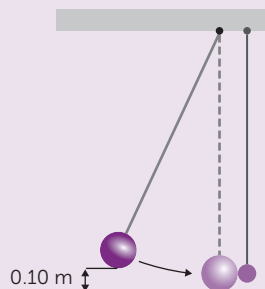


Fig. 4.19

a) Calculate

- (i) the gain of potential energy of the heavier ball as a result of drawing it to one side,
- (ii) the speed of the heavier ball just before impact.

b) Calculate

- (i) the gain of potential energy of the lighter ball as a result of gaining a height of 0.05 m ,
- (ii) the speed of the lighter ball just after impact.

c) (i) Use the principle of conservation of momentum to calculate the velocity of the heavier ball just after the impact.

- (ii) Hence calculate the height gain of the heavier ball as a result of the impact.

4.5 A vehicle of mass 1200 kg , travelling at a speed of 30 m s^{-1} , collides with a stationary car of mass 800 kg . The two vehicles lock together on impact.

a) Calculate the speed of the two vehicles immediately after impact.

b) Calculate the loss of kinetic energy due to the impact.

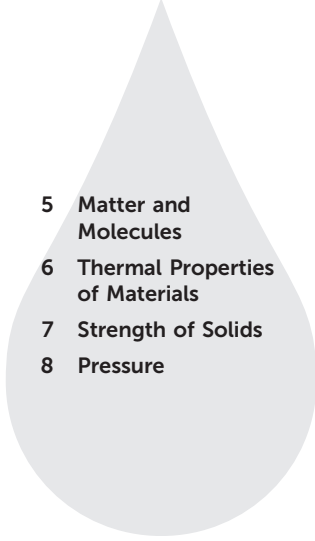
c) The impact causes the 1200 kg vehicle to slow down in a time of 0.12 s . Calculate the force of the impact.

PART

2

Properties of Materials

[Part 2](#) considers the mechanical and thermal properties of materials, and relates these properties to materials at the molecular level. It provides plenty of opportunities for practical work on the measurement of the properties of materials and it builds on topics on force and energy in [Part 1](#). Applications of the properties of materials are also developed, including the strength of solid materials, control of heat transfer and measurement of pressure.

- 
- 5 Matter and Molecules
 - 6 Thermal Properties of Materials
 - 7 Strength of Solids
 - 8 Pressure

Matter and Molecules

OBJECTIVES

After working through this unit, you should be able to:

- describe distinctive properties of each of the three states of matter
- use the kinetic theory of matter to explain the main properties of each state of matter
- classify substances as elements, compounds or mixtures
- understand the nature and composition of atoms and molecules
- define and use the Avogadro constant to calculate atomic masses and sizes
- relate the different types of bonds between atoms and molecules to the distinctive properties of the three states of matter

CONTENTS

- 5.1 States of matter
- 5.2 Elements and compounds
- 5.3 The Avogadro constant, N_A
- 5.4 Intermolecular forces
- Summary
- Revision questions

5.1 States of matter

At a definite temperature, most substances are in one of three states of matter: solid, liquid or gas. The physical characteristics that distinguish the three states of matter are:

- **shape** – a characteristic of a solid substance since a liquid takes the shape of its container, and a gas in a closed container fills its container completely or spreads out into the atmosphere if the container is open. The shape of a solid is due to its inability to flow. Its shape can be changed by applying forces to distort it. Elasticity is the ability of a solid to regain its shape after forces applied to it have been removed. Liquids and gases are fluids because they flow and therefore have no characteristic shape.
- **surfaces** – present only in solids and liquids. A coloured gas released into the atmosphere spreads out and soon becomes invisible as its concentration decreases. The surface of a solid is a definite boundary between the solid and the liquid or gas that surrounds it. There is little or no transfer of matter between a solid and the substance which surrounds it. The open surface of a liquid is a boundary between the liquid and the atmosphere; this is well defined, although transfer of matter across the boundary can and does take place (e.g. water vapour from hot water).



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Fig. 5.1 Solid, liquid and gas

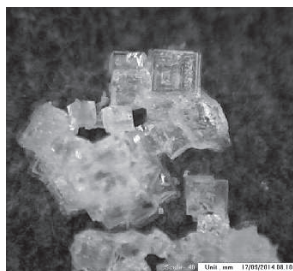


Fig. 5.2 Microscope photograph of common salt showing cubic crystals shortly after formation

Shape and surface tests

- 1 Place a drop of a saturated solution of common salt on a warm glass slide and observe the growth of crystals in the liquid using a microscope. See Fig. 5.2. Repeat the test with a saturated solution of copper sulphate. Crystals of common salt differ in shape to crystals of copper sulphate. As shown in Fig. 5.3, models of crystals made using polystyrene spheres show how different shapes can be produced according to how the spheres are arranged. Crystal shapes depend on how the atoms and molecules of the crystal are arranged.

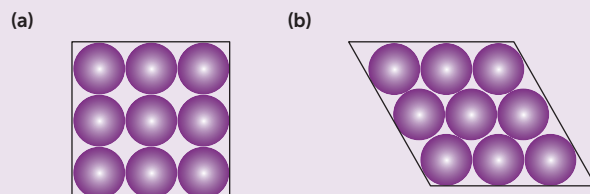


Fig. 5.3 Crystal models
(a) cubic packing, (b) close packing

- 2 Observe some other crystalline substances, including metals if possible, using a microscope if necessary. A metal is composed of tiny crystals called **grains**. The size and nature of the grains in a metal determine its strength. See Fig. 5.4.

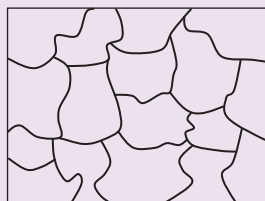


Fig. 5.4 An illustration of the irregular grain structure in a metal



- 3 Heat a glass rod until it melts and you will find it becomes very flexible. Use safety spectacles. When removed from the source of heat, glass quickly solidifies in whatever shape it has at that point. Glass is an example of an **amorphous** solid, which is a solid without a characteristic shape and no long-range order in the arrangement of its particles.
- 4 Place some coloured crystals in a beaker of water and observe the crystals as they gradually **dissolve**. Gradually, the crystals disappear as the particles they are formed from move away into the solution. This is an example of **diffusion**, which is where one substance spreads out into another. Another example is diffusion of a coloured gas in a sealed gas jar into an adjoining gas jar of air when the seal is removed, as in Fig. 5.5.

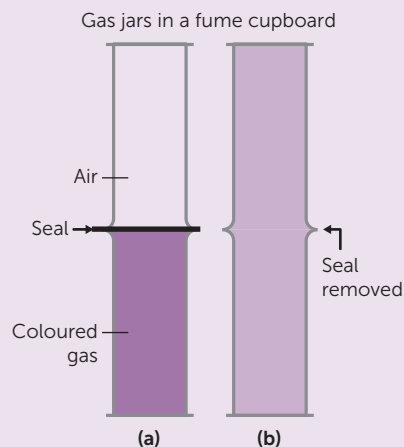


Fig. 5.5 Diffusion of a gas in air
(a) before and (b) after the seal is removed

Changes of state

Pure substances change from one state to another at a well-defined temperature. For example, pure ice melts at 0°C and water boils at 100°C at atmospheric pressure.

- The melting point of a pure substance is the temperature at which it changes from a solid to a liquid when heated, or from a liquid to a solid when cooled.
- The boiling point of a pure substance is the temperature at which it changes from the liquid to the gaseous state as a result of boiling. This is not the same as vaporisation, which occurs from the liquid surface at any temperature.

The energy needed to change the state of a substance is referred to as **latent heat**.

Fig. 5.6 shows the possible changes of state of a pure substance.

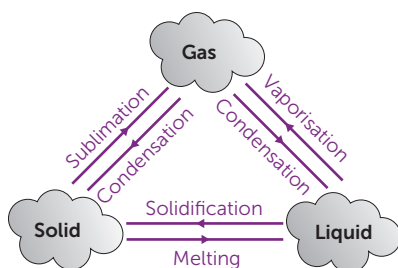


Fig. 5.6 Change of state

5.1 QUESTIONS

- Is jelly a solid or a liquid? Give a reason for your answer.
 - Is an aerosol spray liquid or gas? Give a reason for your answer.
- Describe the changes of state when ice cubes in a beaker are heated until steam is produced,
 - wet clothing on a washing line dries out.

See www.palgrave.com/foundations/breithaupt for liquid crystal displays (LCDs).

5.2 Elements and compounds

Make a list of all the substances within sight. Even without moving, your list is likely to be very long, yet all substances are made from no more than about 90 elementary substances referred to as the **chemical elements**. These cannot be broken down into other substances. There are 92 naturally occurring elements and a few more artificial elements. Each element has its own symbol (e.g. C for carbon, Pb for lead).

A **compound** is a substance composed of two or more elements in fixed proportions. For example, water is composed of the elements hydrogen and oxygen in proportions 1:8 by mass.

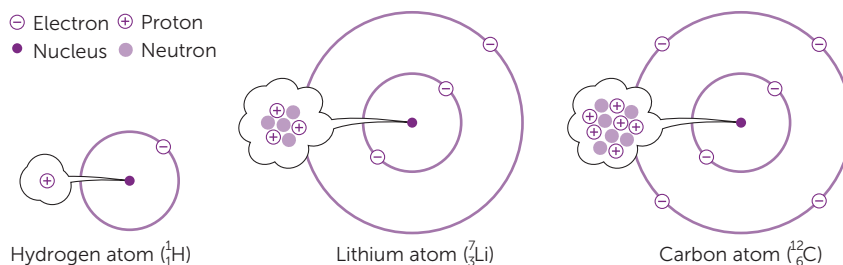
Atoms and elements

An **atom** is the smallest particle of an element which is characteristic of the element. The atoms of a given element are identical to each other but they differ from the atoms of every other element. The atomic theory of matter was developed in the early 1800s. Now, using high-power atomic microscopes, it is possible to 'see' the patterns of atoms on the surface of metals.

Table 5.1 Sub-atomic particles

	Mass	Charge
proton	1 u	+e
neutron	1 u	0
electron	$\frac{1}{1850}$ u	-e

⊖ Electron ⊕ Proton
 • Nucleus ● Neutron

**Fig. 5.7** Inside the atom

Inside every atom, there is a positively charged **nucleus** surrounded by negatively charged particles called **electrons**. The electron is held in orbit round the nucleus by the force of attraction due to its opposite charge. The magnitude of the charge on an electron is represented by the symbol e .

The lightest known atom is the hydrogen atom, which has a single positively charged particle called a **proton** as its nucleus. The proton is approximately 1850 times heavier than the electron. Its charge is exactly equal and opposite to that of the electron. An uncharged atom contains the same number of electrons and protons. The nuclei of all other types of atoms contain uncharged particles called **neutrons** as well as protons. The mass of a neutron is about the same as that of a proton.

Notes

- The atomic number, Z , of an element is the number of protons in the nucleus of an atom of the element. All the atoms of a given element contain the same number of protons in each nucleus. This is sometimes called the proton number of the atom. Atoms of the same element can have different numbers of neutrons in the nuclei.
- Isotopes are atoms of the same element with different numbers of neutrons. For example, deuterium is an isotope of hydrogen in which each nucleus consists of a proton and a neutron. An isotope is represented by the symbol ${}_Z^AX$, where X is the chemical symbol of the element, A is the number of neutrons and protons in the nucleus, and Z is the proton number.
- One unit of atomic mass (1 u) is defined as one-twelfth of the mass of a carbon-12 atom, equal to 1.66×10^{-27} kg.

5.2A QUESTIONS

- Determine the number of protons and the number of neutrons in one atom of each of the isotopes below.
 - ${}^3_1\text{H}$
 - ${}^{16}_8\text{O}$
 - ${}^{23}_{11}\text{Na}$
 - ${}^{206}_{82}\text{Pb}$
 - ${}^{238}_{92}\text{U}$
- Write down the symbol for
 - the isotope of helium (He), which contains 2 neutrons and 2 protons,
 - the isotope of carbon (C), which contains 6 protons and 6 neutrons.

Molecules and compounds

A **molecule** is a combination of atoms joined together. For example, an oxygen molecule consists of two oxygen atoms joined together. The chemical symbol for this molecule is therefore written O_2 . Another example is the carbon dioxide molecule, which consists of two oxygen atoms joined to a carbon atom. The chemical symbol for a carbon dioxide molecule is CO_2 .

A **compound** consists of identical molecules, each with the same atomic composition. This is why a compound is composed of elements in **fixed** proportions. For example, carbon dioxide is a compound consisting of carbon dioxide molecules. For every atom of carbon, there are two atoms of oxygen.

The proportion of each element present in a compound can be determined from the molecular formula if the atomic masses are known. For example, the atomic mass of carbon is approximately 12 u and the atomic mass of oxygen is approximately 16 u. Therefore, the mass of a carbon dioxide molecule is approximately 44 u $[= 12 + (2 \times 16)]$ and the proportion of carbon to oxygen by mass is 0.375 $(= 12 \div 32)$. Molecules composed of chains of atoms can be very long.

Estimating the size of an oil molecule

A tiny oil drop placed on a clean water surface spreads out into a circular patch just one molecule thick. A V-shaped thin wire is dipped into the oil and shaken so just one droplet remains on the wire. The diameter, d , of this droplet can be estimated by using a magnifying glass and a millimetre scale. The water surface is then sprinkled with very light powder after being cleaned. When the droplet is placed on the water surface, the oil spreads into a circular patch, pushing the powder away. The diameter, D , of the patch can then be measured.

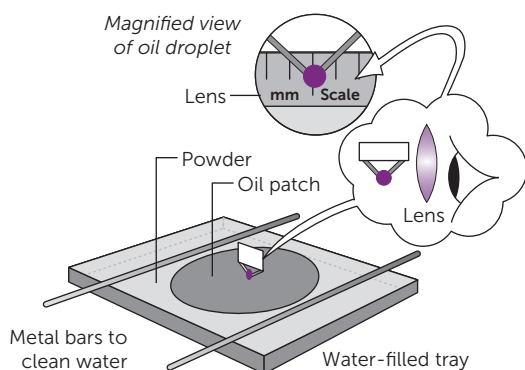


Fig. 5.8 Estimating the size of an oil molecule

The thickness, t , of the patch can be estimated by assuming the volume of the droplet ($= 4\pi r^3 \div 3$, where its radius, $r = D \div 2$) equals the volume of the patch ($= \pi D^2 t \div 4$).

5.1 WORKED EXAMPLE

An oil droplet of diameter 1.0 mm spreads out to form a circular patch of diameter 650 mm when placed on a clean water surface. Use this information to estimate the length of an oil molecule.

Solution

Volume of oil patch $= \frac{\pi D^2}{4} t = \frac{\pi (0.65)^2 t}{4}$, where t is the thickness of the patch

Volume of oil droplet $= \frac{4\pi r^3}{3} = \frac{4\pi (0.5 \times 10^{-3})^3}{3}$ $\therefore \frac{\pi (0.65)^2 t}{4} = \frac{4\pi (0.5 \times 10^{-3})^3}{3}$

Estimate of size of oil molecule $= t = \frac{4\pi (0.5 \times 10^{-3})^3}{3\pi (0.65)^2} = 1.6 \times 10^{-9} \text{ m}$

5.2B QUESTIONS

1 Use the molecular formula given to calculate the mass of one molecule of each of the following compounds or elements in atomic mass units. The relative atomic mass of each element is listed below in atomic mass units.

- water H_2O ,
- carbon monoxide CO ,
- methane CH_4 ,

- ammonia NH_3 ,
- copper oxide CuO .
($\text{H} = 1$, $\text{C} = 12$, $\text{N} = 14$, $\text{O} = 16$, $\text{Cu} = 64$)

2 An oil droplet of diameter 0.8 mm spreads out on water to form a circular patch of diameter 350 mm. Estimate the size of an oil molecule from this data.

5.3 The Avogadro constant, N_A

When atoms combine to form molecules, they do so only in fixed ratios. For example, every carbon dioxide molecule is composed of one carbon atom and two oxygen atoms. The mass ratio of carbon to oxygen in a single molecule is always 12:32 because the ratio of the mass of a carbon atom to the mass of an oxygen atom is always 12:16. For any number of carbon dioxide molecules, the mass ratio of carbon to oxygen is the same as that in a single molecule because all carbon dioxide molecules are alike.

The Avogadro constant is defined as the number of atoms in exactly 12 grams of the carbon isotope $^{12}_6\text{C}$.

Originally, the Avogadro constant was defined as the number of atoms in 1 g of hydrogen. However, the discovery of isotopes of hydrogen (e.g. deuterium) and the difficulty of separating them, eventually led to the carbon 12 scale of atomic mass and the definition of the Avogadro constant in terms of carbon 12. The accepted value of N_A is 6.022×10^{23} .

Note

If m is in atomic mass units, then the molar mass in grams = m because the conversion factor from atomic mass units to grams is $1 \div N_A$. For example, the mass of 1 atom of oxygen = $16 \text{ u} = 16 \div N_A$ grams. Hence the molar mass of N_A oxygen atoms = 16 g.

- One **mole** of a substance is defined as N_A particles of the substance. For example, 1 mole of copper atoms is 6.022×10^{23} copper atoms.
- The **molar mass** of a substance is the mass of 6.022×10^{23} particles of the substance. By definition, the molar mass of carbon 12 is exactly 12 g. Since 1 atom of carbon 12 has a mass of 12 u because 1 u is defined as 1/12th of the mass of a $^{12}_6\text{C}$ atom, then $1 \text{ u} = 1.66 \times 10^{-24} \text{ g} (= 1 \div N_A \text{ A})$ or $1.66 \times 10^{-27} \text{ kg}$. For a substance consisting of particles of mass m , its molar mass is equal to mN_A .

Density and atomic size

The atoms in a solid element are in direct contact with each other.

- The mass of 1 atom of an element = $M \div N_A$, where M is its molar mass.
- The volume of 1 atom of an element = $\frac{\text{mass of 1 atom}}{\text{density, } \rho} = \frac{M}{N_A \rho}$.

Assuming each atom occupies a cube of volume d^3 , where d is the diameter of an atom, then the diameter of an atom can be estimated from the equation

$$d^3 = \frac{M}{N_A \rho}$$

In practice, atoms are packed together in a number of different ways, depending on the type of atom. Nevertheless, the above formula gives a reasonable estimate of atomic size.

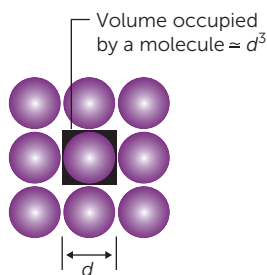


Fig. 5.9 Estimate of atomic size

Note

To calculate d , the cube root of $M \div N_A \rho$ must be calculated. This is written

$$\left(\frac{M}{N_A \rho} \right)^{1/3}$$

Use the button marked y^x on your calculator as follows:

Step 1 Work out $M \div N_A \rho$ and keep the answer on your calculator display.

Step 2 Press the y^x button and key in $(1 \div 3)$; Press = and the calculator will then display the value of $(M \div N_A \rho)^{1/3}$.

5.2 WORKED EXAMPLE

Copper has a density of 8000 kg m^{-3} and an atomic mass of 64. (Avogadro constant, $N_A = 6.02 \times 10^{23}$)

- Calculate the mass of a copper atom in kg,
- Estimate the diameter of a copper atom.

Solution

- a) 64 g of copper contains N_A copper atoms.

$$\therefore \text{the mass of 1 copper atom} \approx \frac{64 \text{ g}}{6.02 \times 10^{23}} = 1.1 \times 10^{-22} \text{ g} = 1.1 \times 10^{-25} \text{ kg}$$

- b) Volume of a copper atom = $\frac{\text{mass of 1 copper atom}}{\text{density}} = \frac{1.1 \times 10^{-24}}{8000} = 1.32 \times 10^{-29} \text{ m}^3$

$$\text{Diameter of atom, } d = (1.32 \times 10^{-29})^{1/3} = 2.4 \times 10^{-10} \text{ m} = 0.24 \text{ nm}$$

5.3 QUESTIONS

- Calculate the number of atoms in 1 kg of (i) carbon, (ii) uranium.
 - Calculate the number of molecules in 1 kg of (i) carbon dioxide (CO_2), (ii) methane (CH_4). (Relative atomic masses: H = 1, C = 12, O = 16, U = 238.)
- The density of aluminium is 2700 kg m^{-3} and its relative atomic mass is 27.

 - Calculate the mass of an aluminium atom.
 - Estimate the diameter of an aluminium atom. The Avogadro constant, $N_A = 6.02 \times 10^{23}$.

5.4 Intermolecular forces

Electrons in atoms

The electrons in an uncharged isolated atom surround the nucleus in 'shells', each shell corresponding to a fixed energy level and capable of holding a certain number of electrons. The innermost shell can hold two electrons, the next shell eight electrons, the third shell eight electrons, the next shell ten electrons. See Fig. 5.10a overleaf. The number of electrons in each shell was first worked out from the rows and columns of the **Periodic Table**. This is a table of the elements, listed in order of increasing atomic mass in such a way that elements with similar chemical properties are in the same vertical column. See Fig. 5.10b overleaf.

Bonds

Atoms interact to form molecules, and molecules interact to form solids and liquids. The different types of forces that hold atoms and molecules together depend on the electron structure of the atoms. The underlying principle is that an atom has minimum energy when its electron shells are complete. Each type of bond achieves this in a different way.

An **ionic bond** is formed when one atom gives up one or more electrons to another atom. An ion is a charged atom, formed as a result of an uncharged atom gaining or losing one or more electrons. For example, sodium chloride consists of positive sodium ions and negative chlorine ions. Each sodium atom has lost its outer electron and each chlorine atom has gained an electron in its outer shell. In this way, both types of atoms have full electron shells.

Examples

- The uncharged carbon atom has 6 electrons, 2 in its innermost shell and 4 in the next shell.
- The uncharged oxygen atom has 8 electrons, 2 in its innermost shell and 6 in the next shell.
- The uncharged sodium atom has 11 electrons, 2 in its innermost shell, 8 in the next shell and a single electron in the third shell.
- The uncharged chlorine atom has 17 electrons, 2 in its innermost shell, 8 in the second shell and 7 in its third shell.

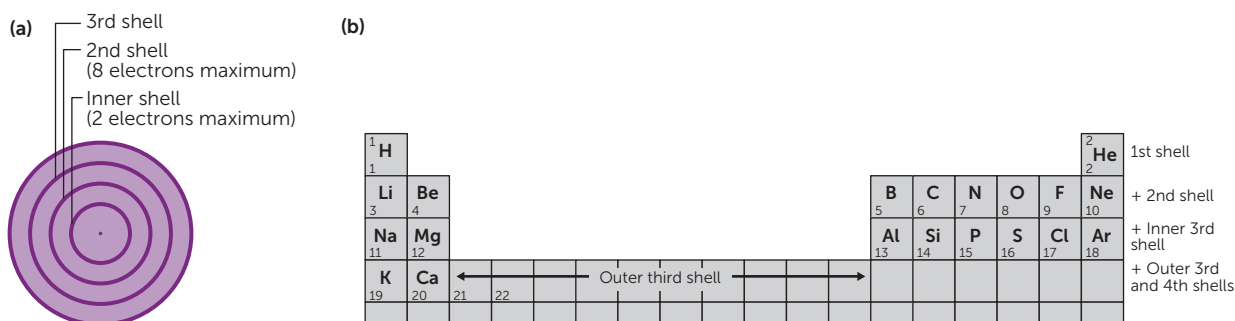


Fig. 5.10 (a) electron shells, (b) the Periodic Table

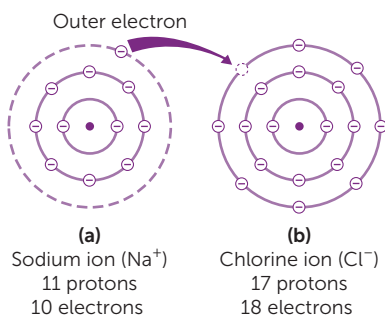


Fig. 5.11 The ionic bond

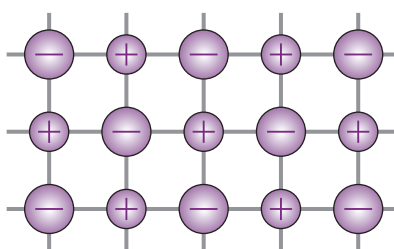


Fig. 5.12 A cubic lattice

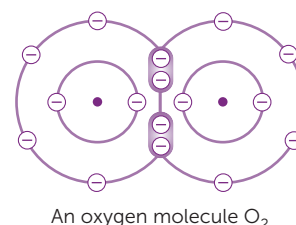


Fig. 5.13 Covalent bonds

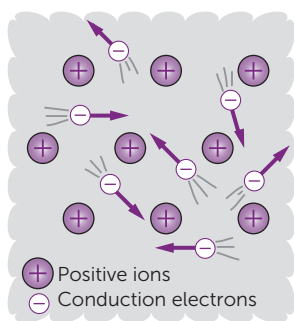


Fig. 5.14 Metallic bonds

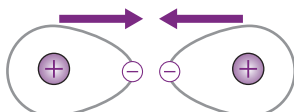


Fig. 5.15 Molecular bonds

Sodium chloride crystals are cubic in shape because the positive ions and negative ions arrange themselves in a cubic lattice, each ion held in place by the ionic bonds between it and the neighbouring, oppositely charged ions, as shown in Fig. 5.13.

A **covalent bond** is formed as a result of two atoms with incomplete electron shells sharing electrons to achieve complete shells. Each bond consists of two electrons, one donated by each atom. For example, an oxygen molecule consists of two oxygen atoms held together by two covalent bonds, as shown in Fig. 5.14. Each oxygen atom in the molecule has eight electrons in its outer shell.

Metallic bonding is where atoms lose their outer electrons to form a structure of positive ions surrounded by a 'sea' of electrons that holds the structure together. The electrons that break free are referred to as **conduction** electrons because they are responsible for electrical conduction when a voltage is applied. This type of bonding exists only in metals because uncharged metal atoms possess outer-shell electrons.

Molecular bonds, sometimes called van der Waals bonds, are weak attractive bonds between uncharged atoms and molecules. Two uncharged molecules attract each other because the electrons of each molecule are attracted by the nuclei of the atoms of the other molecule. Uncharged molecules in a liquid are prevented from leaving the liquid by these bonds, which have a range of no more than a few molecular diameters. In comparison, the ionic bond has a much greater range.

In general, the force between two atoms or molecules or ions attracts the two particles together but prevents them from becoming too close. At very short-range, the electron shells of one particle repel the electron shells of the other particle. The variation of force with separation is shown in Fig. 5.17.

- The **equilibrium separation**, r_0 , is the point where attraction is balanced by repulsion.
- The **bond energy**, ϵ , is the energy needed to separate two molecules from equilibrium separation to infinity.

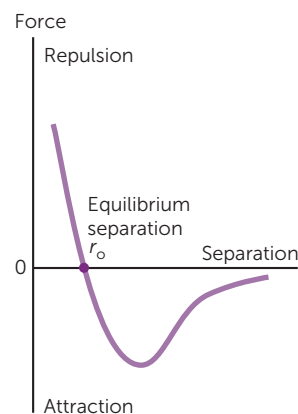


Fig. 5.16 Force versus separation for two molecules

Molecules and materials

Solids and liquids do not diffuse into the atmosphere because of the presence of attractive forces between the molecules of the substance. The molecules are in contact with each other, locked together in a solid and free to move about in a liquid.

In a gas, intermolecular forces are either very weak or non-existent, so gas molecules can move away from each other. The molecules in a gas move about at random, independently of each other, coming into contact only when they collide and bounce off each other.

Molecules composed of more than one atom are held together by covalent bonds. These bonds are localised and in general do not affect the interaction between molecules.

5.4 QUESTIONS

- 1 a) State the type of bond that exists between (i) the atoms of a carbon dioxide molecule, (ii) the atoms in solid copper, (iii) the atoms in a sodium chloride crystal, (iv) water molecules, (v) air molecules.
b) Describe the arrangement of electrons in (i) a carbon atom, (ii) a carbon dioxide molecule.
- 2 State whether or not the molecules in each of the following materials (i) are in contact, (ii) move about at random:
a) air,
b) water,
c) wood,
d) ice.

SUMMARY

- **Solids** possess shape and surfaces; are composed of molecules locked together and in contact.
- **Liquids** possess surfaces but no shape; are composed of molecules in random motion and in contact.
- **Gases** possess no surfaces and no shape; are composed of molecules in random motion not in contact.
- **Atoms** consist of protons (mass = 1 u, charge = +1) and neutrons (mass = 1 u, charge = 0) in the nucleus, surrounded by electrons (mass = 0.005 u approx, charge = -1).
- **1 atomic mass unit** (u) = 1/12th of the mass of a carbon 12 atom.
- The **Avogadro constant**, N_A , is the number of atoms in 12 g of carbon 12.
- **1 mole of a substance** = N_A particles of the substance.
- **Types of bonds:** ionic, covalent, metallic, molecular.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

The Avogadro constant, $N_A = 6.02 \times 10^{23}$

5.1 Use your knowledge of atoms and molecules to explain

- why a solid possesses its own shape,
- why a liquid takes the shape of its container,
- why carbon dioxide gas produced in a chemical reaction in a beaker stays in the beaker for a long time.

5.2 What change of state occurs when

- a hail storm occurs,
- a car windscreen mists up,
- solid carbon dioxide gas evaporates.

5.3 Chlorine gas contains two isotopes of chlorine, $^{37}_{17}\text{Cl}$ and $^{35}_{17}\text{Cl}$.

- How many protons and how many neutrons are there in a single atom of each isotope?
- Chlorine gas contains 3 times as many $^{35}_{17}\text{Cl}$ atoms as the other type. Calculate the mass of 1 mole of chlorine gas.

5.4

- Describe the arrangement of electrons in an uncharged atom of
 - carbon,
 - hydrogen.
- The molecular formula of methane is CH_4 .
 - What is the molar mass of methane?
 - Calculate the mass, in kg, of 1 molecule of methane.
 - Describe the arrangement of electrons in a molecule of methane.

5.5

- $^{235}_{92}\text{U}$ is an unstable isotope of uranium. How many protons and how many neutrons are present in the nucleus of an atom of this isotope?
- Calculate the number of atoms present in 1 g of this isotope.

5.6 The density of water is 1000 kg m^{-3} and it has a molar mass of 0.018 kg.

- Calculate the mass of a single water molecule.
- Estimate the size of a water molecule.

5.7 The density of lead is 11340 kg m^{-3} and its relative atomic mass is 207.

- Calculate the mass of a lead atom.
- Estimate the diameter of a lead atom.

5.8

- Air is approximately 80% nitrogen (molar mass 28 g) and 20% oxygen (molar mass 32 g). Show that the molar mass of air is 29 g.
- The density of air at atmospheric pressure and room temperature is about 1.2 kg m^{-3} .
 - Estimate the average distance between air molecules.
 - Comment on this answer in the light of the knowledge that the diameter of an air molecule is about 0.3 nm.

Thermal Properties of Materials

OBJECTIVES

After working through this unit, you should be able to:

- convert temperature in $^{\circ}\text{C}$ to kelvins and vice versa
- calculate the thermal expansion of a solid for a given temperature rise
- measure the specific heat capacity of different substances
- measure the specific latent heat of fusion and of vaporisation of different substances
- carry out specific heat capacity calculations and specific latent heat calculations
- describe conduction, convection and radiation as heat transfer mechanisms
- carry out thermal conduction and radiation calculations

CONTENTS

- 6.1 Thermal expansion
- 6.2 Specific heat capacity
- 6.3 Specific latent heat
- 6.4 Heat transfer
- 6.5 More about thermal conduction
- Summary
- Revision questions

6.1 Thermal expansion

Temperature and heat

Heat is energy transferred due to a temperature difference. When a substance is heated, its atoms and molecules gain energy. When the temperature rises, the molecules vibrate faster in a solid, or move about more rapidly in a liquid or a gas. When a solid is heated at its melting point or a liquid is heated at its boiling point, the energy supplied is used by the molecules to break free from each other.

Temperature is the degree of hotness of an object. To define a scale of temperature, numerical values are assigned to reproducible and reliable degrees of hotness.

- The **Celsius scale** (in $^{\circ}\text{C}$) is defined by:
 - 1 Ice point (0°C) which is the temperature of pure melting ice.
 - 2 Steam point (100°C) which is the temperature of steam at atmospheric pressure.

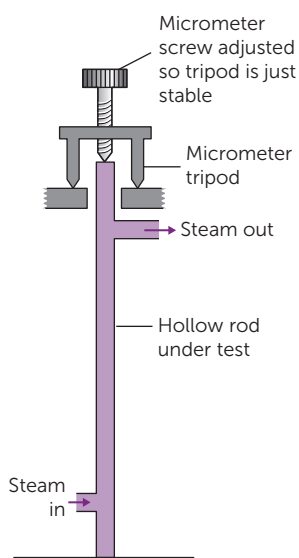


Fig. 6.1 Thermal expansion

Table 6.1 Coefficient of thermal expansion values

Material	Coefficient of thermal expansion K^{-1}
Steel	1.1×10^{-5}
Brass	1.9×10^{-5}
Aluminium	2.6×10^{-5}
Invar*	0.1×10^{-5}

* Nickel–steel alloy

- The **absolute scale** in kelvins (K) is defined by:

- 1 Absolute zero (0 K) which is the lowest possible temperature, equal to -273°C (see p. 413).
- 2 The triple point of water (273 K) which is the point at which ice, water and water vapour are in thermal equilibrium. This is a more reliable standard than ice point which varies slightly with pressure. The temperature of the triple point is assigned a numerical value of 273 K so that the difference between the two scales for any given degree of hotness is 273 degrees.

$$\text{temperature in kelvins} = \text{temperature in } ^{\circ}\text{C} + 273$$

The subject of thermometry is developed further in [Unit 25](#).

Thermal expansion

A solid expands when it is heated and contracts when it cools. [Fig. 6.1](#) shows how this can be demonstrated and measured. Heating a solid makes the atoms in the solid vibrate more and causes the average separation between atoms to increase. The change of length, ΔL , due to thermal expansion of a length of a solid, L , is

- 1 proportional to its length L ,
- 2 proportional to the increase of temperature ΔT ,
- 3 dependent on the material.

The coefficient of thermal expansion, α , of a solid rod is defined as

$$\frac{\text{change of length } \Delta L}{\text{length } L \times \text{temperature rise } \Delta T}$$

[Table 6.1](#) gives values of α for some different materials. The unit of α is K^{-1} (per kelvin). The equation to calculate the change of length, ΔL , given the length, temperature change and coefficient of thermal expansion is:

$$\Delta L = \alpha L \Delta T$$

6.1 WORKED EXAMPLE

Calculate the change of length of a steel girder of length 45.0 m, when its temperature rises from 5°C to 30°C . The coefficient of thermal expansion of steel is $1.1 \times 10^{-5} K^{-1}$.

Solution

$$L = 45.0 \text{ m},$$

$$\Delta T = (30 + 273) - (5 + 273) = 25 \text{ K},$$

$$\alpha = 1.1 \times 10^{-5} K^{-1}.$$

$$\Delta L = \alpha L \Delta T$$

$$= 1.1 \times 10^{-5} \times 45.0 \times 25$$

$$= 1.2 \times 10^{-2} \text{ m}$$

$$= 1.2 \text{ cm}.$$

Applications of thermal expansion

- 1 The **bimetallic strip** consists of a strip of brass bonded to a strip of steel. See Fig. 6.2. The strip bends towards the steel when its temperature rises. This happens because brass expands more than steel for a given rise in temperature. The bimetallic strip is used as a temperature-controlled switch in alarms and heating appliances.
- 2 **Expansion gaps** in bridges, railway tracks and buildings are necessary to allow for thermal expansion in summer. These gaps are usually filled with a soft material like rubber, to prevent chunks of rock falling in.
- 3 A **radiator thermostat** contains oil in a sealed container. A rod attached to a valve protrudes into the oil. When the oil heats up, it expands and forces the rod partly out to open the valve. The valve is closed when the engine is cold and opens when the engine has warmed up.

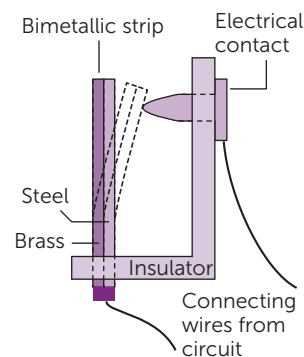


Fig. 6.2 A bimetallic strip

6.1 QUESTIONS

- 1 With the aid of a diagram, explain how a bimetallic strip can be used to switch an electric heater off when the temperature rises too much.
- 2 Use the data in Table 6.1 to calculate the change of length of
 - a) a brass rod of length 0.80 m when its temperature rises from 15°C to 20°C,
 - b) an aluminium bar of length 1.50 m when its temperature falls from 20°C to -5°C.

6.2 Specific heat capacity

Energy and power

The **power** of an electrical appliance is a measure of the rate at which the appliance uses electrical energy. The unit of power is the **watt** (W). This is used for any situation where energy is being transferred. For example, a 1000 W electric heater uses ten times as much energy in a given time as a 100 W light bulb.

The electrical energy used by an electrical appliance is expressed in kilowatt hours (kW h). Thus a 2 kW electric heater which runs for 4 hours uses 8 kW h of electrical energy or 8 'units'. If the unit cost of electricity is 12 p, the cost of running this electric heater for 4 hours is therefore 96 p.

The scientific unit of energy is the **joule** (J) which is equal to the electrical energy used by a 1 W electrical appliance in 1 second. In other words, 1 W is equal to a rate of energy transfer of 1 J s⁻¹. Prove for yourself that 1 kW h of electrical energy is 3.6 MJ.

$$\text{power (in W)} = \frac{\text{energy transferred (in J)}}{\text{time taken (in seconds)}}$$

The electrical energy supplied to a low voltage heater may be measured using a **joulemeter**, as shown in Fig. 6.3. The joulemeter may be used to determine the power of an electric heater by measuring the energy transferred in a given time, then using the equation above.

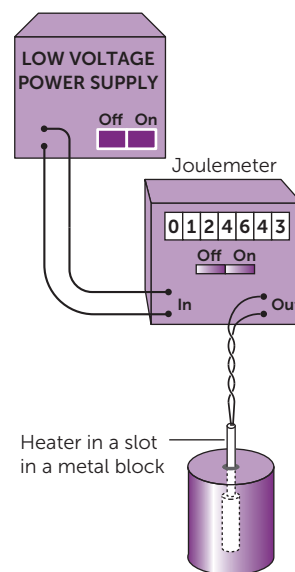


Fig. 6.3 Using a joulemeter

Table 6.2 Specific heat capacity values

Material	Specific heat capacity / $\text{J kg}^{-1}\text{K}^{-1}$
Aluminium	900
Brass	370
Copper	390
Glass	700
Steel	470
Oil	2100
Water	4200
Concrete	850

Note

The symbol Q denotes the quantity of heat transfer.

Heating things up

Some materials heat up more easily than others. For example, in summer, a car in sunlight becomes much hotter than a large swimming pool. The temperature rise of an object depends on:

- the energy supplied to the object; the more energy that is supplied, the greater the temperature rise
- the mass of each substance in the object; the greater its mass, the smaller its temperature rise
- the nature of the substance.

The **specific heat capacity**, c , of a substance is the energy required to raise the temperature of unit mass of the substance by unit rise of temperature. The unit of c is $\text{J kg}^{-1}\text{K}^{-1}$. The value of c for some common substances is shown in Table 6.2. For example, the specific heat capacity of water is $4200 \text{ J kg}^{-1}\text{K}^{-1}$, which means that 4200 J of energy must be supplied to 1 kg of water to raise its temperature by 1 K.

In general, to heat mass, m , of a substance from temperature T_1 to temperature T_2 , the energy needed, Q , can be written as

$$Q = mc(T_2 - T_1) \quad \text{where } c \text{ is the specific heat capacity of the substance.}$$

6.2 WORKED EXAMPLE

A 3 kW electric kettle contains 1.5 kg of water at 15°C .

- Calculate the energy needed to raise the temperature of this amount of water to 100°C .
- How long does it take for a 3 kW heater to supply this amount of energy?
- The time taken to heat this amount of water from 15°C to 100°C in the kettle is longer. Why?

Solution

a) Energy needed = $mc(T_2 - T_1) = 1.5 \times 4200 \times (100 - 15) = 5.4 \times 10^5 \text{ J}$

b) Rearranging the power equation gives

$$\text{time taken} = \frac{\text{energy}}{\text{power}} = \frac{5.4 \times 10^5}{3000} = 180 \text{ seconds}$$

c) Heat is lost from the kettle; the kettle uses some energy as the kettle body becomes hot.

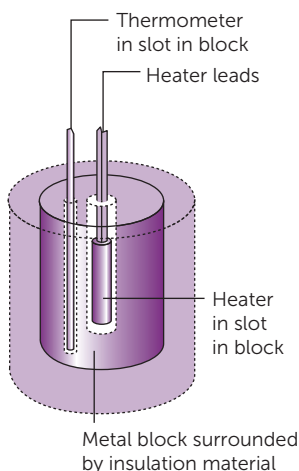


Fig. 6.4 Measuring the specific heat capacity of a metal block

Measuring specific heat capacity

- For a metal, an insulated metal block is heated using a low voltage heater inserted into a slot in the block. A thermometer is inserted in a different slot. The mass of the block alone is measured using a top pan balance. The block is then arranged as in Fig. 6.4. Its initial temperature is measured and the joulemeter is used to measure the energy supplied in a given time. The highest temperature is measured when the heater is switched off. The specific heat capacity is then

$$\frac{\text{energy supplied}}{\text{mass} \times \text{temperature rise}}$$

- 2 **For a liquid**, a low voltage heater is used to heat a measured mass of the liquid in an insulated metal calorimeter, as in Fig. 6.5. The mass of the calorimeter must be measured separately. The joulemeter is used to measure the energy supplied. A thermometer is used to measure the initial and highest temperature of the liquid.

Assuming no heat loss occurs, the energy supplied to the liquid = electrical energy supplied – energy supplied to heat the calorimeter.

$$\therefore m_l c_l (T_2 - T_1) = E - m_{\text{cal}} c_{\text{cal}} (T_2 - T_1)$$

where m_l = mass of liquid, m_{cal} = mass of the calorimeter, c_l = specific heat capacity of the liquid, c_{cal} = specific heat capacity of the calorimeter, T_2 = highest temperature, T_1 = initial temperature, E = electrical energy supplied.

Using this equation, the specific heat capacity of the liquid can therefore be calculated. This method assumes no heat loss, which can be tested by measuring the temperature of the liquid after the heater is switched off.

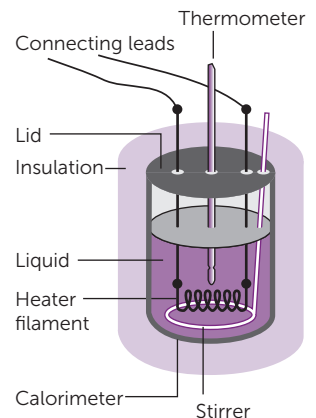


Fig. 6.5 Measuring the specific heat capacity of a liquid

6.3 WORKED EXAMPLE

An insulated copper water tank of mass 27 kg contains 160 kg of water.

- Calculate the energy needed to heat the water and the tank from 15°C to 45°C. The specific heat capacity of copper is 390 J kg⁻¹ K⁻¹ and of water is 4200 J kg⁻¹ K⁻¹.
- The water in the tank is heated by a 3.0 kW electric heater. Calculate the time taken by the heater to supply the energy in (a).

Solution

$$\begin{aligned} \text{a) Energy needed to heat the copper tank} &= 27 \times 390 \times (45 - 15) \\ &= 3.2 \times 10^5 \text{ J} = 0.32 \text{ MJ} \end{aligned}$$

$$\begin{aligned} \text{Energy needed to heat the water} &= 160 \times 4200 \times (45 - 15) \\ &= 2.02 \times 10^7 \text{ J} = 20.2 \text{ MJ} \end{aligned}$$

$$\text{Total energy needed} = 20.2 + 0.3 = 20.5 \text{ MJ}$$

$$\begin{aligned} \text{b) Time taken} &= \frac{\text{energy needed}}{\text{power}} \\ &= \frac{20.5 \times 10^6}{3000} = 6800 \text{ s.} \end{aligned}$$

6.2 QUESTIONS

- Use the data in Table 6.2 to calculate the energy needed to raise the temperature of
 - 5.0 kg of oil from 10°C to 50°C,
 - a concrete block of mass 1500 kg from 5°C to 35°C,
 - an aluminium tank of mass 15 kg containing 95 kg of water from 20°C to 80°C.
- The following measurements were made in an experiment to determine the specific heat capacity of a liquid by heating the liquid in an insulated calorimeter:
 - Mass of empty copper calorimeter = 55 g
 - Mass of calorimeter + liquid = 143 g
 - Initial temperature of the liquid = 115°C
 - Final temperature of the liquid = 62°C
 - Energy supplied = 9340 J
 Use this data and the information in Table 6.2 to calculate the specific heat capacity of the liquid.

6.3 Specific latent heat

Melting and solidification

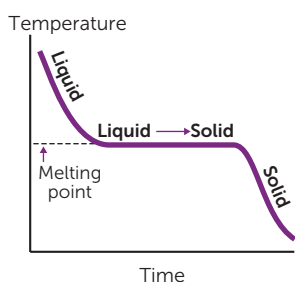


Fig. 6.6 At the melting point

If a pure solid is heated, its temperature rises until it melts. At its melting point, no further change of temperature occurs even though the solid continues to be heated (Fig. 6.6). The energy supplied at the melting point is called **latent heat** because it does not cause a rise of temperature. The reason is that the energy is used to enable the molecules of the solid to break the bonds that hold them in a rigid structure. When a liquid is cooled sufficiently, it solidifies at its melting point. Energy is released when this occurs as the molecules bind each other together in a rigid structure.

The **specific latent heat of fusion** of a substance, l , is the energy required to melt unit mass of substance without change of temperature. The unit of specific latent heat is J kg^{-1} .

- 1 The quantity of heat supplied to melt mass, m , of a substance can be calculated from the equation

$$Q = ml$$

- 2 The same equation is used to calculate the energy removed when mass m solidifies.

6.4 WORKED EXAMPLE

Calculate the energy needed to melt 120 g of ice at 0°C and heat the melted ice to 40°C . The specific latent heat of ice is $340\,000 \text{ J kg}^{-1}$ and the specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$.

Solution

$$\begin{aligned}\text{Energy needed to melt the ice} &= 0.12 \times 340\,000 \\ &= 41\,000 \text{ J}\end{aligned}$$

$$\begin{aligned}\text{Energy needed to heat the melted ice} &= 0.12 \times 4200 \times (40 - 0) \\ &= 20\,200 \text{ J}\end{aligned}$$

$$\text{Total energy needed} = 41\,000 + 20\,200 = 61\,200 \text{ J}$$

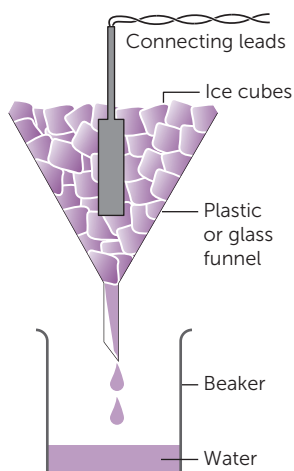


Fig. 6.7 Measuring the specific latent heat of fusion of ice

Measuring the specific latent heat of fusion of ice

A low voltage heater is surrounded by ice cubes packed into a large funnel, as in Fig. 6.7.

- 1 Without the heater switched on, the mass of melted ice, m_0 , from the funnel is measured in a certain time.
- 2 The measurement is repeated with the heater on, connected to the power supply via a joulemeter.

If m_1 represents the mass of melted ice collected with the heater on, the mass of ice melted due to the heater is $(m_1 - m_0)$. The specific latent heat of ice can therefore be calculated by dividing the energy supplied by $(m_1 - m_0)$.

Vaporisation and condensation

When a liquid is heated, its temperature rises until it reaches its boiling point. Energy supplied at the boiling point enables the molecules throughout the liquid to break free from each other and enter the vapour state. Because no change of temperature occurs at the boiling point, the energy supplied is referred to as **latent heat**. See Fig. 6.6. This energy is released when a vapour condenses as the molecules form bonds.

The specific latent heat of vaporisation of a substance, l , is the energy required to vaporise unit mass of liquid without change of temperature. The unit of specific latent heat is J kg^{-1} .

The quantity of heat supplied to turn mass, m , of a liquid into a vapour or vice versa can be calculated from the equation $Q = ml$, where l is the specific latent heat of vaporisation. For example, the specific latent heat of vaporisation of water is 2.3 MJ kg^{-1} . Therefore, the energy needed to boil away 0.5 kg of water is 1.15 MJ ($= 0.5 \times 2.3 \text{ MJ}$).

6.5 WORKED EXAMPLE

A 2.5 kW electric kettle contains 2.0 kg of water.

- Calculate the energy needed to boil away 1.5 kg of water.
- (i) Estimate the time taken for this amount of water in the kettle to boil away.
ii) What assumption have you made in your estimate?

The specific latent heat of vaporisation of water is 2.3 MJ kg^{-1}

Solution

$$\begin{aligned} \text{a) Energy needed} &= 1.5 \times 2.3 \text{ MJ} \\ &= 3.45 \text{ MJ} \end{aligned}$$

$$\begin{aligned} \text{b) (ii) Time taken} &= \frac{\text{energy needed}}{\text{power}} \\ &= \frac{3.45 \times 10^6}{2500} = 1380 \text{ s} \end{aligned}$$

(ii) There is no heat loss from the kettle.

Measuring the specific latent heat of vaporisation of a liquid

The liquid is heated to its boiling point in the apparatus shown in Fig. 6.8. At its boiling point, the vapour from the liquid surrounds the liquid container and enters the vertical condenser tube where it condenses and is collected in a beaker at the bottom of the tube. An empty beaker of known mass is used to collect the condensed liquid for a certain time. The joulemeter is used to measure the energy supplied in this time. The specific latent heat of vaporisation is calculated from energy supplied $\div$ mass of liquid collected.

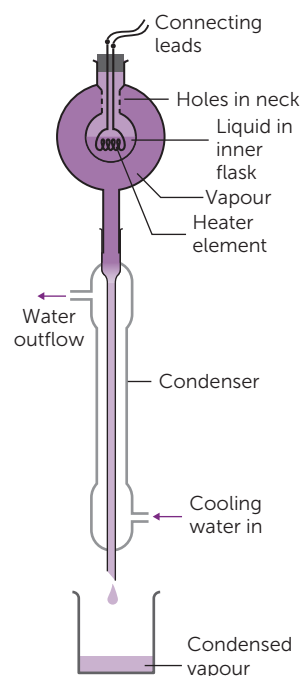


Fig. 6.8 Measuring the specific heat of vaporisation of a liquid

6.3 QUESTIONS

- 1 Calculate the energy needed to turn 0.20 kg of ice at -10°C into steam at 100°C . The specific latent heat of ice is $340\,000\text{ J kg}^{-1}$, the specific latent heat of steam is 2.3 MJ kg^{-1} , the specific heat capacity of ice is $2100\text{ J kg}^{-1}\text{ K}^{-1}$ and the specific heat capacity of water is $4200\text{ J kg}^{-1}\text{ K}^{-1}$.
- 2 A jet of steam is directed into a polythene beaker containing 120 g of water at 20°C . The steam heats the water to 52°C . Assuming no heat loss from the beaker, calculate
 - a) the energy needed to heat the water from 20°C to 52°C ,
 - b) the mass of steam that condensed in the beaker.
 The specific latent heat of steam is 2.3 MJ kg^{-1} and the specific heat capacity of water is $4200\text{ J kg}^{-1}\text{ K}^{-1}$.

6.4 Heat transfer

Heat is energy transfer due to a temperature difference. The three methods by which heat transfer occurs are thermal conduction, convection and radiation. Conduction takes place in solids, liquids and gases; convection takes place in liquids and gases only. For example, heat from a hot radiator in Fig. 6.9 reaches other parts of a room due to thermal conduction through the radiator panel which heats the air and causes convection currents, and also radiates infra-red radiation throughout the room.

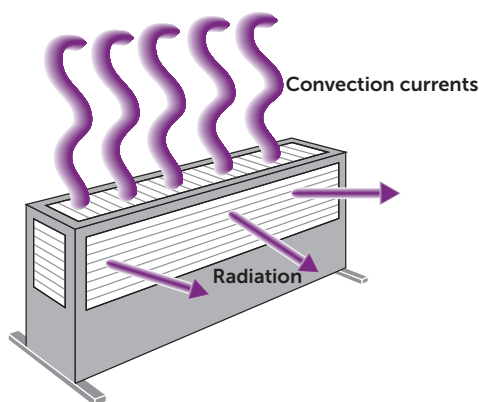


Fig. 6.9 Home heating

Convection

When a fluid is heated it becomes less dense and it rises because it is less dense than the cold fluid. In a closed space, the fluid circulates as the hot fluid rises and cold fluid is drawn into the source of heating, where it heats up and rises. This process is known as **convection**. Some examples are listed below.

- 1 Hot gases from the flames of a gas fire rise, drawing air into the gas fire. The products of combustion include carbon monoxide, which is lethal if allowed to build up. For this reason, gas fires must be well-ventilated so that the products of combustion escape into the atmosphere and fresh air is drawn into the room where the heater is.

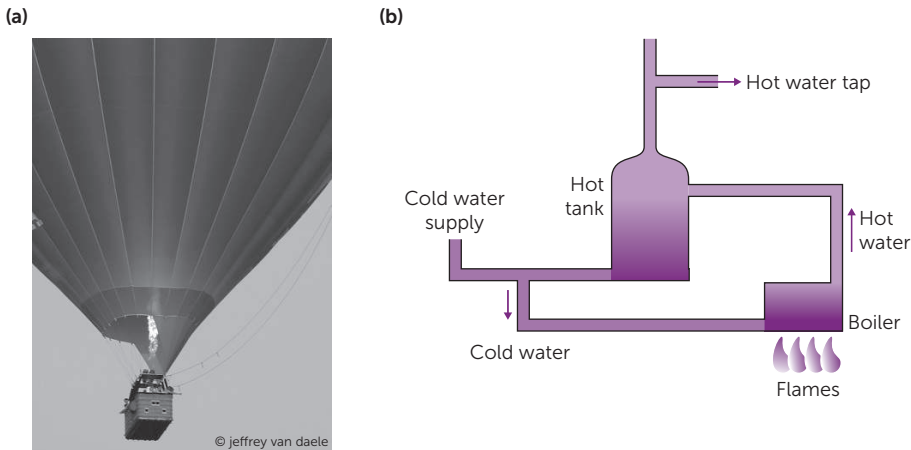


Fig. 6.10 Convection examples (a) a hot air balloon, (b) a hot water system

- 2 A hot air balloon rises because a gas burner heats the air in the balloon and makes it less dense than the surrounding air. Without repeated bursts of heating, the hot gas in the balloon would cool and the balloon would sink as the air in it became more dense. See Fig. 6.10a.
- 3 Water in a hot water tank is usually drawn off near the top of the tank. The hot water from the boiler enters the tank and rises to the top. If the feed pipe for the hot water was near the bottom of the tank, it would draw cold water, unless all the water in the tank was hot. See Fig. 6.10b.

Investigation of convection in air

- 1 **Natural convection** Measure the temperature of a hot object at regular intervals as it cools naturally in air. The measurements may be plotted as a temperature versus time graph, as in Fig. 6.11. The gradient of this graph at any point is the rate of temperature loss of the object at that point. The results show that the rate of temperature loss decreases as the excess temperature of the object above its surroundings decreases.

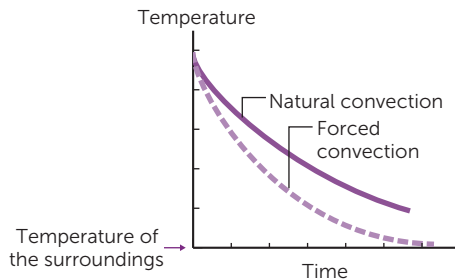


Fig. 6.11 Cooling curves

- 2 **Forced convection** Repeat the measurements in the presence of an airstream from a hair dryer or fan. The temperature decreases more rapidly than without the fan.

See www.palgrave.com/foundations/breithaupt for more information about this investigation

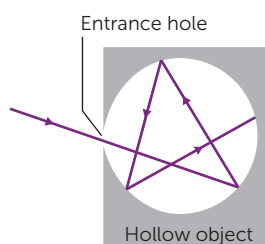


Fig. 6.12 A black body

Note

Fig. 12.3 shows how the energy radiated per second varies with wavelength for different surface temperatures. The wavelength, λ_0 , at which each curve peaks decreases as the absolute temperature, T , rises in accordance with the relationship

$$\lambda_0 T = 0.0029 \text{ K m}$$

This relationship is known as **Wien's law of radiation**.

Radiation

Every object emits electromagnetic radiation due to its temperature. This radiation is known as **thermal radiation** and consists mostly of infra-red radiation, although it includes visible radiation if the temperature is sufficiently high.

Thermal radiation is absorbed most effectively by matt black surfaces and least effectively by shiny, silvered surfaces. A **black body** is a body that absorbs all the radiation incident on its surface. For example, a small hole in the surface of a hollow object would act as a black body as any radiation entering the hole would be completely absorbed by the surface of the cavity. See Fig. 6.12. The Sun and the stars may be considered as black bodies since any radiation incident on the surface would be absorbed.

A surface that is a good absorber of thermal radiation is also a good emitter. The radiation from a black body is referred to as **black body radiation**. The energy per second radiated from a surface depends on

- 1 the area, A , of the surface; the greater the area, the more energy radiated each second
- 2 the surface temperature, T ; the hotter an object's surface is, the more energy it radiates per second
- 3 the nature of the surface; the surface of a black body radiates more energy per second than any other surface of the same area and at the same temperature.

Stefan's law of radiation states that the energy per second, W , radiated from a surface of area A at absolute temperature T is given by the equation

$$W = e\sigma AT^4$$

where σ is a constant known as the Stefan constant, and e , the emissivity of the surface, is the ratio of the energy per second radiated from the surface to energy per second radiated by a black body surface of equal area at the same temperature. The value of σ is $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

6.6 WORKED EXAMPLE

Calculate the energy per second radiated from a glowing fuse wire of length 20 mm and diameter 0.3 mm at a surface temperature of 1600 K, given its surface emissivity is 0.2.

Solution

$$\text{Surface area } A = 2\pi rL,$$

where $r = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}$, and $L = 20 \text{ mm} = 0.020 \text{ m}$

$$\text{Hence } A = 2\pi \times 1.5 \times 10^{-4} \times 0.020$$

$$= 1.89 \times 10^{-5} \text{ m}^2$$

Using Stefan's Law, energy per second radiated from the surface $= e\sigma AT^4$

$$= 0.2 \times 5.67 \times 10^{-8} \times 1.89 \times 10^{-5} \text{ m}^2 \times (1600)^4$$

$$= 1.40 \text{ J s}^{-1}$$

6.4 QUESTIONS

- Fig. 6.13 shows a cross-section of a vacuum flask designed to keep a liquid hot.
 - Why is the glass vessel in the flask silvered?
 - Why is there a vacuum between the inner and outer surfaces of the vessel?
 - Why is a lid essential?
 - What would be a suitable material to surround the vessel? Give a reason for your choice.
- Estimate the heat radiated per second from a hot plate of diameter 0.10 m which is at a surface temperature of 1200 K.

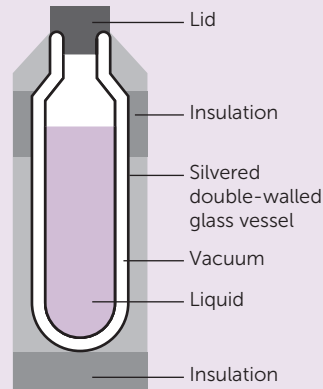


Fig. 6.13

6.5 More about thermal conduction

Some materials conduct heat much more effectively than others. Fig. 6.14 shows how rods of different materials can be compared in terms of thermal conductivity.

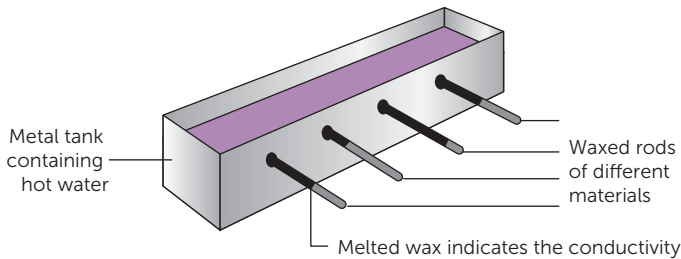


Fig. 6.14
Comparing thermal conduction

Metals are better conductors of heat than non-metals because of the presence of conduction electrons in metals. When a metal is heated, the electrons gain energy and move about faster, transferring energy to atoms and electrons elsewhere.

Non-metals do not contain conduction electrons and therefore do not conduct as well as metals. Thermal conduction in a non-metal takes place as a result of vibrations of the atoms spreading throughout the non-metal.

Heating a non-metal makes the atoms at the point of heating vibrate more and these vibrations cause atoms to vibrate in other parts of the non-metal. This process does occur in a metal but much less than energy transfer due to conduction electrons.

Consider the heat flow along a conductor of uniform cross-sectional area, A , and of length, L , which has one end at fixed temperature, T_1 , and the other end at a lower fixed temperature, T_2 . Provided the conductor is in an insulated jacket so there is no heat loss from its sides, the temperature falls uniformly along its length from T_1 to T_2 , as shown in Fig. 6.15. This shows that the temperature gradient along the bar, the temperature drop per unit length, is constant.

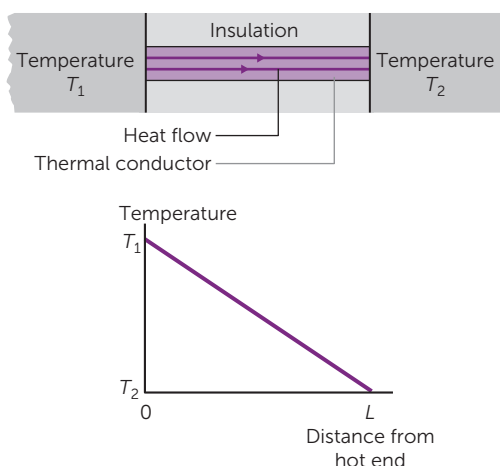


Fig. 6.15 Temperature versus position along an insulated thermal conductor

Table 6.3 Thermal conductivity values

Material	Thermal conductivity / $\text{W m}^{-1} \text{K}^{-1}$
Aluminium	210.0
Copper	390.0
Glass	0.7
Cardboard	0.2
Felt	0.04

The energy per second conducted along the bar (i.e. the heat flow), Q/t is

- 1 proportional to the area of cross-section, A
- 2 proportional to the temperature gradient $\frac{(T_1 - T_2)}{L}$
- 3 dependent on the thermal conductivity of the material.

Hence the heat flow

$$\frac{Q}{t} = kA \frac{(T_1 - T_2)}{L}$$

where k is the coefficient of thermal conductivity of the material. The unit of k is $\text{W m}^{-1}\text{K}^{-1}$. Table 6.3 gives values of k for different materials.

6.7 WORKED EXAMPLE

Calculate the heat loss per second through a rectangular 6 mm glass pane of dimensions $1.20 \text{ m} \times 0.60 \text{ m}$, for a temperature difference of 15 K. The thermal conductivity of glass is $0.72 \text{ W m}^{-1}\text{K}^{-1}$.

Solution

$$\frac{Q}{t} = kA \frac{(T_1 - T_2)}{L} = \frac{0.72 \times (1.20 \times 0.60) \times 15}{6.0 \times 10^{-3}} = 1.3 \times 10^3$$

See www.palgrave.com/foundations/breithaupt for an experiment to measure the thermal conductivity of copper.

U-values

Heat loss calculations from buildings can be very complicated because all three heat transfer mechanisms (i.e. conduction, convection and radiation) are usually involved, cavities may be present and there are many different types of surfaces. For each type of wall, window, floor or roof, a standard value for the heat loss per second per square metre for a 1 K temperature difference is used to calculate the heat loss for a given surface area and a given temperature difference. These standard values, known as **U-values**, are determined by direct measurements. For example, if

the U-value for a certain type of window is 3.0, then a window of this type which has a surface area of 4.0 m^2 would transmit heat at a rate of 12.0 W K^{-1} . In a location where the temperature difference between the interior and the exterior of the building was 10 K, the rate of heat loss would therefore be 120 W. In general terms, for a wall or window or floor or roof:

$$\begin{array}{ccccc} \text{the heat loss per second} & = & \text{U-value} \times & \text{surface area} & \times \text{temperature difference} \\ \text{(in watts)} & & & \text{(in square metres)} & \text{(in kelvins or } ^\circ\text{C)} \end{array}$$

where the **U-value** (of the wall or window or roof or floor) is defined as **the heat loss per second per square metre for a 1 kelvin temperature difference**. U-values are often expressed in watts per square metre per $^\circ\text{C}$ (abbreviated $\text{W m}^{-2} ^\circ\text{C}^{-1}$) rather than watts per square metre per kelvin (abbreviated $\text{W m}^{-2}\text{K}^{-1}$). The numerical value is the same whichever of the above two units is used because a temperature difference of 1 K is the same as a temperature difference of 1°C .

The total heat loss per second from a building can be calculated for a given temperature difference if the U-value for every different type of external wall, window, roof, floor and door in the building is known and the relevant areas are measured. Table 6.4 lists some typical U-values.

Table 6.4 Typical U-values	
Type of surface	U-value / $\text{W m}^{-2} ^\circ\text{C}^{-1}$
Cavity brick wall without cavity wall insulation	1.6
Cavity brick wall with insulation	0.6
Tiled roof with no loft insulation	1.9
Tiled roof with loft insulation	0.6
Single-glazed window	4.3
Double-glazed window	3.2
Floor	0.5

6.5 QUESTIONS

- Calculate the heat flow per second per square metre through a single-brick wall of thickness 120 mm, when the temperature on one side is 20°C and the temperature on the other side is 5°C .
The thermal conductivity of brick is $0.40 \text{ W m}^{-1}\text{K}^{-1}$.
- An aluminium pan of diameter 120 mm is used to boil water on a gas cooker. In 2 minutes, 0.10 kg of water boils away.
 - Calculate the energy per second needed to boil water away at this rate.
 - Estimate the temperature of the underside of the base of the pan.
The specific latent heat of steam is 2.3 MJ kg^{-1} and the thermal conductivity of aluminium is $210 \text{ W m}^{-1}\text{K}^{-1}$.
- A portable box-shaped building has four windows, each 1.0 m high and 1.5 m wide. Use the U-value in Table 6.4 to calculate
 - the total window area,
 - the total heat loss per second through the windows for (i) single-glazed windows, (ii) double-glazed windows, when the temperature difference is 10°C
 - the cost per day of maintaining the temperature difference in (a) using electrical heating at a cost of 1 p per MJ for each type of window.

6.8 WORKED EXAMPLE

A house has a cavity brick wall of total area 280 m^2 with no cavity insulation, double glazed windows of total area 18 m^2 , a floor of area 110 m^2 and an insulated roof of area 130 m^2 .

Use the data in Table 6.4 to calculate the total rate of loss of heat from the house on a day when the temperature outdoors is 15°C higher than the temperature indoors.

Solution

$$\begin{aligned}\text{Heat loss per second from the walls} &= \text{U-value} \times \text{surface area} \times \text{temperature difference} \\ &= 0.6 \times 280 \times 15 = 2520 \text{ W}\end{aligned}$$

$$\text{Heat loss per second from the windows} = 3.2 \times 18 \times 15 = 864 \text{ W}$$

$$\text{Heat loss per second from the roof} = 0.6 \times 130 \times 15 = 1170 \text{ W}$$

$$\text{Heat loss per second from the floor} = 0.5 \times 110 \times 15 = 825 \text{ W}$$

$$\text{Total heat loss per second} = 2520 + 864 + 1170 + 825 = 5400 \text{ W (to 2 significant figures)}$$

SUMMARY

- **Temperature** in kelvins (K) = temperature in $^\circ\text{C} + 273$.
- **Thermal expansion**
Change of length = $\alpha L \Delta T$,
where L = length, ΔT = temperature change, α = coefficient of thermal expansion.
- **Energy** in joules = power in watts $\times$ time taken in seconds.
- **Specific heat capacity**
Energy transferred = $mc(T_2 - T_1)$
where m = mass, c = specific heat capacity, T_2 = final temperature, T_1 = initial temperature.
- **Specific latent heat**
Energy needed or released = ml ,
where m = mass, l = specific latent heat of fusion or of vaporisation.
- **Heat transfer** is due to conduction, convection and radiation.
- **Stefan's law of radiation**
Power radiated, $W = e\sigma AT^4$,
where σ = Stefan constant, e = emissivity of the surface, A = surface area, T = absolute temperature.
- **Surfaces**
A good absorber is a good emitter; a poor absorber is a poor emitter. A black body is a body that absorbs all the radiation incident on it.
- **Thermal conduction**
Metals are the best conductors of heat because of the presence of conduction electrons.
Heat transfer per second, $\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$
where A = area of cross-section, $T_1 - T_2$ is the temperature difference across length L .
- **U-value** (of the wall or window or roof or floor) is defined as the heat loss per second per square metre for a 1 K temperature difference.
heat loss per second = U-value $\times$ surface area $\times$ temperature difference.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

6.1

- a) Convert each of the following temperatures into kelvins:
 (i) 100°C ,
 (ii) -30°C .
- b) Convert each of the following temperatures into $^{\circ}\text{C}$:
 (i) 100 K
 (ii) 1000 K .

6.2

- a) (i) Explain why expansion gaps are necessary in between adjacent concrete spans of road on a bridge.
 (ii) When a steel tyre is fitted on a locomotive wheel, the tyre is heated, placed on the wheel and then allowed to cool. Explain why this procedure is necessary to ensure it fits tightly on the wheel.
- b) Calculate the thermal expansion of a steel cable of length 120 m when its temperature changes from 5°C to 35°C . The coefficient of thermal expansion of steel is $1.1 \times 10^{-5}\text{ K}^{-1}$.

6.3 In an experiment to determine the specific heat capacity of a piece of metal of mass 235 g , the metal object is suspended in steam at 100°C then transferred to an insulated copper calorimeter of mass 65 g containing 185 g of water at 15°C . As a result, the temperature of the water increases to 27°C . Calculate

- a) the energy needed to raise the temperature of
 (i) the water,
 (ii) the copper calorimeter from 15°C to 27°C ,
 b) the specific heat capacity of the metal.
 The specific heat capacity of copper is $390\text{ J kg}^{-1}\text{ K}^{-1}$ and the specific heat capacity of water is $4200\text{ J kg}^{-1}\text{ K}^{-1}$.

6.4 A copper calorimeter of mass 80 g containing 120 g of water at 16°C was placed in a refrigerator. The water in the calorimeter took 35 minutes to freeze solid. Calculate

- a) the energy removed from the calorimeter and the water when the water
 (i) cools to 0°C ,
 (ii) freezes,
 b) the rate at which energy was removed. (Use the specific heat capacity values in question 6.3, specific latent heat of ice = $340\text{ J kg}^{-1}\text{ K}^{-1}$.)

6.5 When an object at temperature T_1 is in an enclosure at a lower temperature T_0 , the net rate of heat transfer due to radiation = $e\sigma AT_1^4 - e\sigma AT_0^4$, where A is the surface area of the body and e is its emissivity.

- a) Calculate the net heat transfer due to radiation from a radiator of surface area 1.6 m^2 when its surface temperature is 45°C , in a room at a temperature of 25°C . The surface emissivity of the radiator is 0.60 . The Stefan constant, $\sigma = 5.67 \times 10^{-8}\text{ W m}^{-2}\text{ K}^{-4}$.
- b) Calculate the heat loss per second per square metre from the ground on a clear night when the ground temperature is 10°C and the air temperature is 0°C . Assume the surface emissivity of the ground is 1 .

6.6 The insulation jacket of a copper tank has a surface area of 0.95 m^2 and is 15 mm thick. When the outer surface of the copper tank was 54°C , the temperature on the outside of the insulation jacket was 26°C .

- a) Calculate
 (i) the temperature gradient across the insulation jacket,
 (ii) the heat per second conducted through the insulation.
- b) The base of the tank rested on a cardboard pad of thickness 10 mm . The underside of the cardboard was at 18°C and the upper side was at 22°C . Calculate the heat per second conducted through the cardboard pad. The area of the pad in contact with the tank was 0.12 m^2 . The thermal conductivity values for the insulation material and for cardboard are 0.037 and $0.2\text{ W m}^{-1}\text{ K}^{-1}$, respectively.

6.7 A householder proposes to fit loft insulation in her home which has a total ceiling area of 80 m^2 . This insulation reduces the U-value of the roof from $2.0\text{ W m}^{-2}\text{ }^{\circ}\text{C}^{-1}$ to $0.5\text{ W m}^{-2}\text{ }^{\circ}\text{C}^{-1}$. Using the U-values in Table 6.4, calculate for a temperature difference of 20°C , the heat loss per second

- a) without loft insulation,
 b) with loft insulation.

6.8 A mobile home of length 9.0 m , width 3.0 m and height 2.5 m has a flat roof and single glazed windows of total area 4.0 m^2 .

- a) Calculate
 (i) the total non-window area of the sides and roof,
 (ii) the floor area,
 (iii) the roof area.
- b) (i) Calculate the heat loss per second when the external temperature is 5°C lower than the internal temperature. U-values; window = $4.3\text{ W m}^{-2}\text{ }^{\circ}\text{C}^{-1}$, sides and roof = $2.5\text{ W m}^{-2}\text{ }^{\circ}\text{C}^{-1}$, floor = $2.0\text{ W m}^{-2}\text{ }^{\circ}\text{C}^{-1}$.
 (ii) Calculate the cost of maintaining the temperature in (b)(i) for 1 week using heating costing 2 p per MJ.

UNIT

7

Strength of Solids

CONTENTS

- 7.1 Measuring force
- 7.2 Force and materials
- 7.3 Stress and strain
- 7.4 Stress versus strain curves
- 7.5 Elastic energy
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- calibrate and use a spring to measure force
- describe the effect of forces on different materials
- appreciate the meaning of strength, stiffness, elasticity, hardness, toughness and brittleness
- calculate the breaking stress of a wire
- measure and carry out calculations on the stress, strain and Young modulus of a material
- describe the general features of the stress versus strain relationship for different materials
- relate the features of a stress versus strain curve to the structure of the material

7.1 Measuring force

Mass and weight

The **weight** of an object is proportional to its mass. Mass is the amount of matter an object has and is measured in kilograms. Weight is a force and is therefore measured in **newtons (N)**, as explained on p. 42. The weight of a 1 kg mass is 9.8 N on the surface of the Earth. This relationship can be used to calculate the weight of any object, given its mass in kilograms. The weight, W , of an object of mass, m , is calculated from the equation:

$$W = mg$$

where g = the force of gravity per unit mass on the object.

The value of g decreases with height above the Earth and is 9.8N kg^{-1} on the surface of the Earth.

Using a spring balance as a force meter

A steel spring suspended freely from a fixed point extends when a weight is hung on its lower end. Its change of length from its initial unstretched length is referred to as its extension. When the spring is stretched beyond its unstretched length, it is said to be under tension. When it supports a weight at rest as in Fig. 7.1, the tension in the spring is equal and opposite to the weight.

The arrangement in Fig. 7.1 can be used to measure the extension of a spring for different weights. The measurements may be plotted as a graph of tension on the vertical axis against extension on the horizontal axis.

Safety note

Always wear impact-resistant safety spectacles when carrying out stretching tests in case the weight drops off suddenly and the spring flies up.

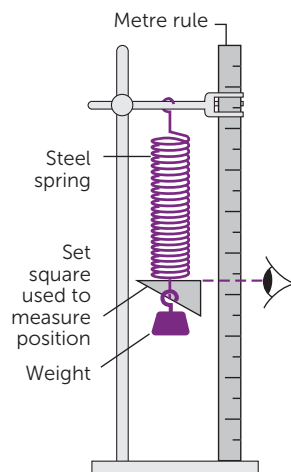


Fig. 7.1 Investigating the extension of a spring

Hooke's Law

For a steel spring, the measurements define a straight line that passes through the origin, as in Fig. 7.2. The weight of an object can be determined by measuring the extension of the spring when it supports the object, then reading the tension off the graph to give the object's weight.

The graph shows that the tension is proportional to the extension because the line is straight and it passes directly through the origin. This relationship, known as **Hooke's Law**, may be expressed as an equation

$$T = ke$$

where T = tension (in N), e = the extension (in m) and k is referred to as the spring constant (in N m^{-1}).

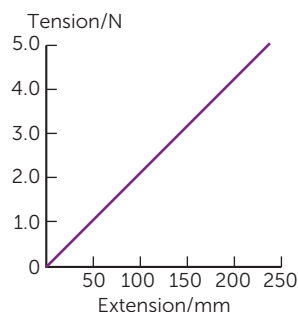


Fig. 7.2 A graph of tension versus extension

Note

Rearranging the equation for Hooke's Law gives $k = T \div e$, which is the gradient of the line in Fig. 7.2 because the line passes through the origin. Show that $k = 20 \text{ N m}^{-1}$ for the spring that was used to generate the graph.

7.1 QUESTIONS

- A steel spring was suspended vertically from a fixed point. Without any weight hung from its lower end, its length was 300 mm.
 - When a 0.40 kg mass was suspended from its lower end, it stretched to a length of 420 mm. Calculate (i) the weight of a mass of 0.40 kg, (ii) the extension of the spring, (iii) the spring constant.
 - The 0.40 kg mass was replaced by an object of unknown mass which caused the spring to stretch to a length of 390 mm. Calculate (i) the weight of the object, (ii) the mass of the object.
- The following measurements were obtained in an experiment to determine the spring constant of a steel spring:

Weight/N	0.0	2.0	4.0	6.0	8.0
Spring length/cm	25.0	30.2	35.0	40.2	44.8

 - Make a table to show the extension for different values of tension.
 - (i) Plot a graph of tension on the vertical axis against extension on the horizontal axis. (ii) Calculate the spring constant of the spring.

7.2 Force and materials

Material properties

The effect of forces on solid materials varies widely from one material to another. For example, a biscuit cracks suddenly when subjected to excessive force; in comparison, a bar of toffee will bend if subjected to a slowly increasing force. The same material will, however, behave like a biscuit if subjected to an impact force of short duration.

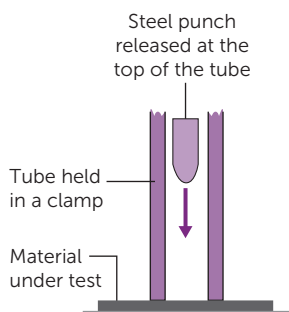


Fig. 7.3 Hardness testing

Safety note



Always wear impact-resistant safety spectacles when carrying out hardness tests in case bits fly up from the material under test.

- **Strength** is a measure of how much force is needed to break an object. Suitable heat treatment of steel objects makes them stronger. Some objects become weaker when subjected to repeated bending.
- **Stiffness** is a measure of the difficulty of stretching or bending an object. For example, a steel strip is much stiffer than a rubber strip. The spring constant of a spring is a measure of its stiffness. Strength and stiffness depend on the dimensions of the material as well as its nature.
- **Elasticity** is the property of a material that enables it to regain its shape after being distorted. For example, a rubber band regains its original length when it is released after being stretched. The **elastic limit** of an object is the limit to which it can be distorted and still be able to regain its shape. A strip of polythene does not regain its shape after being stretched because its elastic limit is very small. An object stretched beyond its elastic limit is said to exhibit plastic behaviour.
- **Hardness** is the resistance of the surface of a material to scratches and dents. The hardness of two surfaces may be compared by dropping a metal punch onto each surface from the same height and comparing the diameters of the dents created. See Fig. 7.3.
- **Toughness** is the ability of a material to withstand impact forces without cracks developing, which would weaken the material when a material is distorted. For example, the sole of a sports shoe must be very tough to withstand repeated flexing and impacts. Once a crack develops in the sole of a sports shoe, it widens and the sole splits. In contrast, a brittle material snaps suddenly without 'giving' when subjected to excessive force.

Investigating stiffness and elasticity

The apparatus in Fig. 7.1 may be modified as shown in Fig. 7.4 to investigate how easily a strip of material stretches and whether or not it regains its shape easily. The results for these tests are plotted with tension on the vertical axis and extension on the horizontal axis, as in Fig. 7.5.

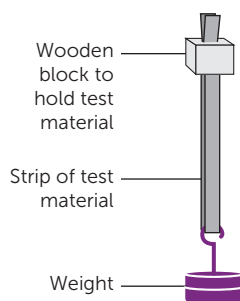


Fig. 7.4 Testing materials

- 1 A rubber band stretches easily then becomes much stiffer. The extension is not proportional to the weight. It regains its original length when all the weights are removed. If the weight is reduced in steps, its length for a given weight is greater than when it was loaded with that weight. This effect is referred to as **mechanical hysteresis** because the extension 'lags' behind the weight. The area of the loop is a measure of the energy used when the material is deformed and allowed to regain its shape.

2. A polythene strip also stretches easily then becomes much stiffer. Its extension is not proportional to the weight. However, unlike rubber, polythene does not regain its original length, as its elastic limit is very small and it undergoes plastic behaviour when it is extended noticeably.

Polymers

Rubber and polythene are examples of polymers, which are materials composed of long 'chain' molecules. Rubber is a natural polymer, unlike polythene. A polymer is formed when much smaller identical molecules called 'monomers' link together, forming covalent bonds as they join end to end like the links in a chain. In rubber and polythene, the polymer molecules become tangled up. When rubber or polythene is stretched, the molecules are straightened and the material becomes difficult to extend further. Rubber molecules tend to curl when released, which is why a rubber band regains its original length when released.

7.2 QUESTIONS

- Describe the ideal mechanical properties of
 - a biscuit,
 - a running shoe,
 - a dustbin.
- Discuss whether or not a rubber band would make a suitable force meter instead of a steel spring.

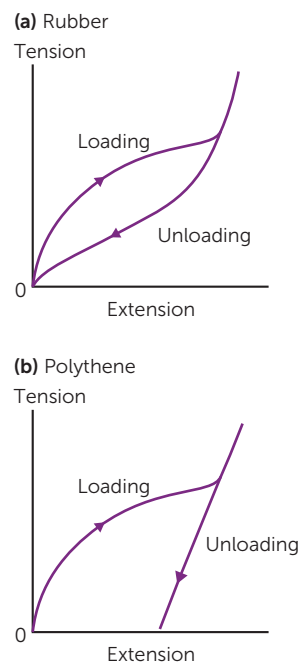


Fig. 7.5 Tension versus extension graphs for (a) rubber and (b) polythene

7.3 Stress and strain

Breaking stress

The force needed to break a wire depends on the diameter of the wire and the material of the wire. Four times as much force is needed to break a wire of diameter 1mm than is needed to break a wire of diameter 0.5 mm, made of the same material. This is because the area of cross-section of the thicker wire is four times as much as the area of cross-section of the thinner wire.

The **breaking stress** of a wire is defined as the breaking force per unit area of cross-section.

If F is the force required to break a wire of cross-sectional area A ,

$$\text{the breaking stress, } \sigma_b = \frac{F}{A}$$

7.1 WORKED EXAMPLE

Calculate the force needed to break a nylon wire of diameter 0.80 mm. The breaking stress of nylon is 5.0×10^7 Pa.

Solution

Area of cross-section, $A = \pi(0.8 \times 10^{-3})^2/4 = 5.0 \times 10^{-7} \text{ m}^2$

Rearranging the equation for breaking stress gives

$$F = \sigma_b A = 5.0 \times 10^7 \times 5.0 \times 10^{-7} = 25 \text{ N}$$

Safety note



Always wear impact-resistant safety spectacles when carrying out stretching tests in case the weight drops off and the material under test flies up.

Notes

- The unit of stress is the pascal (Pa), equal to 1 N m^{-2} . The symbol for stress is σ (pronounced 'sigma').
- For a wire of diameter d , its cross-sectional area $A = \frac{\pi d^2}{4}$
- Safety note: always wear impact-resistant safety spectacles when carrying out strength tests on materials.

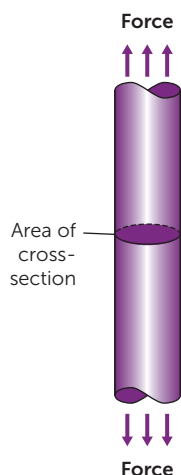


Fig. 7.6 Stress

Measuring stress and strain

Stress is defined as the force per unit area acting perpendicular to the area. See Fig. 7.6. The unit of stress, the pascal (Pa) is equal to 1 N m^{-2} . For a wire of cross-sectional area A , under tension T , the stress, σ , is calculated from the equation

$$\sigma = \frac{T}{A}$$

For a wire suspended vertically from a fixed point, supporting a weight W , the tension is equal to the weight.

Strain is defined as the change of length per unit length. Since strain is a ratio, there is no unit for strain. For a wire of unstretched length L , which extends to a length $L + e$, where e is its extension, as shown in Fig. 7.7, the strain is calculated from the equation

$$\text{strain} = \frac{e}{L}$$

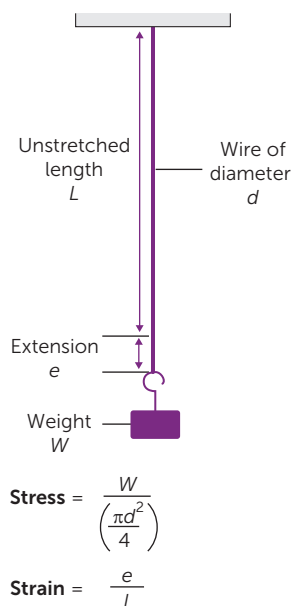


Fig. 7.7 Measuring stress and strain

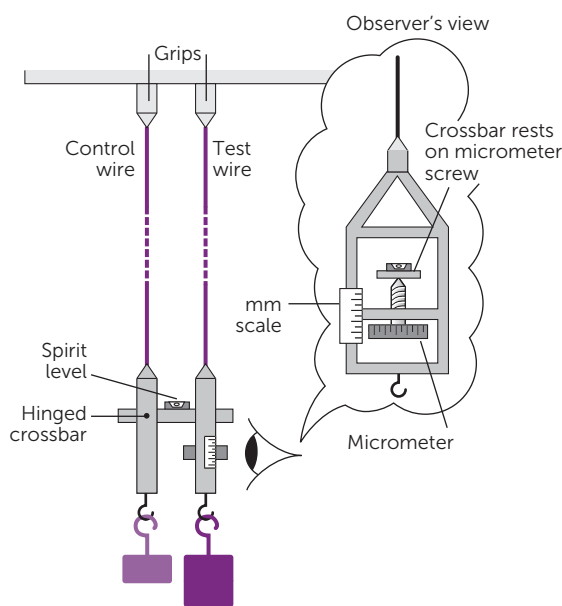


Fig. 7.8 A wire under test

Safety note



Always wear impact-resistant safety spectacles when carrying out stretching tests in case the wire snaps and flies up.

The relationship between stress and strain for a wire can be investigated using the arrangement in Fig. 7.8. The control wire supports a micrometer which is used to measure the change of length of the test wire when the test wire is loaded and unloaded. Each time a measurement is made, the micrometer is adjusted so the spirit level is horizontal before the micrometer reading is taken.

- 1 The two wires are loaded sufficiently to ensure each wire is straight. The micrometer is adjusted as explained above and its reading noted.
- 2 The initial length, L , of the test wire is then measured using a metre rule with a millimetre scale. The diameter of the wire is also measured using a micrometer at several points along the wire to give an average value, d , for the diameter of the wire.

- 3 The test wire is then loaded with a known weight, W , its micrometer is adjusted as above and its reading noted. The procedure is repeated as W is increased in steps and then decreased in steps to zero.
- 4 The extension, e , from the initial length, L , is calculated from the test wire micrometer readings. The strain is then calculated for each reading by dividing the extension by the initial length. The area of cross-section, A , of the wire is calculated using $A = \pi d^2 \div 4$ and the stress ($= W \div A$) calculated for each reading.
- 5 The results may be plotted as a graph of stress on the vertical axis against strain on the horizontal axis, as shown by Fig. 7.9. The graph shows that the stress is proportional to the strain. This relationship holds up to a limit referred to as the *limit of proportionality*. Provided the limit of proportionality is not exceeded, the value of stress $\div$ strain is constant. This value is known as the **Young modulus of elasticity** of the material.

$$\text{Young modulus of elasticity, } E = \frac{\text{stress}}{\text{strain}}$$

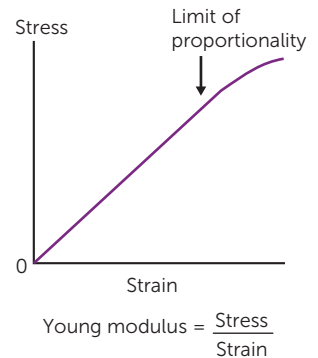


Fig. 7.9 Stress versus strain graph

Notes

- 1 The unit of E is the pascal (Pa), the same as the unit of stress.
- 2 For a wire of unstretched length, L , and area of cross-section, A , under tension, T ,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{e/L} = \frac{TL}{Ae}$$
- 3 A graph of tension, T , on the vertical axis against extension, e , on the horizontal axis also gives a straight line through the origin. See Fig. 7.10. Rearranging the equation for E gives

$$T = \frac{AE}{L}e$$

which is the equation for this graph.

$$\text{The gradient of the line} = \frac{AE}{L}$$

$$\text{Hence } E = \text{gradient (of } T \text{ v. } e \text{ line)} \times \frac{L}{A}$$

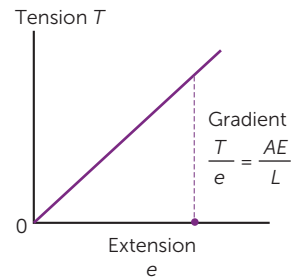


Fig. 7.10 Tension versus extension for a steel wire

7.2 WORKED EXAMPLE

A vertical steel wire of diameter 0.35 mm and unstretched length 1.22 m extends by a distance of 3.2 mm when it is loaded with a 50 N weight. Calculate

- a) (i) the stress and (ii) the strain in the wire,
- b) the Young modulus of steel.

Solution

- a) (i) Area of cross-section, $A = \pi(0.35 \times 10^{-3})^2 \div 4 = 9.62 \times 10^{-8} \text{ m}^2$

$$\therefore \text{Stress} = \frac{T}{A} = \frac{50}{9.62 \times 10^{-8}} = 5.2 \times 10^8 \text{ Pa}$$

(ii) Strain $= \frac{e}{L} = \frac{3.2 \times 10^{-3}}{1.22} = 2.6 \times 10^{-3}$

- b) Young modulus $E = \frac{\text{stress}}{\text{strain}} = 2.0 \times 10^{11} \text{ Pa}$

7.3 QUESTIONS

- 1 A 2.37 m steel wire of diameter 0.38 mm is suspended vertically from a fixed point and used to support a weight of 115 N at its lower end.
 - a) The Young modulus of steel is 2.0×10^{11} Pa. Calculate (i) the stress in the wire, (ii) the extension of the wire.
 - b) The breaking stress of steel is 1.1×10^9 Pa. Calculate the maximum weight this wire can support when vertical without snapping.
- 2 A wire of length 1.28 m and of diameter 0.28 mm was stretched by increasing its tension in steps. The extension of the wire was measured at each step and tabulated as below.

Tension/N	0.0	5.0	10.0	15.0	20.0	25.0
Extension/mm	0.0	0.8	1.6	2.4	3.4	4.6

 - a) Plot a graph of tension on the vertical axis against extension on the horizontal axis.
 - b) (i) Estimate the limit of proportionality of the wire.
(ii) Calculate the Young modulus of elasticity of the wire.

7-4 Stress versus strain curves

A stress-strain curve is characteristic of the material and does not depend on the dimensions of the sample tested. Fig. 7.11 shows a stress-strain curve for a steel wire, with stress on the vertical axis against strain on the horizontal axis. Its features are therefore due to the nature and properties of the material and are determined by the structure of the material.

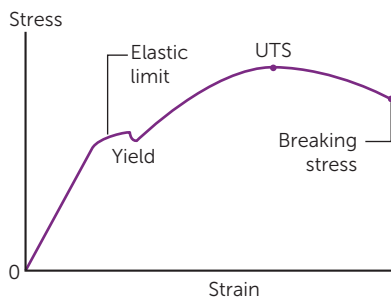


Fig. 7.11 Stress versus strain for steel



Fig. 7.12 A tensometer in use

This type of curve is obtained by using a 'tensometer' which stretches the material and measures the force as it is stretched. See Fig. 7.12.

- **Proportionality:** the stress is proportional to the strain up to the 'limit of proportionality'. The gradient of this straight section is the Young modulus of the material. See p. 99.
- The **elastic limit:** this is the limit beyond which the material becomes permanently stretched.
- **Plastic behaviour:** beyond the elastic limit, the material does not regain its original length and its behaviour is described as plastic.
- **Yield:** the sample suddenly 'gives' a little at a point referred to as the upper yield point. The stress drops slightly to the lower yield point before rising again as the sample is stretched.
- The **ultimate tensile stress (UTS)** of the material is the point where the sample is strongest. The material is unable to withstand stress beyond its UTS.

- The **breaking stress** is the stress at the breaking point of the sample. Stretching a wire beyond its UTS causes it to become narrower at its weakest point where internal cavities form. The effective cross-sectional area is reduced at this point which increases the stress locally, reducing the area of cross-section even more until the wire snaps.

Solid materials and their structure

Solid materials are three-dimensional structures with their atoms arranged in three main categories, as shown in Fig 7.13.

Crystalline solids

The atoms in a crystal are arranged in a regular pattern to give the crystal a distinctive shape. A metal is composed of many tiny crystals called **grains**, each with the same atomic arrangement but aligned at random to each other.

Amorphous solids

The atoms in an amorphous solid are locked together at random. Glass is an example of an amorphous solid.

Polymers

A polymer is composed of long molecules usually tangled together. Polymers that set permanently and cannot be remoulded by heating contain **cross-links** between the molecules.

Application 2

Graphene – the new ‘wonder’ material

Imagine an isolated sheet of material just one atom thick. Such a two-dimensional material was first made using graphite by Andre Geim and Konstantin Novoselov of the University of Manchester in 2004. The importance of their discovery was recognised when they were awarded the 2010 Nobel Prize for physics. Graphite consists of layers of carbon atoms which can slide over each other. Graphene is a single unsupported layer of carbon atoms. Figure 7.13d shows how the atoms are arranged.

Graphene is one of the strongest materials known. It is an excellent conductor of electricity and heat and is transparent. Intensive research is under way in many countries to develop potential applications such as flexible electronic screens, lightweight materials stronger than steel, lightweight batteries and ultra-fast electronic devices.

Molecular interpretation of stress versus strain curves

The general features of the force between two atoms in a solid were explained on pp. 75–6. Fig. 7.14 shows how the force changes with distance apart. The equilibrium separation corresponds to the average separation between the atoms in a solid.

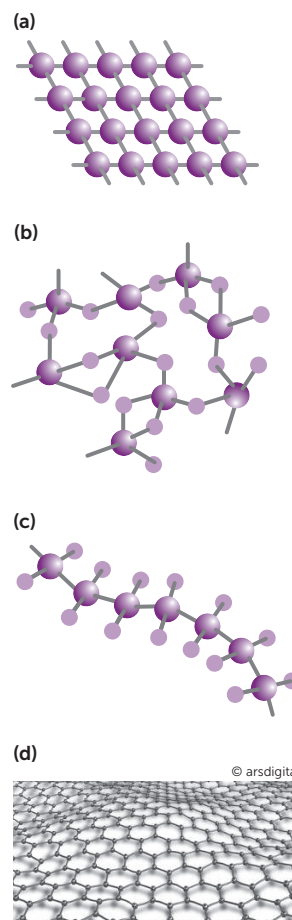


Fig. 7.13 Solid materials (a) crystalline, (b) amorphous, (c) polymer (d) atoms in graphene

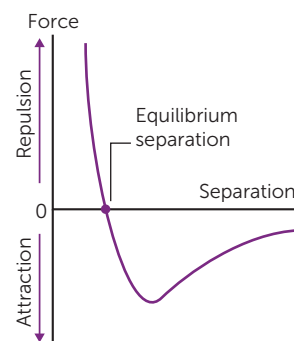


Fig. 7.14 Force versus separation for two atoms

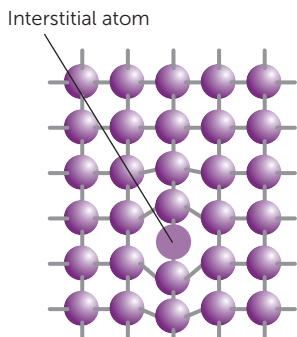


Fig. 7.15 Interstitial atoms

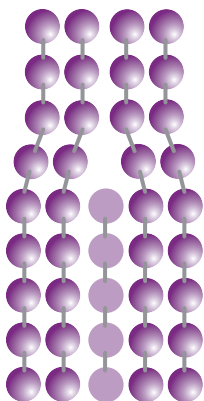


Fig. 7.16 Dislocations

- **Proportionality:** this is because the graph of force versus separation is straight near to the equilibrium separation. Therefore, the force is proportional to the change of separation from equilibrium. This is why the tension in a wire is proportional to its extension.
- **Limit of proportionality:** this is because the force versus separation graph departs from a straight line curve as the separation is increased more and more.
- **Plastic behaviour:** this is because the atoms are made to slip past each other to take up new positions in the structure. This is easily achieved in a polymer such as PVC, but not in an elastic band due to the presence of strong cross-links which prevent the molecules slipping past each other. In a metal, plastic behaviour occurs when planes of atoms are forced to slip past each other. However, because a metal consists of countless grains, the tendency of one grain to slip is opposed by the surrounding grains, so a metal is much stronger than a single crystal. In addition, the presence of **interstitial atoms** strengthens a metal by preventing crystal planes slipping past each other. See Fig. 7.15.
- **Ultimate tensile strength:**
 - (a) Polymer molecules line up with each other when a polymer is stretched as much as possible. To break the polymer, the applied force must be sufficient to break each molecule.
 - (b) In an amorphous solid under increasing stress, tiny cracks appear at the surface and become progressively wider and deeper as the stress becomes concentrated at each crack. Amorphous solids such as glass are brittle because there are no internal boundaries, as in a metal, to prevent surface cracks from propagating into the material.
 - (c) In a metal, the presence of **dislocations** affects its strength. A dislocation is where a plane of atoms is squeezed out by adjacent planes. Little stress is needed to make a dislocation move through a grain and reach the grain boundary. See Fig. 7.16. The grain size affects the strength of the metal because there are more grain boundaries to stop the movement of dislocations if the grains are small. Heat treatment affects grain size, which is why steel is made brittle if it is heated strongly then rapidly cooled by plunging it into water.

7.4 QUESTIONS

- 1 a) (i) Sketch a curve to show how the force between two atoms in a solid varies with separation. (ii) Use your curve to explain why the extension of a wire is proportional to its tension, up to a limit.
b) Explain the difference between elastic and plastic behaviour of a solid.
- 2 Fig. 7.17 shows the stress–strain curves for two different materials, A and B, on the graph. Which material is
a) stronger
b) stiffer?
In each case, give a reason for your answer.

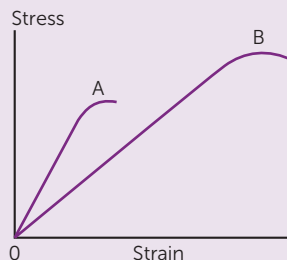


Fig. 7.17

7.5 Elastic energy

Force and energy

Energy is the capacity of a body to do work. Work is done when a force moves its point of application in the direction in which it acts. Lift an object and you have transferred energy from your muscles to the object. Stretch a spring and you have transferred energy from your muscles to the spring. Energy is transferred when work is done by a force moving its point of application in the direction of the force.

1 J of energy is transferred when a force of 1 N moves its point of application through a distance of 1 m in the direction of the force.

Work done by a force = force × distance moved in the direction of the force
(in joules) (in newtons) (in metres)

Force versus distance graphs

- 1 For a constant force, F , acting over distance, s , the work done $= Fs$. Fig. 7.18 shows the force versus distance graph for a constant force. The work done is represented by the area under the force line on the graph.
- 2 For a spring, the force increases with extension, as in Fig. 7.19. To extend the spring when under tension, F' , by a small amount, δs , the work done $= F' \delta s$. This is represented on Fig. 7.19 by the area of the strip of width δs under the line.

$\therefore$ the total work done to stretch the spring from zero to extension e
 $=$ total area under the line $= \frac{1}{2} Fe$

where F is the force needed to extend the spring to extension e . The area under the line is a triangle of height, F , and base length, e . The area of this triangle $= \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2} Fe$.

As the work done is stored in the spring as elastic energy, the energy stored in a stretched spring $= \frac{1}{2} Fe$.

Since $F = ke$, where k is the spring constant of the spring, the energy stored $= \frac{1}{2} Fe = \frac{1}{2} ke^2$.

Energy stored in a stretched spring $= \frac{1}{2} ke^2$

where e = extension and k = spring constant.

Energy stored in a stretched wire

The tension, T , in a stretched wire is related to the extension, e , of the wire in accordance with the equation

$$T = \frac{AE}{L} e$$

where L is the unstretched length of the wire, A is its area of cross-section and E is the Young modulus of the material of the wire.

$$\begin{aligned} \therefore \text{the energy stored in the wire at extension } e &= \frac{1}{2} Te \\ &= \frac{\frac{1}{2}(AE)}{L} e^2 \end{aligned}$$

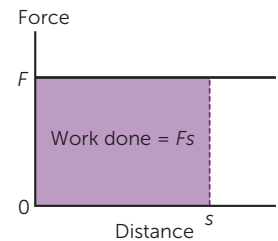


Fig. 7.18 Force versus distance for a constant force

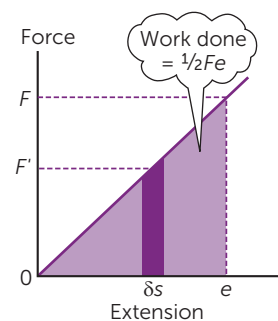


Fig. 7.19 Force versus extension for a spring

Note

Since the volume of the wire $= AL$, the energy stored per unit volume $= \frac{\frac{1}{2} Te}{AL} = \frac{1}{2} \text{stress} \times \text{strain}$

7.3 WORKED EXAMPLE

A 1.25 m steel wire of diameter 0.30 mm is suspended vertically from a fixed point and used to support an 85 N weight. Calculate

- the extension of the wire,
- the energy stored in the wire,
- the energy stored per unit volume in the wire. The Young modulus of steel is 2.0×10^{11} Pa.

Solution

a) Rearranging $E = \frac{TL}{Ae}$ gives $e = \frac{TL}{AE} = \frac{85 \times 1.25}{\frac{1}{4}\pi(0.30 \times 10^{-3})^2 \times 2.0 \times 10^{11}} = 7.5 \times 10^{-3}$ m

b) Energy stored $= \frac{1}{2}Te = \frac{1}{2} \times 85 \times 7.5 \times 10^{-3} = 0.32$ J

c) Volume of wire $= AL = \frac{1}{4}\pi(0.30 \times 10^{-3})^2 \times 1.25 = 8.8 \times 10^{-8}$ m³

Energy stored per unit volume $= \frac{0.32 \text{ J}}{8.8 \times 10^{-8} \text{ m}^3} = 3.6 \times 10^6 \text{ J m}^{-3}$

7.5 QUESTIONS

- The unstretched length of a steel spring is 320 mm. When it was suspended vertically from a fixed point and used to support a weight of 5.0 N at rest, its length extended to 515 mm. Calculate
 - (i) the extension of the spring, (ii) the spring constant.
 - the energy stored in the spring.
- A nylon guitar string of diameter 0.50 mm was stretched from an unstretched length of 750 mm to a length of 785 mm as a result of turning the tension key.
 - Calculate (i) the extension, (ii) the tension in the string at a length of 785 mm.
 - Calculate the energy stored per unit volume in the string at this length.

The Young modulus of nylon $= 3.0 \times 10^9$ Pa.

SUMMARY

- Weight** in newtons $= \text{mass} \times g$.
- Hooke's Law:** For a steel spring, tension, $T = ke$, where e = extension, k = spring constant.
- Material properties:** Strength and stiffness depend on the dimensions and on the material of an object. Elasticity, hardness and toughness are material properties (i.e. do not depend on dimensions).
- Stress** is defined as the force per unit area acting perpendicular to the area.
- Strain** is defined as the change of length per unit length.
- The Young modulus of elasticity, E ,** of a material is defined as stress $\div$ strain.
- The area under a force versus extension graph** represents the work done.
- The energy stored in a stretched spring** $= \frac{1}{2}ke^2$, where e = extension, k = spring constant.
- The energy stored in a stretched wire** $= \frac{\frac{1}{2}(AE)e^2}{L}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

Where necessary, assume $g = 9.8 \text{ N kg}^{-1}$

7.1 A steel spring was suspended vertically from a fixed point. Its length when unloaded was 500 mm.

- When it was loaded with a 6.0 N weight, it extended to a length of 845 mm when at rest. Calculate
 - the spring constant,
 - the energy stored in the spring at this extension.
- The 6.0 N weight was replaced with an unknown weight that made the spring stretch to a length of 728 mm. Calculate
 - the weight of the unknown object,
 - the mass of the unknown object.

7.2

- Fig. 7.20 is a graph of extension versus weight for two different materials, A and B, of the same length, thickness and width. Which material is

- strongest?
- stiffest?

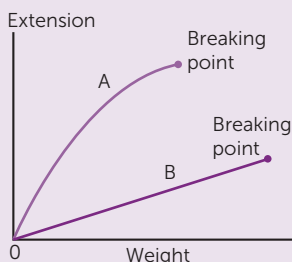


Fig. 7.20

- Sketch curves on the same axes to show how the tension of the following vary with extension
 - an elastic band,
 - a strip of polythene.
- Use your graphs to explain the difference between plastic and elastic behaviour.

7.3 A metal wire of diameter 0.20 mm was suspended vertically from a fixed point and used to support a 5.0 N weight which straightened it out. Its length was then measured at 1.393 m. The weight supported was then increased to 65 N and its length increased to 1.414 m. Calculate

- the extension of the wire,
- the Young modulus of the wire,
- the energy stored in the wire.

7.4 The steel lifting cable of a crane has a diameter of 30 mm and a maximum length of 55 m. The maximum stress in the cable is not allowed to exceed $2.0 \times 10^8 \text{ Pa}$.

- Calculate the maximum weight the cable is allowed to support.
- Calculate
 - the extension of the cable when it supports this maximum weight,
 - the energy stored per unit volume in the cable when extended by this maximum weight. The Young modulus of steel is $2.0 \times 10^{11} \text{ Pa}$.

7.5 The jaws of a vice open by 0.5 mm for each full turn on the handle. A copper cylinder of diameter 10 mm and length 60 mm is placed in the vice which is then tightened up by half a turn. The Young modulus for copper is $1.3 \times 10^{11} \text{ Pa}$. Calculate

- the reduction of length of the copper,
- the force needed to compress it by this amount.

7.6 A passenger lift of weight 1800 N is supported by eight steel cables, each of diameter 5.0 mm and 65 m in length. The maximum allowed stress in the cables is $1.0 \times 10^8 \text{ Pa}$. The Young modulus of steel is $2.0 \times 10^{11} \text{ Pa}$. Calculate

- the maximum weight of passengers allowed in this lift,
- the extension of each lift cable when it supports this weight.

UNIT

8

Pressure

CONTENTS

- 8.1 Pressure and force
- 8.2 Hydraulics
- 8.3 Pressure in a fluid at rest
- 8.4 Measurement of pressure
- 8.5 Flotation
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- carry out calculations relating to pressure, force and area
- describe the principles and applications of hydraulics
- calculate the pressure due to a liquid column
- describe how pressure is measured, including atmospheric pressure
- explain the principle of flotation in terms of pressure

8.1 Pressure and force

Snow shoes are designed with a large contact area so that the wearer does not sink into snow when walking across it. The weight of the wearer is effectively spread out over as large an area as possible. The opposite reason explains why a sharp knife cuts more easily than a blunt knife. The area of the knife in contact with the material it cuts into is less for a sharp knife than for a blunt knife. As a result, the pressure of the knife on the material is greater for a sharp knife than for a blunt knife.

The pressure, p , of a force, F , acting normally (i.e. at right angles) on a surface is defined by the equation

$$p = \frac{F}{A}$$

where A is the area over which the force acts. The unit of pressure is the pascal (Pa), equal to 1 N m^{-2} . See Fig. 8.1.

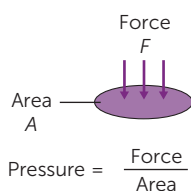


Fig. 8.1 Pressure and force

Note

An area of $1 \text{ m}^2 = 10\,000 \text{ cm}^2$
 $(= 100 \text{ cm} \times 100 \text{ cm}) = 10^6 \text{ mm}^2$
 $(= 1000 \text{ mm} \times 1000 \text{ mm}).$

8.1 WORKED EXAMPLE

A brick of weight 55 N has dimensions 100 mm × 150 mm × 250 mm.

Calculate the pressure of the brick on the ground when it is

a) end-on, b) side-on, c) face-on.

Solution

$$\begin{aligned} \text{a) } A &= 0.100 \times 0.150 = 1.5 \times 10^{-2} \text{ m}^2. & p &= \frac{F}{A} = \frac{55}{1.5 \times 10^{-2}} = 3.7 \times 10^3 \text{ Pa} \\ \text{b) } A &= 0.100 \times 0.250 = 2.5 \times 10^{-2} \text{ m}^2. & p &= \frac{F}{A} = \frac{55}{2.5 \times 10^{-2}} = 2.2 \times 10^3 \text{ Pa} \\ \text{c) } A &= 0.150 \times 0.250 = 3.75 \times 10^{-2} \text{ m}^2. & p &= \frac{F}{A} = \frac{55}{3.75 \times 10^{-2}} = 1.5 \times 10^3 \text{ Pa} \end{aligned}$$

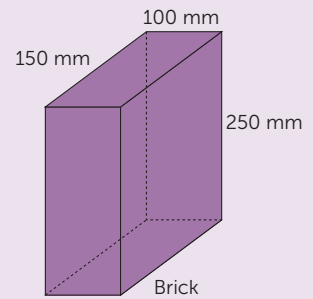


Fig. 8.2

8.1 QUESTIONS

- 1 Explain each of the following:
 - a) A sharp needle is easier than a blunt needle to push through a cloth.
 - b) A farm tractor is fitted with wide tyres to prevent it sinking into earth.
- 2 a) A person of weight 750 N stands upright with both feet on the floor. The contact area of each foot with the floor is $8.5 \times 10^{-3} \text{ m}^2$. Calculate the pressure on the floor.
 b) The four tyres of a car, of total weight 12 000 N, are at a pressure of 250 kPa. Calculate the area of each tyre in contact with the ground.

8.2 Hydraulics

A hydraulic machine is designed to make use of the fact that a liquid is almost incompressible. When a force is applied to a liquid in a sealed system, the liquid is put under pressure. This pressure is transmitted throughout the liquid.

- A **hydraulic press** is designed to squeeze an object using a much smaller force. Fig. 8.3 shows the principle of operation of a hydraulic press. The applied force, F_1 , acts on a piston of area A_1 , creating pressure

$$p = \frac{F_1}{A_1}$$

in the oil in the cylinder. This pressure is transmitted to the wide 'ram' piston to compress the object in the frame.

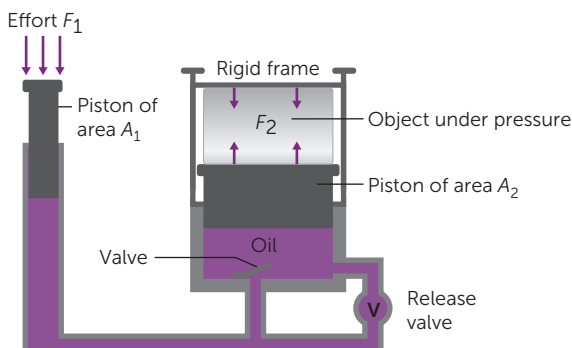


Fig. 8.3 The hydraulic press

The force, F_2 , acting on the wide piston is

$$F_2 = pA_2$$

$$= \frac{F_1 A_2}{A_1}$$

Since A_2 is much greater than A_1 , then force F_2 is much greater than force F_1 . In other words, the applied force is multiplied by a factor equal to the area ratio $A_2 \div A_1$. Air must be absent from the system, otherwise the pressure is used to compress the air instead of being transmitted through the oil. When the release valve is closed, the inlet valve prevents the oil from leaving the wide cylinder. Opening the release valve relieves the pressure in the wide cylinder, allowing the object to be removed.

- **Hydraulic brakes** use the pressure created when the driver applies a force to the foot pedal. This creates pressure on the brake fluid in the master cylinder. This pressure is transmitted to each of the slave cylinders, where it forces the disc pads to grip the wheel disc. Fig. 8.4 shows the idea. No air should be allowed into the system otherwise the force of the brake pedal is used to compress the air instead of creating pressure in the system, making the brakes ineffective.
- A **hydraulic lift** is designed to lift an object using a much smaller force. The platform is supported by four vertical legs, each attached to a piston. Compressed air is used to force oil into the cylinders to raise the pistons and the platform. Fig. 8.5 shows how this is achieved.

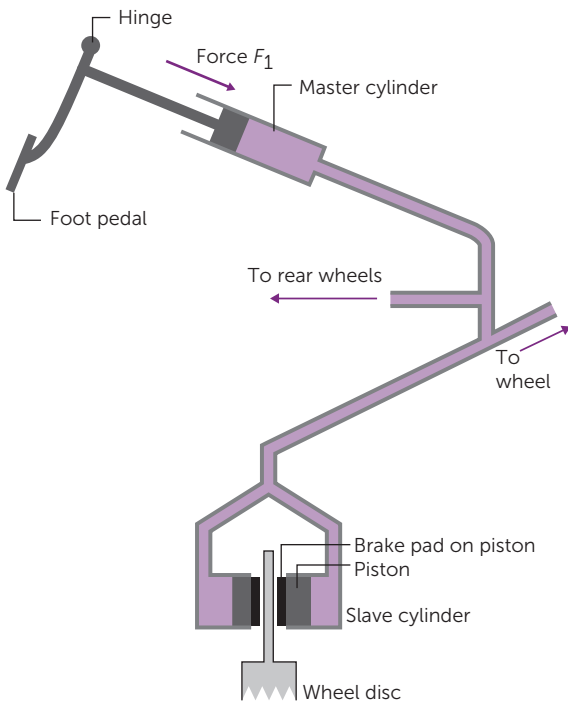


Fig. 8.4 Hydraulic brakes

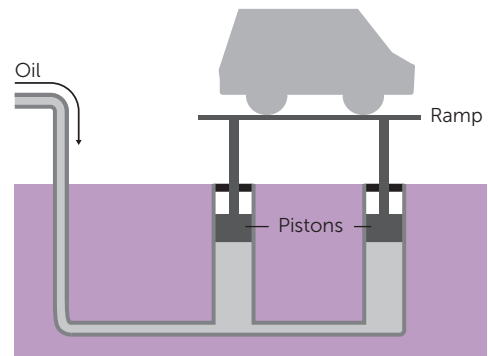


Fig. 8.5 The hydraulic lift

8.2 WORKED EXAMPLE

A hydraulic press has a narrow cylinder of diameter 2.0 cm and a wide 'ram' cylinder of diameter 30 cm. A force of 120 N is applied to the narrow cylinder. Calculate

- the pressure created in the system,
- the force on the 'ram' piston.

Solution

a) Area of the narrow cylinder, $A_1 = \frac{\pi(2.0 \times 10^{-2})^2}{4} = 3.1 \times 10^{-4} \text{ m}^2$

$$\therefore p = \frac{F}{A_1} = \frac{120}{3.1 \times 10^{-4}} = 3.8 \times 10^5 \text{ Pa}$$

b) Force on the ram piston, $F_2 = pA_2$

where $A_2 = \frac{\pi(30.0 \times 10^{-2})^2}{4} = 7.1 \times 10^{-2} \text{ m}^2$

$$\therefore F_2 = 3.8 \times 10^5 \times 7.1 \times 10^{-2} = 2.7 \times 10^4 \text{ N.}$$

8.2 QUESTIONS

- 1 Fig. 8.6 shows the internal construction of a hydraulic car jack. Explain why a small effort applied to the car jack is capable of raising a much greater load.

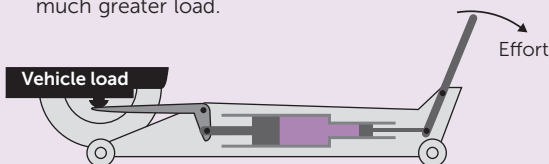


Fig. 8.6

- 2 A hydraulic lift used to raise a vehicle in a workshop has four pistons, each of area 0.012 m^2 . The pressure in the system must not exceed 500 kPa. The total weight of the platform and the pistons is 2500 N. Calculate the maximum additional weight that the platform can support.

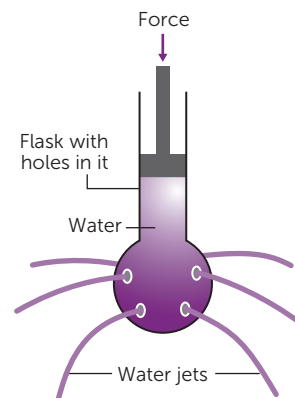


Fig. 8.7 Pressure acts equally in all directions

8.3 Pressure in a fluid at rest

- The pressure in a fluid at rest acts equally in all directions. This can be demonstrated as shown in Fig. 8.7. A water jet emerges from each hole, regardless of where the hole is on the surface, because the water pressure acts in all directions.
- The pressure in a fluid at rest increases with depth. Fig. 8.8 shows how this can be demonstrated. A jet of water emerges through each hole. The further the hole below the water surface, the greater the pressure of the jet. For two holes at the same depth, the pressure is the same. Pressure is unchanged along the same level in a fluid at rest.

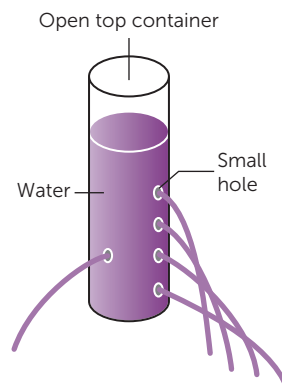


Fig. 8.8 Pressure increases with depth

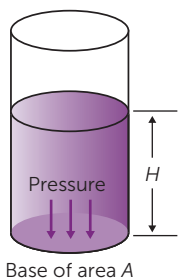


Fig. 8.9 Pressure due to a liquid column

Pressure of a liquid column

The pressure, p , at the base of a column of liquid of height H is given by the equation

$$p = H\rho g$$

where ρ = the liquid density. To prove this equation, consider the liquid column in Fig. 8.9. The volume of liquid in the cylinder = HA , where A is the cross-sectional area of the cylinder.

$$\therefore \text{the mass of liquid in the cylinder} = \text{volume} \times \text{density} = HA\rho$$

$$\text{the weight of liquid in the cylinder} = \text{mass} \times g = HA\rho g$$

$$\text{the pressure on the base} = \frac{\text{weight of liquid}}{\text{base area}} = \frac{HA\rho g}{A} = H\rho g$$

8.3 WORKED EXAMPLE

- Calculate the pressure in water at a depth of 10.0 m.
- What would be the height of a column of mercury with the same pressure at its base as at a depth of 10.0 m in water?

Density of water = 1000 kg m^{-3} , density of mercury = $13\,600 \text{ kg m}^{-3}$

Solution

- $p = H\rho g = 10.0 \times 1000 \times 9.8 = 9.8 \times 10^4 \text{ Pa}$
- Rearrange $p = H\rho g$ to give $H = \frac{p}{\rho g} = \frac{9.8 \times 10^4}{13\,600 \times 9.8} = 0.74 \text{ m}$

8.3 QUESTIONS

- Calculate the pressure at a depth of 20.0 m in sea water of density 1030 kg m^{-3} .
- The mean value of atmospheric pressure at sea level is 101 kPa. Calculate the depth of sea water that would give a pressure equal to 101 kPa.

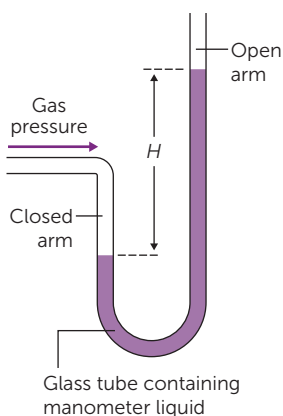


Fig. 8.10 The U-tube manometer

8.4 Measurement of pressure

Blood pressure, gas pressure and tyre pressure are just three examples of the many situations where pressure measurements need to be made.

The U-tube manometer

This is used to measure the **excess pressure** of gas above atmospheric pressure, as shown in Fig. 8.10. The height difference, H , between the liquid surfaces in each arm of the U-tube is measured. The gas pressure to be measured acts on the liquid surface in the closed arm. This is equal to the pressure on the same level in the open arm, which is due to the column of liquid of height H and the atmosphere.

$$\therefore \text{the gas pressure} = H\rho g + \text{atmospheric pressure,}$$

where ρ is the density of the manometer liquid.

$$\therefore \text{the 'excess' pressure of the gas above atmospheric pressure} = H\rho g$$

The Bourdon gauge

This is more robust than the U-tube manometer. The pressure of the gas forces the copper tube to unbend slightly, which makes the pointer move across the scale, as shown in Fig. 8.11. The scale is usually calibrated to measure the absolute pressure of the gas. To avoid confusion, the word *absolute* is introduced here as the sum of the excess pressure and atmospheric pressure.

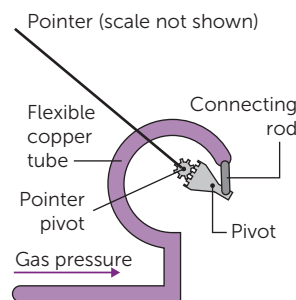


Fig. 8.11 The Bourdon gauge

The electronic pressure gauge

An electronic pressure gauge contains a piezoelectric pressure sensor that generates a voltage when pressure is applied to it. The sensor is part of the inner surface of a sealed hollow container which the pressure is applied to. If there is a vacuum in the container, an electronic pressure gauge is usually calibrated to give readings of the absolute pressure of the fluid.

The barometer

Barometers measure atmospheric pressure. At sea level, atmospheric pressure varies from day to day between about 100 and 102 kPa.

- The **Fortin barometer** is an inverted glass tube with its lower end immersed in a mercury reservoir. See Fig. 8.12. The space at the top of the tube is a vacuum and exerts no pressure on the mercury column in the tube. The pressure at the base of the mercury column, at the same level as the open surface of mercury in the reservoir, is the same as atmospheric pressure. Hence the pressure of the mercury column is equal to atmospheric pressure. Atmospheric pressure is calculated using the equation

$$p = H\rho g$$

where ρ is the density of mercury and H is the height of the mercury column.

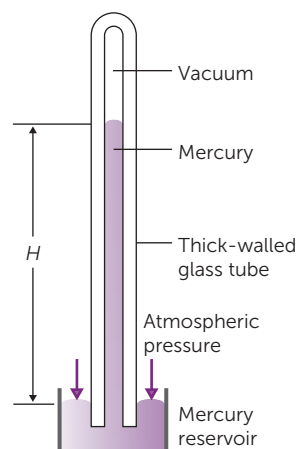


Fig. 8.12 The Fortin barometer

Notes

- The average height of the mercury column of a Fortin barometer at sea level is 760 mm. Using $p = H\rho g$ and $13\,600 \text{ kg m}^{-3}$ for the density of mercury gives 101 kPa for the mean value of atmospheric pressure at sea level ($= 0.760 \text{ m} \times 13\,600 \text{ kg m}^{-3} \times 9.8 \text{ N kg}^{-1}$). This value is referred to as 'standard' pressure.
- Pressure is sometimes quoted in 'mm of mercury' (mm Hg) instead of pascals. The formula $p = H\rho g$ is used to convert from mm of mercury to pascals.

- The **aneroid barometer** is more robust than the Fortin barometer. A partially evacuated hollow box with flexible sides is acted upon by atmospheric pressure. See Fig. 8.13. This operates a pointer via a lever system. When atmospheric pressure changes, the outer face of the box moves and makes the pointer move across the scale of the instrument.

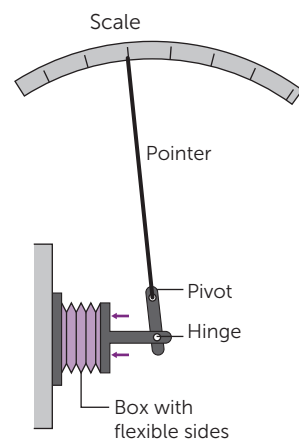


Fig. 8.13 The aneroid barometer

The sphygmomanometer

A sphygmomanometer is used to measure blood pressure. An inflatable cuff is connected to a manometer and fitted round the arm of patient just above the elbow, as shown in Fig. 8.14. A stethoscope is placed over an artery in the lower arm. The cuff is inflated manually to restrict the blood flow in the upper arm. The cuff pressure is gradually released and the manometer reading is noted when the blood flow can be heard again. This pressure is referred to as the **systolic pressure**. The flow of blood becomes quieter when turbulence ceases at a lower pressure, known as the **diastolic pressure**. Blood pressure is usually measured and quoted in 'mm of mercury', even where the mercury manometer is replaced by an electronic pressure gauge.

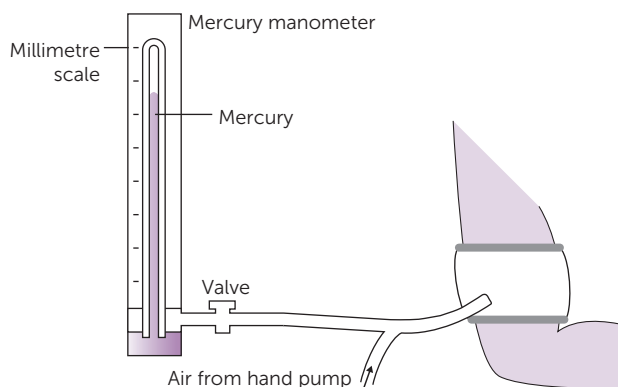


Fig. 8.14 The sphygmomanometer

8.4 QUESTIONS

- 1 a) A U-tube manometer containing water is used to measure the pressure of the domestic gas supply, giving a reading of 25 cm. Calculate the excess pressure of the gas supply (i) in kPa, (ii) as a percentage of standard pressure (101 kPa).
b) Why is it important that the pressure of the domestic gas supply is not (i) too low, (ii) too high?
- 2 a) In a Fortin barometer, explain why the mercury does not drop out of the barometer tube.
b) The blood pressure of a healthy young adult is normally 80 mm of mercury for the diastolic pressure and 120 mm of mercury for the systolic pressure.
(i) Calculate each of these pressures in kPa.
(ii) Why is mercury rather than water used in the manometer of a sphygmomanometer? (density of water = 1000 kg m^{-3} , density of mercury = $13\,600 \text{ kg m}^{-3}$)

8.5 Flotation

An object in equilibrium in a fluid experiences an upthrust equal and opposite to its weight. The upthrust is due to the pressure of the fluid on the object.

- 1 If the upthrust is greater than the weight, the object will rise (e.g. a ball released under water).
- 2 If the upthrust is equal to the weight, the object will float (e.g. a boat).
- 3 If the upthrust is less than the weight, the object will sink (e.g. an overloaded raft).

Archimedes' Principle

The upthrust on an object in a fluid is equal to the weight of fluid displaced by the object.

To prove this statement, consider a uniform cylinder of cross-sectional area, A , immersed upright in a liquid of density, ρ , as in Fig. 8.15. The pressure of the liquid on the base of the cylinder acts upwards on the cylinder, causing an upward force on the base equal to p_1A , where p_1 is the pressure due to the liquid at the base.

The pressure of the liquid on the top of the cylinder acts downwards on the cylinder, causing a downward force on the base equal to p_2A , where p_2 is the pressure due to the liquid at the top.

$$\begin{aligned} \therefore \text{the upthrust on the cylinder} \\ &= \text{upward force on the base} - \text{downward force on the top} \\ &= p_1A - p_2A = (p_1 - p_2)A \end{aligned}$$

Since $p_1 = H_1\rho g$ and $p_2 = H_2\rho g$, where H_1 is the depth of the base below the surface and H_2 is the depth of the top below the surface,

$$\begin{aligned} \therefore \text{upthrust } U &= (H_1\rho g - H_2\rho g)A = (H_1 - H_2)A\rho g \\ &= V\rho g \text{ because } V, \text{ the volume of the object} = (H_1 - H_2)A \end{aligned}$$

The mass of liquid displaced by the object, $m = V\rho$, hence

$$V\rho g = mg = \text{weight of liquid displaced}$$

upthrust, U = weight of liquid displaced

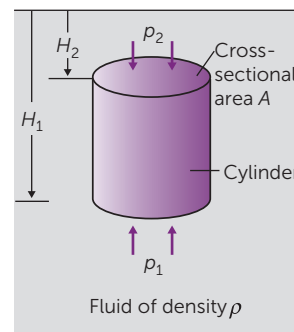


Fig. 8.15 Archimedes' Principle

Floating and sinking

A rowing boat becomes lower and lower in the water as more and more people get into it. It displaces more and more water as it floats lower in the water. The upthrust on it therefore increases, enabling the extra weight to be supported. However, if the water level reaches the upper edge of the hull, no further upthrust is possible because the boat cannot displace any more water. Any additional weight added to the boat at this stage would sink it.

8.4 WORKED EXAMPLE

An empty, flat-bottomed iron barge of weight 450 kN floats with the base of its hull 1.5 m below the water line. Calculate

- the area of the base of the boat,
 - the maximum weight of cargo it can carry if it can float no more than 2.0 m lower in the water.
- The density of water = 1000 kg m^{-3}

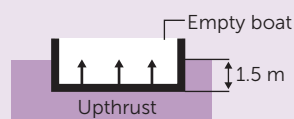


Fig. 8.16 Empty boat

Solution

- a)** Upthrust = weight of boat = 450 kN $\therefore$ Weight of water displaced = 450 kN.

Weight of water displaced = $HA\rho g$, where H is the depth of the boat in the water and A is its base area,

$$\therefore HA\rho g = 450\,000 \text{ and } A = \frac{450\,000}{H\rho g} = \frac{450\,000}{1.5 \times 1000 \times 9.8} = 31 \text{ m}^2$$

- b)** At 2.0 m lower, the additional depth $\Delta H = 2.0 \text{ m}$

$$\therefore \text{the additional weight of water displaced} = \Delta HA\rho g = 2.0 \times 31 \times 1000 \times 9.8 = 6.0 \times 10^5 \text{ N}$$

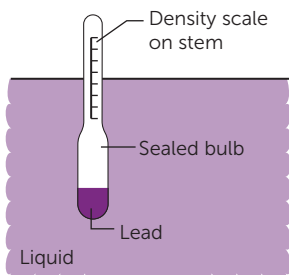


Fig. 8.17 The hydrometer

The hydrometer

This is used to measure the density of a liquid. It consists of a weighted bulb of air with a stem which is vertical when the hydrometer is floating at rest, as shown in Fig. 8.17. The scale is linear provided the stem has a uniform diameter. The scale may be calibrated using liquids of known density.

8.5 QUESTIONS

- 1 A person of weight 700 N floats on an air bed of dimensions $2.0 \text{ m} \times 0.60 \text{ m}$ in a swimming pool. Calculate
 - a) the pressure on the underside of the air bed,
 - b) the depth of the air bed in the water.
 The density of water $= 1000 \text{ kg m}^{-3}$.
- 2 A flat-bottomed ferry boat floats at rest in sea water of density 1050 kg m^{-3} with the underside of its hull 1.20 m below the water line. When it carries its maximum permitted load of $40\,000 \text{ N}$, it floats 0.05 m lower in the water. Calculate
 - a) the weight of the ferry boat when unloaded,
 - b) the depth of the hull below the water line in lake water of density 1000 kg m^{-3} when fully loaded.

SUMMARY

- **Pressure** = force per unit area acting perpendicular to the area.
- **Equation for pressure** $p = \frac{F}{A}$
- **Pressure of a liquid column** of height $H = H\rho g$, where ρ is the density of the liquid.
- **Pressure-measuring devices:** the U-tube manometer, the Bourdon gauge, the electronic pressure gauge, the Fortin barometer, the aneroid barometer (for measuring atmospheric pressure), the sphygmomanometer (for measuring blood pressure).
- **The upthrust** on an object in a fluid is equal to the weight of fluid displaced.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

8.1 A rectangular paving stone has dimensions $450 \text{ mm} \times 300 \text{ mm} \times 25 \text{ mm}$ and has a density 2600 kg m^{-3} . Calculate

- a) the weight of the stone,
- b) the pressure of the stone on the ground when it rests
 - (i) horizontally on its flat side,
 - (ii) on its shortest edge with this edge horizontal and its flat surface in a vertical plane.

8.2 The ram piston of a hydraulic press is in a cylinder of diameter 45 cm and is capable of applying a force of 120 kN .

- a) Calculate the pressure in the cylinder when the ram piston exerts maximum force.

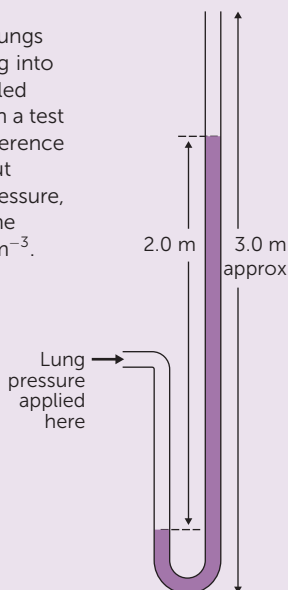
- b) The effort is applied to a narrow piston of diameter 2.5 cm . Calculate the effort necessary to make the ram piston exert a force of 120 kN .

8.3

- a) Atmospheric pressure at sea level is 101 kPa on average. The density of air at sea level is 1.2 kg m^{-3} . Calculate the height of an atmosphere of constant density equal to 1.2 kg m^{-3} .
- b) The atmosphere extends to a height of more than 100 km above the Earth. What does your calculation in (a) tell you about the atmosphere?

8.4

- a) The pressure of a person's lungs can be measured by blowing into one side of a long, water-filled manometer, as in Fig. 8.18. In a test with an adult, the height difference in the water levels was about 2.0 m. Calculate the lung pressure, in pascals, of this person. The density of water = 1000 kg m^{-3} .

**Fig. 8.18**

- b) Why is it very difficult to breathe through a long hollow tube under water at a depth below about 2.0 m?

8.5

- a) With the aid of a labelled diagram, describe the construction of a Fortin barometer and explain how it is used to measure atmospheric pressure.
- b) When the blood pressure of a person is measured, the cuff should normally be wrapped round the upper arm above the elbow and level with the heart. How would the blood pressure readings be affected if the arm were raised so the cuff was significantly above the heart?

8.6.

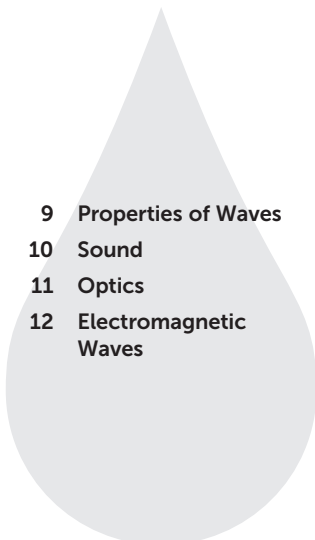
- a) Explain why a submerged submarine rises to the surface when compressed air is used to force water from its ballast tanks.
- b) In an experiment to test a loaded test tube as a hydrometer, the test tube was floated upright in water so 40 mm of its length was above water and 60 mm of its length below water. It was then placed in a liquid of unknown density and its length above water was measured at 32 mm. Calculate the density of the liquid.

PART

3

Waves

Part 3 introduces you to the basic principles of wave motion and shows you how these basic principles are used in a wide range of applications including sound and sight, imaging techniques, communications and energy transfer. The section provides plenty of opportunities for practical work and it offers further practice on the use of mathematics in Physics.

- 
- 9 Properties of Waves
 - 10 Sound
 - 11 Optics
 - 12 Electromagnetic Waves

Properties of Waves

OBJECTIVES

After working through this unit, you should be able to:

- describe different types of waves and their uses
- state the meaning of the amplitude, wavelength, frequency and speed of a wave
- explain what is meant by phase difference
- relate the speed to the frequency and the wavelength of a wave
- describe reflection, refraction and diffraction as properties of all waves
- describe what is meant by a longitudinal wave and give examples
- describe what is meant by a transverse wave and give examples
- describe polarisation as a property of a transverse wave

CONTENTS

- 9.1 Types of waves
- 9.2 Measuring waves
- 9.3 Properties of waves
- 9.4 Longitudinal and transverse waves
- 9.5 Polarised light
- Summary
- Revision questions

9.1 Types of waves

Waves are used to transfer energy or to carry information. For example, microwaves are used for heating food in a microwave oven or for carrying TV or phone channels around the world via satellite links. Here is a list of different types of waves you will meet in this section. Some of the uses of each type of wave are also listed.

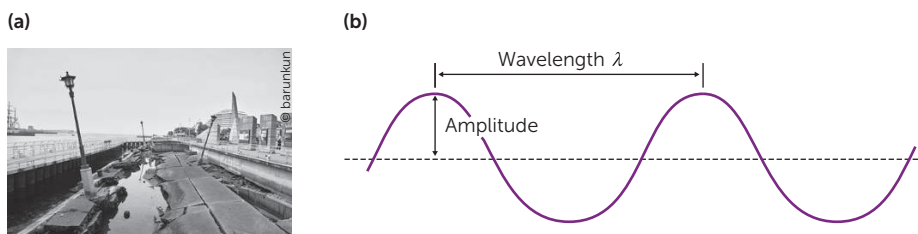
- **Sound waves** are vibrations in a medium that travel through the medium. For example, when you speak, your vocal cords make the surrounding air vibrate and the vibrations spread out. The speed of sound through air at room temperature is about 340 m s^{-1} . In general, sound waves travel faster through solids and liquids than through air. Sound waves above the range of the human ear are called **ultrasonics**. Ultrasonic imaging systems are used in hospitals for pre-natal scans. See Fig. 9.1.
- **Electromagnetic waves** are vibrating electric and magnetic fields that travel through space without the need for a medium. In essence, vibrations of the electric field generate vibrations of the magnetic field in the surrounding space which generate vibrations of the electric field further away, etc. The spectrum of electromagnetic waves is divided according to wavelength, as indicated in Table 9.1 (overleaf). Note that all electromagnetic waves travel at the same speed of $300\,000 \text{ km s}^{-1}$ through space.



Fig. 9.1 Ultrasound scanning is a diagnostic technique which uses high-frequency sound waves. Here, a pregnant woman receives an ultrasound scan of her abdomen to monitor foetal development. The doctor moves a hand-held transducer over the woman's lubricated skin, while two-dimensional images of the foetus can be seen on the screen

Table 9.1 Spectrum of electromagnetic waves

Type of wave	Radio waves	Microwaves	Infra-red	Visible	Ultraviolet	X-radiation	Gamma radiation
Wavelength	>0.1 m	0.1 m–0.1 mm	0.1 mm–700 nm	700 nm–400 nm	400 nm–1 nm	<1 nm	<0.001 nm
Applications	radio and TV	radar, heating	heating, security	the eye, photography	UV ink driers, sun lamps	radiography	gamma therapy, sensors

**Fig. 9.2** (a) Earthquake damage, (b) Snapshot of a travelling wave

- **Seismic waves** are vibrations of the Earth, usually created by earthquakes (Fig. 9.2a). By recording seismic waves from earthquakes, the internal structure of the Earth has been determined.
- **Water waves** are useful for studying the general properties of waves. The ripple tank in Fig. 9.5 shows how the general properties of waves can be investigated. Electrical power from off-shore floating wave machines could make a significant contribution to national power supplies.

9.2 Measuring waves

Fig. 9.2b shows a snapshot of a wave on a rope travelling from left to right. The peaks and crests propagate (i.e. travel) to the right. Suppose the rope was invisible except for a spot at one point. How would you describe the motion of that spot?

The spot moves up and down along a line at right angles to the direction of propagation. If this seems odd, watch a small object floating on water when some waves pass it. The object bobs up and down.

- The **amplitude** of a wave is the maximum distance of a point from its equilibrium position. In Fig. 9.2b, it is the height of a wave from the middle.
- The **wavelength** of a wave is the distance from one peak on a wave to the next peak. The symbol for wavelength is the Greek letter λ (pronounced 'lambda').
- The **frequency** of a wave is the number of complete cycles of oscillation of a given point per second. This is the same as the number of wavelengths that pass a point in one second. The symbol for frequency is f , or the Greek letter ν (pronounced 'nu'). The unit of frequency is the hertz (symbol Hz), equal to 1 cycle per second.

Note

The time period, T , of a wave is the time taken for one complete cycle to pass a point. For a wave of frequency f , $T = 1/f$.

Wave speed

Fig. 9.3 shows the wave in Fig. 9.2b again and then one cycle later, at time T from its initial position. Each peak has moved a distance of one wavelength in time T .

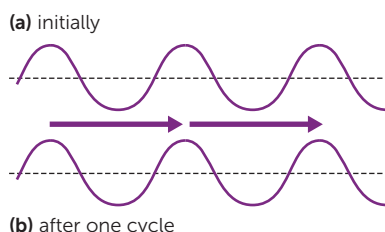


Fig. 9.3 One cycle later

$$\begin{aligned}\text{The speed of propagation, } v &= \frac{\text{distance moved}}{\text{time taken}} \\ &= \frac{\lambda}{T} = \frac{1}{(1/f)} = f\lambda \\ v &= f\lambda\end{aligned}$$

9.1 WORKED EXAMPLE

The speed of light in air is $3.0 \times 10^8 \text{ m s}^{-1}$. Calculate the frequency of light waves of wavelength 590 nm.

Solution

$$v = 3.0 \times 10^8 \text{ m s}^{-1} \quad \lambda = 590 \text{ nm} = 5.90 \times 10^{-7} \text{ m} \quad v = f\lambda \quad \therefore f = \frac{v}{\lambda} = \frac{3.0 \times 10^8 \text{ m s}^{-1}}{5.90 \times 10^{-7} \text{ m}} = 5.1 \times 10^{14} \text{ Hz}$$

Phase difference

In Fig. 9.2b, any two points separated by a distance equal to a whole number of wavelengths move up and down together. They vibrate **in phase** which means that at any instant they are at the same distance and direction from equilibrium and are moving in the same direction.

Consider the wave in Fig. 9.4. Point D is in phase with point O. As shown in Table 9.2 the **phase difference** between any two points in the fraction of a complete cycle between the two points. The symbol for phase difference is $\Delta\phi$ (pronounced ‘delta phi’). Phase difference is usually expressed either in:

- **degrees**, on a scale where 360° corresponds to a complete cycle, or
- **radians**, which is an angle scale in which 2π radians = 360° .

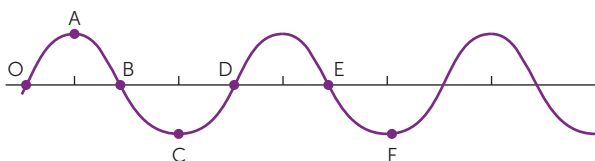


Fig. 9.4 Phase difference

Table 9.2 Phase differences relating to Fig. 9.4

Point	Phase difference relative to O in	
	degrees	radians
A	90	$\pi/2$
B	180	π
C	270	$3\pi/2$
D	360	2π
E	180	π

Note

The phase difference, $\Delta\phi$, between any two points may be calculated in radians using the formula

$$\Delta\phi = \frac{2\pi x}{\lambda},$$

where x is the distance between the two points.

9.2 QUESTIONS

- Use a millimetre rule to measure
 - the amplitude,
 - the wavelength of the wave in Fig. 9.2b.
- In Fig. 9.2b, the frequency of the wave was 5.0 Hz. Calculate
 - the time period of the wave,
 - the speed of propagation of the wave.
- The speed of sound in air at room temperature is 340 m s^{-1} . Calculate
 - the frequency of sound waves in air of wavelength 0.10 m,
 - the wavelength of sound waves in air of frequency 3000 Hz.
- The speed of radio waves in air is $3.0 \times 10^8 \text{ m s}^{-1}$. Calculate
 - the frequency of radio waves of wavelength 1500 m in air,
 - the wavelength of radio waves in air of frequency 1.2 MHz.
- In Fig. 9.4, what is the phase difference in
 - degrees,
 - radians between (i) A and F, (ii) B and D, (iii) C and F, (iv) O and F?

9.3 Properties of waves**Using a ripple tank**

Fig. 9.5 shows how to set up and use a ripple tank. This can be used to investigate reflection, refraction and diffraction of waves as outlined below. Note that each wavefront joins points that vibrate in phase with one another. The wavefront shadows on the screen can be thought of as wavecrests.

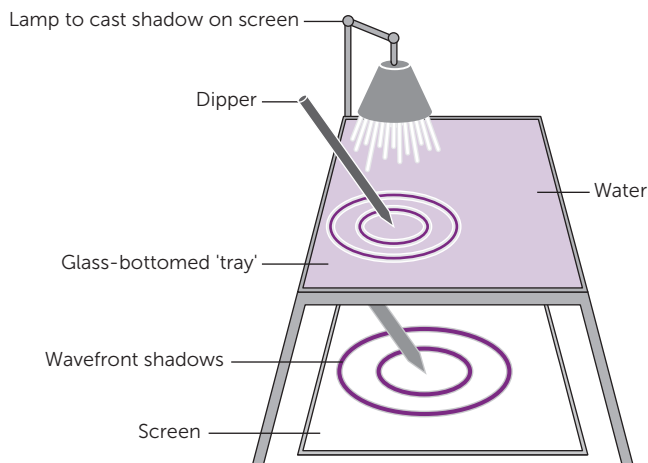


Fig. 9.5 A ripple tank

Reflection

Investigate the reflection of straight and circular waves from different-shaped surfaces. For example, Fig. 9.6 shows how to generate and direct straight waves at a plane (i.e. straight) reflector. If possible, try this and other tests such as:

- directing straight waves at a concave or a convex reflector
- producing circular waves and reflecting them from straight and curved reflectors.

Compare your observations with the patterns described on the following pages.

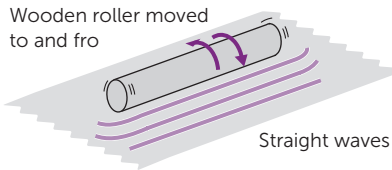


Fig. 9.6 Producing straight waves

Straight waves reflected by a straight reflector

The angles between the reflector and the incident wavefronts and between the reflector and the reflected wavefronts are the same. See Fig. 9.7. The same effect is observed when a ray of light is reflected off a mirror.

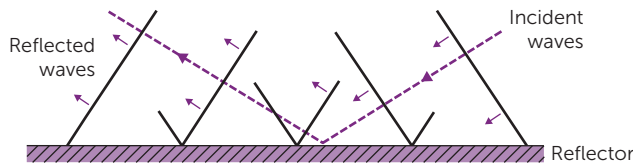


Fig. 9.7 Straight waves reflected by a straight reflector

Straight waves reflected by a concave reflector

The reflected waves converge at a point which is referred to as the **focal point** of the reflector. See Fig. 9.8. The detector of a satellite dish must be positioned at the focal point to pick up the strongest signal.

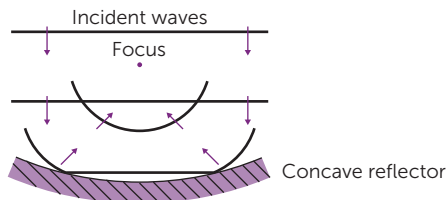


Fig. 9.8 Straight waves reflected by a concave reflector

Circular waves reflected by a straight reflector

The waves reflect as if they originated at the same distance behind the reflector as the point source is in front. See Fig. 9.9. This effect is the same as when you look in a plane mirror and observe your image at the same distance behind the mirror as you are in front.

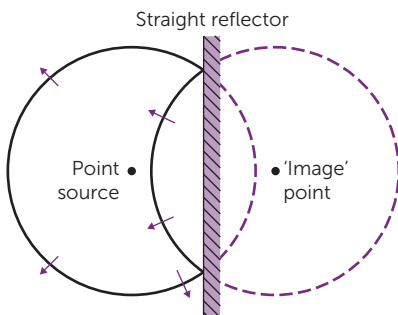


Fig. 9.9 Circular waves reflected by a straight reflector

Refraction

Water waves change direction when they travel across a boundary between deep and shallow water at a non-zero angle to the boundary. See Fig. 9.10. This change of direction is called **refraction**. It occurs because water waves travel faster in deep water than in shallow water. Refraction also takes place when a light ray travels from one transparent medium to another. See p. 148. Fig. 9.11 also shows how refraction explains the formation of an image by a convex lens.

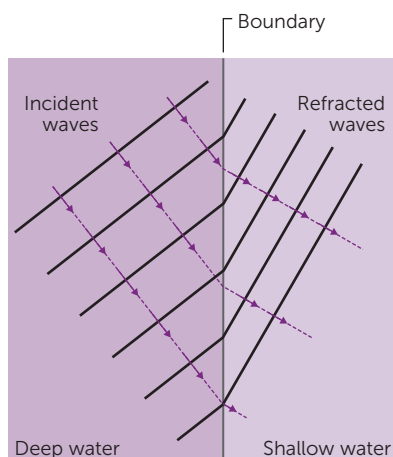


Fig. 9.10 Refraction of water waves in a ripple tank

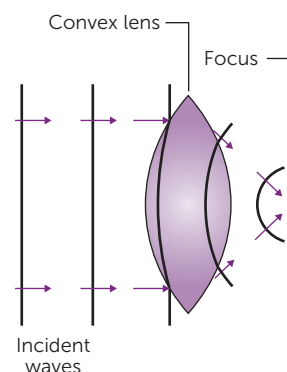


Fig. 9.11 Action of a lens

Diffraction

Waves spread out when they pass through a gap or around the edge of an obstacle. This effect is known as **diffraction**. Figs. 9.12 and 9.13 show plane waves passing through a wide and a narrow gap. The narrower the gap, the greater the amount of diffraction. Diffraction of light, on passing through an aperture, limits the detail of an image seen through a microscope or a telescope. The reduced depth of field in a high-speed photograph is caused by diffraction. This is because the aperture of a camera must be as wide as possible to enable a high-speed photograph to capture as much light as possible. Light from background objects is diffracted less if the aperture is widened, forming images in focus in front of the film.

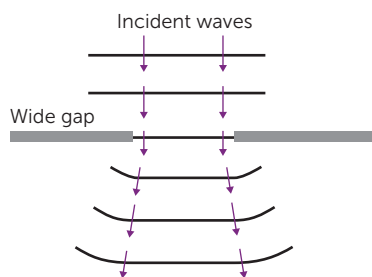


Fig. 9.12 Diffraction at a wide gap

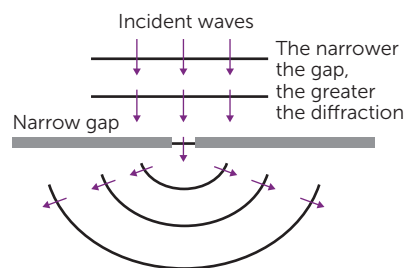


Fig. 9.13 Diffraction at a narrow gap

Interference

A vibrating beam suspended in water, as in Fig. 9.14, creates plane waves that are directed towards two narrow gaps created by metal barriers. The waves diffracted by each gap overlap and produce a pattern of cancellations and reinforcements. In other words, the two sets produce an interference pattern.

- Each reinforcement is where the waves from one gap arrive in phase with the waves from the other gap (e.g. a crest from one gap meets a crest from the other gap to produce a supercrest or a trough from one gap meets a trough from the other gap to produce a supertrough).
- Each cancellation is where the waves from one gap arrive exactly out of phase with the waves from the other gap (e.g. a crest from one gap meets a trough from the other gap).

The wave nature of light was deduced from the fact that light from a point source passing through two narrow, closely spaced slits produces an interference pattern consisting of alternate bright and dark fringes. Each dark fringe is where light from one slit cancels light from the other slit. As explained in Unit 11, this effect is used to deduce the wavelength of light.

Note

The waves from one gap pass through the waves from the other gap. Each crest or trough that meets another crest or trough produces a cancellation or a reinforcement for an instant.

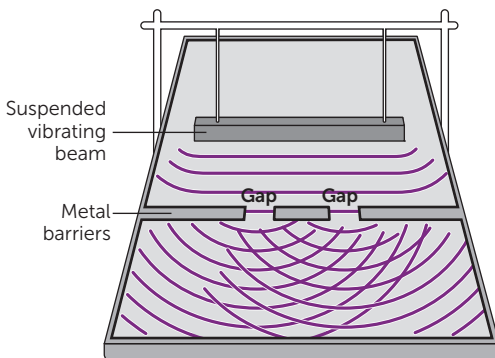


Fig. 9.14 Interference of water waves in a ripple tank

9.3 QUESTIONS

- 1 Copy and complete the diagrams in Fig. 1.15 to show the shape and direction of the reflected wavefronts.

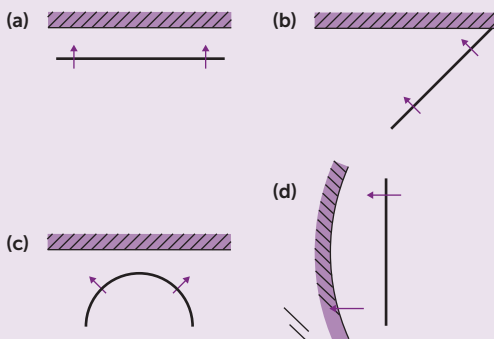


Fig. 9.15

- 2 Which wave property: reflection, refraction or diffraction, is responsible for
 - a) being able to see clearly using contact lenses,
 - b) glare off a road surface due to sunlight,
 - c) being able to hear someone talking around a corner?

9.4 Longitudinal and transverse waves

When waves pass through a medium, the particles of the medium vibrate. The frequency of vibration of the particles is the same as the frequency of the waves. Most types of waves may be classified as either **longitudinal** or **transverse**, depending on whether the direction of vibration of the particles is along or perpendicular to the direction of propagation of the waves.

Longitudinal waves

These are waves in which the direction of vibration of the particles is parallel to (i.e. along) the direction of propagation. Fig. 9.16 shows how to create longitudinal waves in a 'slinky' coil by moving one end to and fro. This generates a series of compressions which travel along the slinky.

- **Sound waves** are longitudinal waves. A loudspeaker supplied with alternating current creates sound waves because the diaphragm of the loudspeaker is forced to move to and fro. See Fig. 9.17. The diaphragm compresses the surrounding air in front of it as it moves forward, then it moves back before creating another compression. Effectively, the air vibrates to and fro as the sound waves pass through it.

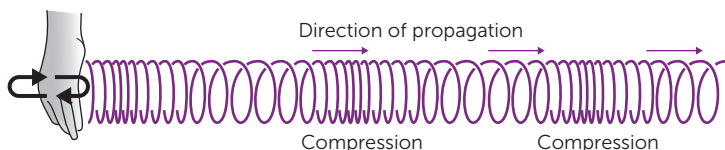


Fig. 9.16 Longitudinal waves on a slinky

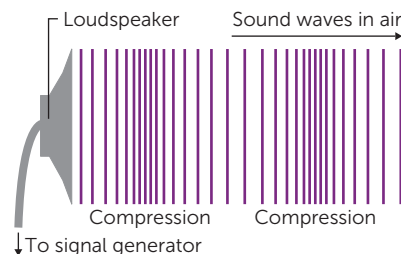


Fig. 9.17 Using a loudspeaker to create sound

- **Primary seismic waves** are longitudinal waves. They travel faster than the secondary or long waves which are the other types of seismic waves created in an earthquake. See Fig. 9.18. Primary seismic waves can travel through solids and liquids as they push and pull on the medium they travel through.

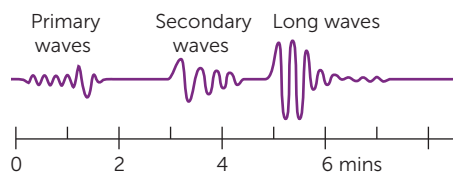


Fig. 9.18 Seismic waves

Transverse waves

Transverse waves are said to be **polarised** if the direction of vibration remains in the same plane, as in Fig. 9.19. If the direction of vibration changes, the waves are said to be **unpolarised**. Seen end-on, polarised waves would appear as a single line as in Fig. 9.20 whereas unpolarised waves would not.

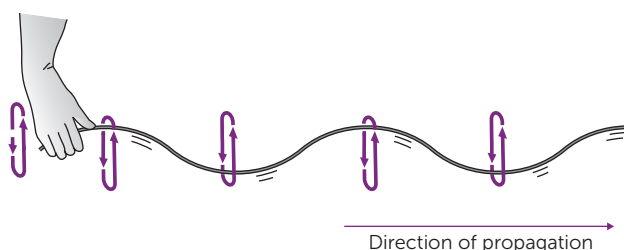


Fig. 9.19 Producing transverse waves on a rope

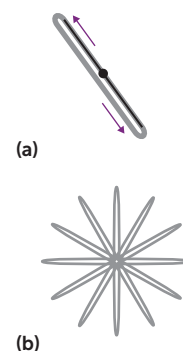


Fig. 9.20 (a) polarised and (b) unpolarised waves seen end-on

- **Electromagnetic waves** are transverse waves. The electric field and the magnetic field of an electromagnetic wave vibrate at right angles to the direction of propagation and at right angles to each other. Because they are transverse in nature, electromagnetic waves can be polarised or unpolarised. The plane of polarisation of an electromagnetic wave is defined as the plane of vibration of its electric field. Radio waves from a transmitter aerial are polarised, which is why the aerial of a radio receiver must be aligned correctly to pick up maximum signal strength. Light can be polarised using polaroid filters, as described on p. 128.
- **Secondary seismic waves** are transverse waves. They travel more slowly than primary seismic waves but faster than surface long waves. Secondary seismic waves shake the material they travel through from side to side. Transverse waves cannot pass through a liquid because liquid molecules slide past each other.
- **Waves on a string or wire** are transverse in nature. If a string is fixed at one end, waves travelling towards that end reflect back along the string, reversing their phase on reflection. See Fig. 9.21.

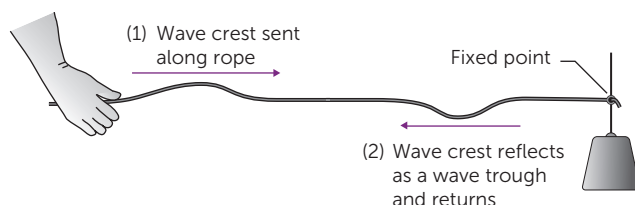


Fig. 9.21 Reflection of waves on a string

Notes

- 1 **Waves on water** are neither transverse nor longitudinal. Observe a cork on water as waves pass it. The cork bobs up and down and also moves to and fro. Its motion takes it round and round in a vertical circle.
- 2 **Seismic waves** comprise three types: primary, secondary and surface (or long) waves. Surface waves travel more slowly than primary or secondary waves but they cause much more damage. They are like waves on water because they shake the ground from side to side as well as making it move to and fro.

See www.palgrave.com/foundations/breithaupt for experiments on wave properties using microwaves.

9.4 QUESTIONS

- 1 a) How would you make transverse waves on a 'slinky' coil?
b) State whether each of the following type of wave is longitudinal or transverse:
(i) ultrasonic waves, (ii) microwaves, (iii) X-rays.
- 2 Use your knowledge of longitudinal and transverse waves to explain
a) how sound waves make the ear drum vibrate
b) why the picture on a portable TV fades if the aerial is turned through 90° .

9.5 Polarised light

Sunlight and light from a filament lamp are unpolarised. If a filament lamp is viewed through a polaroid filter, the brightness of the filament lamp does not change as the filter is rotated about the direction of the light rays. However, if a second polaroid filter is placed in the path of the beam, as in Fig. 9.22, the brightness changes as this filter is rotated.

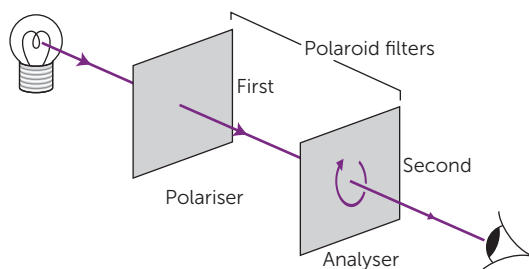


Fig. 9.22 Using polaroid filters

The filament lamp produces unpolarised light, which means the plane of vibration of the light waves along a light ray changes at random, as outlined on p. 126. In Fig. 9.22:

- the first polaroid filter, referred to as the 'polariser', allows light waves through only if the plane of polarisation of the waves is in a certain direction. The light transmitted by the first filter is therefore polarised. This occurs because the molecules in the filter are aligned with each other and they only transmit light waves that are polarised in the same direction as the molecules are aligned.
- the second filter, referred to as the 'analyser', cuts out all the light from the first filter if the two filters are aligned at 90° to each other. Fig. 9.23 shows how the intensity of light changes as the second filter is rotated from the 'crossed' position. The intensity rises, then falls to zero again as a result of rotating the second filter by exactly one-half turn. The two filters are then once again aligned at 90° to each other.

See www.palgrave.com/foundations/breithaupt for experiments on polarised light.

Fig. 9.24 shows a simple analogy of the action of two polaroid filters on an unpolarised light ray. Unpolarised waves on a rope are polarised by the first 'letterbox'. The waves fully pass through the second 'letterbox' only if they are aligned with the first. If the two 'letterboxes' are at 90° to each other, the waves are unable to pass through the second letterbox.

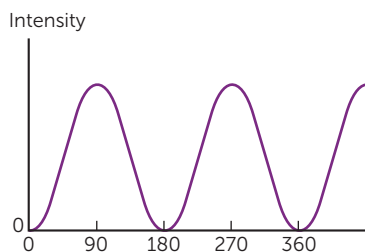


Fig. 9.23 Variation of transmitted intensity with angle of rotation

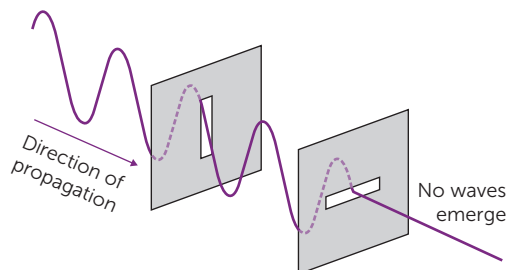


Fig. 9.24 A polarisation analogy

Polarisation applications

- **Polaroid sunglasses** cut out glare due to reflected sunlight. This is because light is polarised (partly or completely, depending on the angle of incidence of the light) when it reflects from water. Someone underwater in an outdoor swimming pool can be seen more easily through polaroid sunglasses. This is because the polaroid lenses cut out sunlight reflected from the water surface. See Fig. 9.25.
- A **calculator display** makes use of the polarisation of light. The display consists of 'pixels', each comprising a liquid crystal sandwiched between two polaroid filters. The molecules of the liquid crystal are long, spiral molecules that can rotate the plane of polarisation of light when they line up with each other. With no voltage applied, light is reflected from the cell by a mirror under the pixel. When a voltage is applied between the two filters, the molecules line up and rotate the plane of polarisation of light passing through the cell by exactly 90° , so no light can pass through the cell. See Fig. 9.26.

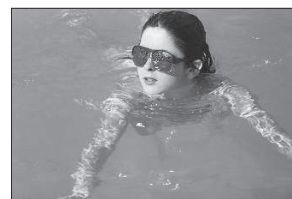


Fig. 9.25 Using polaroid sunglasses

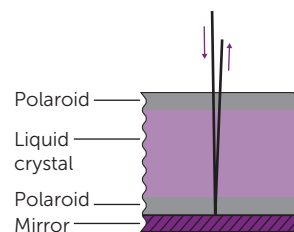


Fig. 9.26 Inside an LCD display

SUMMARY

- **Transverse waves** vibrate at 90° to the direction of propagation. Examples of transverse waves include electromagnetic waves, secondary seismic waves and waves on strings and wires.
- **Longitudinal waves** vibrate along the direction of propagation. Examples of longitudinal waves include sound waves and primary seismic waves.
- The **amplitude** of a wave is the displacement of a point from equilibrium to peak displacement.
- The **phase difference** between any two points at distance x apart $= 2\pi x/\lambda$.
- The **frequency** of a wave is the number of complete cycles that pass a point each second.
- The **wavelength** of a wave is the distance from one peak to the next.
- The **speed** of a wave $=$ wavelength $\times$ frequency.
- **Reflection, refraction** and **diffraction** are properties of all types of waves.
- **Polarisation** is a property of transverse waves only. Unpolarised light is polarised when it passes through a polaroid filter.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

9.1

- List the main parts of the electromagnetic spectrum of waves, in order of increasing wavelength.
- State with which part of the electromagnetic spectrum each of the following wavelengths is associated.
 - 10 m
 - 600 nm
 - 1 mm
 - 10^{-14} m

9.2

- Describe the difference between a transverse wave and a longitudinal wave.
- State which of the following types of waves are transverse and which are longitudinal:
 - sound waves
 - ultrasonics
 - microwaves
 - light
 - primary seismic waves

REVISION QUESTIONS

9.3 Fig. 9.27 shows a snapshot of a transverse wave travelling from left to right.

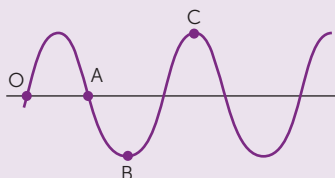


Fig. 9.27

- Use a millimetre rule to measure
 - the amplitude,
 - the wavelength of the wave.
- Calculate the phase difference in radians between point O and
 - point A,
 - point B,
 - point C.
- The frequency of the waves is 0.5 Hz. What is the displacement of:
 - point O 0.1 s later,
 - point O 3.0 s later,
 - point A 1.0 s later,
 - point A 2.5 s later?

9.4

- The speed of light in air is $3.0 \times 10^8 \text{ m s}^{-1}$. Calculate
 - the frequency of light waves of wavelength 500 nm
 - the wavelength of light waves of frequency $5.0 \times 10^{14} \text{ Hz}$.
- The speed of sound in air at room temperature is 340 m s^{-1} . Calculate
 - the frequency of sound waves of wavelength 0.05 m,
 - the wavelength of sound waves of frequency 12 kHz.

9.5

- Fig. 9.28 shows waves directed at different shaped reflectors.

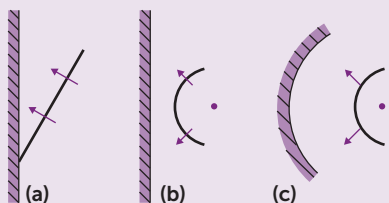


Fig. 9.28

Copy each diagram and show the shape and direction of each wavefront after reflection.

- Explain why a satellite dish is
 - concave in shape,
 - fitted with a detector above the dish surface.
- Explain why water waves do not reflect from a sloping beach.

9.6

- Fig. 9.29 shows straight waves about to pass across a boundary between deep and shallow water. The waves travel faster in deep water than in shallow water.

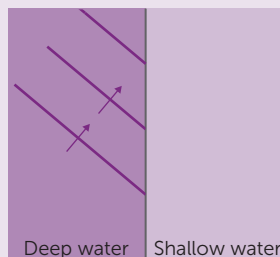


Fig. 9.29

- Copy the diagram and indicate the shape and direction of the waves in the shallow water.
 - What can you say about the wavelength and direction of the waves in the shallow water compared with the deep water?
- Explain why waves running onto a beach are usually parallel to the beach.

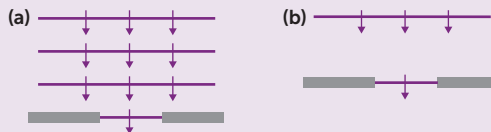
9.7

Fig. 9.30

- Copy and complete the two diagrams in Fig. 9.30 showing the waves after passing through each gap.
- Explain why a large-diameter satellite dish is more difficult to align than a small-diameter dish.

9.8

- Describe how you would use a polaroid filter to find out if light from a particular light source is polarised.
- Light from a filament lamp is observed through two polaroid filters.
 - Explain why the filament lamp cannot be seen when one filter is aligned in a certain direction relative to the other filter.
 - Describe what is observed when the polaroid nearer the observer is rotated through 360° from the position in (i).
 - How would the observation differ if the other filter had been turned through 360° instead?

Sound

UNIT

10

OBJECTIVES

After working through this unit, you should be able to:

- describe the nature of sound waves and measure the speed of sound in air
- describe the main properties of sound, including absorption, reflection, refraction and diffraction
- describe the structure of the ear and recall the main characteristics of human hearing
- describe the main properties and uses of ultrasonics
- relate the fundamental frequency of a vibrating wire or resonance tube to the length of the wire or tube
- explain the relationship between the overtones and the fundamental frequency for a vibrating wire or resonance tube

CONTENTS

- 10.1 The nature and properties of sound
- 10.2 The human ear
- 10.3 Ultrasonics
- 10.4 Vibrating strings
- 10.5 Acoustic resonance
- Summary
- Revision questions

10.1 The nature and properties of sound

Producing sound

An object vibrating in air creates sound waves in the surrounding air because the surface of the vibrating object pushes and pulls on the air.

Sound waves are longitudinal, as explained on p. 126, and compression waves spread out from the object. A sound wave may therefore be thought of as a series of compressions (where the air is at higher pressure than normal) and rarefactions (where the air is at lower pressure than normal). The density of the air is higher than normal at a compression and lower than normal at a rarefaction.

Fig. 9.17 (p. 126) represents how the density varies along the line of propagation at an instant of time. The areas of maximum compression and rarefaction correspond to zero displacement of the layers of air, as represented by the transverse wave diagram in Fig. 10.1.

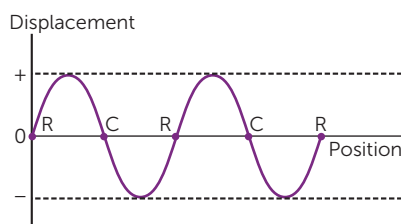


Fig. 10.1 Transverse representation

Note

Fig. 10.1 is a more convenient way to represent a sound wave and will be used from now on. However, don't forget that sound waves are longitudinal and diagrams like Fig. 10.1 are for convenience.

Measuring sound waves

- 1 Generate sound waves at constant frequency using a small loudspeaker connected to a signal generator. Use a microphone connected to an oscilloscope to display the waveform of the sound waves from the loudspeaker on the oscilloscope screen, as in Fig. 10.2, which shows a waveform called a 'sine wave'. This is a waveform of sound waves of a single frequency only. Use the oscilloscope to measure the sound frequency, as shown in Fig. 10.2.

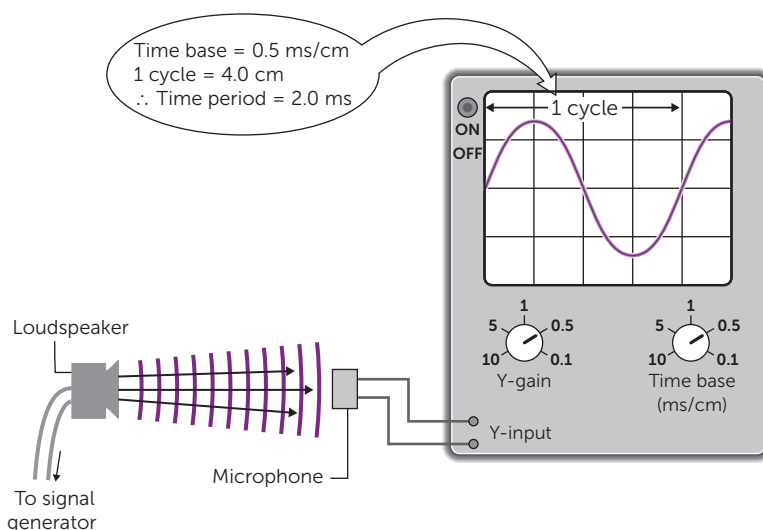


Fig. 10.2 Using an oscilloscope to measure the frequency of sound waves

- 2 Display the sound waveforms of different sources of sound (e.g. tuning fork, whistle, musical instruments, voice) on the oscilloscope. Each sound source except noise has a characteristic waveform. The tuning fork produces a sine wave but the other waveforms are more complicated. Noise is unpleasant to listen to because the waveform varies at random, not rhythmically.

Echoes

Sound waves reflect off hard surfaces. Clap your hands in an empty sports hall and you may hear an echo of the clap due to sound waves reflected from the walls, if the wall surface is smooth. If the surface is rough, the reflected sound waves are broken up and the original waveform is lost.

Echo sounders are used for depth tests on board ships. See Fig. 10.3. Ultrasonic pulses are directed from a transmitter (called a transducer) on board the ship to the sea bed. The pulses reflected from the sea bed are detected by the transducer. The time, t , between transmitting and detecting a pulse is measured and used to calculate the distance to the sea bed. Since the time taken for a pulse to travel from the detector to the sea bed is $\frac{1}{2}t$,

$$\text{the distance to sea bed, } s = \text{speed} \times \text{time} \div 2 = vt \div 2$$

where v = the speed of sound in sea water.

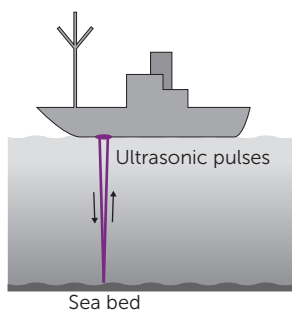


Fig. 10.3 Depth testing

The speed of sound

Sound travels faster in solids and liquids than in gases. The speed of sound in a gas is faster, the lighter the gas molecules are and the higher the temperature. Fig. 10.4 shows how to measure the speed of sound in air in a laboratory.

- The signal from the signal generator is displayed on one beam of the oscilloscope. The other beam is used to display the microphone signal which is delayed because of the time taken by sound to travel from the loudspeaker to the microphone. As a result, the microphone trace is out of phase with the loudspeaker trace.
- If the distance between the microphone and the loudspeaker is increased by exactly one wavelength, the microphone trace moves through exactly one cycle relative to the other trace. This can be used to measure the wavelength of the sound waves. The frequency can be measured from the signal generator. The speed of sound is then calculated from $\text{speed} = \text{wavelength} \times \text{frequency}$.

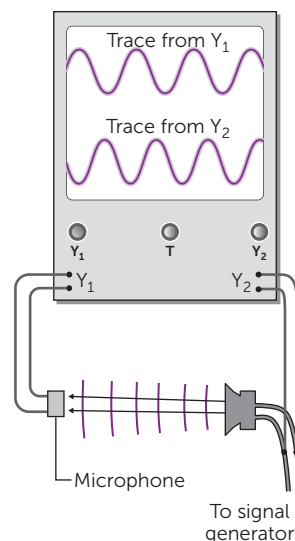


Fig. 10.4 Using a dual beam oscilloscope

10.1 QUESTIONS

- Fig. 10.5 shows a sound waveform displayed on an oscilloscope screen.
 - Calculate the frequency of the sound waves if the time base control was set at 5 ms cm^{-1} .
 - Copy the display and make further sketches to show how the waveform changes if (i) the loudness of the sound is increased, (ii) the frequency of the sound is reduced.
- The speed of sound in water is 1500 m s^{-1} . Fig. 10.6 shows pulses from a depth-finder on an oscilloscope screen. The pulses are produced at a rate of 2 per second.
 - Use Fig. 10.6 to estimate the time taken for each pulse to travel from the ship to the sea bed.
 - Calculate the depth of the sea bed at this position.

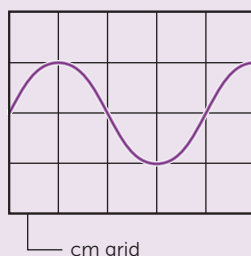


Fig. 10.5



Fig. 10.6

See www.palgrave.com/foundations/breithaupt for experiments on the absorption of sound by materials.

10-2 The human ear

The faintest sound that the normal human ear can detect is about 1 million million times quieter than the loudest sound it can withstand without damage. The ear is a remarkable organ but it needs to be protected from extremely loud sounds.

Fig. 10.7 shows a cross-section through the human ear. Sound waves make the tympanic membrane (ear drum) vibrate and these vibrations are transmitted to the oval window via the bones of the middle ear. These bones amplify the force of the vibrations and filter out unwanted noise (e.g. physiological sounds). They also protect the inner ear by switching to a less sensitive mode of vibration if the sound

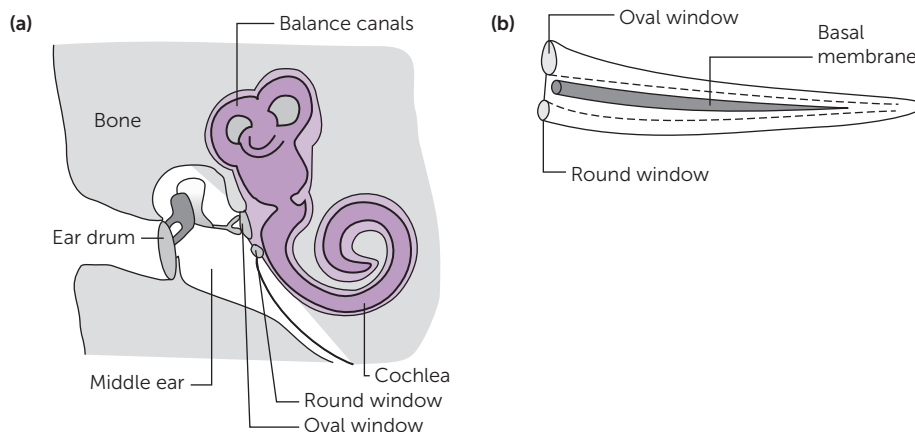


Fig. 10.7 The human ear (a) cross-section, (b) cochlea 'unrolled'

becomes excessive. The vibrations are transmitted through the fluid of the cochlea in the inner ear where they are detected by very sensitive 'hair cells' attached to the basilar membrane. These are nerve cells and when stimulated they send electrical signals via the auditory nerve to the brain.

Notes

- 1 The muscles that hold the bones of the middle ear together slacken when the ear is subjected to excessive sound. This prevents all the sound energy transmitted by the ear drum from arriving at the oval window. When someone leaves a very noisy event, the muscles take a little while to become taut again. This is why a person leaving a noisy nightclub has temporary difficulty with hearing. Frequent over-exposure results in permanent hearing loss, as the muscles weaken permanently and the bones wear away.
- 2 Frequency discrimination is due to the hair cells on the basilar membrane. The pattern of stimulation of the hair cells along the basilar membrane depends on the sound frequency and it is thought that the nerve impulses are triggered by different patterns.

The decibel scale

This is a 'times ten' scale for loudness, similar to the Richter scale for earthquakes in which every further point corresponds to ten times as much energy released. However, on the decibel (dB) scale, an increase of 10 dB (or 1 'bel') corresponds to a tenfold increase in sound energy. Fig. 10.8 shows decibel levels of some everyday sounds.

- Zero dB is defined as the faintest possible sound that can be heard.
- Every 10 dB increase in loudness corresponds to a tenfold increase in sound energy.

Hearing tests

- 1 Use the apparatus in Fig. 10.2 to determine the frequency range of your ears. Take care, as loud sounds can damage your ears. The upper frequency limit, about 18 kHz for a normal young adult, decreases with age. Fig. 10.9 shows the frequency response curve of the ear. At what frequency are your ears most sensitive?

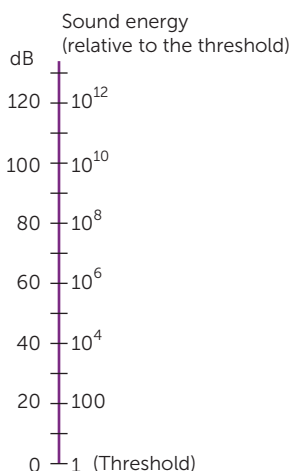


Fig. 10.8 The decibel scale

Least detectable loudness
/dB

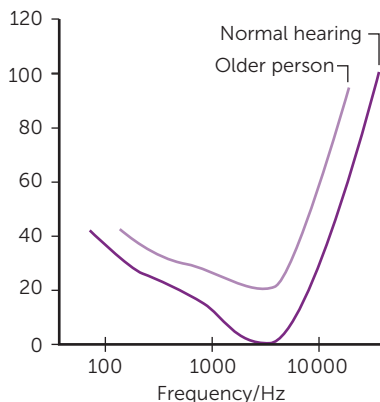


Fig. 10.9 Frequency response of the ear

- 2 Use a decibel meter to measure the loudness of different sounds. At its most sensitive, it responds to the faintest sound detectable. At its least sensitive, it scarcely responds unless the sound is very, very loud.

10.2 QUESTIONS

- 1
 - a) What are the functions of the bones of the middle ear?
 - b) Compare a recording of your own voice with what you hear when you speak. Describe and account for the differences.
 - c) A student walks from a room where the loudness level is 40 dB to a hall where the loudness level is 70 dB. (i) What is the increase in loudness in decibels? (ii) How much more sound energy is the student subjected to in the hall in comparison with the room?
- 2 For the frequency response curve in Fig. 10.9:
 - a) What frequency is the young person most sensitive to?
 - b) What is the hearing loss in dB of the older person at the most sensitive frequency?
 - c) How does the hearing of the older person compare with that of the younger person?

10.3 Ultrasonics

Ultrasonics are sound waves of frequencies too high for the human ear to detect (i.e. above about 18 kHz).

- Ultrasonic scanners are used in hospitals to produce images of babies in the womb. X-rays would damage the baby because they ionise matter which they pass through, whereas ultrasonics are non-ionising and are therefore used instead of X-rays. Ultrasonic devices are also used for the detection of cracks inside metals and for depth location at sea.
- The power of ultrasonics is used to clean street lights by immersing the lighting unit in a tank of water and using ultrasonic waves to dislodge the dust particles from the surfaces of the unit. In hospitals, ultrasonic power is used to pulverise kidney stones, thus avoiding the need for surgical removal.

Producing and detecting ultrasonic waves

An **ultrasonic probe**, sometimes referred to as a transducer, is designed to produce and detect ultrasonic pulses. It contains a thin slice of a certain ceramic material that vibrates strongly when an alternating voltage is applied between its two faces, as

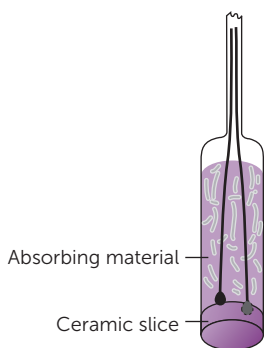


Fig. 10.10 Inside an ultrasonic probe



Fig. 10.11 An ultrasound scan

shown in Fig. 10.10. The applied frequency is matched to the natural frequency of vibration of the slice so it **resonates** when the alternating voltage is applied. This is the same principle as pushing a child on a swing; if the frequency of the pushes matches the natural frequency of the swing, the amplitude of oscillation of the swing becomes very large.

Probes in ultrasonic medical scanners produce ultrasonics at a frequency of about 1.5 MHz. The speed of ultrasonic waves in the ceramic used is about 3.8 km s^{-1} . Hence the wavelength of ultrasonics in the crystal is about 2.5 mm ($= \text{speed/frequency} = 3800 \text{ m s}^{-1} / 1.5 \times 10^6 \text{ Hz}$). For resonance, the thickness of the slice needs to be half a wavelength, corresponding to ultrasonics travelling from one surface and back in one complete cycle.

In an **ultrasonic scanner**, the pulses are produced by repeatedly switching the alternating voltage on for a brief interval. Between transmission of successive pulses, the probe is automatically connected to a circuit that detects the alternating signal produced when reflected ultrasonic pulses hit the slice. Ultrasonic waves travel at a speed of about 1000 m s^{-1} in body tissue and therefore a pulse would travel a typical distance of 1 m through the body (across and back) in about 1 ms. Internal boundaries in the body (e.g. bone-tissue boundaries, boundaries between different types of tissue) reflect the pulses.

- In the A-scan system, the reflected pulses are displayed on an oscilloscope screen and the time taken for each pulse to travel from the probe and back again can be measured. The depth of the boundary causing a reflected pulse can then be calculated in the same way as for depth finding.
- In the B-scan system, an image is built up on the display screen as the probe is moved over the body surface. See Fig. 10.11. Sensors attached to the probe determine the direction of the display trace and each reflected pulse makes the trace brighter.

Notes

- 1 The greater the difference in the density on either side of an interface, the stronger the reflected pulse. The probe is applied to the skin via a paste. This is necessary to prevent strong reflection occurring at the air–skin boundary, otherwise the pulses would not enter the body.
- 2 The ultrasonic frequency needs to be high, otherwise the pulses would spread out from the probe due to diffraction. At 1.5 MHz, the ultrasonic wavelength in the body is about 2.5 mm, which is much smaller than the probe width, therefore minimising diffraction.

10.3 QUESTIONS

- 1 a) In an ultrasonic scanning system, why is it essential that the ultrasonic pulses should be very brief in comparison with the rate at which the pulses are produced?
b) Calculate the time taken for a 1.5 MHz transducer to produce 10 cycles of an ultrasonic wave.
c) The time between successive 1.5 MHz pulses of a scanner system is 1 ms. Calculate the ratio of the width of a pulse consisting of 10 cycles to the time interval between successive pulses.
- 2 a) An ultrasonic transducer in a cleaning unit emits an ultrasonic beam at a frequency of 40 kHz from a flat transducer of width 1 cm. Calculate the wavelength of 40 kHz ultrasonic waves in water. The speed of sound in water is 1500 m s^{-1} .
b) Diffraction is not significant if the width of the source of waves is much greater than the wavelength. Discuss whether or not diffraction is
(i) significant,
(ii) desirable in an ultrasonic cleaning unit.

10-4 Vibrating strings

Loudness

Musical instruments produce rhythmical sounds provided they are played correctly. The notes produced by a string or a wind instrument are characteristic of the type of instrument. Any waveform of a musical note can be analysed to find the frequencies from which it is composed and the relative loudness of each frequency present. See Fig. 10.12.

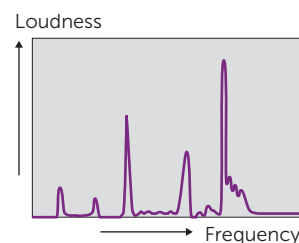


Fig. 10.12 The frequency spectrum of a musical note

Stationary waves on a string

A stationary wave pattern in which the wave peaks and troughs do not travel along the string can be set up on a string or wire under tension, using an arrangement as shown in Fig. 10.13. The mechanical oscillator is driven from a signal generator and it vibrates with a small amplitude, sending waves along the string. These waves reflect at the fixed end of the string and they return to the oscillator in phase with the waves being produced at that instant. Just like pushing a child on a swing at the right instant, the amplitude of vibration builds up as the reflected waves add to the waves being produced to create a stationary wave pattern.

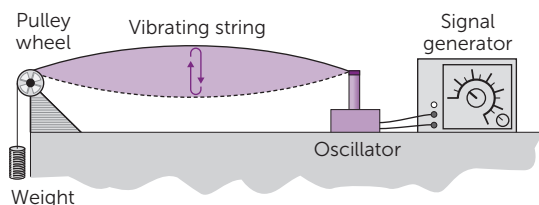


Fig. 10.13 Stationary waves on a string

The fundamental frequency

The simplest pattern is called the **fundamental** mode of vibration of the string, as shown in Fig. 10.13. The amplitude of vibration is zero at the ends and maximum in the middle of the string. This pattern occurs when the oscillator goes through exactly one cycle in the time taken for a wave to travel from the oscillator to the fixed end and back. In other words, the time for one cycle, $T = 2L \div v$, where L is the string length and v is the speed of the waves on the string.

Since the fundamental frequency, $f_0 = 1 \div T$, then

$$f_0 = \frac{v}{2L}$$

Overtones

Further patterns called **overtones** are produced at frequencies $2f_0$, $3f_0$, $4f_0$, etc., corresponding to the oscillator going through 2 or 3 or 4, etc. cycles in the time taken for a wave to travel the length of the string and back. In general, a stationary wave pattern occurs if $2L \div v = mT$, where m is equal to a whole number. Fig. 10.14 shows the pattern of vibration for the fundamental and the overtones.

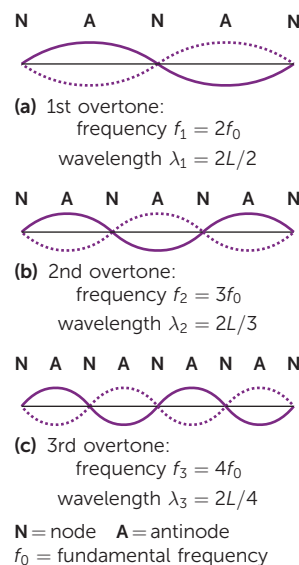


Fig. 10.14 Stationary wave patterns

Note that:

- 1 The frequency, $f = 1/T = mv/2L = mf_0$.
- 2 The wavelength, $\lambda = v/f = 2L/m$. In other words, the string length, $L = m\lambda/2$ (i.e. a whole number of half wavelengths).
- 3 The fundamental frequency increases with tension in the string and decreases with length.

Nodes and antinodes

- The points of no displacement, referred to as **nodes**, are fixed in position.
- The distance between adjacent nodes is exactly one-half wavelength.
- The points of maximum displacement, referred to as **antinodes**, are exactly midway between the nodes. No energy transfer occurs because the antinodes do not move along the string.
- Two points between adjacent nodes, or separated by an even number of nodes, vibrate in phase.
- Two points separated by an odd number of nodes vibrate out of phase by 180° .
- Each stationary wave pattern is due to the reflected waves combining with the waves from the oscillator. In other words, a stationary wave pattern is produced when two sets of waves of the same frequency and amplitude pass through each other.

Sound waves from a string instrument

When a wire or string of a musical instrument produces sound, its vibrations are usually a combination of its fundamental mode of vibration and the overtones. The loudness of each frequency component depends on exactly how the string is made to vibrate. The vibrating string makes the body of the instrument vibrate, creating sound waves in the surrounding air. The shape of the body affects the quality of the note produced since it causes some frequencies to be suppressed and enhances others. See Fig. 10.15.

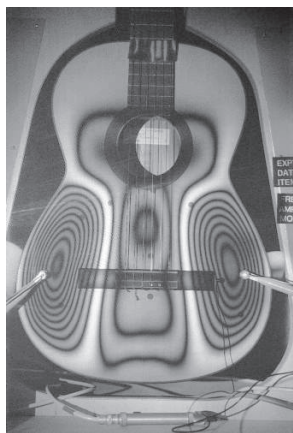


Fig. 10.15 Photograph of the fringe patterns formed by light reflecting from a vibrating guitar

10.4 QUESTIONS

- 1 **Fig. 10.16** shows a wire under tension which is vibrating at its fundamental frequency of 120 Hz.
 - a) Sketch the pattern of vibration of the wire when it vibrates at its second overtone frequency of 360 Hz.
 - b) What would be the fundamental frequency of this wire if its length is (i) doubled, (ii) halved, without change of tension?
- 2
 - a) Sketch the pattern of vibration of a wire vibrating at twice its fundamental frequency.
 - b) Describe (i) how the amplitude of vibration varies with position along the wire, (ii) how the phase difference between any two points on the wire varies with distance apart.



Fig. 10.16

10-5 Acoustic resonance

Wind instruments create sound as a result of resonance in tubes. See Fig. 2.17. For example, the note from a trumpet is due to vibrations of the air created by the lips at the mouthpiece, causing the air in the tube of the trumpet to resonate. An organ pipe resonates with sound when air is blown in at one end over a sharp edge, creating eddy currents that make the air in the pipe resonate. In a reed instrument such as a clarinet, the vibrations of a reed in the mouthpiece cause the air in the tube to vibrate and resonate with sound. The general principle of resonance was outlined in the previous section. Think about a child on a swing again. The swing has a natural frequency of oscillation; if a force is applied to it periodically, the amplitude of swing becomes very large if the forcing frequency matches the natural frequency of the swing, because the periodic force is applied at the same point in each cycle of the swing, causing the amplitude to build up. The same is true when acoustic resonance occurs. The air column has its own natural frequencies of vibration. If the air is made to vibrate at one of these frequencies, the amplitude of vibration builds up and the sound becomes very loud.



Fig. 10.17 The saxophone, a wind instrument, creates sound as a result of resonance in tubes

Resonance in a pipe closed at one end

Fig. 10.18 shows how this can be demonstrated using a small loudspeaker connected to a signal generator. Stationary wave patterns are established in the air column at resonance, as a result of sound waves from the loudspeaker reinforcing sound waves reflected from the closed end. In every case, there is a node at the closed end of the pipe and an antinode at the speaker, a short distance beyond the open end of the pipe. The lowest frequency at which the pipe resonates is its **fundamental frequency**, f_0 . Further resonances occur at frequencies $3f_0$, $5f_0$, $7f_0$, etc.

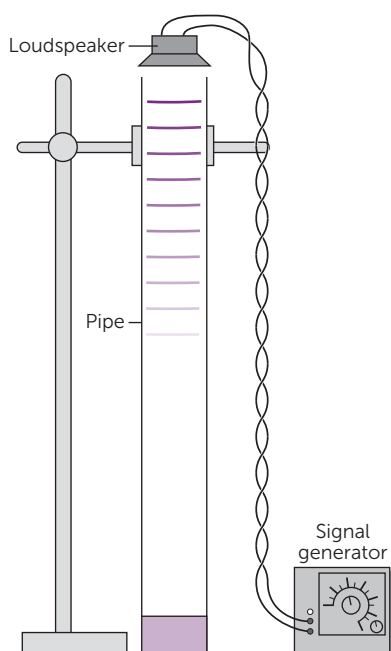


Fig. 10.18 Testing resonance in a pipe

- 1 At the **fundamental frequency**, f_0 , the stationary wave pattern corresponds to a distance of $\frac{1}{4}\lambda$ (the distance from a node to the nearest antinode) between the speaker and the closed end of the pipe. In other words,

$$L + e = \frac{1}{4}\lambda$$

where e (called the 'end correction') is the distance between the speaker and the open end of the pipe, v is the speed of sound in the pipe and λ_0 is the fundamental wavelength. See Fig. 10.19.

$$\text{Hence } f_0 = \frac{v}{\lambda_0} = \frac{v}{4(L + e)}$$

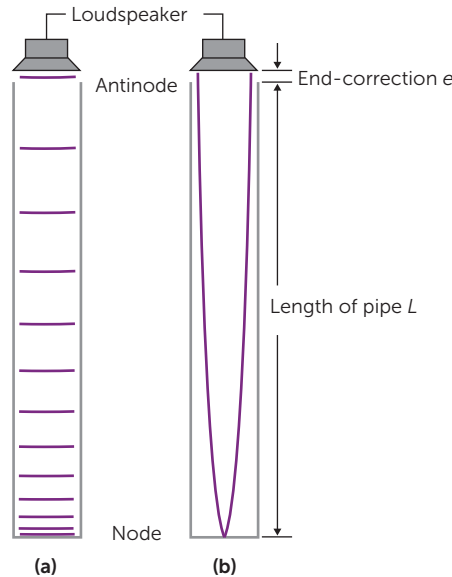


Fig. 10.19 At the fundamental frequency
(a) fundamental vibrations at frequency f_0 ,
(b) transverse representation

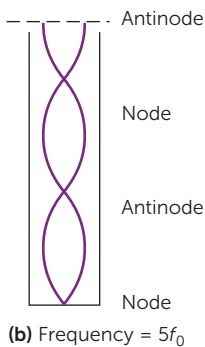
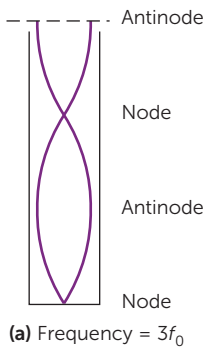


Fig. 10.20 Overtones in a pipe closed at one end

- (a) 1st overtone,
(b) 2nd overtone

- 2 **Overtones** at frequencies $3f_0$, $5f_0$, $7f_0$, etc. are due to stationary waves in which there are an odd number of quarter wavelengths between the speaker and the closed end, corresponding to a node at the closed end and an antinode at the speaker. See Fig. 10.20. In other words,

- $L + e = \frac{3}{4}\lambda_1$ for the first overtone, where $\lambda_1 = 1\text{st overtone wavelength}$, so

$$\begin{aligned} \text{frequency} &= \frac{v}{\lambda_1} = \frac{3v}{4(L + e)} \\ &= 3f_0 \end{aligned}$$

- $L + e = \frac{5}{4}\lambda_2$ for the second overtone, where $\lambda_2 = 2\text{nd overtone wavelength}$, so

$$\begin{aligned} \text{frequency} &= \frac{v}{\lambda_2} = \frac{5v}{4(L + e)} \\ &= 5f_0 \end{aligned}$$

- $L + e = \frac{7}{4}\lambda_3$ for the third overtone, where $\lambda_3 = 3\text{rd overtone wavelength}$, so

$$\begin{aligned} \text{frequency} &= \frac{v}{\lambda_3} = \frac{7v}{4(L + e)} \\ &= 7f_0 \end{aligned}$$

10.1 WORKED EXAMPLE

A pipe of length 0.845 m, closed at one end, is known to resonate at frequencies of 100 Hz and 300 Hz. The speed of sound in the pipe is 340 m s^{-1} .

a) Calculate the fundamental frequency of this pipe.

b) Calculate (i) the wavelength of sound in the pipe at its fundamental frequency, (ii) the end-correction.

Solution

a) The resonant frequencies fit the pattern 1 ; 3 ; 5 ; etc. The fundamental frequency is therefore 100 Hz.

b) (i) Fundamental wavelength, $\lambda_0 = \frac{v}{f_0} = \frac{340}{100} = 3.40 \text{ m}$

(ii) End-correction, $e = \frac{1}{4}\lambda_0 - L = \frac{3.40}{4} - 0.845 = 0.005 \text{ m}$.

Resonance in a pipe open at both ends

Fig. 10.21 shows a loudspeaker used to make an open pipe resonate with sound. This occurs because sound from the loudspeaker partially reflects when it reaches the other end. The result is that the reflected waves reinforce the waves from the speaker to set up a stationary wave pattern in the pipe. Each pattern has an antinode at either end.

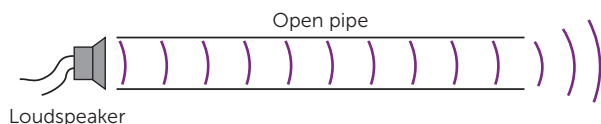


Fig. 10.21 Resonance in an open-ended pipe

See www.palgrave.com/foundations/breithaupt to measure the speed of sound in a pipe.

- At the fundamental frequency, f_0 , the lowest frequency for resonance, the length of the air column is exactly equal to one-half wavelength. This is the distance between adjacent antinodes. The condition for this resonance is therefore $\frac{1}{2}\lambda = (L + 2e)$, since there is an end-correction at either end, where λ_0 is the fundamental wavelength.

$$\text{Hence } f_0 = \frac{v}{\lambda_0} = \frac{v}{2(L + 2e)}$$

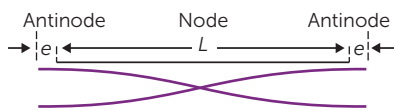
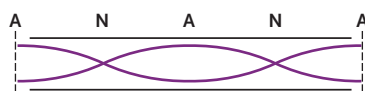
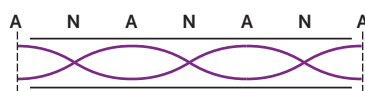


Fig. 10.22 At the fundamental frequency

- Overtone occurs at frequencies $2f_0$, $3f_0$, $4f_0$, etc. In general, resonance occurs if there are a whole number of half wavelengths between the antinodes at the ends as shown in Fig. 10.22 and Fig. 10.23.



(a) Frequency = $2f_0$



(b) Frequency = $3f_0$

Fig. 10.23 Overtones in a pipe open at both ends

- (a) 1st overtone,
(b) 2nd overtone

In other words,

- $L + 2e = \frac{2}{2} \lambda_1$ for the first overtone, where $\lambda_1 = 1\text{st overtone wavelength}$, so

$$\text{frequency} = \frac{v}{\lambda_1} = \frac{v}{(L + 2e)} = 2f_0$$
- $L + 2e = \frac{3}{2} \lambda_2$ for the second overtone, where $\lambda_2 = 2\text{nd overtone wavelength}$, so

$$\text{frequency} = \frac{v}{\lambda_2} = \frac{3v}{2(L + 2e)} = 3f_0$$
- $L + 2e = \frac{4}{2} \lambda_3$ for the third overtone, where $\lambda_3 = 3\text{rd overtone wavelength}$, so

$$\text{frequency} = \frac{v}{\lambda_3} = \frac{4v}{2(L + 2e)} = 4f_0$$

10.2 WORKED EXAMPLE

Calculate the fundamental frequency of sound in an open-ended pipe of length 1.40 m. Neglect end-corrections. The speed of sound in the pipe is 340 m s^{-1} .

Solution

Ignoring end-corrections, the length of the pipe is one-quarter of the fundamental wavelength, λ_0 .

Hence $\lambda_0 = 2 \times 1.40 \text{ m} = 2.80 \text{ m}$.

$\therefore$ Using $f_0 = \frac{v}{\lambda_0}$ gives $f_0 = \frac{340}{2.80} = 121 \text{ Hz}$.

10.5 QUESTIONS

- 1 A pipe of length 0.815 m is closed at one end. The lowest frequency at which it resonates is 104 Hz.
 - a) Calculate the frequency of the first overtone.
 - b) The speed of sound in the pipe is 340 m s^{-1} . Calculate (i) the wavelength of sound of frequency 104 Hz in the pipe, (ii) the end-correction.
- 2 A car silencer box is open at both ends and has a length of 0.60 m. The speed of sound in the pipe is 340 m s^{-1} . Estimate (i) the wavelength of sound in the box when it resonates at its fundamental frequency, (ii) the fundamental frequency.

SUMMARY

• Nature and properties of sound waves

- 1 Sound waves are longitudinal.
- 2 Soft materials absorb sound waves; smooth hard surfaces cause echoes.
- 3 Sound travels faster in solids and liquids than in gases.

• Ultrasonics: sound waves of frequency above the range of the human ear.

• The human ear is most sensitive at a frequency of about 3000 Hz.

• The decibel scale (dB) is a 'times ten' scale used to measure loudness.

• Stationary waves

- 1 on a vibrating wire occur at frequencies $f_0, 2f_0, 3f_0$, etc.
- 2 in a pipe closed at one end occur at frequencies $f_0, 3f_0, 5f_0$, etc.
- 3 in a pipe open at both ends occur at frequencies $f_0, 2f_0, 3f_0$, etc.

• Comparison of travelling waves and stationary waves

1 Travelling wave	Amplitude Same at all positions
2 Stationary wave	Max at antinodes; zero at nodes

Phase difference between any two points

Increases steadily with distance
 $180^\circ \times \text{no. of nodes between the two points}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

10.1 Fig. 2.24 shows an oscilloscope screen displaying the waveform of sound from a loudspeaker connected to a signal generator.

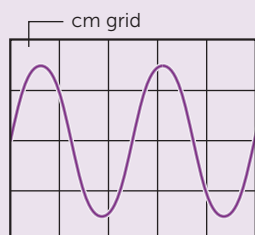


Fig. 10.24

Time base = 10 ms/cm^{-1}

- (i) Measure the time period of the waveform,
(ii) Calculate the frequency of the sound.
- The speed of sound in air at room temperature is 340 m s^{-1} . Calculate the wavelength of the sound waves in (a).
- Sketch the trace on the screen if the frequency is halved without changing the time base setting.

10.2

- (i) Which of the materials A–D listed below is most effective at absorbing sound?
(A) cardboard (B) brick
(C) water (D) cushion material
(ii) Which of the above materials is most effective at reflecting sound?
- (i) A ship at sea in a fog is near some cliffs. The captain sounds a short blast of a siren and hears an echo 1.2 s later. How far is the ship from the cliffs? Assume the speed of sound as for Q10.1(b).
(ii) How could the captain determine if the ship is moving towards or away from the cliffs?

10.3

- (i) State the upper frequency limit of hearing of a young person with normal hearing.
(ii) Give one reason why the upper frequency limit could be lower.
- What is the increase in sound energy in a room if the loudness increases from
(i) 40 dB to 50 dB,
(ii) 50 dB to 60 dB,
(iii) 60 dB to 100 dB?

- A person with poor hearing has a hearing loss of 20 dB at the most sensitive frequency. How many motor bikes, each producing the same noise, would need to be present for the same effect as one motor bike for normal hearing?

10.4

- In an ultrasonic scanner used on a patient, several strong reflected pulses are received by the transducer for every pulse emitted.
(i) Why is there more than one reflected pulse for each emitted pulse?
(ii) Give two reasons why the reflected pulses from each emitted pulse vary in strength.
- In an ultrasonic transducer,
(i) why is it essential that the frequency of the alternating voltage applied to the ceramic slice is the same as the natural frequency of the slice?
(ii) why does the ultrasonic frequency need to be of the order of MHz instead of about 40 kHz?

10.5 A guitar string is tuned to the same frequency as a tuning fork of frequency 256 Hz by altering the tension of the string.

- How would the frequency of the note produced by the guitar string change if
(i) the string is tightened further,
(ii) the string is shortened without change of tension?
- The note from the guitar string is recorded and analysed using a computer. Fig. 10.25 shows the frequency spectrum of the note. Explain
(i) the presence of notes of frequency 256 Hz and 768 Hz,
(ii) the absence of a note of frequency 512 Hz.

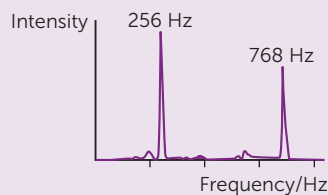


Fig. 10.25

10.6 The fundamental frequency of vibration of a string is inversely proportional to its length. A steel wire of length 0.80 m has a fundamental frequency of vibration of 384 Hz.

- Calculate its fundamental frequency if its length is changed to
(i) 0.40 m,
(ii) 1.20 m.
- What change of length would be needed to raise the frequency by 5% from 384 Hz?

REVISION QUESTIONS

10.7 An open-ended pipe of length 1.10 m resonates with sound at a frequency of 150 Hz. The speed of sound in the pipe is 340 m s^{-1} .

- a) (i) Calculate the wavelength of sound waves in the pipe of frequency 150 Hz.
(ii) Sketch the stationary wave pattern in the pipe when it resonates at 150 Hz.
(iii) Calculate the end-correction of the pipe.
- b) What would be the next highest frequency at which this pipe would resonate?

c) What would be the fundamental frequency of the pipe in (a) if it was to be closed at one end?

2.8 The lengths of the pipes of a certain organ range from 40 mm to 4 m. The speed of sound in the pipes is 340 m s^{-1} .

- a) If each pipe is open at both ends, calculate the range of the fundamental frequencies of the organ.
- b) Calculate the first three overtone frequencies of the shortest pipe.

Optics

UNIT

11

OBJECTIVES

After working through this unit, you should be able to:

- outline the experimental evidence for the wave theory of light
- use the wave theory of light to explain reflection and refraction
- carry out refractive index calculations
- describe total internal reflection and its application to optical fibres
- construct ray diagrams to explain the formation of images by mirrors, lenses, microscopes and telescopes
- outline the factors that govern the quality of the image formed by an optical instrument
- carry out calculations using the lens formula

CONTENTS

- 11.1 Interference and the nature of light
- 11.2 Reflection and refraction of light
- 11.3 Lenses
- 11.4 Optical instruments
- Summary
- Revision questions

11.1 Interference and the nature of light

It isn't obvious that light consists of waves. Over three centuries ago, Isaac Newton put forward the theory that light consisted of tiny particles (he referred to these as 'corpuscles') which were thought to bounce off mirrors and speed up on entering glass or water. About the same time, another less famous scientist called Christiaan Huygens thought that light was composed of waves. His theory also could explain reflection and it predicted that light would slow down on entering glass or water. There was no experimental evidence at the time to resolve the question over the speed of light in glass or water. Since Newton's theory of gravitation and his laws of motion were accepted, the corpuscular theory of light held sway and Huygens' theory was rejected for over a century.

The first experimental evidence for Huygens' theory was produced by Thomas Young, who made a detailed study of light passing through two closely spaced slits. Young observed a pattern of bright and dark fringes which he could only explain using the idea that light waves from one slit interfered with light waves from the other slit. See [Fig. 9.14](#) for a general description of interference.

Interference of light

A parallel beam of light is directed at two closely spaced slits in an opaque plate, as shown in [Fig. 11.1](#). Light is diffracted on passing through each slit. With the aid of a

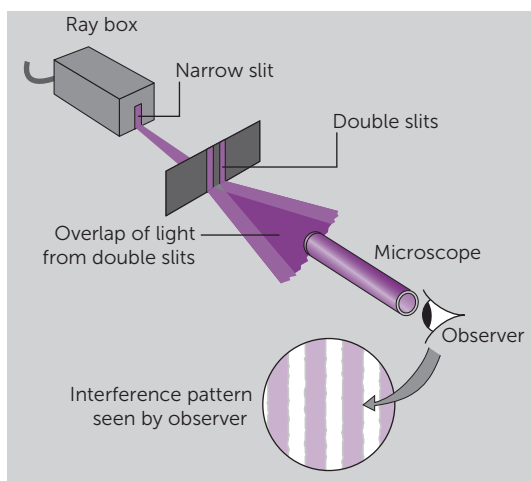


Fig. 11.1 Interference of light

microscope, an interference pattern of bright and dark fringes is observed where light from one slit overlaps light from the other slit. According to Newton's theory of light, there should be just two bright bands, one opposite each slit. In fact, the number of bright and dark fringes depends on how closely spaced and how wide the slits are. With care, more than six or so dark fringes are easily visible. Huygens' wave theory explains the pattern of fringes. In addition, it can be combined with the measurements to enable the wavelength of light to be worked out. This is very small; over 2000 wavelengths can fit into 1 mm.

As explained on p. 125, each dark fringe occurs where light from one slit cancels light from the other slit. Each bright fringe occurs where light from one slit reinforces light from the other slit. The two slits emit waves with a constant phase difference because part of each wavefront from the point light source passes through each slit. The two slits are said to be **coherent** sources of waves because they emit waves with a constant phase difference. In general, at any point, P, where the waves overlap, the waves from one slit travel a different distance to that point in comparison with the waves from the other slit. The phase difference therefore changes with the position of P.

- 1 At any point where a bright fringe is formed, the waves from the two slits arrive in phase and therefore reinforce each other.
- 2 Where a dark fringe is formed, the waves arrive exactly out of phase and therefore cancel each other out.

The theory of the double slits experiment

Fig. 11.2 shows two slits, S_1 and S_2 , and an interference pattern formed along a line parallel to S_1S_2 . For reinforcement (i.e. a bright fringe) at a point R,

$$S_1R - S_2R = m\lambda, \text{ where } m \text{ is a whole number}$$

- 1 $m = 0$ gives the central bright fringe at O.
- 2 $m = 1$ gives the first bright fringe either side of the central bright fringe; hence for reinforcement at point P, the first bright fringe from the centre

$$S_1P - S_2P = \lambda$$

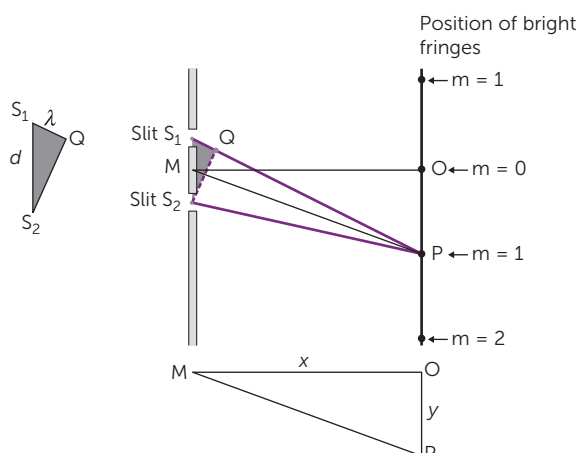


Fig. 11.2 Double slits theory

- 3 Let Q be the point on line S_1P such that $QP = S_2P$; hence

$$QS_1 = S_1P - S_2P = \lambda$$

Let M be the midpoint of S_1S_2 and let O be directly opposite M on the screen. Since triangles MOP and S_1S_2Q are almost the same shape (i.e. similar), then

$$\frac{\text{distance } QS_1}{\text{distance } S_1S_2} \text{ is almost equal to } \frac{\text{distance } OP}{\text{distance } OM}$$

$$\therefore \frac{\lambda}{d} = \frac{y}{X}$$

where d = the slit spacing S_1S_2

y = fringe spacing OP

X = slit-screen distance OM

See www.palgrave.com/foundations/breithaupt to measure the wavelength of laser light using a diffraction grating.

Measurement of the wavelength of light using double slits

Use the arrangement in Fig. 11.1 to measure the fringe spacing. A pin in focus, positioned in front of the microscope, will give the plane of the fringes. The microscope can also be used to measure the slit spacing. A metre rule is used to measure X . Coloured filters may be used to measure the wavelength of light of each colour. Fig. 11.3 shows how the wavelength of light varies with colour. Use the above formula to calculate λ .

11.1 WORKED EXAMPLE

A laser beam is directed at two closely spaced slits 0.4 mm apart. The fringe pattern is observed on a screen placed 1.50 m from the slits. The fringe spacing was measured at 2.4 mm. Calculate the wavelength of the light used.

Solution

$$d = 0.4 \text{ mm}, X = 1.50 \text{ m}, y = 2.4 \text{ mm}$$

Rearranging $\frac{\lambda}{d} = \frac{y}{X}$, gives

$$\lambda = \frac{yd}{X} = \frac{2.4 \times 10^{-3} \times 0.4 \times 10^{-3}}{1.50} = 6.4 \times 10^{-7} \text{ m} = 640 \text{ nm}.$$

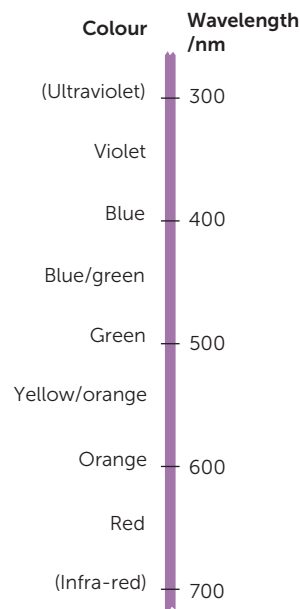


Fig. 11.3 Wavelength and colour

11.1 QUESTIONS

- 1 A double slits experiment was set up using two slits at a spacing of 0.50 mm. The fringes were observed at a distance of 0.80 m from the slits.
 - a) The fringe spacing was 0.90 mm. (i) Calculate the wavelength of the light used, (ii) What colour was the light?
 - b) The light source was replaced by a different light source producing light of wavelength 590 nm. Calculate the fringe spacing produced by this light source.
- 2 Describe and explain how the fringe pattern would change in Q1 if
 - a) one of the two slits was blocked,
 - b) the two slits were replaced by a pair of slits which were (i) the same width but at a greater spacing, (ii) wider but at the same spacing.

11.2 Reflection and refraction of light

Reflection at a plane mirror

The law of reflection

Fig. 11.4 shows a ray of light reflecting off a plane mirror. The ripple tank experiment on p. 122 shows plane waves reflecting off a straight reflector. This is what happens with light except the scale is much smaller. Note that the incident ray is at the same angle to the normal as the reflected ray. This is the **law of reflection**.

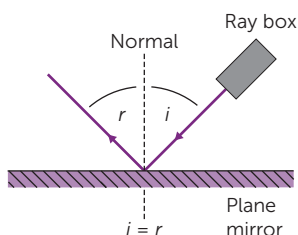


Fig. 11.4 Reflection at a plane mirror

Image formation

The image of an object seen in a plane mirror is formed where the light rays, after reflection, appear to originate from. The image is said to be **virtual** because it cannot be projected onto a screen. Fig. 11.5 shows why the image is at the same distance behind the mirror as the object is in front.

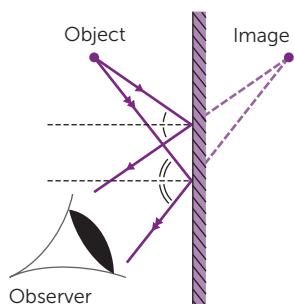


Fig. 11.5 Image formation by a plane mirror

Refraction of light at a plane boundary

A light ray directed into a glass block refracts towards the normal where it enters the block, as shown in Fig. 11.6. Measurements of the angle of incidence, i , and the angle of refraction, r , for different values of i can be used to show that the ratio $\sin i / \sin r$ is a constant. This is known as Snell's law after its discoverer. The constant depends on the material of the block and is referred to as the refractive index of the material.

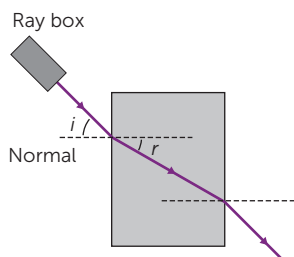


Fig. 11.6 Refraction of light

11.2 WORKED EXAMPLE

A light ray is directed into a glass block of refractive index 1.5 at an angle of incidence of 30° . Calculate the angle of refraction of the light ray.

Solution

$$i = 30^\circ, n = 1.5$$

Rearranging $\frac{\sin i}{\sin r} = n$, gives

$$\sin r = \frac{\sin i}{n} = \frac{\sin 30}{1.5} = \frac{0.5}{1.5} = 0.33$$

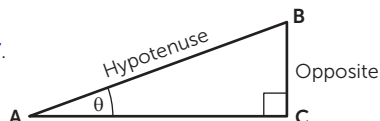
$$\therefore r = 19.3^\circ$$

Note on trigonometry

Consider the right-angled triangle shown in Fig. 11.7. For angle θ (pronounced 'theta'), the sine function is defined by the equation

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB}$$

- To find the sine of an angle, key the angle in degrees on to the display of your calculator, then press the 'sin' button.
- To find the angle for a given value of the sine function (i.e. the inverse of the sine function), key the value on to the display of your calculator, then press the buttons marked 'inv' then 'sin'.

**Fig. 11.7****Using the wave theory to explain refraction**

Light travels more slowly in a transparent substance than in air. Fig. 11.9 shows what happens to plane waves on passing across the boundary between air and a transparent substance.

- 1 Consider triangle WXY; angle WXY = the angle of incidence, i .

WY = wavelength in air, λ_0 (opposite angle i)

$$\therefore XY = \frac{\lambda_0}{\sin i} \quad \left(\text{since } \sin i = \frac{WY}{XY} \right)$$

- 2 Consider triangle XYZ in Figs. 11.8 and 11.9;

angle XYZ = angle of refraction, r ,

and XZ = wavelength in the substance, λ

$$\therefore XY = \frac{\lambda}{\sin r} \quad \left(\text{since } \sin r = \frac{XZ}{XY} \right)$$

$$\text{Hence } \frac{\lambda}{\sin r} = \frac{\lambda_0}{\sin i}$$

Rearranging this equation gives

$$\frac{\sin i}{\sin r} = \frac{\lambda_0}{\lambda}$$

It follows that the refractive index of the substance,

$$n = \frac{\lambda_0}{\lambda}$$

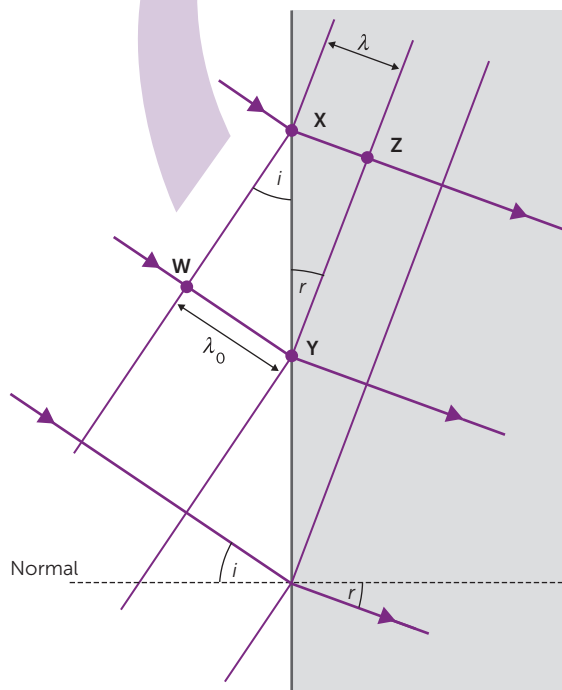
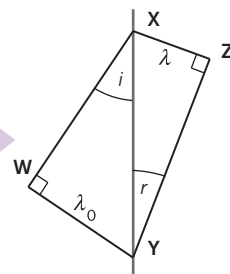
Since wave speed = frequency $\times$ wavelength and the frequency does not change on passing from air into the transparent substance, the refractive index is also equal to

$$\frac{\text{the speed of light in air}}{\text{the speed of light in the substance}}$$

Note

If the light ray travels from the substance into air, it bends away from the normal and the ratio

$$\frac{\sin i}{\sin r} = \frac{1}{n} \quad \left(= \frac{\lambda}{\lambda_0} \right)$$

Fig. 11.8 Using trigonometry**Fig. 11.9** Using wave theory

11.3 WORKED EXAMPLE

The refractive index of glass is 1.5. Calculate the speed of light in glass. The speed of light in air is $3.0 \times 10^8 \text{ m s}^{-1}$.

Solution

$$\frac{\text{speed of light in air}}{\text{speed of light in glass}} = 1.5$$

Rearranging this equation gives

$$\text{speed of light in glass} = \frac{\text{speed of light in air}}{1.5} = 2.0 \times 10^8 \text{ m s}^{-1}.$$

Total internal reflection

When a light ray passes from a transparent substance into air, the light ray refracts away from the normal at the boundary. Suppose the angle of incidence is increased until the refracted ray emerges along the boundary, as shown in Fig. 11.10. This angle of incidence is called the **critical angle**. If the angle of incidence exceeds the critical angle, the light ray undergoes **total internal reflection** at the boundary.

The critical angle, c , can be calculated if the refractive index, n , is known by using Snell's law and the fact that the angle of refraction, r , is 90° when the angle of incidence is equal to the critical angle. Since angle i is in the transparent substance,

$$\frac{\sin i}{\sin r} = \frac{\sin c}{\sin 90} = \frac{1}{n}$$

$$\text{As } \sin 90 = 1, \text{ then } \sin c = \frac{1}{n}$$

Note

Light undergoes total internal reflection at a boundary between two transparent substances, provided:

- the refractive index of the incident medium, $n_i > n_r$, the refractive index of the refracted medium;
- the angle of incidence $>$ the critical angle for the interface which is given by

$$\sin c = \frac{n_r}{n_i}.$$

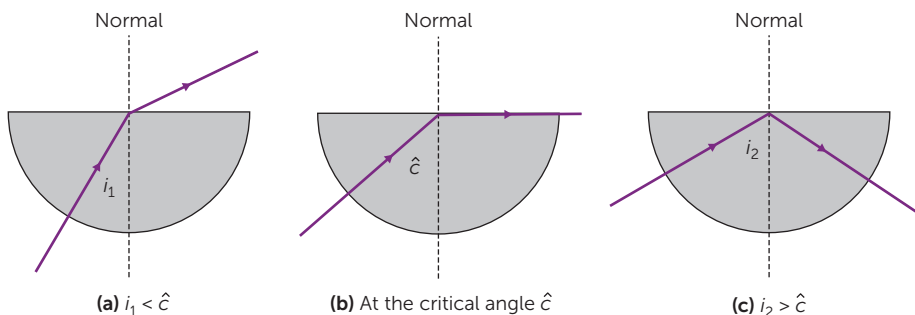


Fig. 11.10 Total internal reflection

Optical fibres

These are used in medicine, to see inside the body and to guide light pulses in digital communication systems. Fig. 11.11 shows a light ray guided by a thin optical fibre. The ray undergoes total internal reflection each time it is incident on the fibre–air boundary, provided the fibre is not too bent. The fibre is usually surrounded by a transparent cladding of lower refractive index than the core. Because some of the light is reflected at the core–cladding boundary, the cladding ensures less light can cross from one fibre to another at points of contact.

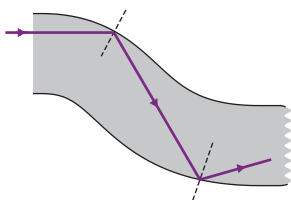


Fig. 11.11 An optical fibre

11.4 WORKED EXAMPLE

A light ray is directed normally into a glass prism of refractive index 1.5, as shown in Fig. 11.12.

- Calculate the critical angle of the glass
- Copy the diagram and complete the path of the light ray through the prism.

Solution

a) $\sin c = \frac{1}{n} = \frac{1}{1.5} = 0.67 \quad \therefore c = 42^\circ$

- b) The light ray passes into the prism normally (i.e. along the normal) through face AB. The angle of incidence at face BC is 45° which exceeds the critical angle. Therefore the ray undergoes total internal reflection at BC and is incident on face AC at 45° . The ray again undergoes total internal reflection at face AC, hence it passes out through AB normally, without refraction.

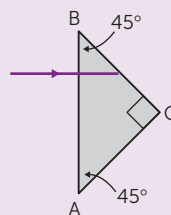


Fig. 11.12

See www.palgrave.com/foundations/breithaupt for the medical endoscope

11.2 QUESTIONS

- Copy and complete the diagram in Fig. 11.13 to show how an image of a point object is formed in a plane mirror.
- An optical fibre consists of a transparent core of refractive index 1.5, surrounded by a transparent layer of cladding of refractive index 1.2.

 - With the aid of a diagram, explain the purpose of the cladding.
 - Calculate the critical angle at the core–cladding boundary.
 - Discuss whether or not a larger refractive index for the cladding is desirable.

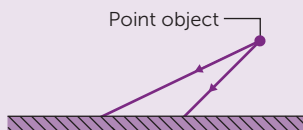


Fig. 11.13

11.3 Lenses

Lenses are used to form real or virtual images.

- A **real image** is one that can be formed on a screen (e.g. a projector image).
- A **virtual image** is formed where the light appears to originate from (e.g. an object viewed in a mirror or using a magnifying glass).

Convex lenses are used in cameras, projectors, microscopes and telescopes and to correct long sight. The eye contains a flexible convex lens which is sometimes too weak to enable near objects to be seen clearly. This sight defect, referred to as long sight, is corrected using a convex lens.

Concave lenses are used in combination with convex lenses to make high-quality lens systems for optical instruments and cameras. In addition, concave lenses are used to correct short sight which is where the eye lens is too strong to see distant objects clearly.

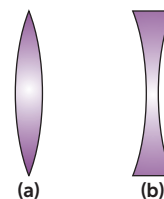


Fig. 11.14 Lens shapes
(a) convex, (b) concave

The convex lens

A convex lens forms a real image by making light from a point object converge to form a point image. Fig. 11.15 shows how this is achieved.

Focal length, f

This is the distance from the lens to the **focal point**, F , of the lens, which is the point where a beam of light parallel to the lens axis is brought to a focus. See Fig. 11.16.

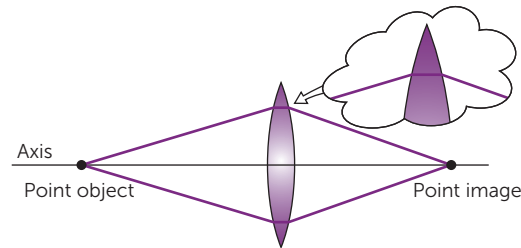


Fig. 11.15 Image formation by a convex lens

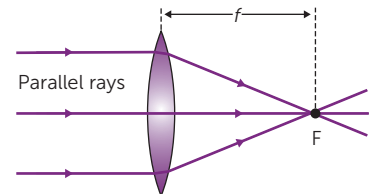


Fig. 11.16 Focal length

Ray diagrams

The position and size of the image formed by a convex lens can be determined by constructing a ray diagram. Fig. 11.17 shows how to do this. The image formed in Fig. 11.17 is real, inverted and magnified (i.e. larger than the object).

The position, nature and size of the image depend on the position of the object in relation to the focal length of the lens, as shown in Figs. 11.17, 11.18, 11.19 and 11.20, and summarised in Table 11.1.

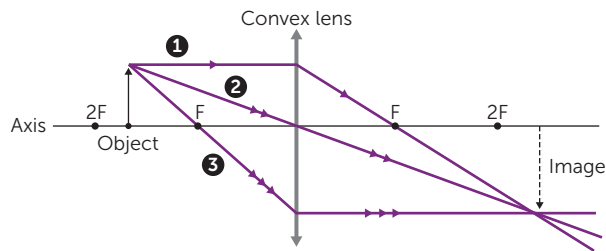


Fig. 11.17 Constructing a ray diagram for a convex lens

Ray ① is parallel to the axis before the lens then through F

Ray ② is through the lens centre without refraction

Ray ③ is through F before the lens then parallel to the axis

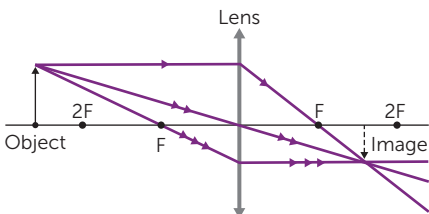


Fig. 11.18 Object beyond $2F$

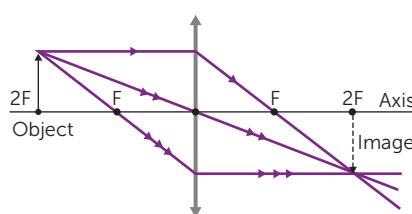


Fig. 11.19 Object at $2F$

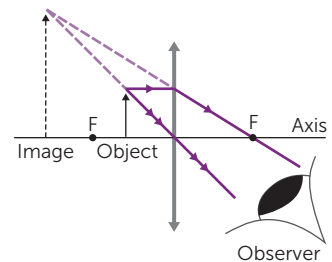


Fig. 11.20 Object between F and the lens

Table 11.1 The convex lens and its applications

Object position	Figure reference	Image position	Nature of image	Size of image	Application
beyond 2F	11.18	between F and 2F on the opposite side	real and inverted	diminished	camera, eye
at 2F	11.19	at 2F on the opposite side	real and inverted	same size	inverter
between F and 2F	11.17	beyond 2F on the opposite side	real and inverted	magnified	projector
at F		no image formed			to produce a parallel beam
inside F	11.20	further away on the same side	virtual and upright	magnified	magnifying glass

The concave lens

A concave lens makes light from a point object diverge more or converge less.

Focal length, f

Fig. 11.21 shows the effect of a concave lens on a parallel beam of light. The focal length is the distance from the lens to the point where a beam of parallel light appears to originate.

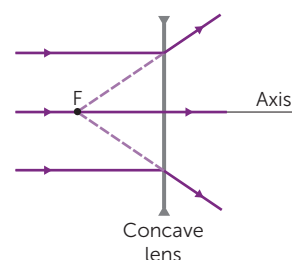


Fig. 11.21 The focal length of a concave lens

Ray diagrams

Fig. 11.22 shows how to construct a ray diagram for a concave lens. The image formed in Fig. 11.22 is virtual, upright and closer to the lens than the object.

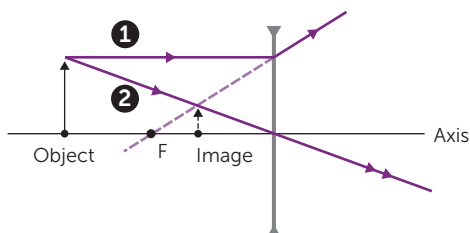


Fig. 11.22 A ray diagram for a concave lens

Ray ① is parallel to the axis before the lens, then diverges as if from F

Ray ② is through the centre of the lens without refraction

The lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

where u = the distance from the lens to the object
 v = the distance from the lens to the image

See www.palgrave.com/foundations/breithaupt for a proof of the lens formula.

Notes

- 1 Real images take positive values and virtual images take negative values in the lens formula.
- 2 The focal length of a convex lens is given a positive value. The focal length of a concave lens is given a negative value.
- 3 Linear magnification = $\frac{\text{image height}}{\text{object height}} = \frac{v}{u}$

11.5 WORKED EXAMPLE

A convex lens of focal length 0.10 m is used to form a magnified image of an object of height 5 mm placed at a distance of 0.08 m from the lens. Calculate the position, nature and size of the image.

Solution

$$u = 0.08 \text{ m}, f = 0.10 \text{ m}$$

$$\text{Using the lens formula gives } \frac{1}{0.08} + \frac{1}{v} = \frac{1}{0.10}$$

$$12.5 + \frac{1}{v} = 10$$

$$\therefore \frac{1}{v} = 10 - 12.5 = -2.5$$

$$\therefore v = -\frac{1}{2.5} = -0.40 \text{ m}$$

The image is virtual and its position is 0.40 m from the lens on the same side as the object. The linear magnification is $(-)(0.40 \div 0.08) = 5$. The image height is therefore 25 mm ($= 5 \times 5 \text{ mm}$).

Determination of the focal length of a convex lens

The focal length of a convex lens can be determined by the plane mirror method, as follows.

Using illuminated cross-wires as the object, a plane mirror is used to reflect light from the lens back towards the object. The position of the object is adjusted to where a clear image of the cross-wires can be seen alongside the cross-wires. The object is then at the focal point of the lens since the beam from the lens to the mirror is exactly parallel and at 90° to the mirror. See Fig. 11.23.

See www.palgrave.com/foundations/breithaupt to measure the focal length of a convex lens by measuring the image distance for different object distances.

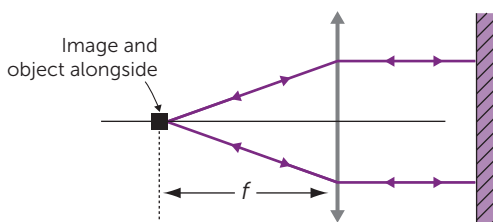


Fig. 11.23 The plane mirror method

11.3 QUESTIONS

- 1 a) By means of a ray diagram, determine the position and nature of the image formed by a convex lens of focal length 0.20 m if the object distance is (i) 0.50 m, (ii) 0.25 m, (iii) 0.15 m.
b) Use the lens formula to calculate the image distance and the linear magnification for each object position in (a).
- 2 a) With the aid of a diagram, explain how a convex lens may be used as a magnifying glass.
b) With the aid of a diagram, explain why the image of an object viewed through a concave lens appears smaller and closer than the object.

11.4 Optical instruments

The quality of the image formed by an optical instrument depends on the quality of the optical components (e.g. lenses) in the instrument. As shown in Figs. 11.24 and 11.25, the two main effects that cause image defects are:

- **spherical aberration**, where the outer rays from a point object are focused to a position different from the inner rays. The shape of the lens or mirror surface determines the extent of this effect.
- **chromatic aberration**, a lens defect due to the splitting of white light when it is refracted. The effect can be minimised using convex and concave lenses of different refractive indices.

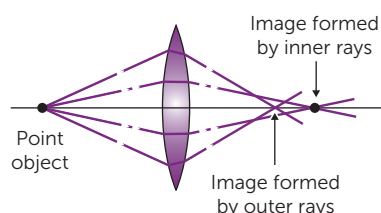


Fig. 11.24 Spherical aberration

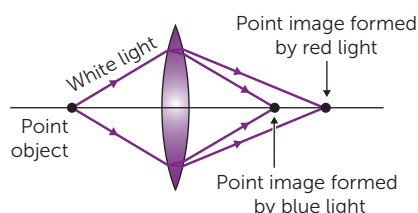


Fig. 11.25 Chromatic aberration

The camera

The convex lens forms a real image on the film inside the camera; see Fig. 11.26. In a digital camera, the film is replaced by a CCD (charge-coupled device).

- The lens position is adjusted for different object distances. The nearer the object, the greater the distance from the lens to the film.
- The aperture width controls the amount of light entering the camera. A wide aperture will produce minimum diffraction to give very clear images but little depth of focus.
- The shutter must open and close rapidly to photograph a fast-moving object. In such a situation, the aperture width must be as wide as possible to let as much light as possible into the camera. As a result, the depth of focus is much reduced in an 'action' photograph.

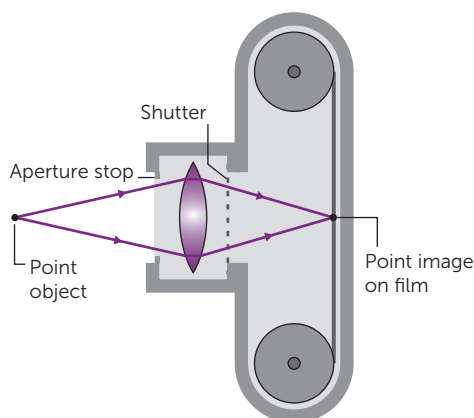


Fig. 11.26 The camera

The optical microscope

The objective lens is used to form a magnified, real image of the object, as shown in Fig. 11.27. The eyepiece lens is used to view this ‘intermediate’ image, enabling a large, magnified, virtual image to be seen by the observer looking into the eyepiece. In normal adjustment, the position of the final image is at the observer’s **near point** of vision.

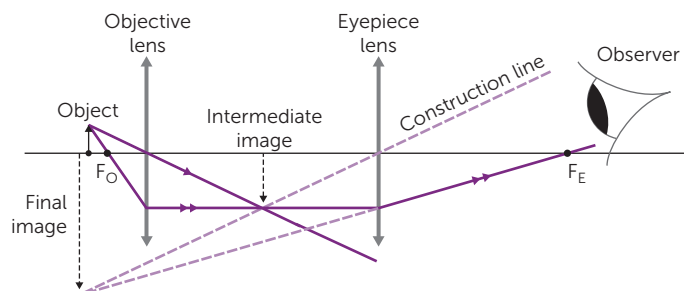


Fig. 11.27 The optical microscope

- The image of a point object is smudged due to diffraction of light as it passes through the objective lens aperture (see Application 3). More image detail can be seen using blue light rather than ordinary white light. Because blue light has a smaller wavelength than all the other components of white light (except violet), there is less diffraction using blue light.
- Cross-wires (or a transparent scale) are seen in focus with the final image, provided the cross-wires are located at the same position as the intermediate image.

Application 3

Diffraction limits

Diffraction is an important feature of any device that emits or receives waves. As explained in Topic 9.3, waves are diffracted when they pass through a gap. A diffraction pattern is formed when light from a point object is focused on a screen after it passes through a gap or hole. For a circular hole, the pattern consists of a central bright spot surrounded by concentric bright and dark rings. The dark rings are formed where the waves interfere destructively and cancel each other out. Fig. 11.28 shows light from two distant point objects passing through a circular hole to form adjacent diffraction patterns. The patterns cannot be resolved (ie. seen separately) if their central spots are too close.

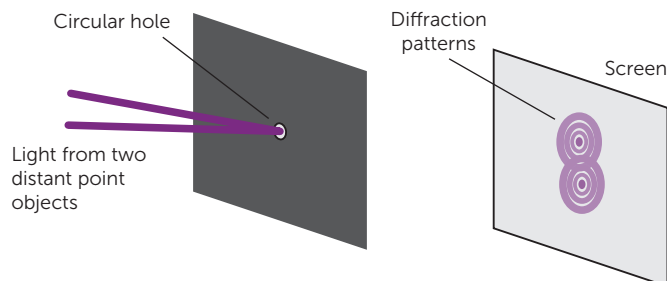


Fig. 11.28
Diffraction by a circular hole

The refracting telescope

The objective lens forms an intermediate, real image of a distant object. This image is magnified using the eyepiece so the observer sees a final virtual image at infinity. The best position for the eye is at the 'eye-ring', as shown in Fig. 11.29.

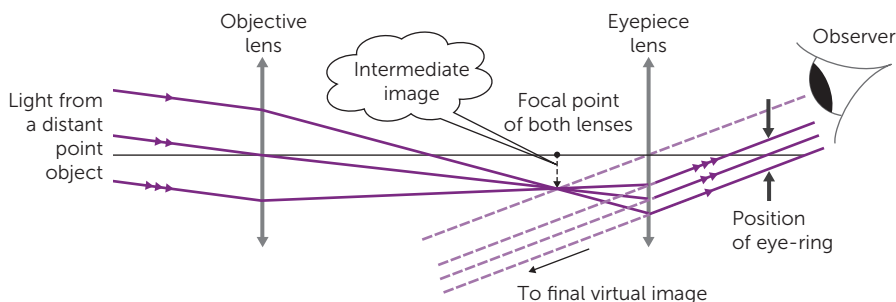


Fig. 11.29 The refracting telescope

- The magnifying power of a telescope is measured by how many times larger the final image appears in comparison with the object. For example, a telescope that makes the Moon appear 10 times wider than if viewed directly has a magnifying power of $\times 10$.
- The wider the objective, the more light is collected by the telescope. This is why stars that cannot be seen directly are visible using a telescope. In addition, diffraction is less and more detail can be seen in extended objects, although beyond a width of about 10 cm, atmospheric smudging limits the detail that can be seen.

Note

It can be shown that the magnifying power of a telescope

$$= \frac{\text{the focal length of the objective}}{\text{the focal length of the eyepiece}}$$

Application 4

Space telescopes

The Hubble Space Telescope (HST) has provided many amazing images of objects in space. Its objective is a concave reflecting mirror which has a diameter of 4.0 m. It can 'resolve' far greater detail in distant objects than much wider telescopes on the ground can, and it can detect faint objects hundreds of millions of light years away. The extent to which HST can resolve detail is limited by the diffraction of light at its reflecting mirror. The wider a telescope is, the better its resolution is because less diffraction occurs.

The limit of resolution of a telescope (sometimes called its 'resolving power') is the angular separation, θ , of two point objects that can just be seen separately. For any telescope, θ is approximately equal to λ/D where λ is the wavelength of light and D is the diameter of the telescope objective. For light of wavelength 600 nm, HST's limit of resolution is therefore about 1.5×10^{-7} radians which is about 0.000001 of a degree.



Fig 11.30 The M101 Pinwheel Galaxy

SUMMARY

● Young's slits equation

$$\frac{\lambda}{d} = \frac{y}{X} \quad \begin{array}{l} \text{where } d = \text{the slit spacing} \\ y = \text{fringe spacing} \\ X = \text{slit-screen distance} \end{array}$$

● The law of reflection

The incident ray is at the same angle to the normal as the reflected ray.

● Snell's law of refraction

$$\text{The refractive index of a substance, } n = \frac{\sin i}{\sin r} = \frac{\lambda_0}{\lambda} = \frac{\text{the speed of light in air}}{\text{the speed of light in the substance}}$$

● Total internal reflection

$$\sin c = \frac{1}{n}$$

● The lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \begin{array}{l} \text{where } u = \text{the distance from the lens to the object} \\ v = \text{the distance from the lens to the image} \end{array}$$

$$\bullet \text{ Linear magnification} = \frac{\text{image height}}{\text{object height}} = \frac{v}{u}$$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

11.1 An interference pattern was observed using two closely spaced slits to view a narrow source of light.

- With the aid of a diagram, explain why the pattern consists of alternate bright and dark fringes.
- The pattern was observed on a white screen at a distance of 0.90 m from the slits. The slit spacing was 0.50 mm.
 - The wavelength of the light observed was 590 nm. Calculate the spacing between adjacent bright fringes.
 - The light source was replaced by a different light source. The bright fringes were observed at a spacing of 0.80 mm. Calculate the wavelength of this light source.

11.2

- State an approximate value for the wavelength of
 - red light,
 - blue light.
- A beam of yellow light of wavelength 590 nm enters a glass lens of refractive index 1.55.
 - Calculate the wavelength of this light in the glass.
 - What change occurs in the speed of this light beam when it travels from air to glass?

11.3

- When a ray of light passes from air into glass, does it refract towards or away from the normal?

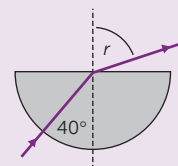


Fig. 11.31

- Calculate the angle of refraction of a ray of light that is incident on a plane surface of a glass block of refractive index 1.50 at an angle of incidence of
 - 25°,
 - 50°.
- Fig. 11.31 shows a ray of light that enters a semi-circular glass block of refractive index 1.50 through the curved face.
 - For an angle of incidence of 40°, calculate the angle of refraction of the light where it leaves the block.
 - Calculate the critical angle of the block.
 - Sketch the path of a light ray in Fig. 11.31 that is incident on the flat face at an angle of incidence of 60°.

REVISION QUESTIONS

11.4

- a) Construct a ray diagram to determine the nature, position and magnification of the image formed by a convex lens of focal length 0.15 m when the object distance from the lens is
- (i) 1.00 m,
 - (ii) 0.20 m,
 - (iii) 0.10 m.
- b) Use the lens formula to calculate the image distance and the linear magnification for each object distance in (a).
- c) Calculate the image distance for an object of height 12 mm at a distance of 0.20 m from a concave lens of focal length 0.30 m, and state the nature and size of the image.

11.5

- a) With the aid of a diagram, explain the function of the convex lens in a camera.

- b) A certain camera is used to take a photograph of a distant scene. What adjustment should be made to the camera to photograph someone indoors? Explain your answer.

11.6

- a) Draw a ray diagram to show the formation of a virtual image by an optical microscope.
- b) Explain why more detail is visible in a microscope image viewed in blue light.
- c) Give one reason why an image observed through a certain microscope is distorted at the edges.

11.7

- a) With the aid of a suitable diagram, explain the function of the lenses in a refracting telescope.
- b) What are the advantages and disadvantages of using a wide objective lens in a telescope?

UNIT

12

Electromagnetic Waves

CONTENTS

- 12.1 The visible spectrum
- 12.2 Types of spectra
- 12.3 Infra-red and beyond
- 12.4 Ultraviolet radiation, X-radiation and gamma radiations
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- explain the splitting of white light by a prism and explain the colour of an object
- describe and explain continuous and line spectra
- describe how to measure optical spectra using a spectrometer and diffraction grating
- state similarities and differences between different parts of the electromagnetic spectrum
- describe the production and common uses of different types of electromagnetic waves

12.1 The visible spectrum**Using a prism to produce a visible spectrum**

White light is split into the colours of the spectrum when it is refracted. Fig. 12.1 shows the splitting of a beam of white light by a prism. The beam is split into colours on entering the prism. The splitting of white light into colours is called **dispersion**. On leaving the prism, the colours are separated further.

The reason why the beam is split into the colours of the spectrum is because the speed of light in glass depends on wavelength and hence on colour. Consequently, the refractive index of glass depends on colour, being greatest for violet and blue and least for red light. A prism bends blue light more than red light (blue bends best!) and therefore the red part of a spectrum produced by a prism is always the part which is least refracted.

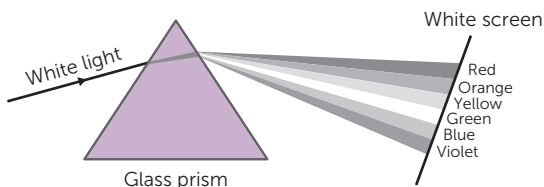


Fig. 12.1 The visible spectrum

Primary and secondary colours

The primary colours of light are red, green and blue. Fig. 12.2 shows the effect of these colours overlapping on a white screen.

- Where all three colours overlap, white light is observed.
- Where two primary colours overlap, a secondary colour is observed. For example, yellow is a secondary colour because it is produced where red and green overlap.
- Adding a secondary colour to the other primary colour produces white light. The secondary colour and the primary colour are said to be complementary. For example, yellow is complementary to blue since blue and yellow overlap to produce white.

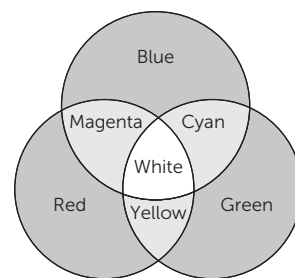


Fig. 12.2 Primary colours overlapping

12.1 QUESTIONS

- Sketch a diagram to show how a glass prism can be used to split a narrow beam of white light into the colours of the spectrum.
 - Describe and explain the appearance of a white light spectrum displayed on a blue screen.
- State the three primary colours and the secondary colour that complements each primary colour.
 - What colour would you expect a shirt that is yellow in normal light to appear in (i) red light, (ii) blue light?

12.2 Types of spectra

Continuous spectra

The white light spectrum is an example of a continuous spectrum because the colours change continuously with change of position across the spectrum, from deep red to violet, corresponding to a continuous range of wavelengths. The Sun, the stars and filament lamps all produce light with a continuous range of wavelengths and therefore give light spectra which are continuous.

As shown in Fig. 12.3, the temperature of the light source determines the distribution of light energy emitted in each part of the spectrum.

- Observe a low voltage filament light bulb when the current through it is gradually increased from zero to the current for normal brightness. The current heats the filament enough to make the filament emit light. The filament glows dull-red at first, then it becomes orange-red and then yellow. This is because the higher its temperature, the smaller the wavelength at which it emits most strongly.

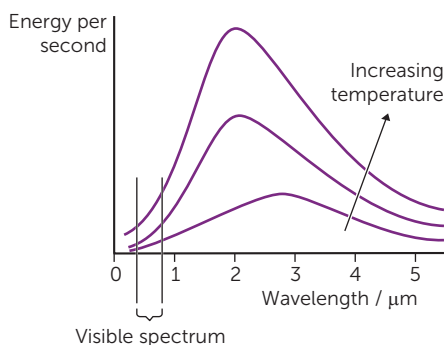


Fig. 12.3 The energy distribution across a continuous spectrum

- 2 A carbon arc lamp glows white because its temperature is so high that it emits blue light as well as the other colours of the spectrum. If its temperature is raised even more, it glows blue-white as it emits mostly in the blue part of the spectrum.
3. The above observations can be applied when comparing the colour of two stars. For example, a blue star is hotter than a red star. The Sun is a yellow star and its surface is therefore hotter than the surface of a red star.

Line emission spectra

The spectrum of light from a discharge tube (e.g. a neon tube) or from a vapour lamp (e.g. a sodium street lamp) is composed of narrow vertical lines, each line being a different colour. See Fig. 12.4. This is because the light source emits light of certain wavelengths only. When a narrow beam of light from such a light source is split by a prism, each colour of light present appears as a discrete line in the spectrum. The pattern of lines makes up the line emission spectrum of the light source. The wavelength of each line can be measured using a diffraction grating as explained below. If the light source is a single element (e.g. neon in a neon tube), the line spectrum is characteristic of the element and can be used to identify the element. If the light source is due to two or more elements, the line spectrum can be used to identify the elements present. This is how astronomers identify the elements present in stars.



Fig. 12.4 Line emission spectrum

The theory of the diffraction grating

The diffraction grating is used to measure the wavelength of light of any colour. It consists of many closely spaced parallel slits on a transparent slide. When a parallel beam of light of a single colour is directed normally at the grating, the effect of the slits is to diffract the beam into certain well-defined directions only, as shown in Fig. 12.5. The action of a diffraction grating is due to light waves diffracting as they pass through the slits and the diffracted waves reinforcing each other in certain directions only.

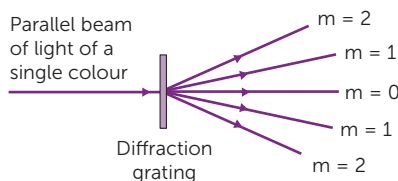


Fig. 12.5 The action of a diffraction grating

The diffracted beams are numbered in order from the centre. Fig. 12.6 illustrates the formation of the diffracted wavefronts.

- 1 To form the first order wavefronts, light from one slit travels a distance of one wavelength before it is in phase with light emerging through the next slit. As shown by Fig. 12.6, this extra distance is equal to $d \sin \theta_1$ where d is the spacing between adjacent slits and θ_1 is the angle of diffraction. Hence the angle of diffraction of the first order is given by the equation,

$$d \sin \theta_1 = \lambda$$

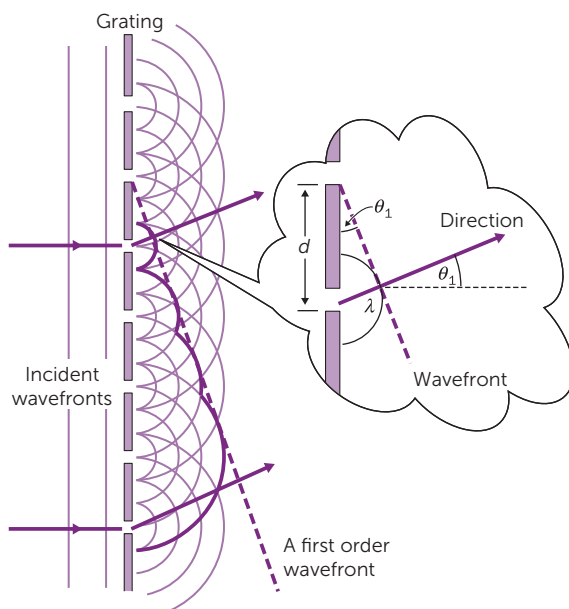


Fig. 12.6 The theory of the diffraction grating

- 2 To form the second order wavefronts, light from one slit travels an extra distance of two wavelengths to be in phase with light emerging through the next slit. Using the same reasoning as above therefore gives

$$d \sin \theta_2 = 2\lambda$$

for the angle of diffraction, θ_2 , of the second order beam.

- 3 In general, the m th order wavefront is due to light from any slit travelling an extra distance of m wavelengths to be in phase with light emerging through the next slit. The angle of diffraction of the m th order wavefront, θ_m , is therefore given by the equation,

$$d \sin \theta_m = m\lambda$$

θ_m = angle of diffraction of m th order
 d = slit spacing (centre-to-centre)
 λ = wavelength of light

Notes

- 1 The number of lines per unit width on the grating, $N = 1/d$. Gratings are usually specified in terms of N rather than d . For example, for a grating that has 300 lines per millimetre, $N = 300 \text{ mm}^{-1}$ and $d = 1/300 = 3.33 \times 10^{-4} \text{ mm}$, since $d = 1/N$.
- 2 The maximum number of orders for light of a certain wavelength, λ , incident normally on a grating of spacing, d , is determined by the condition that the maximum value of $\sin \theta$ is 1 (which is when $\theta = 90^\circ$). To determine the maximum order number, substitute 1 for $\sin \theta_m$ in the diffraction grating equation to give $m = d/\lambda$. The maximum value of m is therefore d/λ rounded down to the nearest whole number.

12.1 WORKED EXAMPLE

A parallel beam of monochromatic light of wavelength 640 nm is directed normally at a grating. The angle of diffraction of the first order is 22.6° . Calculate

- the slit spacing of the grating,
- the number of diffracted beams,
- the angle of diffraction of the highest order.

Solution

a) $\lambda = 640 \text{ nm}$, $m = 1$, $\theta_m = 22.6^\circ$

Using $d \sin \theta_m = m\lambda$ gives $d \sin 22.6 = 1 \times 640 \times 10^{-9}$

$$\therefore d = 1 \times \frac{640 \times 10^{-9}}{\sin 22.6} = 1.67 \times 10^{-6} \text{ m}$$

b) $\frac{d}{\lambda} = \frac{1.67 \times 10^{-6}}{640 \times 10^{-9}} = 2.62 \quad \therefore \text{maximum order number} = 2$

There are two diffracted beams either side of the central beam.

c) For $m = 2$, the angle of diffraction of the second order beam, θ_2 , is given by $d \sin \theta_2 = 2\lambda$

$$\therefore \sin \theta_2 = \frac{2\lambda}{d} = \frac{2 \times 640 \times 10^{-9}}{1.67 \times 10^{-6}} = 0.776 \quad \therefore \theta_2 = 50.9^\circ$$

Measurement of the wavelength of light using a laser

**Note**

The reason why you must never look along a laser beam is that the eye lens would focus it to a tiny spot on the retina, concentrated sufficiently to damage the retina permanently.

A laser produces a narrow, parallel beam of light which is highly monochromatic (i.e. a single colour corresponding to a very narrow wavelength range). Use laser safety goggles and do NOT look along the direct beam or any reflected beam. The beam from the laser is directed normally at a large, white screen so a spot of laser light is observed on the screen. The diffraction grating is then positioned between the laser and the screen, parallel to the screen, as in Fig. 12.7. The diffracted orders are seen as a horizontal row of light spots on the screen.

Measurements and theory

- The perpendicular distance, x , from the grating to the screen is measured and the distance, y , of each spot from the central order spot is measured. The angle of diffraction of each order can be calculated using the trigonometric formula $\tan \theta = y/x$. See Fig. 12.8.
- The diffraction grating equation is used to measure the wavelength of the laser light. Each order should give the same value of wavelength, although errors of measurement may give some variation, in which case the average value should be calculated.

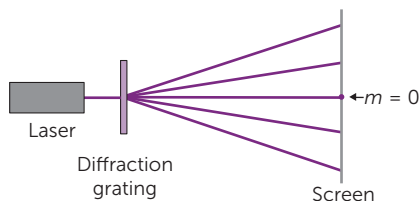


Fig. 12.7 Using a laser

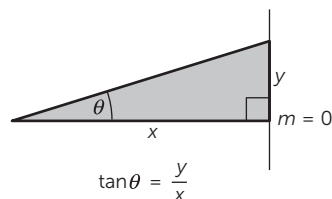


Fig. 12.8 Calculating the angle of diffraction

Measurement of the wavelength of light using a spectrometer

This is used to measure accurately the wavelengths of light in a spectrum. See Fig. 12.9. The light source is used to illuminate a narrow slit at the focal point of the collimator lens so a parallel beam of light emerges from the collimator tube. The diffraction grating is placed on the spectrometer turntable at 90° to the beam. The telescope is used to locate each diffracted beam, observed as a narrow line which is an image of the slit. Each wavelength of light present gives a narrow line of a particular colour for each order number. Thus each order gives a pattern of spectral lines on each side of the central line. The scale attached to the telescope is used to measure the position of each line. From the colour and scale reading of each observed line, the angle of diffraction and the order number can be worked out and the wavelength of each line calculated.

See www.palgrave.com/foundations/breithaupt for more about using a spectrometer.

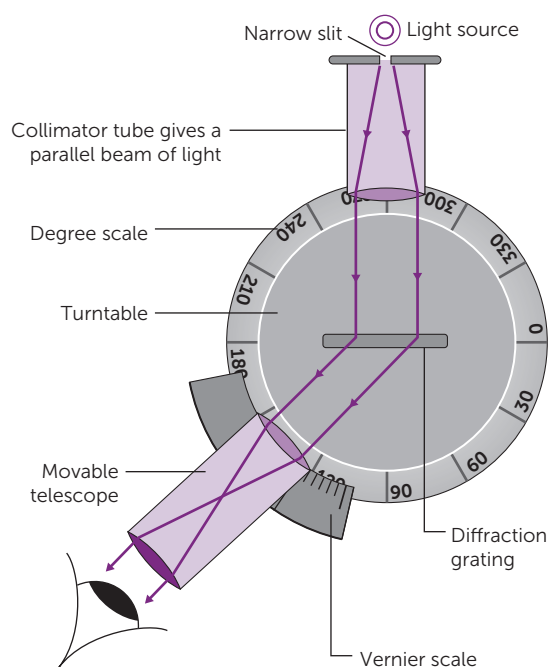


Fig. 12.9 Using a spectrometer

Note

The degree scale of a spectrometer is usually sub-divided into minutes, where 60 minutes (symbol ') equals 1° .

12.2 QUESTIONS

- The spectrum of light from a sodium vapour lamp was observed and measured using a spectrometer and a diffraction grating which had 600 lines per millimetre. The position of the telescope was adjusted until the crosswires were centred on the zero order slit image. The scale reading was then $101^\circ 22'$.
 - The telescope was then used to locate and measure the diffracted orders. A first order yellow line was observed at $122^\circ 06'$. Calculate the wavelength of this yellow line.
 - Calculate the angle of diffraction of each further order of this wavelength on the same side as the observed first order line.
- A spectrometer fitted with a diffraction grating was used to observe the spectrum of light from a certain discharge lamp known to emit red light at a wavelength of 629 nm.
 - A first order red line was observed at an angle of diffraction of $10^\circ 52'$. Calculate the number of lines per millimetre of the grating.
 - Calculate the maximum number of diffracted orders to be expected at 629 nm.

Note

In space, no object can travel as fast as electromagnetic waves. The speed of light in a medium is less than in free space and is equal to c/n where n is the refractive index of the medium.

12.3 Infra-red and beyond

Light is just one part of the spectrum of electromagnetic waves. The other parts of the electromagnetic spectrum are shown in Table 9.1 on p. 120. All electromagnetic waves propagate through space at a speed of $300\,000\text{ km s}^{-1}$. The symbol c is used for this speed.

Infra-red radiation

The visible spectrum extends to no more than 700 nm , fading from red to deep red to black beyond about 700 nm . A blackened thermometer bulb placed just beyond the longer wavelength end of the visible spectrum should show an increase of temperature due to infra-red radiation. See Fig. 12.10. This is electromagnetic radiation in the wavelength range from about 700 nm ($= 0.7\text{ }\mu\text{m}$) to about 0.1 mm ($= 100\text{ }\mu\text{m}$).

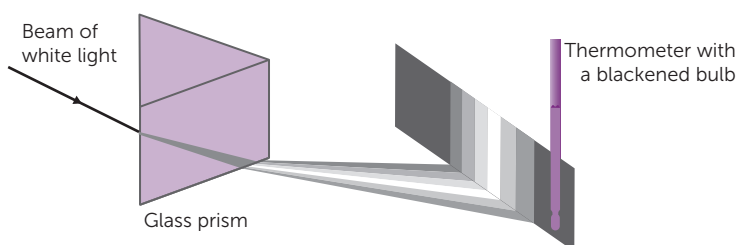


Fig. 12.10
Detecting
infra-red
radiation

Producing infra-red radiation

Infra-red radiation is emitted by every object. The hotter an object is, the more infra-red radiation it emits from its surface. If an object is hot enough to glow, its surface emits visible as well as infra-red radiation. The colder an object is, the less infra-red radiation it emits and the longer the wavelength of the radiation. If you enter a refrigeration room, you soon feel cold because you radiate away much more infra-red radiation to the walls of the room than you get back from the walls. As a result, you lose internal energy.

Absorbing infra-red radiation

The amount of infra-red radiation absorbed by a silvered surface is much less than for a black surface. Also, a surface that is smooth and shiny absorbs much less than a matt, rough surface. A good absorber of infra-red radiation is a good emitter; a poor absorber is a poor emitter. This is why a hot object wrapped in shiny aluminium foil retains internal energy longer than if unwrapped. Fig. 12.11 shows one way of comparing infra-red emission from different surfaces at the same temperature.

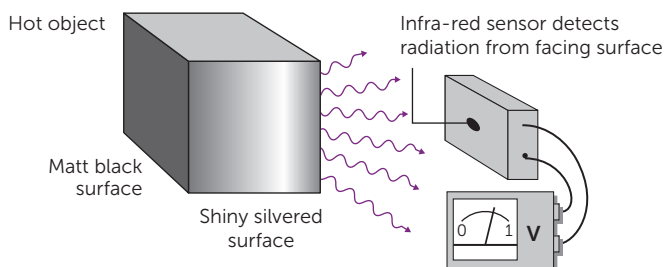


Fig. 12.11 Testing
different surfaces

Detecting infra-red radiation

Infra-red radiation can be detected using electronic sensors, as in an alarm system. A night-vision camera used on a satellite forms an infra-red image because it is fitted with an array of electronic sensors that detects infra-red radiation. See Fig. 12.12. Such cameras at ground level are used to 'see' people and animals in darkness, provided the surroundings are at a different temperature to the surface temperature of the bodies. Special infra-red sensitive photographic film can also be used to take infra-red photographs. Glass absorbs infra-red radiation beyond about $2\text{ }\mu\text{m}$, and other transparent substances such as quartz or certain liquids are used to make lenses and prisms capable of transmitting infra-red radiation beyond about $2\text{ }\mu\text{m}$.

Using infra-red radiation

- 1 Infra-red radiation of wavelength about $1\text{ }\mu\text{m}$ is used to carry digital signals along optical fibres. The frequency of such radiation is of the order of $3 \times 10^{14}\text{ Hz}$, which is about 10 000 times higher than microwave frequencies. Because the wavelength is so small, much more data can be carried in the form of digital signals in comparison with microwaves or radio waves. For example, a microwave link to or from a satellite can carry several TV channels. However, a fibre optic link can carry many more TV channels.
- 2 Infra-red radiation is used for heating purposes. For example, a halogen hob of an electric cooker contains a high-intensity compact bulb that is positioned between a reflector and the transparent plate of the hob. When the hob is switched on, infra-red radiation from the bulb is absorbed by a saucepan on the hob, after passing through the transparent plate. Infra-red heaters are used for industrial drying, for example to dry car bodies after spraying with paint.

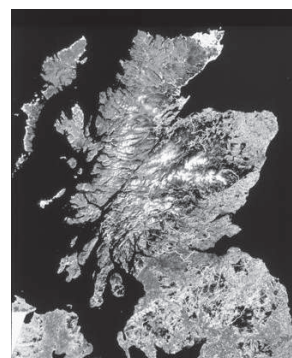


Fig. 12.12 An infra-red image of Scotland, taken from a satellite

12.3A QUESTIONS

1. Satellites fitted with infra-red cameras produce pictures which show populated areas, rural areas, rivers and coastlines. Why is it possible for such a camera to tell the difference between
 - a) sea and land,
 - b) a rural area and an urban area?
- 2 Explain why your hand would feel
 - a) warm if you placed it near a hot plate,
 - b) cold if you placed it near a beaker of ice.

Microwave radiation

Microwave radiation is electromagnetic radiation in the wavelength range from about 1 mm to about 0.1 m. The atmosphere does not absorb microwaves and so they are used for 'line of sight' communications. They are reflected by metals and partly absorbed by non-metals.

Producing and detecting microwaves

The magnetron valve is a vacuum tube which is designed to produce high-intensity microwave radiation. A microwave oven is fitted with a magnetron valve that produces microwaves at a frequency of 2.45 GHz. Electromagnetic waves of this frequency penetrate food and are absorbed by water molecules, causing internal

heating of the food. Low power microwaves are also produced by a different type of valve, known as a klystron valve, and by solid-state devices.

Microwaves can be detected using a point-contact diode in parallel with a sensitive meter or an amplifier.

Using microwaves

- 1 Microwaves are used for heating food and for drying non-conducting materials such as cotton. In a microwave oven, microwaves penetrate the food and heat it up. The food must be in a non-conducting dish, otherwise dangerous voltages are generated in the dish. Also, the food is rotated on a turntable because stationary waves and nodes are set up in the oven by microwaves reflected from different sides of the oven. No microwave power is delivered to food at a node, so the food is rotated in the oven to heat all parts.
- 2 Microwaves are used for communications because they pass through the atmosphere without significant absorption and they can be directed in beams with much less diffraction than radio waves. They are used to carry digital signals from ground stations to satellites and vice versa, and from one part of the country to another via transmitter and receiver dishes on towers and hilltops. Microwave frequencies are of the order of 10 GHz, sufficiently high to carry much more information in digital form than a copper cable or on radio waves.

12.3B QUESTIONS

- 1 What properties of microwaves make them suitable for
 - a) heating food rapidly,
 - b) carrying digital signals to and from a satellite?
- 2 In a microwave oven, why is it essential that
 - a) the food rotates on a turntable,
 - b) there is a safety switch so no microwaves are produced if the door is open?

Radio waves

Radio waves are electromagnetic waves of wavelengths about 0.1 m and longer. They are used for radio and TV broadcasting, mobile phone communications and local emergency channels. The radio spectrum is sub-divided into bands, as shown in Fig. 12.13.

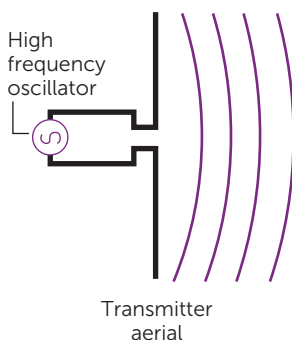


Fig. 12.14 Producing radio waves

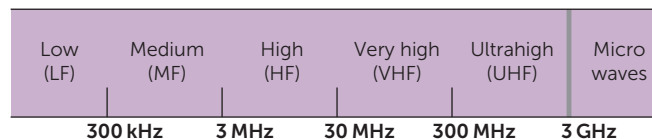


Fig. 12.13 The radio spectrum

Producing and detecting radio waves

Radio waves are generated when a high-frequency, alternating voltage is applied to a suitable transmitter aerial, as shown in Fig. 12.14. The electrons in the aerial are forced to and fro, which makes them generate radio waves in the surrounding space. The radio-wave frequency is equal to the frequency of the alternating voltage. When radio waves pass across a receiving aerial, they force the electrons in the aerial to move to and fro along the aerial, thus generating an alternating voltage which can then be detected.

Using radio waves

Radio waves are used to carry information through the atmosphere and through space. There are three main methods of carrying information using radio waves:

- 1 **Amplitude modulation (AM)** is where an audio or TV signal modulates the amplitude of the carrier wave. See Fig. 12.15. Long wave and medium wave radio channels use AM. A radio channel needs a 'bandwidth' of 4 kHz to carry most of the frequencies in an audio signal. A TV channel has a bandwidth of 8 MHz and is therefore carried by radio waves in the UHF band of frequencies, from 400 to 900 MHz.

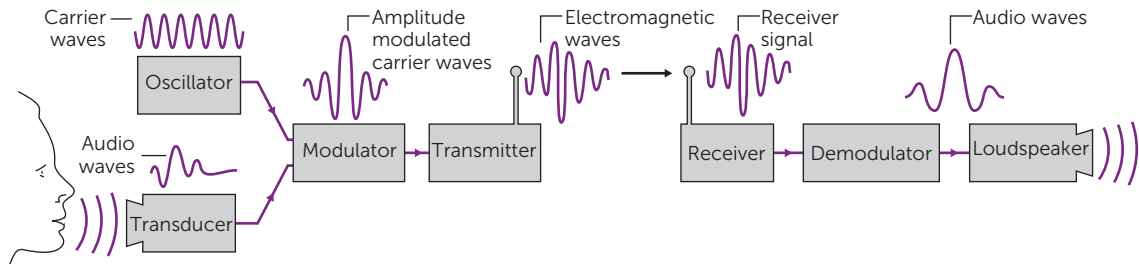


Fig. 12.15 Amplitude modulation

- 2 **Frequency modulation (FM)** is where the frequency of the carrier wave is modulated by the audio signal. See Fig. 12.16. A greater bandwidth is necessary, so carrier frequencies need to be in the VHF band, higher than for AM. FM broadcasts suffer less from distortion than AM.

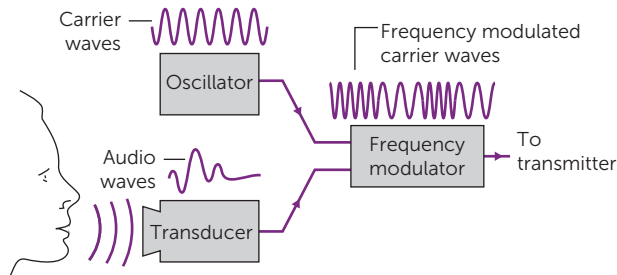


Fig. 12.16 Frequency modulation

3. **Pulse code modulation (PCM)** is where a signal is transmitted in digital form as a stream of pulses, carried by microwaves through the atmosphere or space, or by infra-red radiation in optical fibres. See Fig. 12.17. An audio signal or a video

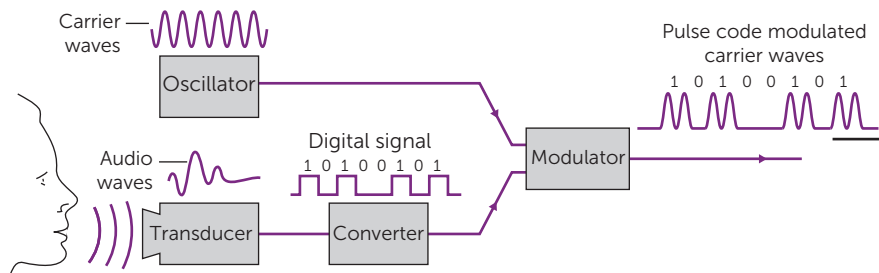


Fig. 12.17 Pulse code modulation

signal has a variable amplitude and is called an analogue signal. This signal is sampled periodically and each sample is converted to a digital signal as a sequence of pulses by an encoder circuit. At the receiver, the digital signal is then decoded to recreate the original analogue signal. PCM is scarcely affected by distortion since the pulses are regenerated along a long transmission link.

12.3C QUESTIONS

- 1 Radio waves of frequencies up to about 300 kHz follow the Earth's curvature. Radio waves from about 300 kHz to 30 MHz travel along the line of sight up to the ionosphere in the upper atmosphere, where they are reflected back to Earth. Beyond 30 MHz, radio waves pass through the atmosphere into space.
 - a) (i) Calculate the frequency of radio waves of wavelength 1500 m. (ii) 1500 m radio waves can be used to broadcast to distant countries. Why?
 - b) Local radio stations broadcast at frequencies between about 80 and 120 MHz. Give two reasons why local radio stations cannot be received in other parts of the country.
- 2 a) The picture quality of a portable TV is affected by altering the orientation of the aerial. What property of electromagnetic waves is responsible for this?
- b) A satellite TV dish points directly to a satellite in orbit in space, high above the Equator. Such dishes are larger and more difficult to align in Northern Europe than in Southern Europe. Why?

12.4 Ultraviolet radiation, X-radiation and gamma radiation

Ultraviolet radiation

Ultraviolet radiation is electromagnetic radiation beyond the violet end of the visible spectrum from wavelengths of about 400 nm to about 1 nm. Ultraviolet radiation is harmful to the eyes and its presence in sunlight causes sunburn and skin cancer.

Producing ultraviolet radiation

The Sun is a natural source of ultraviolet radiation. The ozone layer in the Earth's upper atmosphere filters out ultraviolet radiation from the Sun, but the depletion of this layer due to certain chemicals (e.g. CFCs) released into the atmosphere is a cause of continuing concern. People who stay outdoors in summer are advised to use appropriate skin creams to protect the skin from ultraviolet radiation. Sun lamps and certain types of discharge tubes also produce ultraviolet radiation.

Detecting ultraviolet radiation

Fluorescent dyes glow when exposed to ultraviolet radiation. The molecules in these dyes absorb ultraviolet radiation and emit visible light. Washing powders that contain such dyes makes clothing glow in ultraviolet radiation and give a 'whiter than white' effect in sunlight.

Photographic film is sensitive to ultraviolet radiation down to about 200 nm. Photocells are also used to detect ultraviolet radiation. See Fig. 12.18. In a vacuum photocell connected to a microammeter, ultraviolet radiation causes electrons to be emitted from the metal cathode, which creates a current through the microammeter.

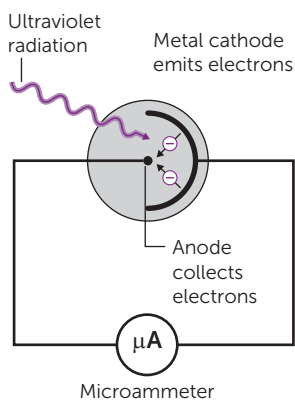


Fig. 12.18 A vacuum photocell

Using ultraviolet radiation

- 1 Security marker pens, used to mark equipment and banknotes, contain invisible ink which glows in ultraviolet radiation. Fluorescent pens can produce outstanding displays in ultraviolet radiation.
- 2 High-power ultraviolet lamps are used to rapidly dry printing ink. The paper is not warmed by the radiation, which is absorbed only by the ink.

12.4A QUESTIONS

- 1 Describe how you could detect whether or not ultraviolet radiation is present beyond the violet part of the visible spectrum.
- 2 Explain
 - a) why a white shirt glows under ultraviolet light,
 - b) how a skin cream designed to protect the skin from ultraviolet radiation works.

X-rays and gamma radiation

Electromagnetic radiation of wavelength less than about 1 nm is either X-radiation or gamma (γ) radiation depending on whether it is produced by an X-ray tube or by radioactive decay. X- and γ radiation both knock electrons off atoms and therefore create ions. Ionising radiation is harmful because it damages living cells. See p. 338 for more information.

- 1 In an X-ray tube, electrons are accelerated through a very high voltage onto a metal target, where they are suddenly stopped. As a result, X-rays are emitted by the electrons at the point of impact. Fig. 12.19 shows the construction of an X-ray tube.

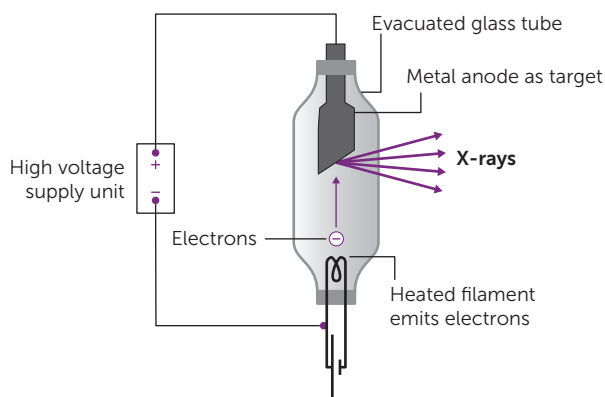


Fig. 12.19 Inside an X-ray tube

- 2 Unstable nuclei become stable or less unstable as a result of emitting one of three types of radiation known as alpha, or beta, or gamma radiation. Alpha radiation is stopped by paper or thin metal foil, whereas beta radiation requires several millimetres of lead to be stopped. Gamma radiation is even more penetrating and is only stopped by several centimetres of lead. See p. 352 for more about radioactivity.

Notes

- 1 Low energy X-rays are removed from the beam by placing a metal plate between the tube and the patient. Such X-rays would be absorbed by body tissue, causing unnecessary cell damage.
- 2 To take an X-ray photograph of organs such as the stomach, the organ is filled with a contrast medium which absorbs X-rays. For example, a barium drink is given to a patient who is to have a stomach X-ray.
- 3 Personnel in an X-ray unit must not be exposed to X-rays. The X-ray tube is surrounded by a thick lead shield with a window that allows a beam of X-rays through.

X-rays in medicine

X-rays are used to take photographs of internal organs and bones. This is possible because X-rays pass through body tissue but not through bone. As shown in Fig. 12.20, when an X-ray photograph is taken of a broken bone in a limb, X-rays are directed through the limb at a light-proof photographic plate or film behind the limb. The X-rays form a shadow of the bone on the film since it absorbs the X-rays whereas the surrounding tissue does not. If the bone is fractured, the fracture can be seen on the X-ray photograph. See Topic 21.5 for more about X-rays in medicine.

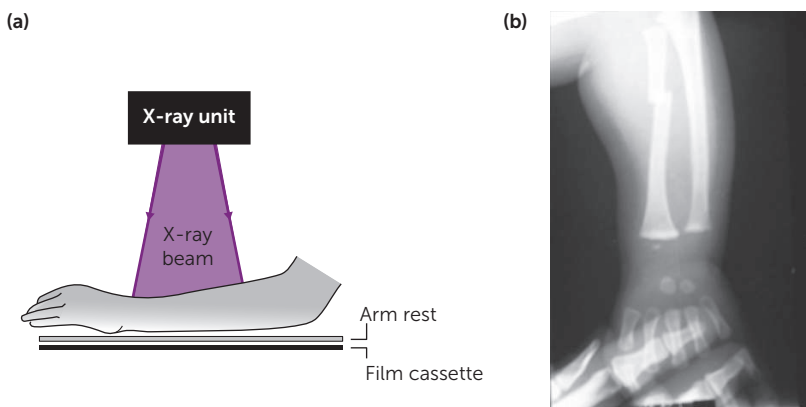


Fig. 12.20 Taking an X-ray picture (a) arrangement, (b) an X-ray photograph showing a fractured radius in the forearm of an infant. (Reproduced with the permission of Wellcome Images.)

12.4B QUESTIONS

- 1 When an X-ray picture is taken, why is it important that
 - a) the X-rays should originate from a small spot on the metal target?
 - b) the photographic film is in a light-proof cover?
- 2 a) Calculate the frequency of gamma radiation of wavelength 1×10^{-14} m. The speed of electromagnetic waves in space is 3.0×10^8 m s⁻¹.
 b) (i) Why is gamma radiation harmful? (ii) Why is gamma radiation used to destroy tumours?

SUMMARY

- Dispersion is the splitting of white light into the colours of the spectrum
- Primary (and complementary secondary) colours are red (cyan), green (magenta), blue (yellow).
- The wavelength range of light varies from about 400 nm for blue light to about 650 nm for red light.
- Spectra
 - a) A continuous spectrum contains all the colours of the spectrum.
 - b) A line spectrum consists of well-defined vertical lines of different colours.
- The diffraction grating equation

$$d \sin \theta_m = m\lambda$$

m = angle of diffraction of m th order
 d = slit spacing (centre-to-centre)
 λ = wavelength of light

● Beyond the visible spectrum

	Wavelength range	Source	Applications
X- and gamma radiation	less than 1 nm	X-ray tube, unstable nuclei	X-ray pictures in medicine, gamma therapy
Ultraviolet radiation	1–400 nm approx.	UV lamps, the Sun	security markers
Infra-red radiation	0.7–100 μm approx.	hot objects	heating, night-viewing
Microwave radiation	0.1–100 mm approx.	magnetron valve	heating, communications
Radio waves	more than 100 mm	radio transmitters	TV and radio broadcasting, mobile phone links

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

12.1 With the aid of a diagram, describe how you would

- split a narrow beam of white light into the colours of the spectrum,
- demonstrate that the critical angle for red light in glass is more than the critical angle for blue light.

12.2 Describe and explain the colour of

- a yellow shirt in blue light,
- a red poster carrying blue print in
 - blue light,
 - yellow light.

12.3 A laser beam was directed normally at a diffraction grating with 300 lines per millimetre. Diffracted beams were observed on a white screen at a distance of 1.50 m from the grating.

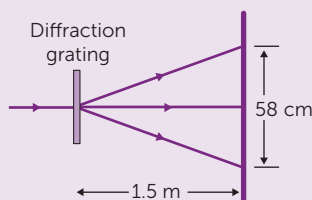


Fig. 12.21

- The first order beams were located on the screen at a distance of 58.0 cm apart as in Fig. 12.21. Calculate
 - the angle of diffraction of the first order beams,
 - the wavelength of the laser light.
- Calculate the maximum number of diffracted orders from this grating with this light.

12.4 A spectrometer fitted with a diffraction grating is used to measure the wavelength of a prominent blue line in the line spectrum of an element. In addition, the angle of diffraction of a first order prominent yellow line at 590 nm was measured. The telescope scale readings were as follows:

1st order blue line;
left of centre = $95^\circ 39'$ right of centre = $125^\circ 33'$
1st order yellow line;
left of centre = $89^\circ 52'$ right of centre = $131^\circ 20'$

- Calculate
 - the angle of diffraction for each line,
 - the number of lines per millimetre of the grating,
 - the wavelength of the blue light.
- Calculate the angle of diffraction for each further order of
 - the blue light,
 - the yellow light.

12.5

- Which part of the electromagnetic spectrum includes electromagnetic waves of wavelength
 - 100 m
 - 0.01 mm
 - 100 nm?
- Calculate the frequency of
 - radio waves of wavelength 1000 m in air,
 - microwaves of wavelength 3 cm in air,
 - light of wavelength $0.60 \mu\text{m}$ in air. The speed of electromagnetic waves in air is $3.0 \times 10^8 \text{ m s}^{-1}$.

12.6 What property or properties make each of the following types of electromagnetic radiation suitable for the application listed?

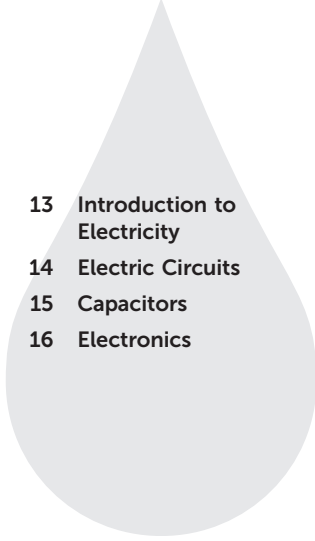
- Infra-red radiation; night surveillance cameras
- Ultraviolet radiation; security marker pens
- Microwaves; satellite links
- X-rays; X-ray imaging
- Gamma rays; gamma therapy
- Radio waves in the UHF range; mobile phone channels.

PART

4

Electricity

Where would you be without electricity? We take electricity for granted – until there's a power cut. Electricity in every home is perhaps the greatest development of the 20th century. In [Part 4](#), we consider the nature of electricity, how it is controlled and stored in electric circuits and how it is measured. The principles of electronics and its applications are also considered. Mathematical skills from earlier parts can be consolidated in this part, through circuit calculations, before moving on to the more demanding later parts.

- 
- 13** Introduction to Electricity
 - 14** Electric Circuits
 - 15** Capacitors
 - 16** Electronics

Introduction to Electricity

UNIT

13

OBJECTIVES

After working through this unit, you should be able to:

- explain the nature and cause of static electricity
- explain the nature of conductors, insulators and semiconductors
- measure an electric current and relate charge and current
- define and measure potential difference, and relate electrical power to current and potential difference
- describe the characteristics of common electrical components
- define resistance and calculate the resistance of a combination of resistors

CONTENTS

- 13.1 Static electricity
- 13.2 Current and charge
- 13.3 Potential difference
- 13.4 Resistance
- Summary
- Revision questions

13.1 Static electricity

Electric charge

Certain insulating materials, such as polythene and perspex, are easily charged with static electricity by rubbing with a dry cloth. Test a plastic ruler by rubbing it with a dry cloth, then see if it can pick up small bits of paper or if it generates a tiny spark when you hold its charged surface near a finger. Static electricity was first discovered in Ancient Greece when it was found that amber, a naturally occurring substance, produced sparks when rubbed. The word 'electricity' is derived from the Greek word for amber.

When investigations using many different materials were carried out more than 200 years ago, scientists discovered that there are two types of electric charge. They discovered that objects carrying the same type of charge always repel each other, whereas objects carrying different charges always attract each other. This simple rule explains many effects of static electricity. See [Fig. 13.1](#) overleaf.

Like charges repel; unlike charges attract.

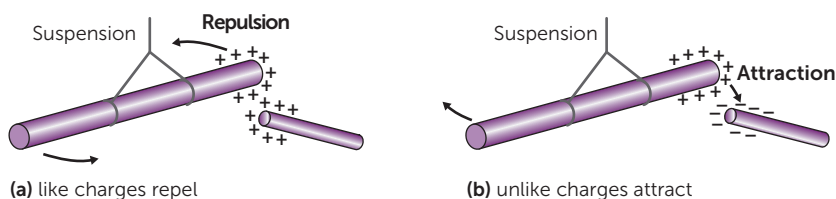


Fig. 13.1 Repulsion and attraction

The two types of charge cancel each other out. For example, if a charged object is placed in an insulated metal can, the can also becomes charged. If a second charged object carrying the other type of charge is also placed in the can, the can becomes uncharged. One type of charge, the charge on glass rubbed with silk, was therefore labelled 'positive' and the other type

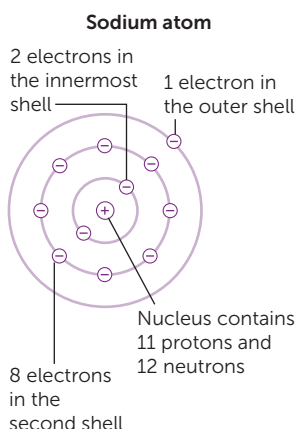


Fig. 13.2 Inside the atom

Inside the atom

The atom is the smallest particle of an element that is characteristic of the element. Every atom contains a positively charged nucleus, where most of the atom's mass is located. The nucleus is surrounded by electrons which are much lighter, negatively charged particles. The electrons are held in the atom by the force of attraction of the oppositely charged nucleus. They occupy certain orbits called 'shells' round the nucleus. Each shell can hold a certain number of electrons, which fill the shells from the innermost outwards. In an uncharged atom, the total negative charge of the electrons is equal and opposite to the positive charge of the nucleus. Fig. 13.2 represents an atom of the metallic element, sodium.

Explaining static electricity

Rubbing a perspex rod with a dry cloth causes the rod to become positively charged. The reason why this occurs is because the friction between the rod and the cloth detaches electrons from the atoms of the rod and they transfer to the cloth. Rubbing a polythene rod with a dry cloth causes the rod to become negatively charged. The reason why this occurs is because the rod gains electrons from the dry cloth.

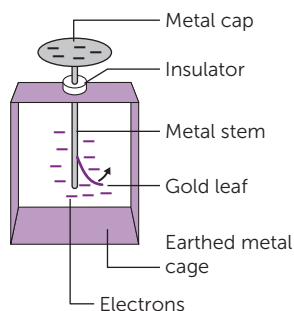


Fig. 13.3 The gold leaf electroscope

Experiment to investigate static electricity

- 1 The gold leaf electroscope in Fig. 13.3 is designed to detect static electricity. Charge placed on the cap spreads to the stem and the gold leaf of the electroscope. Because the stem and leaf acquire the same charge, the leaf is repelled by the stem. If the cap is touched by hand, the leaf returns to its original position because the charge on the electroscope leaks to earth through the body. Test different insulating materials to find out how easily each material can be charged by friction.
- 2 To determine the type of charge on a charged object, charge the electroscope with a known charge (e.g. negative charge from a charged polythene rod). If the leaf rises further when an unknown charged object is held above the cap, the unknown object must carry the same type of charge as the known charge.

3 Charging by induction An insulated metal object can be charged without direct contact with a charged body. Fig. 13.4 shows how this occurs. In (a), a negative rod held near an uncharged insulated conductor forces electrons to the other side of the conductor. Touching the conductor briefly allows these electrons to flow to earth. When the rod is removed, the conductor is left with a positive charge. In (b), a positive rod held near the conductor attracts electrons in the conductor. When the cap is touched briefly, electrons flow from earth onto the conductor. When the rod is removed, the conductor is left negatively charged.

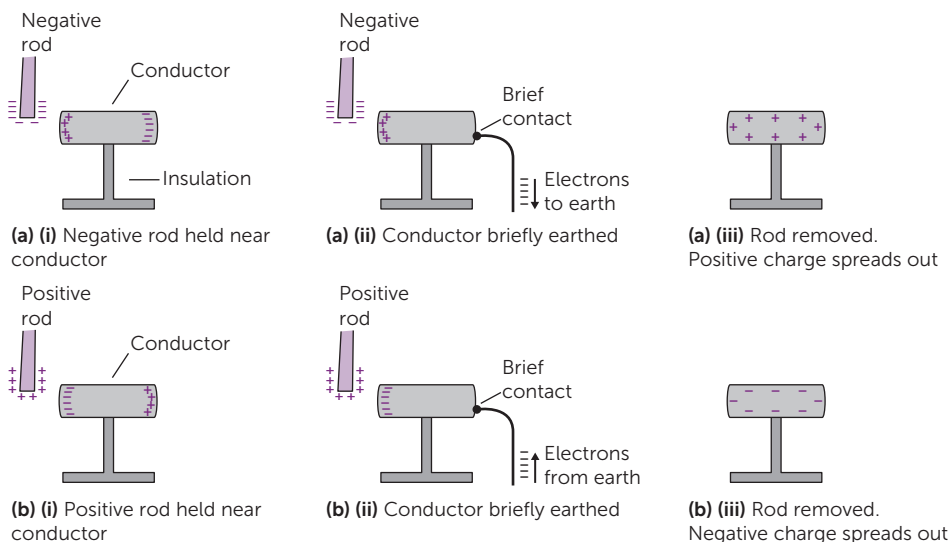


Fig. 13.4 Charging by induction (a) positively (b) negatively

Static hazards

Sparks generated by static electricity can cause flammable vapours and powders to explode. Antistatic flooring in an operating theatre is necessary to prevent any static electricity building up, as anaesthetic gases are flammable. Oil tanks and pipes are earthed to prevent static electricity building up. Earthing an object allows electrons to transfer between it and the ground, which prevents static electricity building up.

13.1 QUESTIONS

- 1 a) Explain in terms of electrons why a polythene rod becomes negatively charged when it is rubbed with a dry cloth.
b) A charged polythene rod is held near one end of a charged rod that is freely suspended in a horizontal position. The suspended rod is repelled by the polythene rod. What type of charge does the suspended rod carry?
c) Given a polythene rod, a perspex rod, a dry cloth and a gold leaf electroscope, describe how you would find out the type of charge carried by a rod of unknown material that can be charged by rubbing with the dry cloth.
- 2 a) Integrated circuits can be damaged by static electricity. Describe how the metal pins of a microcomputer chip could become accidentally charged without direct contact with a charged body.
b) When oil is delivered from an oil tanker to a storage tank, why is it necessary to earth the tanker and the delivery pipe?

13.2 Current and charge

Conductors and insulators

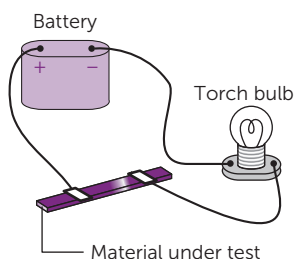


Fig. 13.5 Testing conductors and insulators

Metals and graphite are electrical conductors. Wood, glass, air and most plastic materials are examples of electrical insulators. Electricity is supplied through copper wires because copper is a very good conductor of electricity. Electrical wires are usually insulated using PVC, since PVC is flexible, a good insulator and it does not deteriorate with age. Fig. 13.5 shows how to test a material to find out if it is an electrical conductor. The light bulb can be replaced with an **ammeter**, an electrical meter used to measure current.

Metals conduct electricity because they contain free electrons which are not fixed to the atoms in the metal. As explained in Unit 5 (p. 72), an atom consists of a positively charged nucleus where most of its mass is located. The nucleus is surrounded by much lighter electrons which move about the nucleus in allowed orbits called 'shells'. The electrons are negatively charged. In an uncharged atom, the negative charge of all its electrons is equal to the positive charge of the nucleus. Atoms become negatively or positively charged ions as a result of gain or loss of electrons.

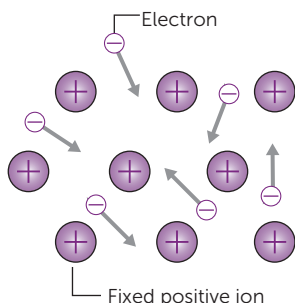


Fig. 13.6 Electrical conduction

- In a **metal**, the atoms lose their outermost electrons to form a lattice of positive ions. The outermost electrons move about at random between the ions, acting like a glue which holds the ions in place as shown in Fig. 13.6. These electrons are referred to as **conduction** (or free) electrons and they are responsible for the conduction of electricity by the metal. In an electric circuit, the electric current in a metal is the flow of charge due to free electrons being attracted towards the positive end of the metal.

- In an **electrical insulator**, all the electrons are fixed rigidly to the atoms and cannot move throughout the material. As explained in Unit 5 (p. 76), the bonds that hold atoms together in non-metallic substances are due to the transfer or sharing of electrons between the atoms. These electrons are referred to as **valence** electrons. If an insulator is connected in an electric circuit, no current is possible in any part of the circuit because the electrons in the insulator cannot move towards the positive end of the insulator.

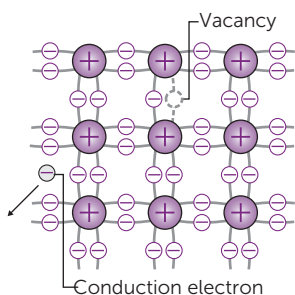


Fig. 13.7 Semiconductors

- **Semiconductors** are materials that are insulators only at a temperature of absolute zero. All the electrons in a semiconductor at absolute zero are rigidly held by individual atoms in the structure. Above absolute zero, a semiconductor conducts electricity because some of the electrons break free from the atoms as shown in Fig. 13.7. The higher the temperature of a semiconductor, the more easily it conducts electricity. This is because more and more electrons break free from individual atoms as the temperature of the semiconductor is raised. Semiconductors, such as the elements silicon and germanium, are used to make integrated circuits used in microcomputers. Compound semiconductors, such as gallium arsenide (GaAs), are used to make opto-electronic devices such as laser diodes.

A simple electric circuit

An electric current is a flow of charge. When a torch bulb is connected to a battery, the torch bulb lights because electrons flow around the circuit, transferring energy from the battery to the bulb. Work is done on the electrons inside the battery and they are then forced from the negative pole of the battery, through the bulb and back to the battery via the positive pole. The electrons repeatedly collide with the atoms of the bulb filament as they are forced through the filament, transferring their energy to the atoms as a result.

Electricity flows one way only in the electric circuit shown in Fig. 13.5. This is an example of a direct current circuit. Later, you will meet alternating current circuits, in which the current direction repeatedly reverses. The one-way nature of a direct current was first discovered by André Ampère over two centuries ago. Before Ampère's discovery, it was thought that positive electricity from the positive pole of a battery reacted in the filament with negative electricity from the negative pole to produce heat and light. Ampère discovered that an electric current along a wire created a magnetic field around the wire. He did this by showing that the electric current caused the needle of a nearby plotting compass to deflect. More significantly, he showed that reversing the connections of the wire to the battery made the needle deflect in the opposite direction. This can only happen if current flows one way only around the circuit.

Ampère did not know whether an electric current was a flow of positive charge or a flow of negative charge. He thought it was a flow of positive charge, so he marked the direction of the current from 'positive to negative'. Even though we now know that negatively charged particles (i.e. electrons) are responsible for the current in a metal, the 'positive to negative' rule is still the standard convention. The importance of Ampère's work was recognised by naming the S.I. unit of electric current as the ampere (A). This is defined in terms of the magnetic effect of an electric current, as explained on p. 274.

Electric current effects

- 1 The **heating effect** of an electric current can be demonstrated using the arrangement in Fig. 13.8. The current is increased by adjusting the variable resistor. As the current increases, the thin strand of wire heats up and glows red then orange before it melts. This is how a fuse works. If the current becomes too large, the fuse wire melts and the circuit is then broken. Take care not to touch the wire when it is hot. The element of an electric heater is a wire that heats up when an electric current passes through it.

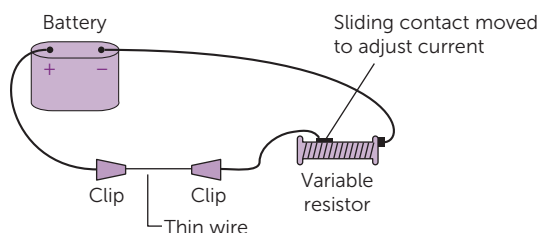
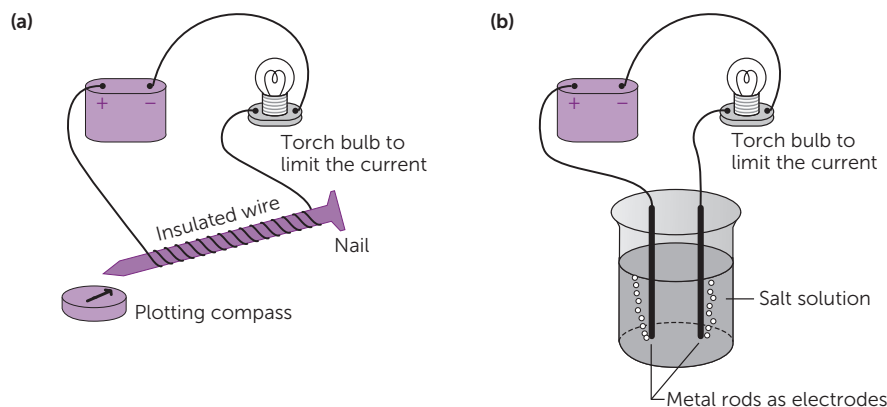


Fig. 13.8 The heating effect of an electric current

Fig. 13.9 More effects of an electric current **(a)** magnetic effect, **(b)** chemical effect



- 2 The **magnetic effect** of an electric current can be demonstrated using a plotting compass, or by winding an insulated wire around a nail as shown in Fig. 13.9a. When connected to a suitable battery, the magnetic effect of the current magnetises the nail, which can then attract small iron or steel objects such as paper clips. The nail loses its magnetism when the current is switched off. An electromagnet used to move vehicles in a scrapyard is no more than a coil of insulated wire wrapped round an iron core.
- 3 The **chemical effect** of an electric current can be demonstrated by passing an electric current through a beaker of salt water. The two metal rods in the solution connected to the circuit are called **electrodes**. Gas is evolved at the electrodes, demonstrating that the solution is decomposed by the electric current. This is an example of **electrolysis**. See Fig. 13.9b.

Application 6

DNA sequencing

The nucleus of a human cell has 23 pairs of chromosomes, all containing deoxyribonucleic acid (DNA) molecules. A DNA molecule is a long thin double-stranded chain of molecules with cross-links like the rungs of a rope ladder. Each rung is a pair of 'base' molecules, either adenine (A) joined to thymine (T) or cytosine (C) joined to guanine (G). Genes, our genetic characteristics, are determined by the sequence of the bases in different sections of the DNA molecule. These sections carry the codes for making proteins which are the 'building block' molecules of the human body. The Human Genome project has now established the complete sequence of the 3 billion bases in a human DNA molecule and the sequence for some of our genes.

To map the Human Genome, fragments of DNA are tagged with a C, G, A or T base containing a dye. Each tagged fragment carries a negative charge. A voltage is applied across a strip of gel with a spot of the DNA fragments on. The fragments move to the positive electrode. The smaller they are, the faster they move so they separate out according to size. The fragments pass through laser spot causing each one to fluoresce. Light sensors linked to a computer detect the glow from each fragment. The computer is programmed to work out and display the sequence of bases in the DNA fragment.



Fig 13.10 Mapping the human genome

Current and charge

In the circuit in Fig. 13.5, the amount of electric charge passing through the torch bulb is proportional to the number of electrons that pass through it. Each electron carries the same tiny amount of negative charge. The current through the torch bulb is a measure of the charge or the number of electrons that pass through the bulb each second.

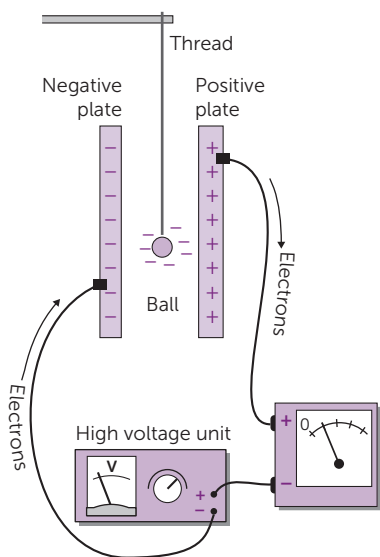


Fig. 13.11 The shuttling ball experiment

The link between current and charge is demonstrated in Fig. 13.11 where electrons are transferred from the negative to the positive plate, causing a reading to register on the meter. Each time the ball touches the negative plate, it gains electrons to become negative. It is then attracted to the positive plate where it releases electrons to become positively charged due to a shortage of electrons. It is then attracted back to the negative plate to continue the cycle, transferring the same amount of negative charge each time it moves from the negative to the positive plate. If the plates are moved closer together, the ball shuttles back and forth between the two plates at a faster rate, causing the current to increase. The current is therefore a measure of the rate of transfer of charge.

One coulomb of charge is defined as the charge transferred in one second when the current is exactly one ampere.

In more general terms, the charge, Q , transferred in coulombs (abbreviation C) is given by the equation

$$Q = It$$

where I = current in amperes,

t = time in seconds.

13.1 WORKED EXAMPLE

- a) Calculate the charge that passes a point in a circuit in 1 minute when the current is 0.5 A.
- b) Calculate the current that would transfer, in 10 s, the same amount of charge as a current of 0.5 A in 1 minute.

Solution

- a) Charge $Q = It = 0.5 \text{ A} \times 60 \text{ s} = 30 \text{ C}$
- b) Rearrange $Q = It$ to give $I = \frac{Q}{t} = \frac{30 \text{ C}}{10 \text{ s}} = 3.0 \text{ A}$.

Current rules

- Current is measured using an ammeter.** To measure the current in a wire, an ammeter must be connected in series with the wire. An ideal ammeter has no electrical resistance so it does not affect the current to be measured.
- The current leaving a component is equal to the current entering a component.** Fig. 13.12a shows a circuit that can be used to test this. Fig. 13.12b shows the corresponding circuit diagram. Further circuit symbols are shown on p. 188. In any component through which a steady current passes, the number of electrons leaving the component is equal to the number of electrons entering the component in the same time. In other words, the current out of a component is equal to the current into the component.

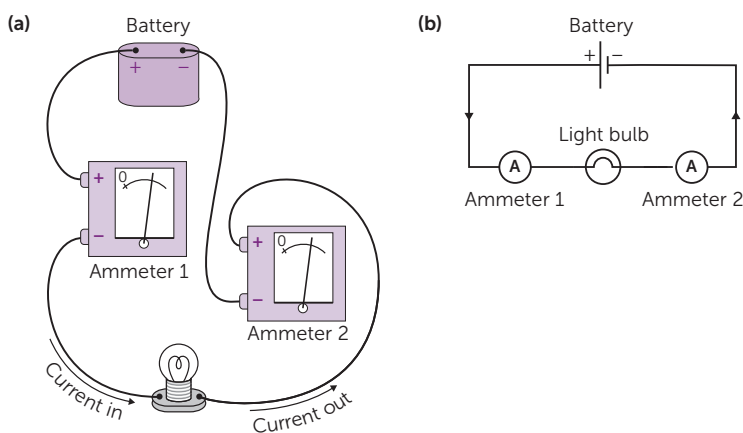


Fig. 13.12 Current entering and leaving a component
(a) circuit connections,
(b) circuit diagram

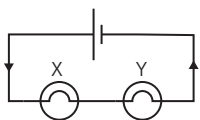


Fig. 13.13 Components in series

- Components in series pass the same current.** Fig. 13.13 shows two torch bulbs in series connected to a battery. An electron from the battery passes through one bulb then through the other bulb before returning to the battery. The number of electrons per second passing through one bulb is the same as through the other bulb. In other words, the same current passes through each bulb.
- At a junction in a circuit, the total current entering the junction is equal to the total current leaving the junction.** Fig. 13.14 shows a circuit in which two torch bulbs, X and Y, are connected to a battery. Each bulb can be switched on or off

without affecting the other one. The two bulbs are said to be in parallel with each other. An electron from the battery passes either through X or through Y before returning to the battery. Consequently, the flow of electrons from the battery divides at junction Q and recombines at junction P. Measurements show that the reading of the ammeter in series with the battery is equal to the sum of the readings of the ammeters in series with the torch bulbs.

$\therefore$ current from the battery = current through X + current through Y

More generally, the total current entering a junction is equal to the total current leaving the junction.

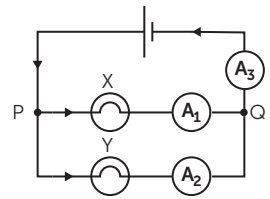


Fig. 13.14 Components in parallel

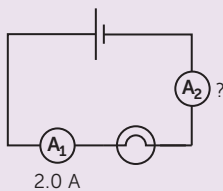
13.2 QUESTIONS

- a) Which of the following materials are not electrical conductors?
air aluminium glass germanium graphite

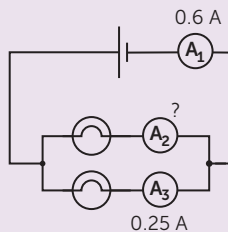
b) Copy and complete the table opposite by inserting the missing values.
- Calculate the missing current in each of the circuits in Fig. 13.15.

Current/A	(i)	0.60	15.00	(iv)
Charge/C	15.00	(ii)	45.00	25.00
Time/s	3.00	0.50	(iii)	10.00

(a)



(b)



(c)

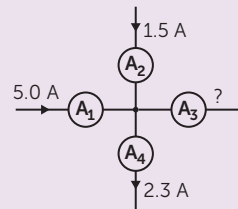


Fig. 13.15

13.3 Potential difference

Electrical energy and potential difference

In Fig. 13.9, an electric current passes through the torch bulb filament only if the filament is part of a complete circuit and the circuit includes a battery. The battery converts chemical energy into electrical energy and is therefore a source of electrical energy. The torch bulb converts electrical energy into heat and light and is therefore a sink of electrical energy.

The potential difference between two points in an electrical circuit is the electrical energy delivered per unit charge when charge flows from one point to the other.

The unit of potential difference is the **volt** (V) which is equal to one joule per coulomb. For example, when a 6.0 V battery is connected to a 6.0 V torch bulb, every coulomb of charge from the battery delivers 6.0 J of electrical energy to the bulb. Potential difference, sometimes abbreviated as p.d., is often referred to as **voltage**.

$$V = \frac{E}{Q}$$

where E = energy delivered and Q = charge.

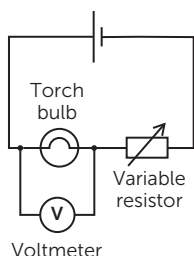


Fig. 13.16 Using a voltmeter

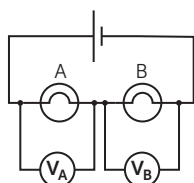


Fig. 13.17 Adding voltages

See www.palgrave.com/foundations/breithaupt for an experiment to calibrate a voltmeter

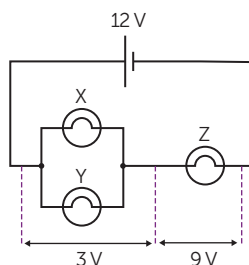


Fig. 13.18 Parallel components have the same voltage

Voltage rules

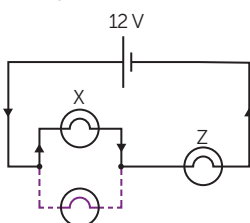
- 1 **A voltmeter is used to measure the potential difference between two points in a circuit.** An ideal voltmeter has infinite resistance so it does not draw any current from the circuit to which it is connected. Fig. 13.16 shows a voltmeter connected across a torch bulb which is in series with a variable resistor. The voltmeter is connected in parallel with the bulb so it measures the potential difference across the bulb only. Charge from the battery delivers some of its electrical energy to the bulb and some to the variable resistor.
- 2 **For two or more components in series, the total voltage is equal to the sum of the individual voltages.** In Fig. 13.17, the two light bulbs are in series and therefore pass the same current. Each coulomb of charge passing through bulb A also passes through bulb B. The potential difference across each light bulb is the electrical energy delivered per coulomb of charge.

Suppose the p.d. across light bulb A is 3 V and the p.d. across light bulb B is 9 V. Each coulomb of charge delivers 3 J of electrical energy to A and 9 J to B. Therefore the total electrical energy delivered per coulomb is 12 J ($= 9 \text{ J} + 3 \text{ J}$). Hence the total p.d. is 12 V across A and B.

$$V = V_A + V_B$$

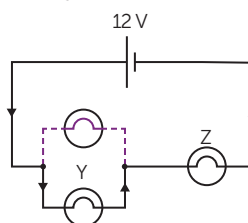
- 3 **For components in parallel, the voltage is the same.** In Fig. 13.18, bulbs X and Y are in parallel with each other and in series with bulb Z and a battery. Each coulomb of charge from the battery flows around the circuit through Z then through X or Y as shown in Fig. 13.19. Suppose the p.d. across Z is 9 V and the battery p.d. is 12 V. Each coulomb of charge therefore delivers 12 J to the light bulbs as a result of delivering 9 J to Z and 3 J to either X or Y. Thus the p.d. across either X or Y is 3 V.

(a) through X and Z



Each coulomb of charge passing through X and Z delivers 3 J to X and 9 J to Z.

(b) through Y and Z



Each coulomb of charge passing through Y and Z delivers 3 J to Y and 9 J to Z.

Fig. 13.19 Two routes for charge flow

Electrical power

As explained on p. 56, power is the rate of transfer of energy. Thus a heating element with a rating of 1000 W dissipates heat at a rate of 1000 J s^{-1} . If the potential difference across the torch bulb in Fig. 13.16 is represented by V , then the energy, E , delivered to the bulb as a result of charge Q passing through the bulb is QV . In time t , the charge flow, $Q = It$, where I is the current. Hence the energy delivered is given by the equation

$$E = QV = ItV$$

Since the power, P , supplied to the torch bulb is the energy delivered per second,

$$P = \frac{E}{t} = \frac{ItV}{t} = IV$$

Energy delivered in time t , $E = ItV$

Power supplied, $P = IV$

where I = current, V = potential difference.

Fuse ratings

A fuse wire is designed to melt if excessive current passes through it. The current rating of a fuse is the maximum current the fuse is capable of conducting without melting. See Fig. 13.20. The current rating of a fuse intended to protect an appliance from excessive current can be calculated from the power and potential difference (i.e. voltage) using the equation $I = P \div V$.

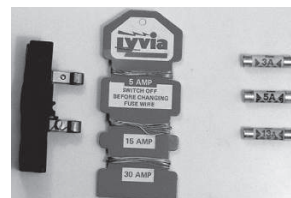


Fig. 13.20 Domestic fuses

13.2 WORKED EXAMPLE

An electric kettle operates at 240 V and has a power rating of 3000 W.

a) Calculate

- the energy supplied to the kettle in exactly 5 minutes,
- the current through the kettle element.

b) Which of the following fuses is suitable for this appliance: 1 A, 3 A, 5 A, 13 A?

Solution

$$\begin{aligned} \text{a) (i) Energy supplied} &= \text{power} \times \text{time} \\ &= 3000 \text{ W} \times 300 \text{ s} \\ &= 9.0 \times 10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{(ii) Rearrange } P &= IV \text{ to give } I = \frac{P}{V} \\ &= \frac{3000}{240} \\ &= 12.5 \text{ A} \end{aligned}$$

b) The 13 A fuse.

13.3 QUESTIONS

1 A 12 V, 36 W electric heater is connected to a 12 V battery. Calculate

- the current,
- the charge flow through the heater in 10 minutes,
- the energy delivered to the heater in this time.

2 In a test to determine the power and current supplied to a 230 V electric oven, all the appliances in the household

were switched off and the electricity meter reading noted. The electric oven was then switched on for exactly 30 minutes and the meter reading was found to have increased by 2.8 kWh. Calculate

- the electrical energy supplied in joules,
- the charge flow in this time,
- the current.

Notes

- SI units must be used throughout. Current is always in amperes, potential difference in volts, time in seconds, energy in joules and power in watts.
- Prefixed commonly used for numerical values include mega (M) for 10^6 , kilo (k) for 10^3 , milli (m) for 10^{-3} and micro (μ) for 10^{-6} .
- Domestic electricity meters record the electrical energy used in kilowatt hours (kWh). 1 kWh is the electrical energy supplied at a constant rate of 1 kW for 1 hour. Prove for yourself that $1 \text{ kWh} = 3.6 \text{ MJ}$.
- The power equation can be rearranged as

$$I = \frac{P}{V}$$

to find I , given values for P and V , or as

$$V = \frac{P}{I}$$

to find V , given values for P and I .

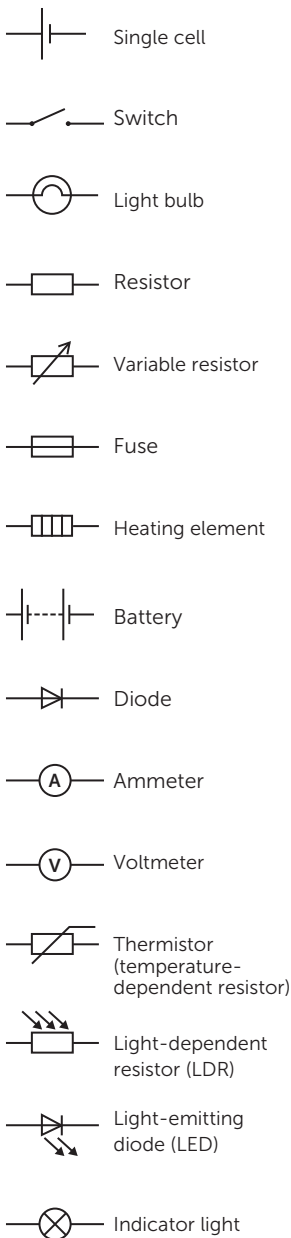


Fig. 13.21 Circuit symbols

13.4 Resistance

Symbols and diagrams

You need to learn the standard symbols used for each type of component in circuit diagrams. Major problems could arise if a motorist who needs to change a fuse in a vehicle lighting circuit confuses the symbols for a fuse and an electric heater. The standard symbols for common electrical components are shown in Fig. 13.21. In addition to recognising these symbols, you need to know the general characteristics of each type of component.

Resistors

The current in a circuit depends on the voltage of the power supply or battery and on the resistance of the circuit components. The electrical resistance of a conductor is due to the opposition by the atoms of the conductor to the flow of electrons through the conductor. This is not unlike a central-heating system, where the flow of water depends on the width of the pipes and radiators as well as the pump which forces water around the system. The pipes and radiators resist the flow of water. The narrower the pipes, the greater the resistance to flow. A resistor is designed to limit the current in a circuit by resisting the flow of charge through it.

The **resistance** R of a component is defined by the equation

$$R = \frac{V}{I}$$

where V = the potential difference across the component, and I = the current through the component.

The unit of resistance is the ohm (symbol Ω pronounced 'omega') which is equal to 1 volt per ampere.

Notes

- 1 The equation $R = V \div I$ can be rearranged to $V = IR$ to calculate V from values of I and R or to $I = V \div R$ to calculate I from values of V and R .
- 2 The power dissipated in a resistor, $P = IV = I^2R$ since $V = IR$.
- 3 Resistors are colour-coded to show the value of resistance. Fig. 13.22 shows the resistor colour code.

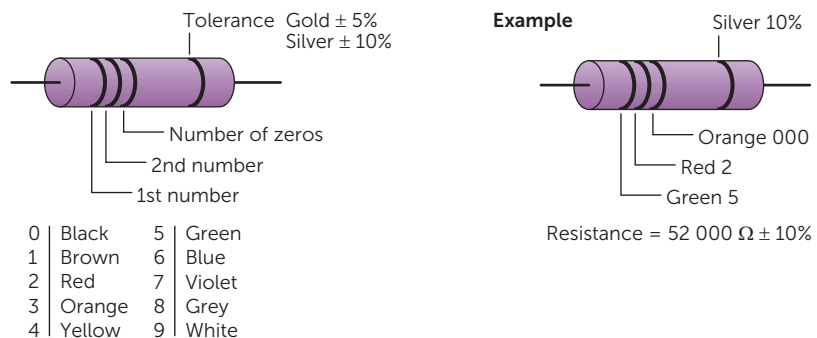


Fig. 13.22 Resistor colour code

13.4A QUESTIONS

1 Copy and complete the table:

Current/A	2.00	0.30	?	2.5×10^{-3}	?
Potential difference/V	15.00	?	1.50	6.00	5.0×10^4
Resistance/D	?	6.8×10^2	2.2 k	?	100 M

2 Fig. 13.23 shows the colour-coding of a certain resistor in a circuit.

- What is the resistance of this resistor?
- What current would pass through this resistor if it was connected across a 6.0 V battery?

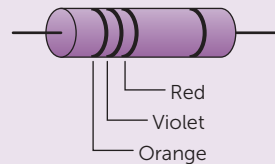


Fig. 13.23

Experiment to investigate the characteristics of circuit components

A wire-wound resistor

Fig. 13.24a shows how the current through a wire-wound resistor can be measured for different potential differences applied across the resistor. The variable resistor is adjusted to alter the current. The measurements may be plotted on a graph of y = potential difference against x = current, as in Fig. 13.24b.

- The results plotted on the graph define a straight line passing through the origin. The potential difference, V , is therefore proportional to the current, I .
- The resistance is constant, the same value being obtained for $V \div I$ whatever value for I is used, provided the corresponding value of V is used.
- Because the line is straight and passes through the origin, the gradient of the line is equal to the resistance. A different resistor would give a steeper line if its resistance was greater.

A light bulb filament

The resistor in Fig. 13.24 is replaced with a light bulb. The results plotted on a graph of y = potential difference against x = current define a curve of increasing steepness, as in Fig. 13.25. This shows that the resistance of the filament increases as the current increases.

The reason why this happens is that the filament becomes hotter as the current increases. The resistance of any metal increases with increase of temperature, hence the filament resistance increases as its temperature rises with increasing current.

A silicon diode

A diode conducts electricity in one direction only, referred to as the 'forward' direction. Fig. 13.26a (overleaf) shows a diode connected into a circuit in the forward direction. Reversing the diode in the circuit enables it to be tested in the reverse direction.

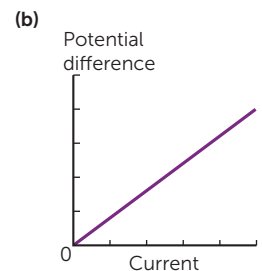
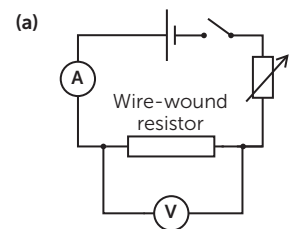


Fig. 13.24 Investigating component characteristics
(a) circuit diagram, (b) typical results

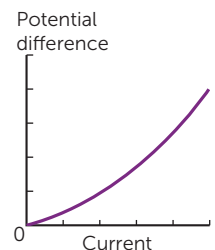


Fig. 13.25 Potential difference versus current for a light bulb filament

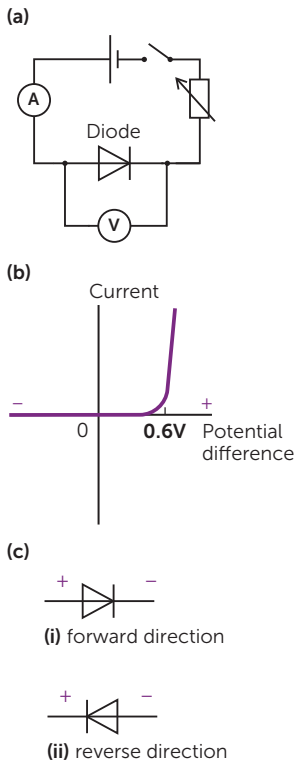


Fig. 13.26 Testing a diode
(a) circuit diagram, (b) typical results, (c) diode directions

The results are usually plotted on a graph of $y = \text{current}$ against $x = \text{potential difference}$, as in Fig. 13.26b. This graph shows that

- the forward resistance of the diode is very low when it conducts
- the diode has a very high resistance in the reverse direction.

In addition, a silicon diode only conducts if the applied potential difference exceeds a threshold of about 0.6 V. The current increases sharply with little change of potential difference beyond about 0.6 V.

If the current through the diode is allowed to become too great, the diode overheats and stops conducting permanently. For this reason, a diode in a circuit is always connected in series with a resistor, which therefore limits the current passing through the diode.

Light-emitting diodes (LEDs) emit light when they conduct. For this reason, they are used as indicators in electronic circuits. The threshold voltage for an LED is higher than that for an ordinary diode.

13.3 WORKED EXAMPLE

A light-emitting diode with a maximum current rating of 40 mA and a threshold voltage of 2.0 V is to be used in a circuit where the p.d. is 5.0 V, as in Fig. 13.27. Calculate the resistance of a suitable resistor that should be connected in series with the LED.

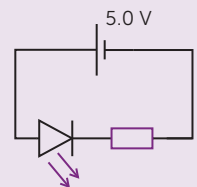


Fig. 13.27

Solution

The p.d. across the resistor = $5.0 - 2.0 = 3.0 \text{ V}$

Current = 0.04 A

$$\therefore \text{resistance of resistor, } R = \frac{V}{I}$$

$$= \frac{3.0 \text{ V}}{0.04 \text{ A}} = 75 \Omega$$

Resistor combination rules

Resistors in series

Fig. 13.28 shows two resistors in series. The current, I , is the same through two or more resistor in series.

Hence the p.d. across resistor R_1 , $V_1 = IR_1$

the p.d. across resistor R_2 , $V_2 = IR_2$

the p.d. across resistor R_3 , $V_3 = IR_3$, etc.

Since the total p.d. $V = V_1 + V_2 + V_3 + \dots$

the total resistance, $R = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \dots}{I}$

$$\therefore R = \frac{V_1}{I} + \frac{V_2}{I} + \frac{V_3}{I} + \dots$$

$R = R_1 + R_2 + R_3 + \dots$ for resistors in series

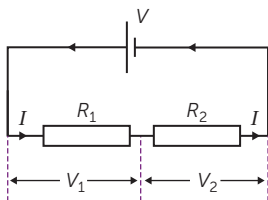


Fig. 13.28 Resistors in series

13.4 WORKED EXAMPLE

A $12\ \Omega$ resistor is connected in series with a $6\ \Omega$ resistor and a 9 V battery. Sketch the circuit diagram and calculate

- the resistance of the two resistors in series,
- the current and p.d. for each resistor.

Solution

a) $R = 12 + 6 = 18\ \Omega$

b) $I = \frac{V}{R} = \frac{9\text{ V}}{18\ \Omega} = 0.5\text{ A}$ for both resistors

$$V_1 = IR_1 = 0.5\text{ A} \times 12\ \Omega = 6.0\text{ V}; \quad V_2 = IR_2 = 0.5\text{ A} \times 6\ \Omega = 3.0\text{ V}$$

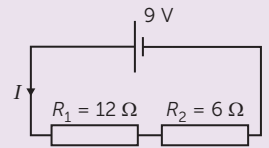


Fig. 13.29

Resistors in parallel

Fig. 13.30 shows two resistors in parallel. The potential difference across two or more resistors in series is the same.

Hence the current through resistor R_1 , $I_1 = \frac{V}{R_1}$

the current through resistor R_2 , $I_2 = \frac{V}{R_2}$

the current through resistor R_3 , $I_3 = \frac{V}{R_3}$, etc.

Since the total current through the combination $I = I_1 + I_2 + I_3 + \dots$

and the total resistance $R = \frac{V}{I}$

then $\frac{1}{R} = \frac{I}{V} = \frac{I_1 + I_2 + I_3 + \dots}{V}$

hence $\frac{1}{R} = \frac{I_1}{V} + \frac{I_2}{V} + \frac{I_3}{V} + \dots = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$ for resistors in parallel.

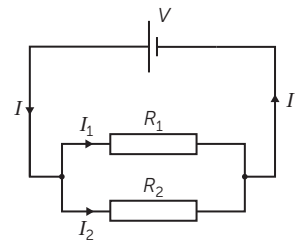


Fig. 9.30 Resistors in parallel

13.5 WORKED EXAMPLE

A $5\ \Omega$ resistor is connected in parallel with a $20\ \Omega$ resistor and the combination is connected to a 12 V battery. Sketch the circuit diagram and calculate

- the resistance of the two resistors,
- the current from the battery,
- the current through each resistor,
- the power dissipated in each resistor.

Solution

The circuit diagram is shown in Fig. 13.31.

a) Using $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{20} = 0.20 + 0.05 = 0.25$

$$\therefore R = \frac{1}{0.25} = 4.0\ \Omega$$

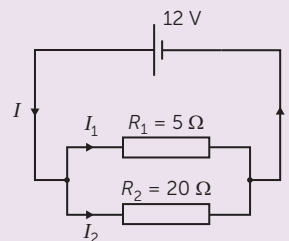


Fig. 13.31

13.5 WORKED EXAMPLE *continued*

b) Using $I = \frac{V}{R}$ gives $I = \frac{12}{4.0} = 3.0 \text{ A}$

c) $I_1 = \frac{V}{R_1} = \frac{12}{5} = 2.4 \text{ A}; \quad I_2 = \frac{V}{R_2} = \frac{12}{20} = 0.6 \text{ A}$

d) Power, $P = IV$

$\therefore P_1 = 2.4 \times 12 = 28.8 \text{ W}; \quad P_2 = 0.6 \times 12 = 7.2 \text{ W}$

13.4B QUESTIONS

- 1 a) A 3Ω resistor is connected in series with a 6Ω resistor and a 9 V battery. Sketch the circuit diagram and calculate (i) the resistance of the two resistors in series, (ii) the current through each resistor, (iii) the p.d. across and power dissipated in each resistor.
- b) The two resistors in (a) are connected in parallel to the same battery. Sketch the circuit diagram and calculate (i) the resistance of the two resistors in parallel, (ii) the current through each resistor, (iii) the p.d. across and power dissipated in each resistor.
- 2 a) Sketch the possible resistance combinations of a 2Ω resistor, a 3Ω resistor and a 6Ω resistor.
- b) Calculate the resistance of each combination in (a).

SUMMARY

- **Law of force between charged objects**

Like charges repel; unlike charges attract.

- **Equations**

Charge, $Q = It$, where I = current, t = time

Potential difference, $V = \frac{E}{Q}$ where E = electrical energy transferred by charge Q

Power, $P = IV$

Resistance, $R = \frac{V}{I}$

Resistor combination rules $R = R_1 + R_2$ for two resistors in series

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ for two resistors in parallel}$$

- **Units**

Current in amperes (A), charge in coulombs (C), p.d. in volts (V), power in watts (W), resistance in ohms (Ω).

- **Circuit rules**

1 **Components in series:** same current; total p.d. = sum of individual p.d.s.

2 **Components in parallel:** same p.d.; total current = sum of individual currents.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

13.1 Explain the following observations:

- If held briefly on the ceiling, a balloon will stick to the ceiling if it has been rubbed beforehand.
- A gold leaf electroscope is charged by direct contact with a perspex rod. The leaf of the electroscope rises further when a plastic ruler is rubbed, then held over the cap of the electroscope.
- A person sitting on a plastic chair receives an electric shock when she stands up and touches a door handle.

13.2

- Explain why the resistance of a semiconductor falls when it is heated.
- Explain why the resistance of a metal wire rises when it is heated.

13.3 A 1.5 V, 0.25 A torch bulb is connected to a 1.5 V battery for exactly 10 minutes.

- Calculate
 - the charge passing through the bulb in this time,
 - the power supplied to the bulb,
 - the electrical energy delivered to the bulb in 10 minutes.
- The battery is capable of lighting the bulb for 1 hour. Calculate
 - the charge passing through the bulb in 1 hour, (ii) the total electrical energy the battery can deliver.

13.4 Fig. 13.32 shows two 3 V, 0.1 A torch bulbs, X and Y, in parallel with each other. The combination is in series with a variable resistor and a 4.5 V battery. The variable resistor is adjusted until the two bulbs light normally.

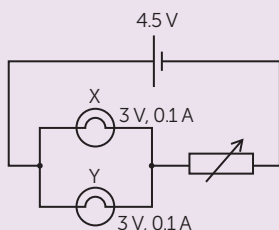


Fig. 13.32

- Calculate
 - the current from the battery,
 - the p.d. across the variable resistor,
 - the power supplied to each bulb,
 - the power supplied by the battery to the circuit.
- Account for the difference between the power supplied by the battery and the power supplied to the two bulbs.

13.5

- A hair drier is rated at 230 V, 750 W maximum. Calculate
 - the maximum current,
 - the number of units of electricity it would use in 30 minutes.

- Which of the following fuse ratings would be suitable for this hairdrier:

1 A 3 A 5 A 13 A

13.6 In an experiment to measure the resistance of a heating element, the following measurements were made:

Current / A	0.00	0.25	0.38	0.48	0.57	0.66
Potential difference / V	0.00	2.00	4.00	6.00	8.00	10.00

- Plot a graph of $y = \text{potential difference}$ against $x = \text{current}$.
- Use your graph to determine the resistance of the resistor at
 - 5.0 V,
 - 10.0 V.
- Explain why the resistance changed as the potential difference was increased.

13.7 A 2.0 A diode with a threshold voltage of 0.6 V is to be connected in its forward direction in series with a 9.0 V battery, and a resistor chosen to limit the current to 2.0 A.

- Sketch the circuit diagram.
- Calculate
 - the potential difference across the resistor,
 - the resistance of the resistor.

13.8 A $6\ \Omega$ resistor and a $4\ \Omega$ resistor are connected in series with each other and a 6.0 V battery.

- Sketch the circuit diagram.
- Calculate
 - the resistance of the two resistors in series,
 - the battery current,
 - the p.d. across each resistor,
 - the power dissipated in each resistor.

13.9 A $6\ \Omega$ resistor and a $12\ \Omega$ resistor are connected in parallel with each other and a 12 V battery.

- Sketch the circuit diagram.
- Calculate
 - the resistance of the two resistors in parallel,
 - the battery current,
 - the current through each resistor,
 - the power dissipated in each resistor.

13.10 In the circuit in Q13.8, a $3\ \Omega$ resistor is connected in parallel with the $6\ \Omega$ resistor.

- Sketch this circuit diagram.
- Calculate
 - the total resistance of the combination,
 - the battery current,
 - the p.d. across each resistor,
 - the current through each resistor,
 - the power dissipated in each resistor.

UNIT

14

Electric Circuits

CONTENTS

- 14.1 Cells and batteries
- 14.2 The potentiometer
- 14.3 The Wheatstone bridge
- 14.4 Resistivity
- 14.5 Electrical measurements
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- define and measure the e.m.f. and internal resistance of a cell
- describe a potential divider and use a potentiometer to compare the e.m.f.s of two cells
- use a Wheatstone bridge to measure resistance
- define and measure resistivity and conductivity
- explain how to extend the range of an ammeter or a voltmeter
- use a multimeter to make electrical measurements

14.1 Cells and batteries**Inside a cell**

Cells and batteries are used whenever a portable source of electrical power is needed. In addition, certain cells are used to calibrate voltmeters and to check other electrical measurements. One of the first batteries was made by Volta in 1800, who stacked discs of zinc and silver alternately in a pile, interleaved with paper soaked in brine as shown in Fig. 14.1. He discovered that the battery produced an electric current through a wire connected between the ends of the pile. He also found that an insulated metal object was charged positively when connected to the silver disc at the end of the pile, with the lead disc at the other end earthed. When the connections were reversed, the object charged negatively.

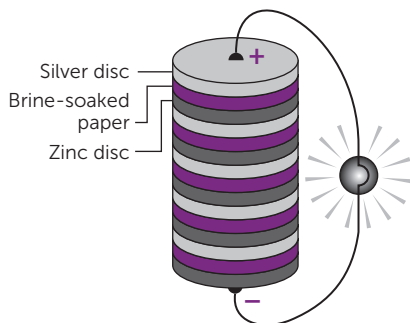


Fig. 14.1 A voltaic pile

Primary cells are not rechargeable. Once a primary cell has run down, it must be replaced by a fresh cell. Torch batteries and non-rechargeable calculator batteries consist of one or more primary cells in series.

Fig. 14.2 shows a simple primary cell consisting of a copper plate and a zinc plate as electrodes, with dilute sulphuric acid as the electrolyte. Zinc is more reactive than copper in dilute sulphuric acid. Zinc atoms leave the plate and go into the solution as positive zinc ions, making the zinc plate negative. Hydrogen ions in the acid are attracted to the copper plate, making the copper plate positive. In a complete circuit, electrons would be forced around the circuit, leaving the cell via the zinc plate and returning via the copper plate.

In a torch battery or a calculator battery, the electrolyte is a paste and the metal case of the cell is one of the electrodes.

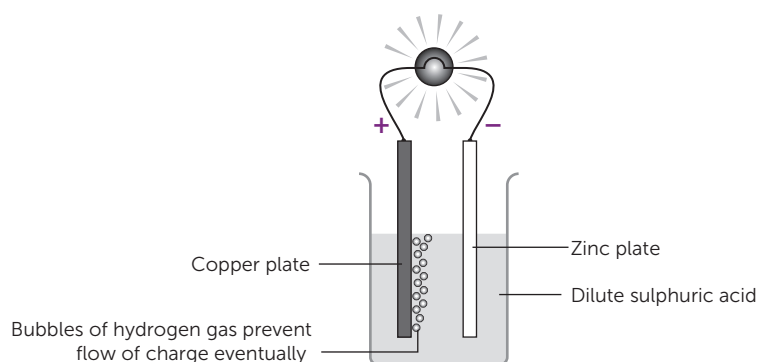


Fig. 14.2
A simple cell

Secondary cells are rechargeable. Most car batteries consist of six 'lead-acid' cells in series. Each cell consists of a set of lead plates as one electrode, interleaved with a set of lead dioxide plates as the other electrode, as shown in Fig. 14.3. Lead from the lead plates dissolves in the dilute sulphuric acid, each lead atom from the plate going into solution as a positive ion, leaving behind electrons on the lead plate. Hydrogen ions attracted to the lead dioxide plate combine with oxygen atoms in the lead dioxide to form water molecules. The electrolyte becomes weaker, therefore, as the lead dissolves more and more.

To recharge the cell, a battery charger is used to force electrons into the cell via the lead plate. The lead ions in the solution are attracted back onto the lead plates, lead dioxide forms on the other plate and the strength of the acid is increased.

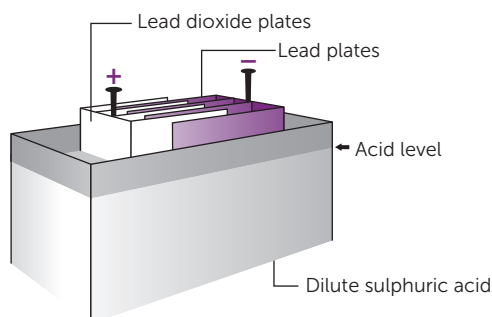


Fig. 14.3 A lead-acid battery
(cap not shown)

Electromotive force

A cell forces electrons around a complete circuit, from negative to positive. The electrons are supplied with electrical energy in the cell as a result of chemical reactions inside the cell. They are then forced out of the cell at the negative terminal, provided the cell is part of a complete circuit. As the electrons pass through each component in the circuit, the electrons do work in the component, converting their electrical energy into other forms of energy.

The electromotive force (e.m.f.) of a cell is the electrical energy per unit charge created in the cell.

The unit of e.m.f. is the volt, equal to 1 joule per coulomb. Note that the volt is also the unit of potential difference. However, potential difference is defined as electrical energy delivered per unit charge, whereas e.m.f. is electrical energy created per unit charge. Thus a source of e.m.f. is a source of electrical energy, whereas a potential difference is a 'sink' of electrical energy.

Table 14.1 Types of cells

Type of cell	e.m.f / V	Rechargeable
alkaline cell	1.5 V	No
mercury cell	1.35 V	No
nickel–cadmium cell	1.2 V	Yes
lead–acid cell	2.0 V	Yes

The e.m.f. of a cell depends on the substance used as the electrolyte and on the materials which the electrodes are made from. Table 14.1 gives the e.m.f.s of some different types of cells.

When a cell is in a circuit with current passing through the cell, energy is wasted in the cell because of the cell's internal resistance. The electrons use electrical energy to move through the cell against the resistance of the electrolyte and the electrodes. The electrical energy per coulomb delivered to the circuit components is less than the e.m.f. of the cell because electrical energy is used inside the cell. The potential difference across the cell terminals (i.e. the electrical energy delivered to the circuit components) is therefore less than the e.m.f. of the cell. The difference between the e.m.f. and the cell p.d. is in effect 'lost voltage' inside the cell. The internal resistance of the cell is the lost voltage per unit current.

Electrical energy per coulomb created in the cell (the cell's e.m.f.)
 = electrical energy per coulomb delivered to the circuit (cell p.d.)
 + electrical energy per coulomb used inside the cell (lost voltage)

In the circuit shown in Fig. 14.4, a cell of e.m.f. E , and internal resistance, r , is connected to a resistor, R . Hence

$$E = IR + Ir$$

where I is the current, the term IR is the cell p.d. and the term Ir is the lost voltage.

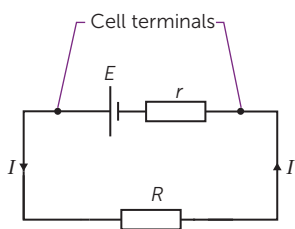


Fig. 14.4 Internal resistance

14.1 WORKED EXAMPLE

A cell of e.m.f. 1.5 V and internal resistance 0.5Ω is connected to a 4.5Ω resistor. Calculate

- the total resistance of the circuit,
- the current through the 4.5Ω resistor,
- the potential difference across the cell terminals.

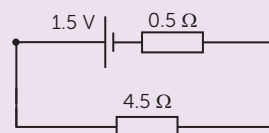


Fig. 14.5

Solution

- Total resistance $= 4.5 + 0.5 = 5.0\Omega$ since the internal resistance and the external resistance are in series.
- Using $I = \frac{V}{R}$ gives $I = \frac{1.5}{5.0} = 0.3\text{ A}$
- The cell p.d. is the same as the p.d. across the 4.5Ω resistor
 $= 0.3\text{ A} \times 4.5\Omega = 1.35\text{ V}$

Note

The lost voltage $= 0.3\text{ A} \times 0.5\Omega = 0.15\text{ V}$, which is the difference between the e.m.f. and the cell p.d.

Experiment to measure the e.m.f. and the internal resistance of a cell

Use the circuit in Fig. 14.6 to measure the cell p.d. for different currents delivered by the cell. Use the variable resistor to alter the current. Note the torch bulb is present to limit the current from the cell. Tabulate your measurements and use them to plot a graph of the cell p.d. on the vertical axis against the current on the horizontal axis. Use the theory below and your graph to calculate the e.m.f. of the cell and its internal resistance.

- Theory** The cell p.d. $V = IR = E - Ir$, where E is the cell e.m.f., r is the cell's internal resistance and I is the current. This is the equation for a straight line in the form, $y = mx + c$, provided V is plotted on the y-axis and I on the x-axis. Thus the gradient, $m = -r$ and the y-intercept, $c = E$ as in Fig. 14.7.

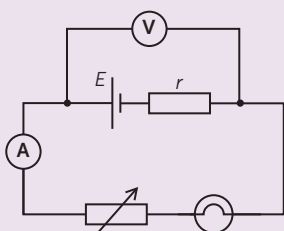


Fig. 14.6 Testing a cell

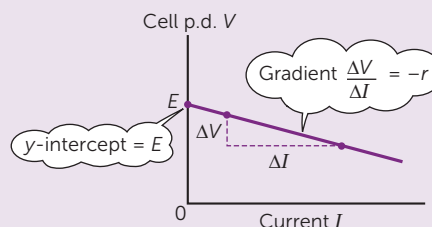


Fig. 14.7 Cell p.d. against current

14.2 WORKED EXAMPLE

A cell of unknown e.m.f., E , and internal resistance, r , was connected in series with a resistance box of resistance, R , and an ammeter, as shown in Fig. 14.8.

When the resistance, R was 8.0Ω , the current was 0.50 A . When the resistance, R was changed to 4.0Ω , the current was 0.90 A . Calculate the e.m.f. and the internal resistance of the cell.

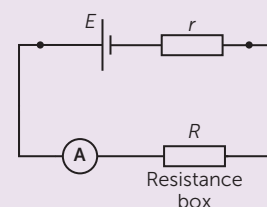


Fig. 14.8

Solution

Use $E = IR + Ir$

When $R = 8.0\Omega$, $I = 0.50\text{ A}$ $\therefore E = (0.5 \times 8.0) + 0.5r = 4.0 + 0.5r$

When $R = 4.0\Omega$, $I = 0.90\text{ A}$ $\therefore E = (0.90 \times 4.0) + 0.9r = 3.6 + 0.9r$

continued overleaf

14.2 WORKED EXAMPLE *continued***Solution**

$$E = 3.6 + 0.9r; \quad E = 4.0 + 0.5r$$

These two equations are a pair of simultaneous equations that can be solved to find E and r as follows:

$$E = 3.6 + 0.9r = 4.0 + 0.5r$$

$$\therefore 0.9r - 0.5r = 4.0 - 3.6$$

$$\therefore 0.4r = 0.4, \text{ so } r = \frac{0.4}{0.4} = 1.0 \, \Omega \text{ and } E = 4.0 + (0.5 \times 1.0) = 4.5 \, \text{V}$$

14.1 QUESTIONS

- 1 A cell of e.m.f. $2.0 \, \text{V}$ and internal resistance $0.5 \, \Omega$ is connected in series with an ammeter and a torch bulb. The ammeter reads $0.25 \, \text{A}$.
 - a) Sketch the circuit diagram, including the internal resistance of the cell.
 - b) Calculate the p.d. across the bulb and the power delivered to the bulb.
 - c) Calculate the power supplied by the cell.
 - d) Explain why more power is supplied by the cell than is delivered to the bulb.
- 2 A cell of unknown e.m.f., E , and internal resistance, r , is connected in series with a resistance box. A voltmeter is then connected across the terminals of the resistance box. When the resistance of the resistance box was $16.0 \, \Omega$, the voltmeter read $1.20 \, \text{V}$. When the resistance of the resistance box was reduced to $8.0 \, \Omega$, the voltmeter reading fell to $1.00 \, \text{V}$.
 - a) Sketch the circuit diagram and explain why the voltmeter reading decreased as a result of increasing the resistance of the resistance box.
 - b) Calculate the e.m.f. and the internal resistance of the cell.

14.2 The potentiometer**The potential divider**

A potential divider is used to supply a specified voltage from a fixed voltage supply. For example, a $5 \, \text{V}$ electronic circuit could be powered from a $9 \, \text{V}$ battery using a potential divider. Fig. 14.9 shows a potential divider consisting of two resistors connected to a fixed voltage supply. The potential difference across each resistor is a fixed fraction of the potential difference across the two resistors.

Since the current, $I = \frac{V}{(R_1 + R_2)}$

then the p.d., V_1 , across resistor $R_1 = IR_1 = \frac{R_1}{(R_1 + R_2)}V$

and the p.d., V_2 , across resistor $R_2 = IR_2 = \frac{R_2}{(R_1 + R_2)}V$

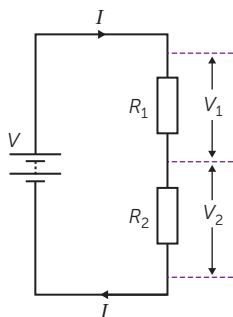


Fig. 14.9 The potential divider

14.3 WORKED EXAMPLE

A potential divider is to be used to supply a fixed potential difference of $5.0 \, \text{V}$ using a $10 \, \text{k}\Omega$ resistor, a $9.0 \, \text{V}$ battery and a resistor R , as shown in Fig. 14.10 opposite.

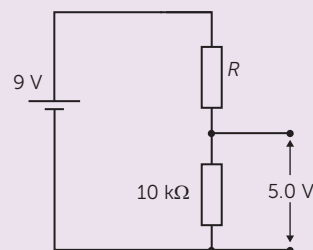
- a) Calculate the current through the $10 \, \text{k}\Omega$ resistor when the p.d. across it is $5.0 \, \text{V}$.
- b) Calculate the potential difference across R when the p.d. across the $10 \, \text{k}\Omega$ resistor is $5.0 \, \text{V}$.
- c) Calculate the resistance of R for a potential difference of $5.0 \, \text{V}$ across the $10 \, \text{k}\Omega$ resistor.

Solution

$$\text{a) } I = \frac{V}{R} = \frac{5.0}{10 \times 10^3} \\ = 5.0 \times 10^{-4} \text{ A}$$

$$\text{b) P.d. across } R = 9.0 - 5.0 = 4.0 \text{ V}$$

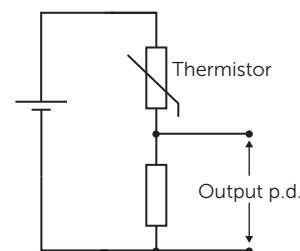
$$\text{c) Use } R = \frac{V}{I} \text{ to give } R = \frac{4.0}{5.0 \times 10^{-4}} \\ = 8000 \, \Omega (= 8 \text{ k}\Omega)$$

**Fig. 14.10**

Using a potential divider in a sensor circuit

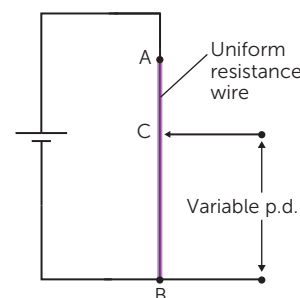
In Fig. 14.10, a different value for R would give a different voltage across each resistor. For example, a value of $10 \text{ k}\Omega$, the same as the other resistor, would give a p.d. of 4.5 V across each resistor. The supply voltage is shared between the two resistors in proportion to their resistances. Altering the resistance ratio alters the proportions in which the supply voltage is shared between the two resistors.

If resistor, R , in Fig. 14.10 is replaced by a thermistor (a temperature-dependent resistor) or a light-dependent resistor (LDR), the p.d. across the other fixed resistor varies according to the temperature or the light intensity respectively. Fig. 14.11 shows the idea. The circuit in Fig. 14.11 supplies an 'output p.d.' across the fixed resistor that increases with temperature (or with light intensity if an LDR is used). The output p.d. can be used to operate a relay or to switch on a warning indicator, as explained in Unit 16.

**Fig. 14.11** Using a thermistor

Supplying variable p.d.

This is achieved by connecting a battery across a fixed length of wire of variable p.d. between the sliding contact and either end of the wire, as in Fig. 14.12. The wire may be coiled around a circular track or wound around an insulated tube or replaced by a uniform, thin-layered track of carbon or other resistive material. In Fig. 14.12, the p.d. between B and C increases from zero up to the battery voltage as the sliding contact C is moved from B to A.

**Fig. 14.12** A variable potential divider

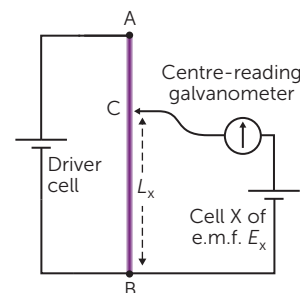
The principle of the potentiometer

A potentiometer is a potential divider used to compare or measure e.m.f.s. It can also be used to measure internal resistance and to calibrate an ammeter or a voltmeter. The slide-wire potentiometer consists of a uniform, straight length of resistance wire with a 'driver' cell or battery connected across its ends, as in Fig. 14.13. The p.d. between the sliding contact, C, and end, B, of the wire, V_{BC} , is therefore in proportion to the length, L , of wire between B and C.

$$V_{BC} = kL$$

where k is a constant.

- 1 A test cell, X, and a centre-reading galvanometer in series are connected between B and C, as in Fig. 14.13. If the sliding contact, C, is moved down the wire from A to B, the meter deflection changes direction at some point between A and B. The

**Fig. 14.13** The slide-wire potentiometer

position of C on the wire where the meter reading is exactly zero, the balance point, is located precisely. At this point, the potential difference between B and C opposes the e.m.f. of cell X exactly.

$$E_X = kL_X$$

where L_X is the length of wire between B and C needed to balance the e.m.f. of X. There is no lost voltage across the internal resistance of X when the meter reads zero because there is no current through X at the balance point.

- The procedure is repeated using a standard cell, S, with an accurately known e.m.f., E_S , in place of X. The new balance point is located and the new balance length, L_S , is measured.

$$E_S = kL_S$$

provided the driver cell continues to supply the same p.d. across the wire. Hence

$$\frac{E_X}{E_S} = \frac{kL_X}{kL_S} = \frac{L_X}{L_S} \text{ to give } E_X = \frac{L_X}{L_S} E_S$$

Thus E_X can be calculated if E_S is known.

14.4 WORKED EXAMPLE

A potentiometer consisting of a metre length of uniform resistance wire with a driver cell connected across the wire was used to measure the e.m.f. of a cell X. The cell was connected between one end of the wire and a sliding contact on the wire, as in Fig. 14.13. The balance point was located and the balance length measured at 480 mm. Cell X was then replaced by a standard cell of e.m.f. 1.50 V and the new balance point was located, giving a balance length of 655 mm. Calculate the e.m.f. of X.

Solution

$$\text{Using } E_X = \frac{L_X}{L_S} E_S = \frac{0.480}{0.655} \times 1.5 = 1.1 \text{ V}$$

Experiment to measure the e.m.f. of a cell

Use a standard cell of known e.m.f. to calibrate a slide-wire potentiometer as shown in Fig. 14.14. Replace the standard cell with the cell under test and measure the balance length for the test cell. Use the standard cell again to check the calibration measurement and confirm the p.d. across the wire has not changed. Use the measurements to calculate the e.m.f. of the test cell.

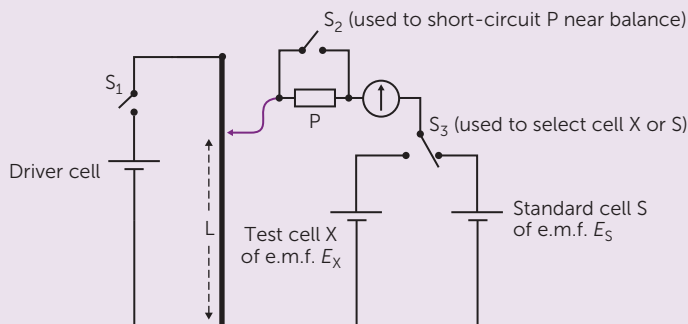


Fig. 14.14 Using a slide-wire potentiometer

Notes

- 1 Include a protective $1\text{ k}\Omega$ resistor, P , in series with the centre-reading meter until the sliding contact is near the balance position. The balance point can then be located precisely by short-circuiting the protective resistor R .
- 2 If the balance length for the standard cell differs when the calibration is repeated, use the average of the two measurements.
- 3 The internal resistance of cell X can be determined by measuring the balance length with a resistor of known resistance connected across X 's terminals.

The p.d. across X 's terminals (cell p.d.), $V = kL_X'$,

where L_X' is the balance length with R across the terminals of cell X .

The current through resistor R at balance, $I = \frac{V}{R} = \frac{kL_X'}{R}$

Since the e.m.f. of X , $E_X = kL_X$, then using $E = V + Ir$ gives

$$r = \frac{(E_X - V)}{I} = \frac{(kL_X - kL_X')}{kL_X'/R} = \frac{(L_X - L_X')}{L_X'} R$$

14.2 QUESTIONS

- 1 For each potential divider shown in Fig. 14.15, calculate the cell current and the p.d. across resistor Y .

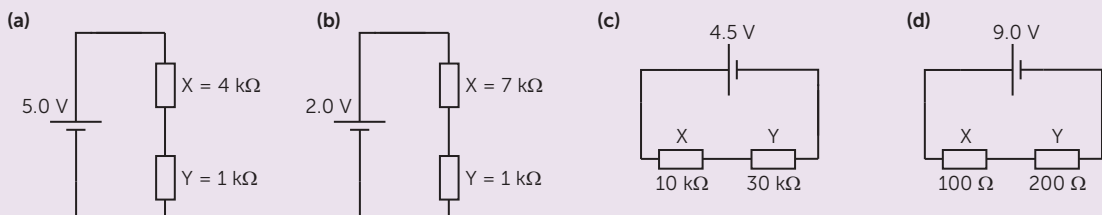


Fig. 14.15

- 2 A standard cell of e.m.f. 1.08 V was used to calibrate a slide-wire potentiometer. The standard cell was then replaced with a cell, X , of unknown e.m.f. and the balance length was then measured with and without a $5.0\text{ }\Omega$ resistor connected across the terminals of the cell. The measurements are listed below.

Balance length for the standard cell = 558 mm

Balance length for the test cell without the $5\text{ }\Omega$ resistor = 775 mm

Balance length for the test cell with the $5\text{ }\Omega$ resistor across its terminals = 692 mm

- a) Explain why the balance length was smaller when the $5\text{ }\Omega$ resistor was connected across X .
- b) Calculate (i) the e.m.f., (ii) the internal resistance of X .

14.3 The Wheatstone bridge

Comparison of two resistances

Fig. 14.16 shows two resistors, R and S , in series with each other, connected in parallel with a fixed length of uniform resistance wire. A driver cell supplies a constant potential difference between points A and B at the ends of the wire and the resistors. A centre-reading meter is connected between a sliding contact, C , on the wire and fixed point, D , the junction of R and S . As the contact is moved along the wire, the meter reading changes, R becoming zero where the p.d. between C and B is balanced by the p.d. between D and B .

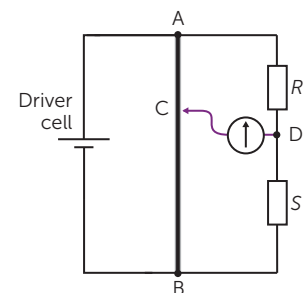


Fig. 14.16 Comparing two resistances

At the balance point,

$$\frac{\text{the p.d. across } R}{\text{the p.d. across } S} = \frac{\text{the p.d. between A and C}}{\text{the p.d. between C and B}}$$

Since R and S form a potential divider, the ratio of the p.d. across R to the p.d. across S is equal to the resistance ratio $R \div S$. Also, the ratio of the p.d. between A and C to the p.d. between C and B is the same as the length ratio $AC \div CB$ since wire ACB is a potential divider.

$$\frac{R}{S} = \frac{L_{AC}}{L_{CB}}$$

If S is a standard resistance, then the value of R can be determined precisely by locating the balance point, measuring the lengths AC and CB , then using the above equation.

The theory of the balanced Wheatstone bridge

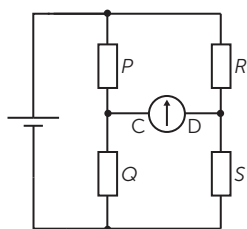


Fig. 14.17 The balanced Wheatstone bridge

The circuit in Fig. 14.16 is an example of a balanced Wheatstone bridge. Fig. 14.17 shows the same circuit, except the wire has been replaced with two resistors, P and Q .

The centre-reading meter reads zero when the p.d. between points C and D is zero. This occurs if

$$\frac{\text{p.d. across } P}{\text{p.d. across } Q} = \frac{\text{p.d. across } R}{\text{p.d. across } S}$$

The ratio of the p.d. across P to the p.d. across $Q = P \div Q$ at balance, as the current through P is the same as the current through Q at balance.

Also, the ratio of the p.d. across R to the p.d. across $S = R \div S$ at balance, as the current through R is the same as the current through S at balance. Therefore the resistance ratio $P \div Q$ is equal to the resistance ratio $R \div S$.

$$\frac{P}{Q} = \frac{R}{S}$$

14.5 WORKED EXAMPLE

An unknown resistance, R , was connected into a Wheatstone bridge circuit consisting of a metre length of uniform resistance wire and a standard 10.0Ω resistor, S , as in Fig. 14.18. The balance point was located and the length of wire between the balance point and the end of the wire to which S was connected was measured at 658 mm.

- Calculate the resistance of R .
- The standard 10Ω resistor was then replaced with a 5.0Ω resistance. Determine the position of the new balance point.

Solution

a) $\frac{R}{S} = \frac{L_1}{L_2}$ where $L_1 = (1000 - 658) \text{ mm} = 0.342 \text{ m}$, and $L_2 = 0.658 \text{ m}$

Hence $R = \frac{0.342}{0.658} \times 10.0 = 5.2 \Omega$

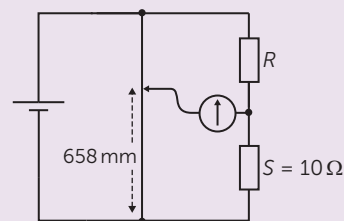


Fig. 14.18

b) Let L = the balance length from the end at S ; let L_w = the length of the wire ($= 1.0$ m).

$$\text{Using } \frac{R}{S} = \frac{(L_w - L)}{L} \text{ gives } \frac{(L_w - L)}{L} = \frac{R}{S} = \frac{5.2}{5} = 1.04$$

$$\text{Hence } L_w - L = 1.04L \quad \therefore 2.04L = L_w, \text{ so } L = \frac{L_w}{2.04} = \frac{1.0 \text{ m}}{2.04} = 0.490 \text{ m}$$

Using the Wheatstone bridge

To measure an unknown resistance, R

As explained on p. 201, P and Q are replaced with a uniform length of resistance wire and S is a standard resistance. The balance point is located and the length of each section, L_P and L_Q , corresponding to P and Q , is measured. The value of S is chosen so that the balance point is in the middle third of the wire. The unknown resistance is calculated using the equation

$$R = \frac{L_P}{L_Q} S$$

To make an output transducer

A transducer produces an output voltage that changes in response to the change of a physical variable. For example, Fig. 14.19 shows an output transducer based on a Wheatstone bridge, in which a thermistor is used in place of resistor P and a variable resistor in place of resistor Q . The centre-reading meter has been replaced by a voltmeter. The variable resistor is adjusted so that the voltmeter reads zero. When the temperature of the thermistor alters, the voltmeter reading becomes either positive or negative, depending on whether the temperature of the thermistor rises or falls. The output voltage could be supplied to an electronic circuit to operate an alarm. Replacing the thermistor with a light-dependent resistor would give a light-dependent output transducer.

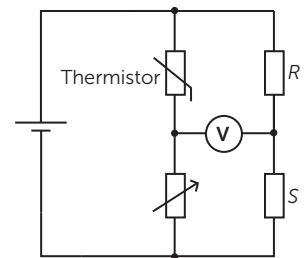


Fig. 14.19 A temperature-dependent output transducer

14.3 QUESTIONS

- A Wheatstone bridge consists of a metre length of uniform resistance wire, a $50\ \Omega$ standard resistor, S , and an unknown resistor, X , as in Fig. 14.20.
 - At balance, the length of wire from the balance point to the end at which X is connected is 0.452 m. Calculate the resistance of X .
 - A second $50\ \Omega$ resistor is connected in parallel with S . Determine the new position of the balance point.
- Fig. 14.21 shows a light-dependent resistor, R , a variable resistor, VR , and two $10\ \text{k}\Omega$ resistors in a Wheatstone bridge. With the LDR exposed to daylight, the variable resistor is adjusted to make the output voltage zero.
 - The light-dependent resistor is then covered. State and explain how this change affects the p.d. between X and Y .
 - State and explain whether the resistance of the variable resistor should be increased or decreased to return the output voltage to zero with the LDR still in darkness.

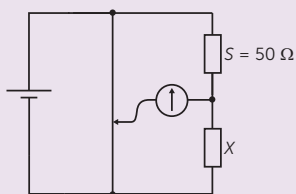


Fig. 14.20

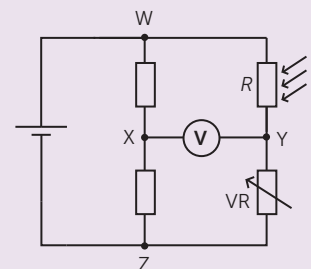


Fig. 14.21

14.4 Resistivity

The resistance of a wire depends on the length and diameter of the wire as well as the material the wire is made from. See Fig. 14.22. Measurements show that the resistance R of a wire is

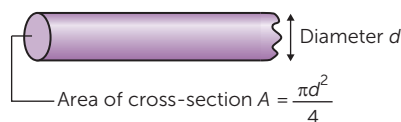


Fig. 14.22 Resistivity

- 1 proportional to its length, L ,
- 2 inversely proportional to its area of cross-section, A .

$$R = \text{constant} \times \frac{L}{A}$$

The constant in this equation depends on the material of the wire and is referred to as the resistivity (symbol ρ , pronounced 'rho') of the material. Rearranging the equation therefore gives the following equation for resistivity:

$$\rho = \frac{RA}{L}$$

Notes

- 1 The unit of resistivity is the ohm metre ($\Omega \text{ m}$).
- 2 The area of cross-section of a wire $= \pi d^2 \div 4$, where d is the diameter of the wire.
- 3 Resistivity values for different materials are given in Table 14.2.
- 4 The conductivity of a substance (symbol σ , pronounced 'sigma') is defined as $1 \div \rho$. The unit of conductivity is $\Omega^{-1} \text{ m}^{-1}$.

Table 14.2 Resistivity values for different materials

Material	Resistivity/ $\Omega \text{ m}$
Copper	1.7×10^{-8}
Constantan	5.0×10^{-7}
Carbon	3.0×10^{-5}
Silicon	2300
PVC	$\sim 10^{14}$

14.6 WORKED EXAMPLE

Calculate the resistance of a copper wire of length 2.50 m and uniform diameter 0.36 mm. The resistivity of copper is $1.7 \times 10^{-8} \Omega \text{ m}$.

Solution

$$\begin{aligned} \text{Area of cross-section, } A &= \frac{\pi d^2}{4} = \frac{\pi(0.36 \times 10^{-3})^2}{4} = 1.0 \times 10^{-7} \text{ m}^2 \\ \therefore R &= \frac{\rho L}{A} = \frac{1.7 \times 10^{-8} \times 2.50}{1.0 \times 10^{-7}} = 0.42 \Omega \end{aligned}$$

Experiment to measure the resistivity of a wire

Use a Wheatstone bridge as in Fig. 14.16 to measure the resistance of different measured lengths of the wire under test. Use a micrometer to measure the diameter of the wire at different points to obtain an average value, d . Calculate the area of cross-section, A , using the equation $A = \pi d^2 \div 4$.

Plot a graph of resistance, R , on the vertical axis against length, L , on the horizontal axis. The graph should be a straight line through the origin, in accordance with the equation

$$R = \frac{\rho L}{A}$$

Since the gradient of the line $= \rho \div A$, the resistivity can be calculated from
 $\rho = \text{gradient} \times A$

See www.palgrave.com/foundations/breithaupt for more information about this experiment.

Application 5

Superconductors

Superconductors have zero electrical resistance. The material of a superconductor has zero electrical resistivity. Superconductivity was discovered in 1911 by the Dutch physicist Kamerlingh Onnes after scientists discovered how to cool objects down to temperatures close to absolute zero. He investigated how the resistance of different metals changed as they were cooled to such low temperatures. To his surprise, he found some metals suddenly lost their resistivity when cooled below a critical temperature which was different for each metal. See Figure 14.23. However, the search for materials that are superconducting at room temperatures has so far been unsuccessful. Such materials would have great technological benefits including superconducting power lines and very strong lightweight electromagnets. Expectations were raised in 1986 when materials were discovered with a critical temperature of 90 K, about 60 K higher than the previous record. Such materials can be made superconducting by cooling them with liquid nitrogen as this has a boiling point of 77 K. Superconducting electromagnets are used in magnetic resonance (MR) medical scanners. In 2014 the ‘critical temperature’ record for a superconductor was 138 K.

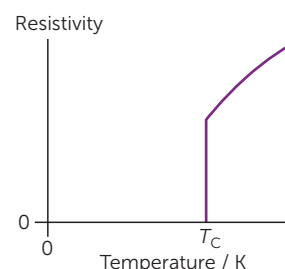


Fig. 14.23

14.4 QUESTIONS

- 1 a) Calculate the resistivity of a wire of diameter 0.24 mm and of length 5.50 m that has a resistance of $58\ \Omega$.
 b) Calculate the resistance of a 1.0 m wire of identical material that has a diameter of 0.35 mm.
- 2 A thin layer of carbon on a flat insulating surface is 1.0 mm wide and 10.0 mm long. The resistance from one end to the other is $15\ \Omega$. Calculate the thickness of the layer. The resistivity of carbon is $3.0 \times 10^{-5}\ \Omega\ \text{m}$.

14.5 Electrical measurements

Most electrical measurements involve using an ammeter to measure current, or a voltmeter to measure p.d., or an oscilloscope to display and measure an electrical waveform. A multimeter is used as either an ammeter or a voltmeter according to its switch settings. All these instruments need to be checked periodically to ensure they give the correct measurement. An analogue ammeter or voltmeter is usually a moving coil meter in which a coil turns against a spring. If the spring gradually weakens, the readings gradually exceed the correct measurements. Digital multimeters and oscilloscopes contain electronic components that might deteriorate and give false readings. Periodic checks might result in an instrument needing to be recalibrated. This section considers how to check electrical instruments.

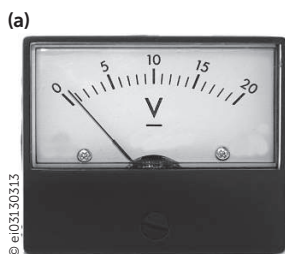


Fig. 14.24 Electrical instruments (a) analogue meter, (b) digital meter

- The **ampere**, one of the base units of the SI system of units, is defined in terms of the force between two current-carrying conductors, as explained on p. 274. An ammeter can be checked by connecting it in series with a current balance which is designed to measure the force between two current-carrying conductors. However, accurate current balances are not readily available and are not easy to use.
- The **volt** is a derived unit, equal to the potential difference between two points in a circuit when 1 coulomb of charge delivers 1 joule of electrical energy on passing from one point to the other. Potential difference can be determined by measuring the electrical energy supplied to an electric heater by a measured amount of charge. The potential difference is the energy supplied per unit charge. Fig. 14.25 shows the idea.

A potential difference determined in this way can then be used to calibrate a potentiometer and thus to measure accurately the e.m.f. of a standard cell. Although this work and other methods of determining potential difference are part of the activity of specialist standards laboratories, standard cells as accurate sources of e.m.f. are readily available as a result. A standard cell can be used to calibrate a potentiometer which can then be used to check the accuracy of a voltmeter, as in Fig. 14.26.

- The ohm is also a derived unit, equal to the resistance of a conductor carrying a current of 1 A when the p.d. across the conductor is exactly 1 V. The resistance of a conductor can be determined by measuring the p.d. across the conductor when a measured current is passed through it. The resistance of a standard resistor can therefore be determined using a calibrated voltmeter and a calibrated ammeter.

The electrical meters in a laboratory can therefore be checked if a standard cell and a standard resistance are available. As explained above, a standard cell may be used to calibrate a voltmeter. Standard resistances do not usually change and can therefore be used with a calibrated voltmeter to check the accuracy of an ammeter. In addition, a standard resistance can be used to measure other resistances using a Wheatstone bridge, as explained on p. 203.

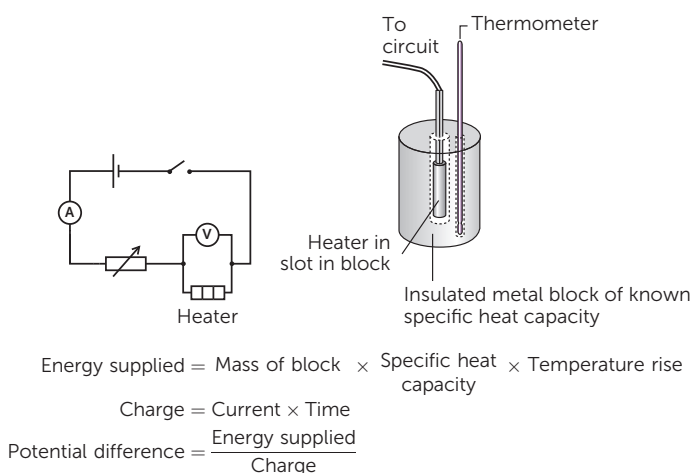


Fig. 14.25 Measuring potential difference

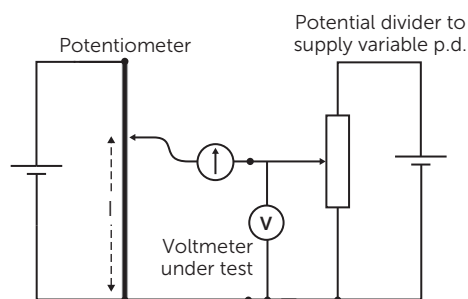


Fig. 14.26 Calibrating a voltmeter

Extending the range of a moving coil meter

A moving coil meter is a current-operated instrument with a linear scale. In other words, the deflection of the pointer from zero is in proportion to the current. If the current through a moving coil meter is increased, the deflection of the pointer increases. If the current exceeds the maximum reading on the scale, referred to as the 'full-scale' deflection, the coil of the instrument could be permanently damaged due to the heating effect of the current.

Extending the range of an ammeter

The full-scale current for a sensitive meter may be of the order of a milliampere or less. For this reason, a 'shunt' resistor, in parallel with the meter, is used to extend the current range of the meter. Fig. 14.27 shows the idea.

Let the full-scale current for the meter = i and the resistance of the meter = r . The maximum p.d. across the meter for full-scale deflection is therefore equal to ir .

To measure maximum current I , the excess current $(I - i)$ must pass through the shunt resistor instead of through the meter. At maximum current, the p.d. across the shunt is equal to the p.d. across the meter ($= ir$).

$$\text{Hence the resistance of the shunt, } R = \frac{\text{p.d.}}{\text{current}} = \frac{ir}{(I - i)}$$

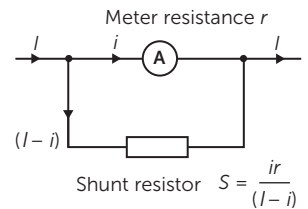


Fig. 14.27 Ammeter conversion

14.7 WORKED EXAMPLE

A moving coil meter has a resistance of 10Ω and a full-scale deflection current of 100 mA. Calculate the shunt resistance needed to extend its range to read currents to 5.0 A.

Solution

The maximum current through shunt $S = 5.0 - 0.1 = 4.9$ A

The maximum p.d. across $S = iR = 0.1 \times 10 = 1.0$ V $\therefore S = \frac{1.0}{4.9} = 0.20\ \Omega$

Extending the range of a voltmeter

The p.d. across a moving coil meter is proportional to the current. The meter may be used to measure p.d. provided the current does not exceed the full-scale current. However, since the resistance of most moving coil meters is low, full-scale current is reached when the p.d. is no more than of the order of millivolts. To extend the range of a moving coil voltmeter, a 'multiplier' resistance is connected in series with the meter, as in Fig. 14.28.

Let the full-scale voltage across the meter = v and the resistance of the meter = r . Hence the maximum current through the meter, $I = v \div r$. This is also the maximum current through the multiplier, since the multiplier is in series with the meter.

To read voltages up to V , the excess voltage $(V - v)$ must be dropped across the multiplier.

$$\text{the multiplier resistance, } R = \frac{(V - v)}{I} = \frac{(V - v)}{v} r$$

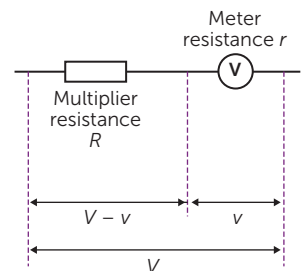


Fig. 14.28 Voltmeter conversion

14.8 WORKED EXAMPLE

A moving coil meter has a resistance of $1000\ \Omega$ and a full-scale deflection current of $0.1\ \text{mA}$. Calculate the resistance of the multiplier that must be connected in series with it to extend its range to read voltages up to $20\ \text{V}$.

Solution

Maximum current, $I = 0.1\ \text{mA}$ $\therefore$ maximum p.d. across meter coil $= 0.1\ \text{mA} \times 1000\ \Omega = 0.1\ \text{V}$

Hence excess voltage across multiplier $= 20 - 0.1 = 19.9\ \text{V}$

$\therefore$ multiplier resistance $= 19.9\ \text{V} \div 0.1\ \text{mA} = 199\ 000\ \Omega$

Multimeters

A multimeter can be used as an ammeter or a voltmeter, according to the position of a switch that connects a resistor in series or in parallel with the meter.

Digital meters

A digital multimeter is a digital voltmeter which can be used to measure voltage directly. If it is connected in series or in parallel with a resistor, it can be used to measure voltage or current, as explained below.

Digital voltmeters

Most digital voltmeters have a very high input resistance and therefore draw negligible current when connected across a component. To extend the range of a digital voltmeter, a suitable resistor is connected in series with the voltmeter, just as for a moving coil voltmeter. For example, suppose a digital voltmeter reads up to $2.0\ \text{V}$ and has an input resistance of $1\ \text{M}\Omega$. The maximum current is therefore $2\ \mu\text{A}$ ($= 2.0\ \text{V} \div 1\ \text{M}$). To read up to $20\ \text{V}$, the p.d. across the series resistor must be $18\ \text{V}$ ($= 20 - 2\ \text{V}$). Hence the resistance of the series resistor must be $9\ \text{M}\Omega$ ($= 18.0 \div 2\ \mu\text{A}$).

Digital ammeters

A digital ammeter is a digital voltmeter with a resistor of low resistance connected across its input terminals, as in Fig. 14.29. For example, a digital voltmeter which reads up to $2.0\ \text{V}$ and has an input resistance of $1\ \text{M}\Omega$ can be converted to read currents up to $2.0\ \text{A}$ by connecting a $1.0\ \Omega$ resistor across its input terminals. The current through the actual meter is negligible, so the entire current of $2.0\ \text{A}$ passes through the $1.0\ \Omega$ resistor to give a p.d. of $2.0\ \text{V}$ across the meter's input terminals.

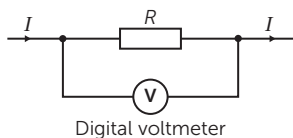


Fig. 14.29 A digital meter used as an ammeter

14.5 QUESTIONS

- 1 A moving coil meter has a resistance of $100\ \Omega$ and a full-scale deflection current of $1.0\ \text{mA}$. How could the meter be adapted to read
 - a) a current up to (i) $100\ \text{mA}$, (ii) $10.0\ \text{A}$?
 - b) voltages up to (i) $10.0\ \text{V}$, (ii) $30.0\ \text{V}$?
- 2 A digital meter has an input resistance of $5.0\ \text{M}\Omega$ and a full-scale reading of $200\ \text{mV}$. How could the meter be adapted to read
 - a) a voltage up to (i) $2.0\ \text{V}$, (ii) $100\ \text{V}$?
 - b) a current up to (i) $200\ \text{mA}$, (ii) $10\ \text{A}$?

SUMMARY

- The **electromotive force (e.m.f.)** of a cell is the electrical energy per unit charge created in the cell.
- The **internal resistance** of a cell is the lost voltage per unit current in the cell.
- The **potential divider equation**

For two resistors R_1 and R_2 in series with each other and a voltage supply V ,

$$\text{the p.d. across } R_1 = \frac{R_1 V}{(R_1 + R_2)}$$

$$\text{and the p.d. across } R_2 = \frac{R_2 V}{(R_1 + R_2)}$$

- **Comparison of two e.m.f.s using the slide-wire potentiometer**

$$\frac{E_1}{E_2} = \frac{L_1}{L_2}$$

- The **Wheatstone bridge equation**

$$\frac{P}{Q} = \frac{R}{S}$$

- **Resistivity**

$$\rho = \frac{RA}{L}$$

- **Meter conversion**

To extend the range of a meter of resistance r and full-scale current i to

$$1 \quad \text{maximum current } I, \text{ shunt resistance } S = \frac{ir}{(I - i)}$$

$$2 \quad \text{maximum voltage } V, \text{ multiplier resistance } R = \frac{(V - v)}{v}r, \text{ where } v = ir.$$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

14.1 A cell of e.m.f. 2.0 V and internal resistance 0.8Ω is connected to a 4.2Ω resistor.

- (i) Sketch the circuit diagram,
(ii) calculate the current and the p.d. across the 4.2Ω resistor.
- Explain why the p.d. across the 4.2Ω resistor is less than the cell e.m.f.

14.2 Two cells, each of e.m.f. 1.5 V and internal resistance 0.5Ω , are connected in series with each other, a variable resistor and a 2.5 V 0.25 A torch bulb.

- Sketch the circuit diagram.
- The variable resistor is adjusted so the bulb is at its normal brightness. Calculate
 - the power delivered to the bulb,
 - the electrical energy per second produced by the two cells.
- Account for the difference between your answers to (b)(i) and (ii).

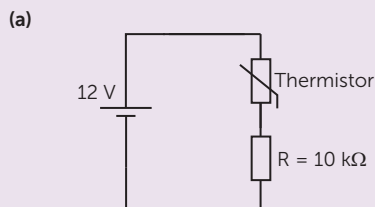
14.3 A cell of e.m.f. E and internal resistance r was connected in series with an ammeter and a resistance box. When the resistance of the resistance box was 15.0Ω , the current was 100 mA. When the resistance of the box was changed to 5.0Ω , the current was 200 mA.

- Calculate the e.m.f. and the internal resistance of the cell.
- Calculate the current from the cell when the resistance of the box was changed to 3.0Ω .

REVISION QUESTIONS

14.4 The potential divider in Fig. 14.30a overleaf consists of a resistor, R , of resistance $10\text{ k}\Omega$ and a thermistor in series connected to a 12.0 V battery. Fig. 14.30b shows how the resistance of the thermistor varies with its temperature.

- Calculate the voltage across R at 20°C .
- Determine the temperature at which the voltage across R is 5.0 V .



(b) Thermistor resistance / $\text{k}\Omega$

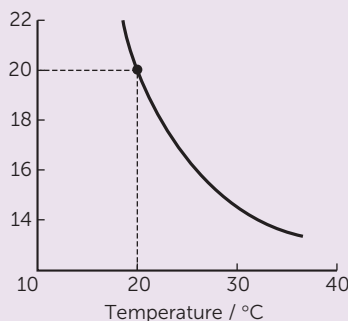


Fig. 14.30

14.5 Two $1\text{ k}\Omega$ resistors are connected in series with each other and a cell of e.m.f. 2.0 V and negligible internal resistance. A voltmeter is connected across one of the resistors.

- Sketch the circuit diagram.
- Calculate the voltmeter reading if the voltmeter resistance is
 - infinite,
 - $1\text{ k}\Omega$.

14.6 In a slide-wire potentiometer experiment to measure the e.m.f. of a cell X , the wire was calibrated using a standard cell of e.m.f. 1.50 V . The balance length readings obtained were as follows:

- Balance length for cell $X = 651\text{ mm}$
- Balance length for the standard cell = 885 mm

- Calculate the e.m.f. of X .
- A resistor of resistance $2.0\text{ }\Omega$ was connected across the terminals of the cell X when it was connected in the circuit, reducing the balance length to 542 mm . Calculate
 - the p.d. across the terminals of cell X at balance with the $2.0\text{ }\Omega$ resistor connected across its terminals,
 - the internal resistance of cell X .

14.7 In the Wheatstone bridge circuit in Fig. 14.31, calculate

- the resistance of R to balance the bridge when $P = 5.0\text{ }\Omega$, $Q = 12.0\text{ }\Omega$ and $S = 50.0\text{ }\Omega$,
- the resistance of Q to balance the bridge when $P = 5.0\text{ }\Omega$, $R = 20.0\text{ }\Omega$ and $S = 50.0\text{ }\Omega$,
- the resistance of P to balance the bridge when $R = 20.0\text{ }\Omega$, $Q = 12.0\text{ }\Omega$ and $S = 50.0\text{ }\Omega$.

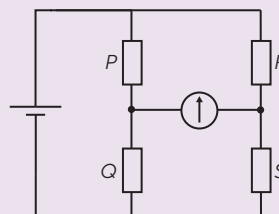


Fig. 14.31

14.8 A Wheatstone bridge consisting of a metre length of uniform resistance wire, a test resistor, R , and a resistance box, S , as in Fig. 14.32, was used to measure the resistance of the test resistor.

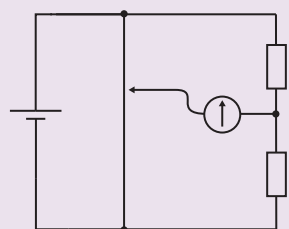


Fig. 14.32

- When $S = 12.0\text{ }\Omega$, the balance point was 438 mm from the end of the wire where S was connected.
 - Calculate the resistance of R .
 - The balance point was located to a precision of 2 mm . Calculate the probable error in the resistance of R .
- Determine the new balance point if a $6.0\text{ }\Omega$ resistor was connected in parallel with S .

14.9

- Calculate the resistance of a wire of length 1.50 m and diameter 0.36 mm which is made from a material of resistivity $4.8 \times 10^{-7}\text{ }\Omega\text{ m}$.
- A 10.0 m length of wire made of the same material as in (a) has a resistance of $105\text{ }\Omega$. Calculate the diameter of this wire.
- Calculate the length of a copper wire of diameter 2.0 mm and resistance $0.040\text{ }\Omega$. The resistivity of copper is $1.7 \times 10^{-8}\text{ }\Omega\text{ m}$.

14.10 A moving coil meter has a resistance of $500\text{ }\Omega$ and a full-scale deflection current of 0.10 mA . Explain how the meter can be adapted to read

- a maximum current of
 - 100 mA ,
 - 5.0 A ,
- a maximum p.d. of
 - 1.0 V ,
 - 15.0 V .

Capacitors

OBJECTIVES

After working through this unit, you should be able to:

- describe how a capacitor stores electric charge
- define capacitance and the unit of capacitance
- measure the capacitance of a capacitor
- calculate the combined capacitance of two or more capacitors in series or in parallel
- calculate the energy stored in a charged capacitor
- calculate the change of p.d. and energy stored when two capacitors are connected together
- describe the factors that determine the capacitance of a parallel plate capacitor
- describe and calculate the change of p.d. across a capacitor charging or discharging through a fixed resistor

UNIT

15

CONTENTS

- 15.1 Storing charge
- 15.2 Capacitor combinations
- 15.3 Energy stored in a charged capacitor
- 15.4 Capacitor design factors
- 15.5 Capacitor discharge
- Summary
- Revision questions

15.1 Storing charge

If you have ever received an electric shock from the body of a car, you will know that an insulated metal object is capable of storing charge. Storage of charge can be achieved by direct contact with a charged insulator or by connecting one terminal of a battery to the object and then connecting the other terminal to an earthed metal pipe. Electrons transfer to or from the insulated metal object, depending on whether it is negatively or positively charged.

A **capacitor** is a device designed to store charge. The simplest type of capacitor consists of two parallel metal plates opposite each other. When a battery is connected to the plates, as in [Fig. 15.1](#) overleaf, electrons from the negative terminal of the battery transfer onto the plate connected to that terminal. At the same time, electrons transfer from the other plate to the positive terminal of the battery, leaving this plate with a positive charge. The flow of electrons decreases as the plates charge up. When the plate p.d. is the same as the battery p.d., the flow of electrons ceases.

The two plates store equal and opposite amounts of charge. If the charge on the positive plate is $+Q$, the charge on the negative plate will be $-Q$. The capacitor is said to store charge Q . The symbol for a capacitor is shown in [Fig. 15.1b](#).

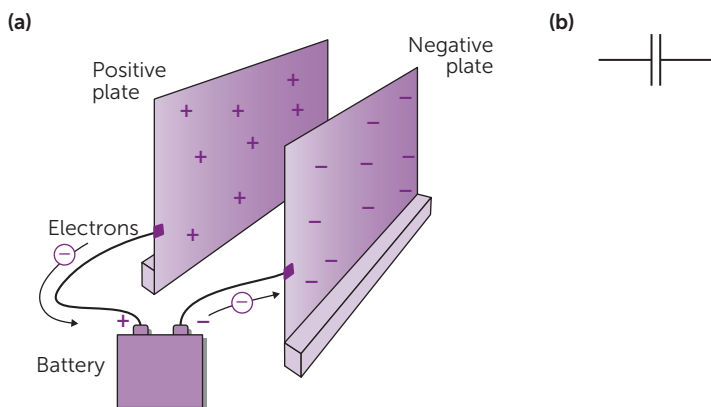


Fig. 15.1 Storing charge on a parallel plate capacitor (a) circuit, (b) symbol for capacitor

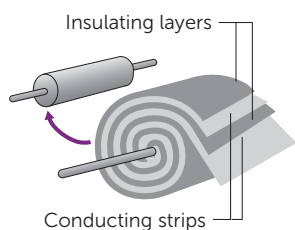


Fig. 15.2 Practical capacitor

Capacitors in practice

A practical capacitor consists of two flexible conducting strips separated by an insulating layer as shown in Fig. 15.2. The strips are like the metal plates of the parallel plate capacitor but they are rolled up to make the capacitor much smaller in physical size. The insulating layer is a **dielectric** substance, which allows the capacitor to store much more charge for a given voltage. The molecules of the dielectric substance become polarised when a voltage is applied between the two conducting strips. This means that the electrons of each molecule are attracted slightly towards the positive plate. This causes the dielectric surface in contact with the positive plate to become negative and the other surface positive. The overall result is that the surface charge on the dielectric surface effectively cancels out some of the charge on the conducting strip. Fig. 15.3 shows the action of a dielectric substance.

A dielectric substance is an insulator because the electrons remain attached to its molecules, even if displaced slightly, when a voltage is between the two conducting strips. However, if the applied voltage exceeds a certain value that depends on the nature of the substance and the thickness of the insulating layer, the insulation breaks down. The maximum rating of a capacitor should never be exceeded for this reason. In an electrolytic capacitor, the dielectric substance is created by electrolysis

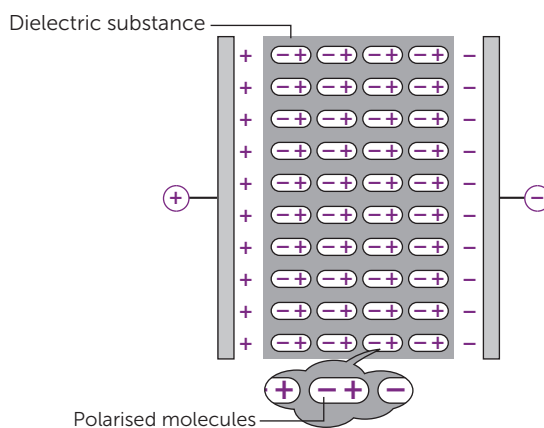


Fig. 15.3 The action of a dielectric substance

as a very thin layer at the interface between a conducting strip and the insulating material when the capacitor is first charged up. An electrolytic capacitor **must** be connected into the circuit according to the polarity indicated at its terminals, otherwise it might explode.

Experiment to investigate charging a capacitor

1 Through a fixed resistance

Fig. 15.4 shows how to charge a capacitor through a fixed resistor. The current decreases to zero as the capacitor charges up. When the capacitor voltage is the same as the battery voltage, no more charge can be stored on the capacitor and the rate of flow of charge is then zero. Using a larger resistance reduces the rate of charging so the charging process takes longer.

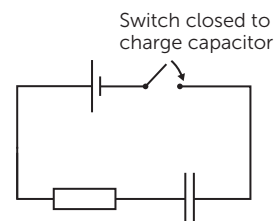


Fig. 15.4 Charging a capacitor through a fixed resistor

2 At constant current

Fig. 15.5 shows how a capacitor can be charged at a constant rate. The variable resistor must be continually reduced to keep the charging current constant. When the variable resistance has been reduced to zero, the current falls to zero as the voltage across the capacitor is then the same as the battery voltage. A digital voltmeter is used to measure the capacitor voltage at fixed intervals as the capacitor charges up at constant current. The ammeter is used to measure the current. The results from such an experiment are shown in Table 15.1. The charge stored has been calculated using the equation 'charge = current $\times$ time'.

The results may be plotted as a graph of y = charge stored, Q , on the vertical axis against x = applied voltage, V , on the horizontal axis. The measurements in Table 15.1 are plotted in Fig. 15.5b. The results show that the charge stored is proportional to the voltage. In other words, the charge stored per unit voltage is constant.

The capacitance, C , of a capacitor is defined as the charge stored per unit voltage by the capacitor.

The unit of capacitance is the farad (F) which is equal to 1 coulomb per volt. In practice, capacitance values are usually expressed in microfarads ($\mu\text{F} = 10^{-6}\text{F}$) or picofarads ($\text{pF} = 10^{-12}\text{F}$). Show for yourself that the capacitance in Fig. 15.5 is $4700\mu\text{F}$.

$$\text{Capacitance } C = \frac{Q}{V}$$

where Q = charge stored, V = voltage.

Mathematical note Rearranging $C = \frac{Q}{V}$ gives $Q = CV$ or $V = \frac{Q}{C}$

Table 15.1 Charging a capacitor at constant current						
Time/s	0.00	5.00	10.00	15.00	20.00	25.00
Capacitor voltage/V	0.00	0.45	0.91	1.37	1.81	2.27
Charge/mC	0.00	2.15	4.30	6.45	8.60	10.75

Current = 0.43 mA

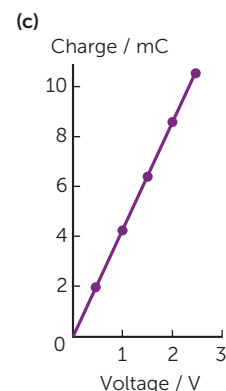
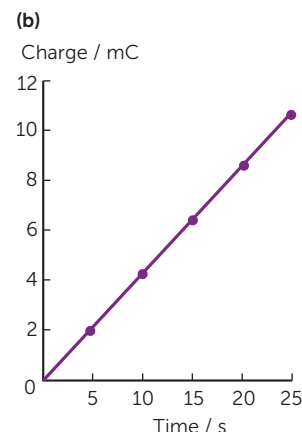
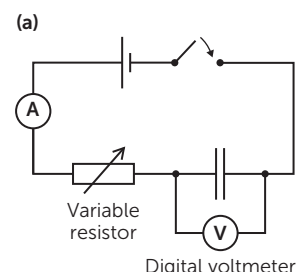


Fig. 15.5 Charging a capacitor at constant current

15.1 WORKED EXAMPLE

A $4700\ \mu\text{F}$ capacitor is connected in series with a switch, a variable resistor, an ammeter and a $3.0\ \text{V}$ battery. When the switch is closed, the variable resistor is continually reduced to keep the current constant at $0.050\ \text{mA}$.

Calculate

- the charge stored by the capacitor when fully charged,
- the time taken to reach full charge at a current of $0.050\ \text{mA}$.

Solution

a) Rearrange $C = \frac{Q}{V}$ to give $Q = CV = 4700 \times 10^{-6} \times 3.0 = 1.4 \times 10^{-2}\ \text{C}$

b) Rearrange $Q = It$ to give $t = \frac{Q}{I} = \frac{1.4 \times 10^{-2}}{0.050 \times 10^{-3}} = 280\ \text{s}$.

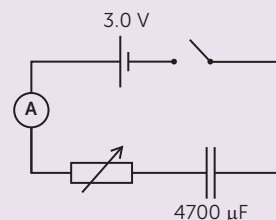


Fig. 15.6

15.1 QUESTIONS

- 1 Copy and complete the table below by calculating the missing value in each column.

Capacitance / μF	10.00	0.22	?	1.00×10^{-3}
Voltage / V	6.00	?	4.50	?
Charge / μC	?	0.33	9.90×10^4	5.00×10^{-2}

- 2 A $2200\ \mu\text{F}$ is charged at a constant current of $2.5\ \text{mA}$ using a $9\ \text{V}$ battery and a variable resistor. Calculate

- the charge stored by this capacitor at $9.0\ \text{V}$,
- the time taken to become fully charged at a constant current of $0.25\ \text{mA}$.

15.2 Capacitor combinations

Capacitors in series

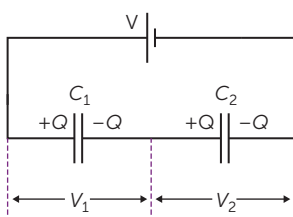


Fig. 15.7 Capacitors in series

Fig. 15.7 shows two capacitors, C_1 and C_2 , connected in series to a battery. Each capacitor stores the same amount of charge, Q . The total charge stored is the same as the charge stored on each capacitor. This is because the charge on the two capacitor plates connected together is due to electrons from one of these two plates being attracted onto the other plate. Just as two resistors in series carry the same current, two capacitors in series carry the same charge.

The p.d. across capacitor C_1 , $V_1 = \frac{Q}{C_1}$

The p.d. across capacitor C_2 , $V_2 = \frac{Q}{C_2}$

The battery voltage, $V = V_1 + V_2$; hence $V = \frac{Q}{C_1} + \frac{Q}{C_2}$

Because the combined capacitance, $C = \frac{Q}{V}$

$$\frac{1}{C} = \frac{V}{Q} = \left(\frac{Q}{C_1} + \frac{Q}{C_2} \right)$$

The combined capacitance, C , of two capacitors, C_1 and C_2 , in series is given by the equation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

15.2 WORKED EXAMPLE

A $2\ \mu\text{F}$ capacitor is connected in series with a $6\ \mu\text{F}$ capacitor and a $6.0\ \text{V}$ battery. Calculate

- a) the combined capacitance of these two capacitors in series,
b) (i) the charge stored by each capacitor, (ii) the voltage across each capacitor.

Solution

$$\text{a) } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{6} = \frac{6+2}{2 \times 6} = \frac{8}{12} \quad \therefore C = \frac{12}{8} = 1.5\ \mu\text{F}$$

$$\text{b) (i) } Q = CV = 1.5\ \mu\text{F} \times 6.0\ \text{V} = 9.0\ \mu\text{C},$$

$$\text{(ii) } 2\ \mu\text{F}; V = \frac{Q}{C} = \frac{9.0\ \mu\text{C}}{2.0\ \mu\text{F}} = 4.5\ \text{V}$$

$$6\ \mu\text{F}; V = \frac{Q}{C} = \frac{9.0\ \mu\text{C}}{6.0\ \mu\text{F}} = 1.5\ \text{V}$$

Note that the voltages add up to the battery voltage.

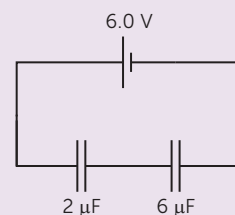


Fig. 15.8

Capacitors in parallel

Fig. 15.9 shows two capacitors, C_1 and C_2 , connected in parallel to a battery. Each capacitor has the same voltage, V , across its terminals because they are in parallel with each other.

Charge on capacitor C_1 , $Q_1 = C_1V$

Charge on capacitor C_2 , $Q_2 = C_2V$

Total charge stored, $Q = Q_1 + Q_2 = C_1V + C_2V$

Hence the combined capacitance, $C = \frac{Q}{V} = \frac{C_1V + C_2V}{V} = C_1 + C_2$

The combined capacitance, C , of two capacitors, C_1 and C_2 , in parallel is given by the equation

$$C = C_1 + C_2$$

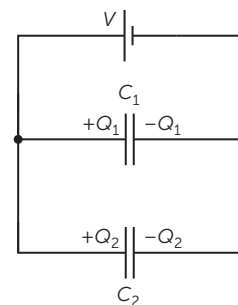


Fig. 15.9 Capacitors in parallel

Notes

For more than two capacitors in parallel, the equation becomes $C = C_1 + C_2 + C_3 + \dots$

Likewise, for more than two capacitors in series, the series combination equation becomes $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$

15.3 WORKED EXAMPLE

A $4\ \mu\text{F}$ capacitor and a $6\ \mu\text{F}$ capacitor in parallel are connected to a $3.0\ \text{V}$ battery. Calculate

- a) the combined capacitance of these two capacitors in parallel,
b) (i) the total charge stored, (ii) the charge stored on each capacitor.

Solution

$$\text{a) } C = C_1 + C_2 = 4 + 6 = 10\ \mu\text{F}$$

$$\text{b) (i) } Q = CV = 10\ \mu\text{F} \times 3.0\ \text{V} = 30\ \mu\text{C}$$

$$\text{(ii) } 4\ \mu\text{F}; Q = CV = 4\ \mu\text{F} \times 3.0\ \text{V} = 12\ \mu\text{C} \quad 6\ \mu\text{F}; Q = CV = 6\ \mu\text{F} \times 3.0\ \text{V} = 18\ \mu\text{C}$$

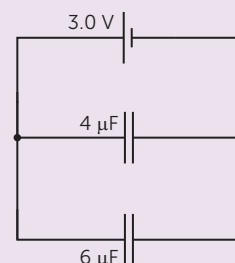


Fig. 15.10

Application 7

Touch screens

Cash machines, mobile phones and tablets are just a few of the devices that have touch screens. The screen of each device is fixed on top of an electronic display which can be seen through the screen. Touching the screen sends an electrical signal to the device to tell where the screen was touched. The various options or features shown on the display are activated in this way.

The touch screen of a mobile phone or a tablet consists of a matrix of transparent capacitors under a protective glass sheet. The matrix is formed by two sets of parallel conducting strips perpendicular to each other and separated by a layer of insulator. A potential difference is applied between the two sets of strips. When someone touches the protective glass screen, their own capacitance alters the electrical charge on the capacitors under the point of contact changes. The flow of charge between the electronic circuit of the device and the affected capacitors causes electrical signals to be sent to the device. The electronic circuit of the device is programmed to work out the location of the point of contact according to which capacitors are affected and consequently to activate the display feature at that point. The circuit is also programmed to scroll the display when the point of contact is moved and expand the display using two points of contact.

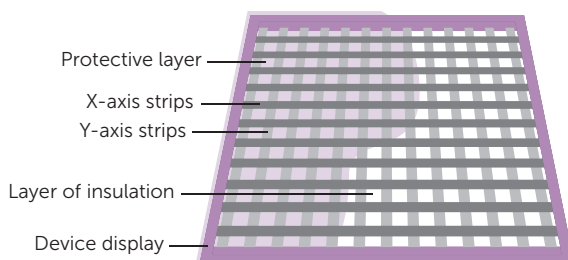


Fig. 15.11 Inside a touch screen

Sharing charge

When an uncharged capacitor is connected in parallel with an isolated charged capacitor, the total charge is shared between the two capacitors. As a result, the voltage across the second capacitor falls.

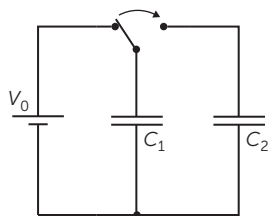


Fig. 15.12 Sharing charge

In Fig. 15.12, a capacitor, C_1 , is charged from a battery of voltage, V_0 , using a two-way switch, S . The switch is then changed over to disconnect the battery and to connect a second capacitor, C_2 , in parallel with C_1 . Let V_f represent the final voltage across the two capacitors.

The initial charge stored on C_1 , $Q = C_1 V_0$

The final charge stored on C_1 , $Q_1 = C_1 V_f$

and the final charge stored on C_2 , $Q_2 = C_2 V_f$

Since the total final charge stored = the initial charge stored, $Q = Q_1 + Q_2$

Hence $C_1 V_0 = C_1 V_f + C_2 V_f = (C_1 + C_2) V_f$

Rearranging this equation gives

$$V_f = \frac{C_1}{(C_1 + C_2)} V_0 = \frac{\text{initial charge } C_1 V_0}{\text{combined capacitance } (C_1 + C_2)}$$

15.4 WORKED EXAMPLE

A 3.0 V battery is briefly connected across a 10 μF capacitor to charge the capacitor. An uncharged 22 μF capacitor is then connected across the charged capacitor. Calculate

- the initial charge stored on the 10 μF capacitor,
- (i) the final p.d. across the two capacitors, (ii) the charge stored on each capacitor.

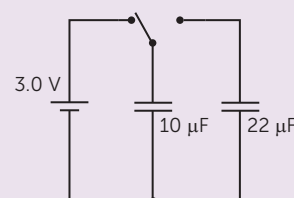


Fig. 15.13

Solution

- $Q = CV = 10 \mu\text{F} \times 3.0 \text{ V} = 30 \mu\text{C}$
- (i) Combined capacitance $= (C_1 + C_2) = 10 + 22 = 32 \mu\text{F}$
 $\therefore$ final p.d. $= \frac{\text{initial charge}}{\text{combined capacitance}} = \frac{30 \mu\text{C}}{32 \mu\text{F}} = 0.94 \text{ V}$
 (ii) 10 μF ; $Q = CV = 10 \mu\text{F} \times 0.94 \text{ V} = 9.4 \mu\text{C}$ 22 μF ; $Q = CV = 22 \mu\text{F} \times 0.94 \text{ V} = 20.6 \mu\text{C}$

15.2 QUESTIONS

- Calculate the combined capacitance of a 3 μF capacitor and a 6 μF capacitor (i) in series, (ii) in parallel.
 - For each circuit shown in Fig. 15.14, calculate (i) the combined capacitance, (ii) the total charge stored, (iii) the charge stored and the voltage for each capacitor.
- A 4.7 μF capacitor was charged using a 6.0 V battery, then disconnected from the battery. An uncharged 10 μF capacitor was then connected across the terminals of the 4.7 μF capacitor. Calculate
 - the total charge stored,
 - the final p.d. across each capacitor,
 - the final charge stored on each capacitor.

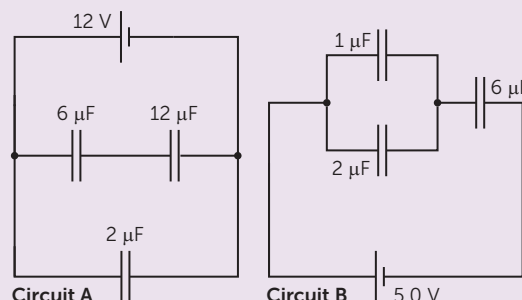


Fig. 15.14

15.3 Energy stored in a charged cell

If a charged capacitor is discharged through a suitable light bulb, the light bulb will light up briefly. If the voltage used to charge the capacitor is doubled, the energy stored is sufficient to make four light bulbs, connected as in Fig. 15.15, light up briefly. This is because four times as much energy is stored in a charged capacitor as a result of doubling the charging voltage. In other words, the energy stored in a charged capacitor is proportional to the square of the voltage.

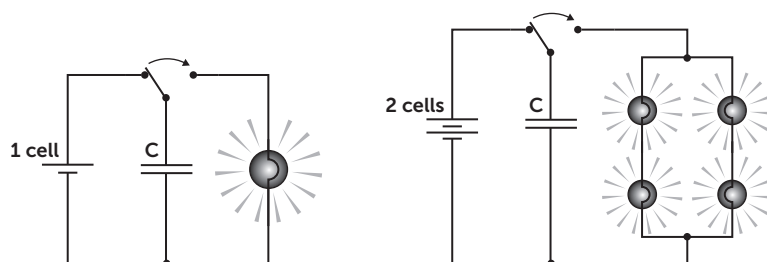


Fig. 15.15
Using capacitor energy

Consider a brief interval of time, δt , during the process of charging a capacitor, C , to final voltage, V . During this brief interval, suppose the voltage across the capacitor rises from v to $v + \delta v$ and the charge increases from $q (= Cv)$ to $q + \delta q (= C(v + \delta v))$. During this time, extra charge, δq , is forced by the battery onto the plates against an average potential difference of $v + \frac{1}{2}\delta v$. Hence the work done on this extra charge,

$$\delta W = \delta q(v + \frac{1}{2}\delta v)$$

since potential difference is the work done per unit charge.

$$\delta W = (v\delta q) + (\delta q \frac{1}{2}\delta v) = v\delta q$$

since the second term is negligible.

Fig. 15.16 shows how the p.d. across the capacitor increases as the charge on its plates rises. The work done in the interval $\delta t (= v\delta q)$ is represented by the area of a narrow strip of width δq and height v on this graph. It therefore follows that the total work done to charge the capacitor is represented by the total area under the line, which is half the area of the rectangle of width Q and height V . Hence the total work done, $W = \frac{1}{2}QV$. This is therefore the energy stored in the capacitor. Since the final charge $Q = CV$, then the energy stored $= \frac{1}{2}QV = \frac{1}{2}CV^2$.

$$\text{Energy stored, } E = \frac{1}{2}QV = \frac{1}{2}CV^2$$

where Q = charge stored, V = voltage and C = capacitance.

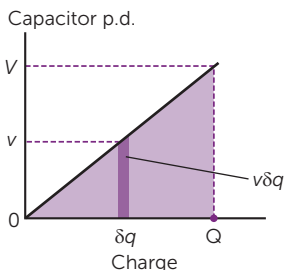


Fig. 15.16 Energy stored in a charged capacitor

15.5 WORKED EXAMPLE

A 10 000 μF capacitor is charged using a 6.0 V battery.

a) Calculate the energy stored in this capacitor at 6.0 V.

b) Estimate for how long this capacitor could light a 6.0 V, 0.36 W torch bulb.

Solution

a) $E = \frac{1}{2}CV^2 = 0.5 \times 10\,000 \times 10^{-6} \times 6.0^2 = 0.18 \text{ J}$

b) Rearrange $\text{power} = \frac{\text{energy}}{\text{time}}$ to give $\text{time} = \frac{\text{energy}}{\text{power}} = \frac{0.18 \text{ J}}{0.36 \text{ J}} = 0.5 \text{ s}$

See www.palgrave.com/foundations/breithaupt for more about energy stored in a capacitor.

15.3 QUESTIONS

- 1 **a)** Calculate the energy stored in (i) a 5.0 μF capacitor charged to a p.d. of 12.0 V, (ii) a 100 μF capacitor containing 1.8 mC of charge, (iii) a capacitor containing 30 mC of charge at a voltage of 6.0 V.
- b)** (i) Calculate the voltage at which a 20 μF capacitor would store 250 μJ of energy. (ii) Estimate the time for which a 50 000 μF capacitor charged to 6.0 V could light a 6.0 V, 0.2 W torch bulb.
- 2 Fig. 15.17 shows a two-way switch used to charge a 100 μF capacitor from a 6.0 V battery. The switch is then used to reconnect the charged capacitor to an uncharged 50 μF capacitor.
 - a)** Calculate (i) the initial charge and energy stored in the 100 μF capacitor, (ii) the final p.d. across the two capacitors, (iii) the final charge and energy stored on each capacitor.
 - b)** Account for the loss of stored energy as a result of reconnecting the switch.

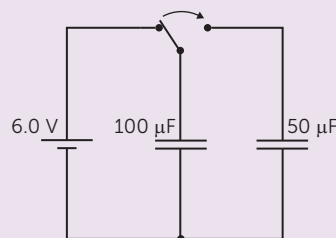


Fig. 15.17

15.4 Capacitor design factors

The capacitance of a capacitor depends on the surface area of the conducting strips, the spacing between the strips and the dielectric substance between the conducting strips. The larger the surface area, the greater the charge the capacitor can store for the same p.d. The closer the plates are together, the more charge the capacitor can store for the same p.d.

Experiment to investigate the factors that determine the capacitance of a parallel plate capacitor

Fig. 15.18 shows how these factors can be investigated using a parallel plate capacitor. The vibrating reed switch repeatedly charges and discharges the capacitor at a known frequency, f . Each time the switch connects the capacitor to the supply voltage V , the capacitor gains charge $Q = CV$. The switch then discharges the capacitor through the microammeter. Since this process occurs repeatedly, the microammeter registers a constant current $I = Q \div t = Qf$, where $t = 1 \div f$. Hence $I = CVf$. Rearranging this equation therefore gives

$$C = \frac{I}{Vf}$$

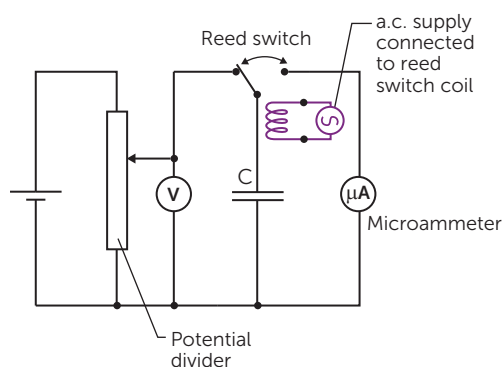


Fig. 15.18 Investigating capacitor factors

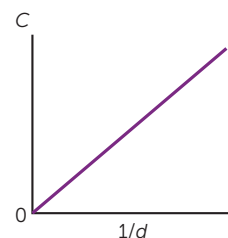


Fig. 15.19 Capacitance versus $1/\text{plate separation}$ for a parallel plate capacitor

- To investigate the effect of the plate separation, d , on the capacitance:** the current, I , is measured at a known p.d., V , with the reed switch operating at a measured frequency, f . Insulating spacers of known thickness may be used to keep the plates at a measured spacing. The capacitance, C , can therefore be calculated from the measurements of I , V and f for different spacings, d .

Fig. 15.19 shows the results plotted on a graph of capacitance C on the vertical axis against $1 \div d$ on the horizontal axis. The results define a straight line through the origin, proving that the capacitance is proportional to $1 \div d$.

- To investigate the effect of surface area, A , on the capacitance:** the overlap area, A , is changed, keeping the p.d. and the plate spacing the same. The current is measured for different measured overlap areas. Fig. 15.20 shows the results plotted on a graph of capacitance, C , on the vertical axis against A on the horizontal axis. The results ought to give a straight line through the origin, proving that the capacitance is proportional to the overlap area, A .

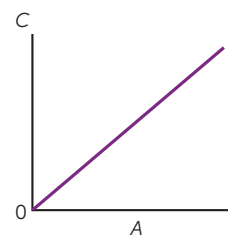


Fig. 15.20 Capacitance versus overlap area for a parallel plate capacitor

Combining the two results gives the following equation for C ,

$$C = \text{constant} \frac{A}{d}$$

The constant of proportionality in this equation is called the **absolute permittivity of free space**. This constant is a measure of the charge per unit area, Q/A , on the plates needed to produce unit potential gradient, V/d , between the plates. The symbol for the absolute permittivity of free space is ϵ_0 (pronounced 'epsilon nought'). The value of ϵ_0 is $8.85 \times 10^{-12} \text{ F m}^{-1}$.

3 To investigate the effect of different materials between the plates on the capacitance: the capacitance is measured without then with different materials between the plates. Any material that increases the capacitance is a dielectric substance. As explained on p. 212, the molecules of a dielectric substance between the plates are polarised as the electrons in each molecule are attracted slightly to the positive plate. The effect is to increase the capacitance of the capacitor. The relative permittivity, ϵ_r , of a dielectric substance can therefore be written as

$$\epsilon_r = \frac{\text{capacitance, } C, \text{ with dielectric filling the plates}}{\text{capacitance, } C_0, \text{ with no substance between the plates}}$$

$$\text{hence } C = \epsilon_r C_0 = \frac{A \epsilon_0 \epsilon_r}{d}$$

where A = surface area, d = plate spacing, ϵ_0 = absolute permittivity of free space, and ϵ_r = the relative permittivity of the dielectric.

15.6 WORKED EXAMPLE

Calculate the capacitance of a pair of conducting strips of metal foil of length 1.20 m and width 0.05 m, separated by a sheet of polythene of thickness 0.20 mm. The relative permittivity of polythene = 2.3. ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$)

Solution

$$\text{Using } C = \frac{A \epsilon_0 \epsilon_r}{d} \text{ gives } \frac{1.2 \times 0.05 \times 8.85 \times 10^{-12} \times 2.3}{0.20 \times 10^{-3}} = 6.1 \times 10^{-9} \text{ F}$$

15.4 QUESTIONS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$$

- 1** A capacitor consists of two strips of metal foil of length 1.5 m and width 0.040 m separated by an insulating layer of thickness 0.01 mm and of relative permittivity 2.5.
 - a)** Calculate the capacitance of this capacitor.
 - b)** (i) The insulator breaks down and becomes a conductor if the p.d. across it exceeds 700 kV per millimetre thickness. Calculate the maximum p.d. allowable across the insulator if it is not to break down. (ii) Calculate the energy stored in the capacitor at maximum p.d.
- 2** A car is capable of storing charge in dry weather if it is insulated from the ground.
 - a)** By considering a car as one plate of a parallel plate capacitor and the ground as the other plate, estimate the capacitance of a car which has a clearance of 0.10 m above the ground and an effective surface area of 6.0 m^2 .
 - b)** Calculate the charge and energy stored by the car for a p.d. of 1000 V.

15.5 Capacitor discharge

The power supply of a computer may be fitted with a large capacitor in parallel with its output terminals. If the power supply is interrupted, the capacitor continues to supply electricity to the computer until the power supply is restored. Without the capacitor, the loss of power to the computer would cause loss of data. Provided the interruption is not too long, the capacitor can supply enough power to enable the computer to operate normally.

Experiment to investigate the discharge of a capacitor through a resistor

An oscilloscope or a digital voltmeter may be used to measure the voltage across a capacitor as it discharges through a resistor. In Fig. 15.21a, the voltage across the capacitor is measured at regular intervals after the switch is opened. The results may be plotted as a graph of voltage on the vertical axis against time on the horizontal axis, as in Fig. 15.21b.

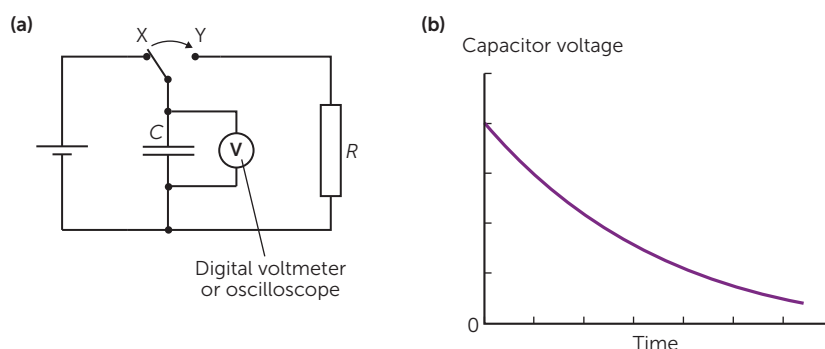


Fig. 15.21 Investigating capacitor discharge (a) circuit diagram, (b) exponential decay

The voltage decreases with time at a decreasing rate. The shape of the curve in Fig. 15.21b is called an **exponential decay** curve. The voltage continues to decrease but in theory never becomes zero. Suppose the time taken for the voltage to drop to 90% of its initial value is t_1 , then the voltage drops to 90% of 90% of the initial voltage in time $2t_1$ and to 90% of 90% of 90% of the initial voltage in time $3t_1$, and so on. The voltage therefore never becomes zero, although in practice it eventually becomes negligibly small.

For example, if the voltage drops from 10.0 V initially to 9.0 V in 5.0 s, then it falls to 8.1 V (= 90% of 9.0 V) in a further 5.0 s and to 7.3 V (= 90% of 8.1 V) 5.0 s later. Table 15.2 shows how the voltage in this numerical example continues to decrease. The calculated values are plotted in Fig. 15.22 to give a curve with the same shape as the experimental curve in Fig. 15.21b.

Table 15.2 Calculated results								
Time from start / s	0.00	5.00	10.00	15.00	20.00	25.00	30.00	35.00
Voltage / V	10.00	9.00	8.10	7.29	6.56	5.90	5.31	4.78

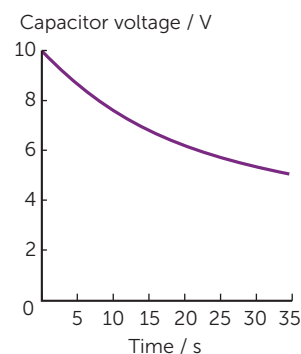


Fig. 15.22 Calculated voltage versus time

Theory of capacitor discharge

Consider the capacitor at time, t , after the discharge has started. Let V represent the voltage at this instant.

- 1 The charge on the capacitor, $Q = CV$, where C is its capacitance.
- 2 The discharge current, I , through the resistor, R , is given by the equation

$$I = \frac{V}{R}$$

Combining the two equations gives

$$I = \frac{Q}{CR}$$

In a small time interval, Δt , the charge flow $\Delta Q = I\Delta t$, hence the current

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

where $\frac{d}{dt}$ means 'rate of change'. See Section 2.6 if necessary.

$$\therefore \frac{dQ}{dt} = -\frac{Q}{CR}$$

where the minus sign indicates loss of charge.

The solution of this equation is

$$Q = Q_0 e^{-t/CR}$$

where the function

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3 \times 2} + \frac{x^4}{4 \times 3 \times 2} + \dots$$

This function is called the **exponential function**. Its shape is shown in Fig. 15.22. Its main property is that its rate of change

$$\frac{d}{dx}(e^x)$$

is proportional to the function itself. This is because the rate of change of any of the terms in this formula equals the adjacent left-hand term. For example,

$$\frac{d}{dx}\left(\frac{x^2}{2}\right) = x$$

Therefore, the exponential function appears in any physical process in which the rate of change of a quantity is proportional to the quantity itself.

Mathematical note on differentiation

The operation of determining an expression for the rate of change of a function is called **differentiation**. The reason why the rate of change of any term in the exponential function is equal to the adjacent left-hand term is because of the general rule that

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

To explain this general rule, consider the function $y = x^n$.

A small change of x to $x + \delta x$ causes a small change of y to $y + \delta y$, where $y + \delta y = (x + \delta x)^n$

Expansion of $(x + \delta x)^n$ gives $x^n + nx^{n-1}\delta x + \text{terms in } \delta x^2, \delta x^3, \text{ etc.}$. Hence $\delta y = (x + \delta x)^n - x^n = nx^{n-1}\delta x + \text{terms in } \delta x^2, \delta x^3, \text{ etc.}$

$$\begin{aligned} \therefore \frac{\delta y}{\delta x} &= \frac{nx^{n-1}\delta x + \text{terms in } \delta x^2, \delta x^3, \text{ etc.}}{\delta x} \\ &= nx^{n-1} + \text{terms in } \delta x, \delta x^2, \text{ etc.} \end{aligned}$$

In the limit $\delta x \rightarrow 0$, all the terms in $\delta x, \delta x^2, \text{ etc.}$ become zero

$$\text{and } \frac{\delta y}{\delta x} \rightarrow \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = nx^{n-1}$$

Notes on the exponential function

- 1 The inverse of the exponential function is called the natural logarithm function, denoted by either 'ln' or 'log_e'. In other words, if $y = e^x$ then $x = \ln y$.
- 2 A scientific calculator will have an exponential function button labelled 'e' or accessed by pressing 'inverse ln'.
- 3 The quantity, CR , is called the **time constant** of the circuit. It is equal to the time taken for the capacitor to discharge to $0.37Q_0$. This is because at time $t = CR$, $Q = Q_0 e^{-1} = 0.37Q_0$.
- 4 Rearranging the equation

$$\frac{dQ}{dt} = -\frac{Q}{CR} \text{ gives } \frac{dQ}{Q} = -\frac{1}{CR} dt$$

This rearranged equation means that the fractional decrease of charge

$$\frac{dQ}{Q}$$

in each interval of time, dt , is a constant, determined only by the product CR . For example,

if $C = 100 \mu\text{F}$, $R = 0.5 \text{ M}\Omega$ (giving $\frac{1}{CR} = 0.02$) and $dt = 5.0 \text{ s}$,

$$\text{then } \frac{dQ}{Q} = -\frac{1}{CR} dt = -0.02 \times 5 = -0.1.$$

The charge therefore decreases by 10% every 5 s. Work out for yourself that if the initial charge is $1000 \mu\text{C}$, the charge falls to $900 \mu\text{C}$ after 5 s, to $810 \mu\text{C}$ after 10 s, to $729 \mu\text{C}$ after 15 s, etc. as in [Table 15.2](#).

See www.palgrave.com/foundations/breithaupt to model capacitor discharge using a spreadsheet.

15.7 WORKED EXAMPLE

A $2.0 \mu\text{F}$ capacitor is charged to a voltage of 5.0 V using the circuit in [Fig. 15.23](#). The capacitor is then discharged through a $10 \text{ M}\Omega$ resistor by reconnecting the switch. Calculate

- a) the initial charge stored by the capacitor,
- b) the charge stored 30 s after the switch was reconnected,
- c) the voltage after 30 s.

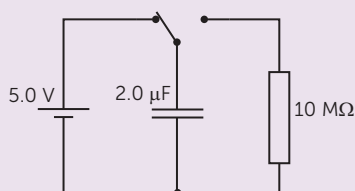


Fig. 15.23

Solution

- a) Initial charge, $Q_0 = CV_0$, where V_0 is the initial voltage.

$$\text{Hence } Q_0 = 2.0 \mu\text{F} \times 5.0 \text{ V} = 10 \mu\text{C}$$

- b) $t = 30 \text{ s}$, $CR = 2.0 \mu\text{F} \times 10 \text{ M}\Omega = 20 \text{ s}$

$$\therefore \frac{t}{CR} = \frac{30}{20} = 1.5$$

$$\text{hence } Q = Q_0 e^{-t/CR} = 10 e^{-1.5} = 2.2 \mu\text{C}$$

- c) $V = \frac{Q}{C} = \frac{2.2 \mu\text{C}}{2.0 \mu\text{F}} = 1.1 \text{ V}$

15.5 QUESTIONS

- 1 A $5.0\ \mu\text{F}$ capacitor, charged using a $12.0\ \text{V}$ battery, is discharged through a $0.5\ \text{M}\Omega$ resistor. Calculate
 - a) the initial charge and energy stored in the capacitor,
 - b) the charge and energy stored $5.0\ \text{s}$ after the discharge started.
- 2 A $2200\ \mu\text{F}$ capacitor is charged from a $6.0\ \text{V}$ battery then discharged through a $100\ \text{k}\Omega$ resistor. Calculate
 - a) the initial charge stored in the capacitor,
 - b) the time constant of the circuit,
 - c) the charge stored and the capacitor voltage after a time equal to the time constant,
 - d) the current through the resistor at this instant.

SUMMARY

- **Capacitance** is defined as charge stored per unit voltage.
- **The unit of capacitance** is the farad (F), equal to 1 coulomb per volt.
- **The time constant** of a capacitor discharge circuit = CR .
- **Equations**

Capacitance equation, $C = \frac{Q}{V}$

Combined capacitance

$C = C_1 + C_2$ for two capacitors in parallel

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ for two capacitors in series

Energy stored, $E = \frac{1}{2}CV^2$

Capacitance of parallel plates, $C = \frac{A\epsilon_0\epsilon_r}{d}$

Capacitor discharge equation, $\frac{dQ}{dt} = -\frac{Q}{CR}$

Solution of the capacitor discharge equation, $Q = Q_0e^{-t/CR}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

15.1

- a) A capacitor is connected in series with a light-emitting diode, a switch and a $1.5\ \text{V}$ cell. When the switch is closed, the LED lights briefly. Explain
 - (i) why the LED lights up initially,
 - (ii) why the LED stops emitting light.
- b) A $5000\ \mu\text{F}$ capacitor in series with a variable resistor, an ammeter and a $3.0\ \text{V}$ battery is charged at a constant current of $0.15\ \text{mA}$. Calculate
 - (i) the charge stored by the capacitor when it is fully charged,
 - (ii) the time taken to charge fully with a constant charging current of $0.15\ \text{mA}$.

15.2

- a) Calculate the combined capacitance of a $3\ \mu\text{F}$ capacitor and a $6\ \mu\text{F}$ capacitor
 - (i) in parallel with the other,
 - (ii) in series with each other.
- b) Calculate
 - (i) the total capacitance,
 - (ii) the total charge stored,
 - (iii) the charge and energy stored in each capacitor for the circuit in Fig. 15.24.

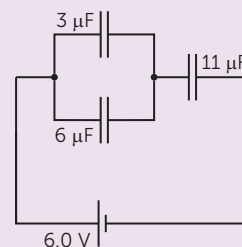


Fig. 15.24

REVISION QUESTIONS

15.3

- a) A parallel plate capacitor consists of two rectangular metal plates of dimensions $0.30 \text{ m} \times 0.25 \text{ m}$, separated by an insulator of thickness 1.5 mm and of relative permittivity 3.5 . Calculate the capacitance of the capacitor.
- b) Calculate the charge and energy stored in the parallel plate capacitor in (a) when a voltage of 50 V is applied across its plates.

15.4

- a) In Fig. 15.25, a $4.7 \mu\text{F}$ capacitor is charged to a p.d. of 9.0 V then discharged through a $10 \text{ M}\Omega$ resistor. Calculate
- the initial charge and energy stored in the capacitor,
 - the time constant of the discharge circuit,
 - the charge and energy stored in the capacitor 100 s after the discharge started.
- b) In Fig. 15.25, the capacitor is allowed to discharge fully and the $10 \text{ M}\Omega$ resistor is replaced by a $2.2 \mu\text{F}$ capacitor. The $4.7 \mu\text{F}$ capacitor is then charged to a p.d. of 9.0 V then isolated from the battery and reconnected to the $2.2 \mu\text{F}$ capacitor. Calculate
- the final p.d. across each capacitor,
 - the energy stored in each capacitor at this p.d.,
 - the loss of stored energy as a result of connecting the capacitors together.

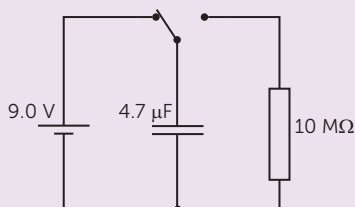


Fig. 15.25

15.5. A security alarm circuit is powered by a 6.0 V voltage supply which has a $50\,000 \mu\text{F}$ capacitor connected across its output terminals. The alarm circuit draws a current of 0.5 mA from the voltage supply when the alarm is not activated.

- a) Calculate the input resistance of the alarm circuit when it is not activated.

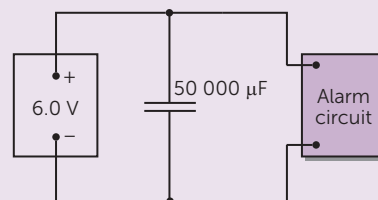


Fig. 15.26

- b) If the voltage supply is interrupted, the capacitor continues to supply current to the alarm circuit, which can operate in its non-activated state provided the voltage supply does not fall below 5.0 V .
- Calculate the time constant of the circuit.
 - Calculate the voltage across the capacitor 100 s after the 6.0 V voltage supply has been disconnected.
 - Calculate how long the alarm circuit in its non-activated state can operate after the voltage supply has been disconnected.

UNIT

16

Electronics

CONTENTS

- 16.1 The systems approach
- 16.2 Electronic logic
- 16.3 Electronics at work
- 16.4 Operational amplifiers
- 16.5 Multivibrators
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- represent an electronic system in block diagram form
- explain the difference between a digital system and an analogue system
- describe the function of different types of logic gates
- explain the operation of simple combinations of logic gates
- describe and explain the operation of different input and output transducers
- describe and explain the operation of an inverting op-amp and a non-inverting op-amp
- describe and explain the operation of a monostable and an astable multivibrator
- use an oscilloscope to display and measure waveforms
- construct and test a simple electronic system

16.1 The systems approach

Make a list of the functions that can be carried out using a calculator watch. In addition to keeping the time very accurately, such a watch can be used to perform calculations or to store phone numbers as well as keeping track of the date and the time in every time zone. All these functions and more are carried out by a tiny electronic chip inside the watch. Thousands of electronic switches inside the chip control and process the input voltages from the key pad and the selector buttons. To understand how the chip works, a block diagram is used to show the function of each part of the chip and how these parts relate to each other. This method of representing electronic circuits is referred to as a 'systems approach'. The function of each block must be known, rather than the circuits the block represents. [Fig. 16.1](#) shows part of such a block diagram in which voltage pulses at exactly 1 pulse per second are supplied to a counter linked to a separate display chip.

Electronic circuits are either **digital** or **analogue**. These terms are used widely in connection with telecommunications, particularly in connection with mobile phones and television broadcasting.

- 1 In a **digital circuit**, the voltage at any point in the circuit can have just two values, referred to as 'high' or 'low' or as 1 or 0. Usually, these two voltages are close to the voltage limits set by the power supply. For example, a digital electronic system connected to a 5.0 V power supply might operate at 4.5 V for its high voltage (1) and 1.0 V for its low voltage (0). No point in the system would be at a voltage between 1.0 and 4.5 V.
- 2 In an **analogue circuit**, the voltage at any point can vary over a continuous range within the limits set by the power supply. For example, the voltage from a microphone to an amplifier increases with loudness and the output voltage from the amplifier is proportional to the input voltage. The amplifier is an analogue circuit because the voltages in it can be at any level between the limits of the power supply, depending on the loudness of sound at the microphone.

16.1 QUESTIONS

- 1 Find out if each of the following electronic devices is digital or analogue.
 - a) A fax machine,
 - b) a microcomputer,
 - c) an audio amplifier,
 - d) a music cassette player.
- 2 Make a block diagram showing a microcomputer, keyboard, printer, mouse and visual display unit, indicating clearly the direction of the signals in each link.

16.2 Electronic logic

Everyone makes decisions every day. Each decision involves making a choice, usually determined by conditions relevant to the decision. A logical decision is one in which the choice made is the same whenever the same conditions arise. For example, suppose you are about to go out and you are thinking about whether or not to wear a coat. Your decision might depend on your answer to two questions:

- Question A Is it raining?
 Question B Is it cold outdoors?

A logical answer to these questions would be YES (i.e. take a coat) if A = YES **OR** B = YES.

The decision could be made for you by an electronic circuit consisting of a temperature sensor and a rain sensor connected to a logic circuit known as an OR gate as in Fig. 16.2. A logic circuit is a digital circuit in which the output voltage is determined by the condition of the input voltage. All voltages are relative to the

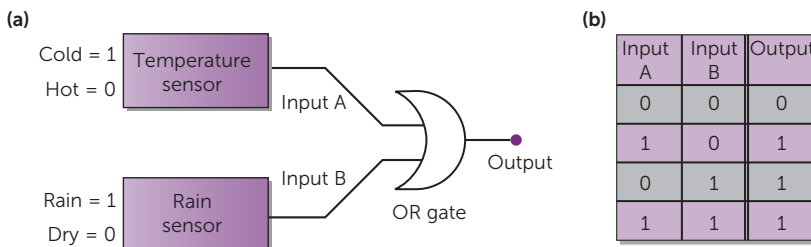


Fig. 16.2 (a) using an OR gate, (b) the truth table of an OR gate

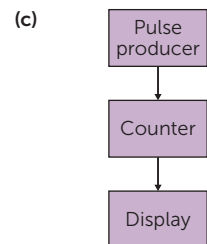
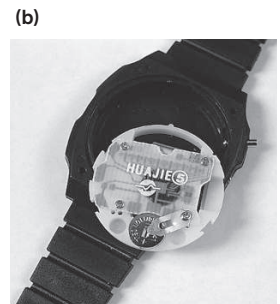


Fig. 16.1 An electronic system (a) a digital watch, (b) inside a digital watch, (c) a block diagram

Type of gate	Symbol	Truth table	
		Input A B	Output
OR		0 0 0 1 1 0 1 1	0 1 1 1
AND		0 0 0 1 1 0 1 1	0 0 0 1
NOR		0 0 0 1 1 0 1 1	1 0 0 0
NAND		0 0 0 1 1 0 1 1	1 1 1 0
NOT		0 1	1 0

Fig. 16.3 Logic gates

negative terminal of the voltage supply which is usually earthed. Each input or output voltage has two possible 'states', which are 'high' (i.e. 1) and 'low' (i.e. 0). The OR gate in Fig. 16.2 is designed to produce an output state of 1 if any of its input states is 1. The truth table in Fig. 16.2b shows how the output state depends on the input states for this type of logic gate.

The circuit symbols and truth tables for some common logic gates are shown in Fig. 16.3. In Fig. 16.2a, what would happen if an AND gate was used instead of an OR gate? You could be very cold on a dry day in winter or very wet on a wet day in summer!

Inside a logic gate

Logic gates are made in chip form. Fig. 16.4 shows a chip which contains four NAND gates, each with two inputs and an output. The chip contains tiny electronic switches called transistors as well as resistors and diodes. All these components are made on a small piece of pure silicon in a process that involves masking the silicon surface at different stages and 'doping' the exposed surface with different types of atoms. It is not necessary to understand the internal circuitry of a chip or the manufacturing process to understand its functions. Table 16.1 shows the main properties of TTL and CMOS chips which are the two main families of chips in common use at present.



Fig. 16.4 A quad 2-input NAND chip. (Reproduced with the permission of Maplin Electronics.)

Table 16.1 Comparison of TTL and CMOS chips

	Supply voltage V_s	Input voltage	Output voltage	Input resistance	Power consumption	Fan-out
TTL	5 V	Low = 0.8 V High = 2.0 V	Low = 0.4 V High = 2.4 V	$10^6 \Omega$	milliwatts	10
CMOS	3–15 V	Low = 0.3 V High = 3.5 V for 5 V supply	Low = 0 High = 5.0 V for 5 V supply	$10^{12} \Omega$	microwatts	50

Notes

- 1 The fan-out is the number of inputs of other gates that can be 'driven' by the output of the gate.
- 2 Unconnected TTL inputs are always high (i.e. logic state = 1) whereas unconnected CMOS inputs can be high or low. Unused CMOS inputs should always be connected either to $+V_s$ or $-V_s$ according to the circuit.
- 3 The output voltage of a logic gate can be indicated using an LED in series with a suitable resistor, as in Fig. 16.5. The LED lights when a '1' is applied to the input terminal of the indicator.

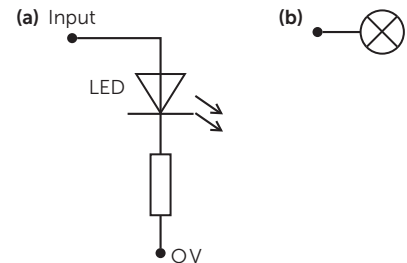


Fig. 16.5 An LED indicator (a) circuit, (b) symbol

Logic gate combinations

In general, a logic system consists of input sensors connected to a logic circuit which drives an output device. Different types of input sensors include switch sensors, pressure sensors, light sensors and temperature sensors. Output devices include warning indicators driven directly by logic gates, and relay-operated electric motors, heaters and sound alarms. The logic circuit is a combination of logic gates linked together to activate the output device when the input sensors are in specified states.

For example, new cars are fitted with a 'seat belt' warning light that indicates if either of the front seat belts is unfastened when the handbrake is on and either seat is occupied. Sensors are attached to each front seat and to the seat belts. These sensors are connected to the logic circuit which switches the warning indicator on or off according to the input conditions. Fig. 16.6 shows a block diagram and the truth table for this system. The warning light is on if the driver's seat is occupied AND the driver's seat belt is unfastened OR the front passenger's seat is occupied AND the passenger seat belt is unfastened.

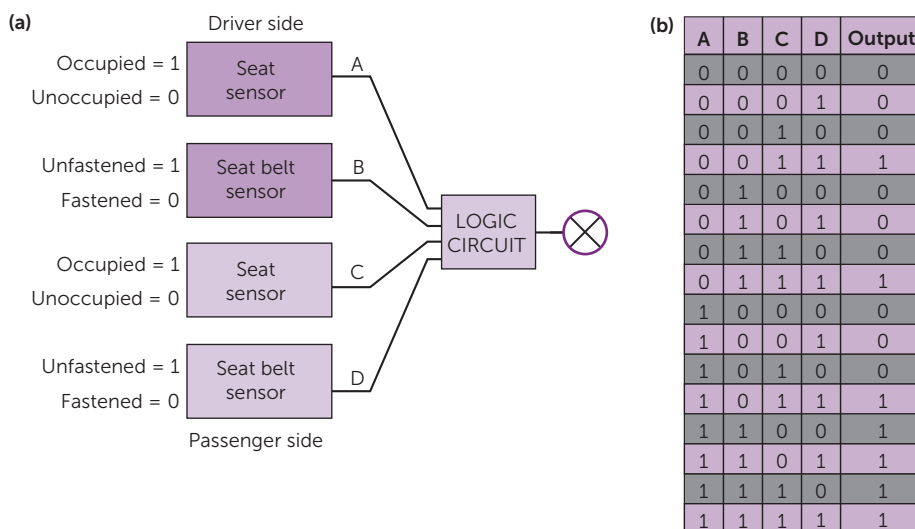


Fig. 16.6 A seat-belt warning indicator (a) block diagram, (b) truth table

A suitable logic circuit for this system is shown in Fig. 16.7. A seat sensor and a seat-belt sensor fitted to each front seat supply the input voltages to an AND gate. The outputs of the two AND gates are supplied to the inputs of an OR gate which is connected to the warning indicator. This logic gate combination activates the warning indicator if either AND gate is supplied with a 1 at each input.

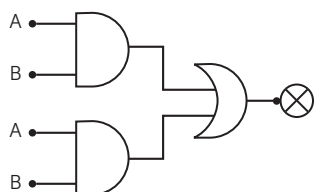


Fig. 16.7 Logic circuit

16.1 WORKED EXAMPLE

A home security alarm system consists of a control panel fitted with a key switch, K, to turn the system on or off, a main door sensor, D, and a window sensor, W. The alarm is activated if the system is on AND the main door is open OR the window sensor is open. Fig. 16.8 shows the block diagram and an incomplete truth table for the system.

- a) Complete the truth table for this system.
- b) Sketch a suitable combination of logic gates for this system.

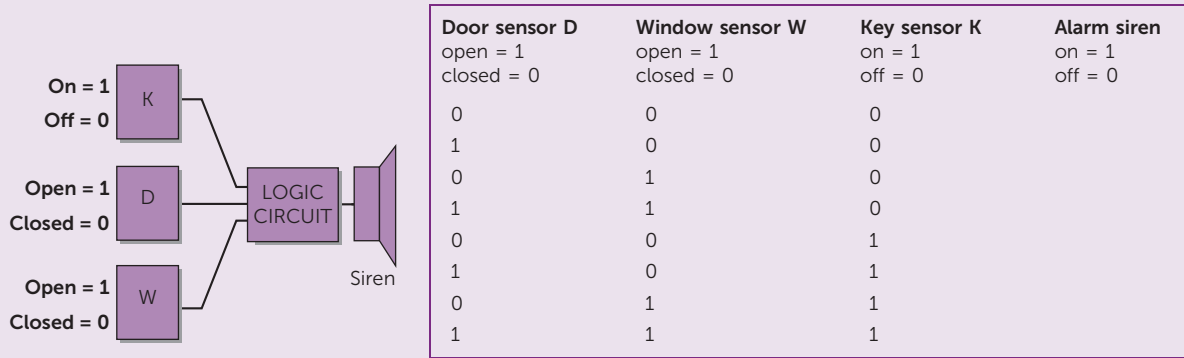


Fig. 16.8

Solution

a)

Door sensor D	Window sensor W	Key sensor K	Alarm siren
open = 1 closed = 0	open = 1 closed = 0	on = 1 off = 0	on = 1 off = 0
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	1

- b) See Fig. 16.9.

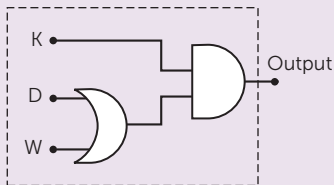
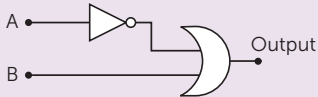


Fig. 16.9

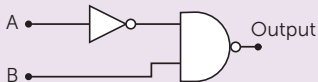
16.2 QUESTIONS

- 1 Create the truth table for each of the combinations of logic gates illustrated in Fig. 16.10.

(a)



(b)



(c)

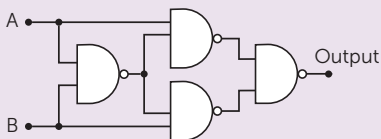


Fig. 16.10

- 2 a) A radio-operated car alarm is designed to operate a siren if the master switch is closed and either of the car's two doors are opened. Design a suitable electronic system including door sensors that supply logic 1 when a door is opened and a test switch that supplies logic 1 when pressed.
- b) A fire alarm system consists of a smoke sensor, a panic button, an on/off master switch and a test switch, as shown in Fig. 16.11. Design a suitable combination of logic gates for this system.

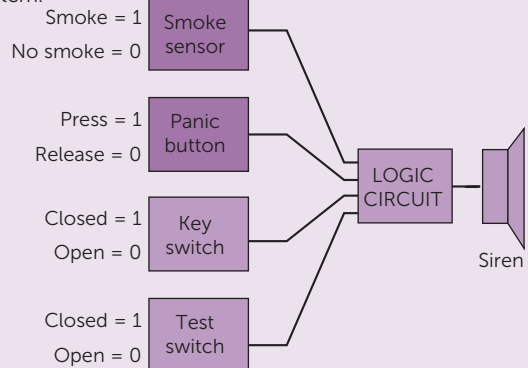


Fig. 16.11

16.3 Electronics at work

Automated doors, card-operated cash machines, pressure-activated air bags and many more systems use logic gates which are supplied with input voltages from sensors and which control devices such as electric motors and valves. All these systems consist of one or more input systems connected to a logic circuit which operates an output device.

Input sensors

Fig. 16.12 shows the construction of some different input sensors.

- **Switch sensor:** when the switch is closed, the output voltage is high. When the switch is open, no current passes through the resistor so the output voltage is low.
- **Pressure sensor:** the switch is a pressure-operated switch that closes only when the pressure is above a certain level.
- **Light sensor:** this is a potential divider consisting of an LDR and a variable resistor. When the LDR is in darkness, its resistance is much greater than that of the variable resistor so the output voltage is low. In daylight, the LDR resistance decreases and the output voltage is high. If the sensor is connected to the input of a NOT gate, the NOT output state will either be 0 or 1, according to whether or not the light intensity is above or below a value determined by the setting of the variable resistor.

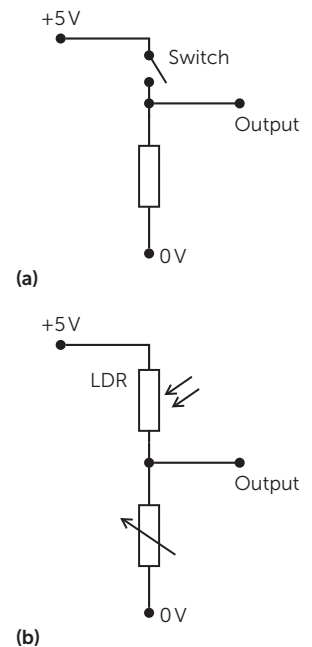


Fig. 16.12 Input sensors
(a) switch sensor, (b) light sensor

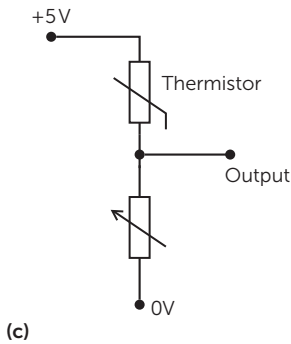


Fig. 16.12 Input sensors
(c) temperature sensor

- **Temperature sensor:** this is the same as a light sensor except the LDR is replaced by a thermistor. When the thermistor temperature is high, the resistance of the thermistor is low so the output voltage is high. If connected to a NOT gate, the NOT output state will be 0 if the thermistor is above a specified temperature and 1 if the thermistor is below that temperature.

Output devices

- A **warning indicator** can be visual (e.g. an LED indicator) or audible (e.g. a sound bleeper). See Fig. 16.13. Provided the current at the input terminal of a warning indicator is low enough to be supplied directly from the output terminal of a logic gate, no current amplifier is needed.

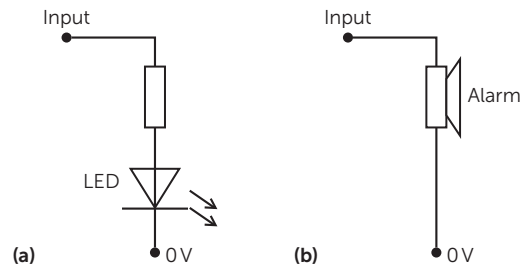


Fig. 16.13 Warning indicators (a) an LED indicator, (b) an audio indicator

- A **relay** is a current-operated switch that can be used to switch on or off a high-current output device such as a mains heater. A relay is necessary if the output device needs more current at its input terminal than a logic gate could supply. See Fig. 16.14. A relay consists of an electromagnet that automatically closes a switch when a current passes through the relay coil. If the necessary coil current is sufficiently low, the relay can be operated directly by the output voltage of a logic gate. If not, the relay itself must be switched on or off by a transistor.

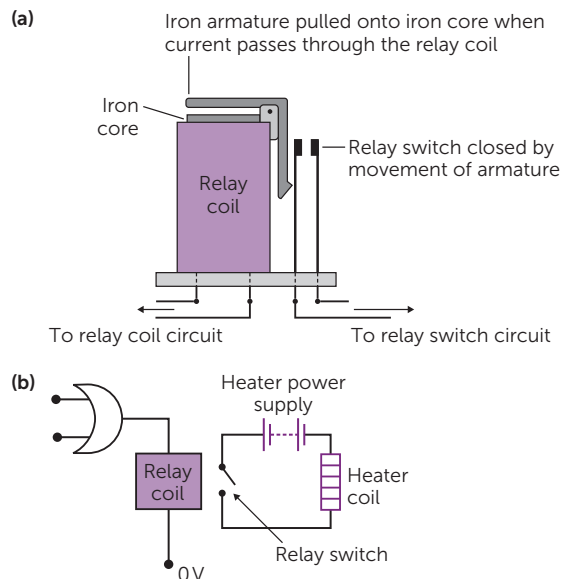


Fig. 16.14 Using a relay directly
(a) construction, (b) in a circuit

- A **transistor** is a semiconductor device that can function as an electronic switch. Essentially, a transistor is a current amplifier in which a small current controls a much larger current. It can therefore be used as a current-operated switch, as shown in Fig. 16.15. A small current which enters the base and leaves at the emitter of the transistor, controls a much larger current which enters the collector and leaves also at the emitter. The transistor may therefore be used to switch on or off a device in series with the collector. If the collector current is insufficient to operate the device, a relay coil in series with the collector can be used to switch on or off the device operated from its own power supply.

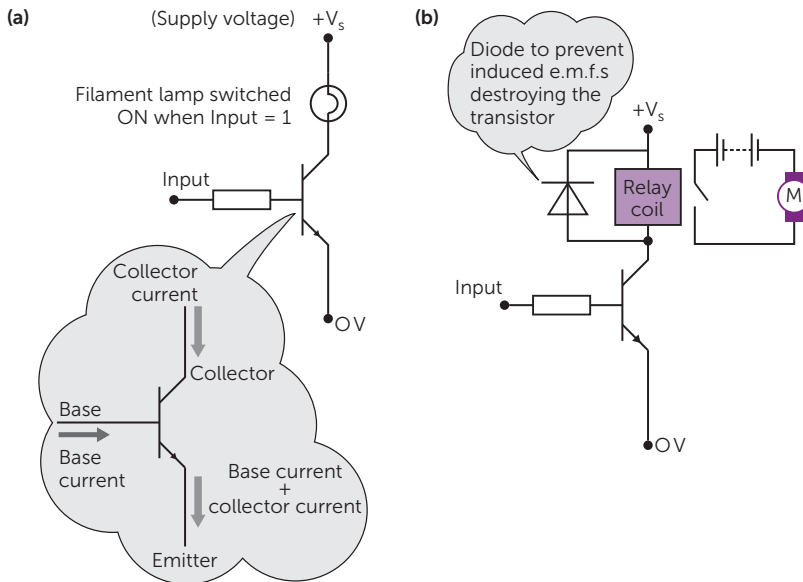


Fig. 16.15 (a) a transistor as a current-operated switch (b) a transistor-operated relay

Experiment to construct a light-operated electric motor

Construct and test the circuit in Fig. 16.16 which is designed to switch an electric motor on when the LDR is illuminated. The variable resistor is used to set the light level at which the motor is switched on.

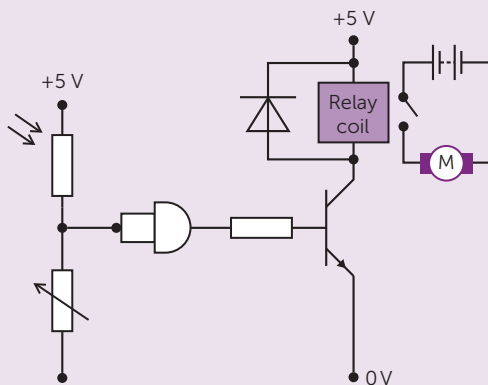


Fig. 16.16 A light-operated motor

16.3 QUESTIONS

- 1 Fig. 16.17 shows a temperature-controlled fan which is fitted with a test switch, S.
 - a) Explain why the fan is switched on if the test switch is closed.
 - b) (i) Why does the fan switch on if the thermistor is above a certain temperature? (ii) Describe and explain the effect of reducing the resistance of the variable resistor.
- 2 Design an electronic system that would switch a mains electric heater on if the temperature in a room falls below a certain value at night. Your system should include a light sensor, a temperature sensor, a test switch, a suitable logic circuit, as well as a heater operated via a relay and a transistor.

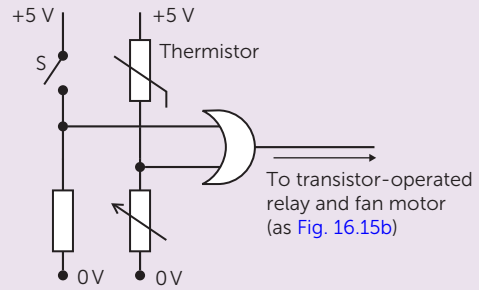


Fig. 16.17

16.4 Operational amplifiers

Amplifiers

An amplifier is an analogue circuit in which the output voltage depends on a smaller input voltage and increases as the input voltage increases. See Fig. 16.18. An amplifier in an audio system is used to make the audio signal stronger. The main characteristics of amplifiers are:

- The **voltage gain** is the ratio of the output voltage to the input voltage. If a graph of the output voltage (on the y -axis) against the input voltage (on the x -axis) is a straight line through the origin, the voltage gain is constant, equal to the gradient of the line.
If an alternating voltage is applied to the input of an amplifier, the output voltage waveform should be the same shape as the input voltage waveform and larger. At very high frequencies, the voltage gain of an amplifier becomes much smaller than at low frequencies. Fig. 16.19 shows how a dual beam oscilloscope can be used to compare the input and output waveforms of an amplifier.
- **Saturation** of the output voltage occurs when the output voltage reaches the limits set by the amplifier power supply. For example, the output voltage of an

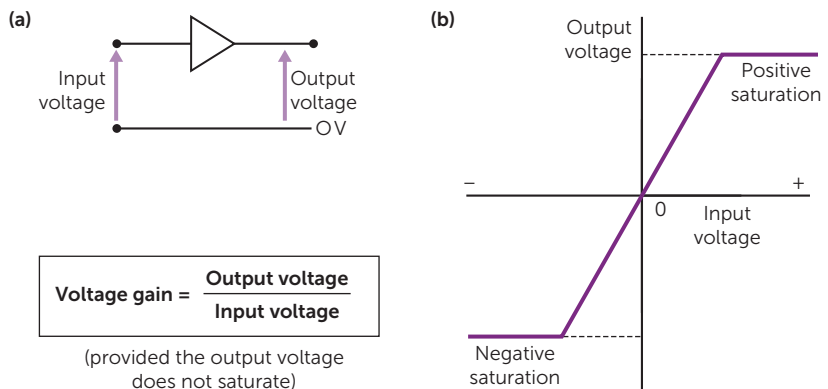


Fig. 16.18 Voltage gain (a) an amplifier, (b) output voltage versus input voltage

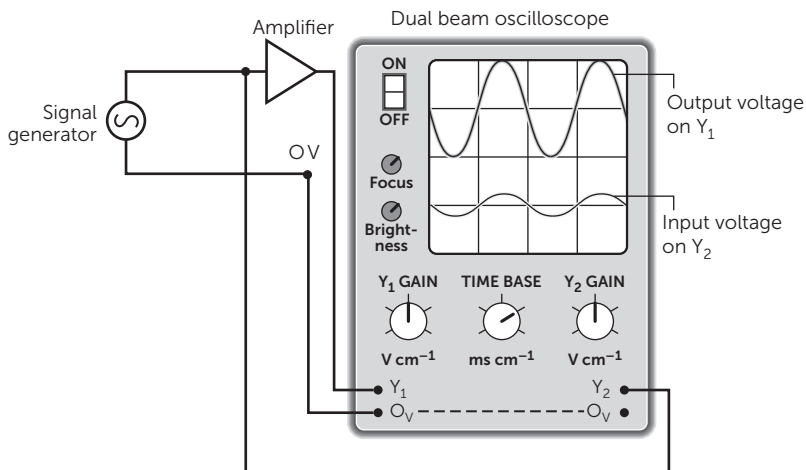


Fig. 16.19 Amplifying an alternating voltage

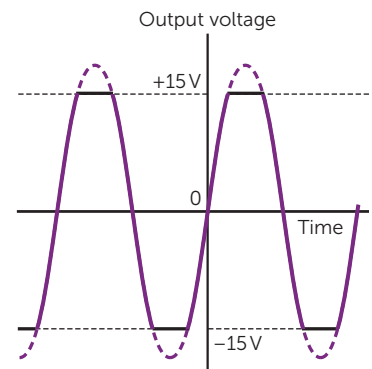


Fig. 16.20 Saturation

amplifier which operates from a 15 V power supply can vary from 0 to 15 V. If the voltage gain of the amplifier is 50, the output voltage would be 15 V when the input voltage is 0.3 V ($= 15 \text{ V} \div 50$). If the input voltage exceeds 0.3 V, the output voltage remains at 15 V and is said to be 'saturated'. Fig. 16.20 shows the effect of saturation on an alternating voltage.

- The **bandwidth** of an amplifier is the frequency range over which the voltage gain is constant. Fig. 16.21 shows how the voltage gain of a typical amplifier varies with the frequency of the alternating voltage at its input. In this example, an audio signal which contains a continuous range of frequencies up to about 18 kHz would be distorted because its frequency components above 10 kHz would not be amplified as much as the components below 10 kHz.
- The **signal to noise ratio** of an amplifier determines how weak an input signal can be and still be detected at the output of the amplifier. Most amplifiers generate internal 'noise' caused by random motion of electrons in the resistors in the circuit. If the noise is too high, the input signal cannot be detected at the output. For example, a hiss due to internal noise is all that can be heard from an untuned radio.
- **Feedback** in an amplifier is where a fraction of the output voltage is fed back to the input terminals of the amplifier as shown in Fig. 16.22. The link between the output terminals and the input terminals of an amplifier is called a feedback loop. If no feedback exists, the amplifier is said to be on open-loop.

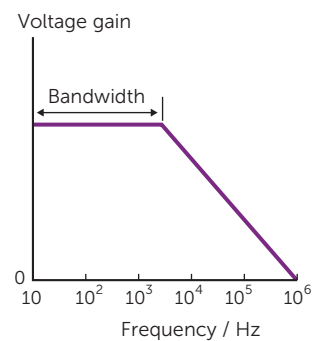


Fig. 16.21 Bandwidth

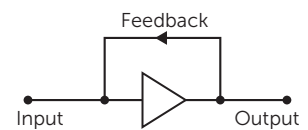


Fig. 16.22 Feedback

- 1 **Positive feedback** is where the fraction of the output voltage fed back to the input terminals reinforces the input voltage, making the output larger. For example, this occurs when a microphone is placed too near the speaker of a sound system, creating an unpleasant screeching noise.
- 2 **Negative feedback** is where the fraction of the output voltage fed back to the input terminals reduces the input voltage, making the output smaller. For example, a negative feedback loop would reduce the gain of an amplifier and make saturation less likely.

Operational amplifiers

An operational amplifier (or op-amp as it is sometimes called) is designed to amplify voltages or to add them together. By letting the voltage in an analogue circuit represent a mathematical variable, certain mathematical operations such as multiplication by a constant (i.e. amplification at constant voltage gain) or summation (i.e. adding two voltages together) can be carried out using operational amplifiers. For example, suppose an amplifier is required with a voltage gain of 10. This is achieved as explained later by connecting appropriate resistors to an operational amplifier. Choosing different resistors would give a different voltage gain. Unlike an audio amplifier, an operational amplifier can be adapted to perform exactly as needed.

An operational amplifier is a high-gain voltage amplifier with two input terminals and an output terminal. It is connected to a power supply with a positive terminal, $+V_S$, a negative terminal, $-V_S$, and an earthed terminal. The most common type of operational amplifier, the 741 chip, needs a ± 15 V power supply with an earthed terminal.

The output voltage, V_{OUT} , of an operational amplifier, the voltage between its output terminal and the zero voltage power supply terminal, can vary from $+V_S$ to $-V_S$, which are its supply voltage limits. The output voltage is proportional to the voltage difference between its two input terminals, which are labelled P and Q in Fig. 16.23.

$V_{OUT} = A(V_Q - V_P)$ where A is the open-loop voltage gain.

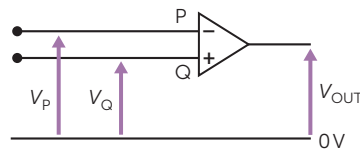


Fig. 16.23 An operational amplifier on 'open-loop'

- **Input Q** is called the **non-inverting input** because if P is earthed, the output voltage is the same polarity as the voltage applied to Q.
- **Input P** is called the **inverting input** because if Q is earthed, the output voltage is the opposite polarity to the voltage applied at P.
- The **input resistance** (i.e. the resistance between P and Q) for a 741 op-amp is of the order of $1\text{ M}\Omega$. Very little current therefore enters the op-amp at its input terminals.
- The **open-loop gain, A**, is very high, typically 100 000, varying slightly from one chip to another. Thus an op-amp on open-loop saturates if the voltage between P and Q exceeds $150\text{ }\mu\text{V}$ ($= 15\text{ V} \div 100\text{ 000}$). The output voltage is therefore at positive saturation (i.e. $+V_S$) when V_Q exceeds V_P by $150\text{ }\mu\text{V}$, or at negative saturation (i.e. $-V_S$) when V_P exceeds V_Q by $150\text{ }\mu\text{V}$. Fig. 16.24 shows how the output voltage depends on the input voltage.
- A **comparator** is an op-amp on open-loop which can be used to compare two voltages by applying one voltage to input P and one voltage to input Q. If $V_{OUT} = +V_S$, then V_Q exceeds V_P ; If $V_{OUT} = -V_S$, then V_P exceeds V_Q . Such comparators are used in computer circuits to open electronic gates and direct data along desired routes.

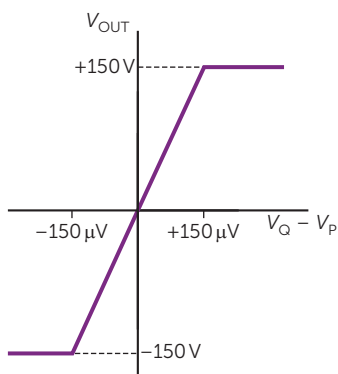


Fig. 16.24 Output voltage versus input voltage for an op-amp on open-loop

- The **voltage follower** shown in Fig. 16.25 is an op-amp in which the output terminal is directly connected to the inverting terminal, P. The input voltage is applied to the non-inverting terminal, Q. Provided the output does not saturate, P and Q are virtually at the same voltage. Hence the voltage at the output is the same as the voltage at the input. In other words, the output voltage follows the input voltage. Because the input resistance between P and Q is very high, the voltage follower draws very little current from the circuit its input terminal is connected to. It can therefore be used with a moving coil voltmeter across its output to measure voltage without drawing current.

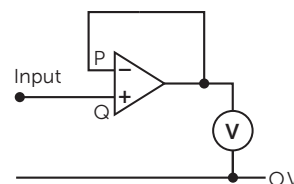


Fig. 16.25 The voltage follower

The non-inverting amplifier

A potential divider between the output terminal and the 0 V line (i.e. earth) is used to supply a fraction of the output voltage to the inverting input, P. The input voltage, V_{IN} , is applied to the non-inverting input. Fig. 16.26 shows the arrangement.

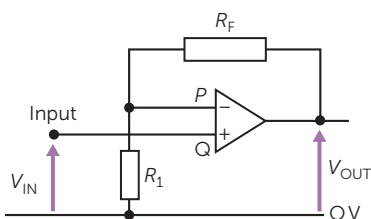


Fig. 16.26 The non-inverting amplifier

Provided the output voltage does not saturate, V_Q and V_P are virtually at the same voltage, since $V_Q - V_P$ does not exceed $150 \mu\text{V}$.

$$\text{Since } V_P = \frac{R_1}{R_F + R_1} V_{OUT}$$

(as R_1 and R_F form the potential divider) and

$$V_Q = V_{IN},$$

then

$$V_P = V_Q \text{ gives } \frac{R_1}{R_F + R_1} V_{OUT} = V_{IN}$$

$$\text{the voltage gain } \frac{V_{OUT}}{V_{IN}} = \frac{R_1 + R_F}{R_1}$$

Note

The voltage gain is therefore determined only by the resistances, R_1 and R_F , and is independent of the open-loop gain of the chip.

16.2 WORKED EXAMPLE

A non-inverting amplifier is required with a voltage gain of 20. Calculate
 a) the feedback resistance if the resistance of R_1 is $0.5\text{ M}\Omega$,
 b) the maximum input voltage if the output voltage is not to exceed 15 V .

Solution

a) Voltage gain $= \frac{R_1 + R_F}{R_1} = 20$, and $R_1 = 0.5\text{ M}\Omega$

$$\therefore \frac{0.5 + R_F}{0.5} = 20, \text{ so } 0.5 + R_F = (20 \times 0.5) = 10$$

$$\text{Hence } R_F = 10 - 0.5 = 9.5\text{ M}\Omega$$

b) For $V_{\text{OUT}} = 15\text{ V}$, $V_{\text{IN}} = \frac{V_{\text{OUT}}}{20} = 0.75\text{ V}$

The inverting amplifier

A potential divider between the output terminal and the input terminal is used to supply a fraction of the output voltage to the inverting input, P. The non-inverting input is connected to the 0 V line. Fig. 16.27 shows the arrangement.

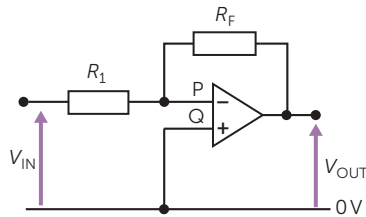


Fig. 16.27 The inverting amplifier

Notes

- 1 The voltage gain depends only on the resistances R_F and R_1 , and is independent of the open-loop gain of the chip.
- 2 The output voltage has the opposite polarity to the input voltage.
- 3 Negligible current enters the op-amp at its input terminals P and Q because its input resistance is very high. Consequently the current through R_1

$$= \frac{V_P - V_{\text{IN}}}{R_1}$$

is equal to the current through R_F

$$= \frac{(V_{\text{OUT}} - V_P)}{R_F}$$

Since $V_P = 0$, then

$$\frac{V_{\text{OUT}}}{R_F} = -\frac{V_{\text{IN}}}{R_1}$$

which gives the voltage gain formula.

Resistors R_1 and R_F form a potential divider between the output terminal to V_{OUT} and the input terminal at V_{IN} .

Hence the voltage between P and the input terminal

$$V_P - V_{\text{IN}} = \frac{R_1}{R_F + R_1}(V_{\text{OUT}} - V_{\text{IN}})$$

Provided the output voltage does not saturate, V_Q and V_P are virtually at the same voltage, since $V_Q - V_P$ does not exceed $150\text{ }\mu\text{V}$. Because Q is earthed (i.e. $V_Q = 0$) then V_P is virtually at earth potential too (i.e. $V_P \approx 0\text{ V}$).

$$\text{Hence } -V_{\text{IN}} = \frac{R_1}{R_F + R_1}(V_{\text{OUT}} - V_{\text{IN}}) \text{ as } V_P = 0$$

Rearranging this equation gives

$$-V_{\text{IN}}(R_F + R_1) = R_1(V_{\text{OUT}} - V_{\text{IN}})$$

$$\therefore -V_{\text{IN}}R_F - V_{\text{IN}}R_1 = R_1V_{\text{OUT}} - R_1V_{\text{IN}}$$

$$\text{hence } -V_{\text{IN}}R_F = R_1V_{\text{OUT}} - R_1V_{\text{IN}}$$

$$\text{the voltage gain } \frac{V_{\text{OUT}}}{V_{\text{IN}}} = -\frac{R_F}{R_1}$$

The summing amplifier

Each voltage to be added is applied to an input terminal connected to the inverting input, P, via a resistor. The non-inverting input is earthed, as in Fig. 16.28.

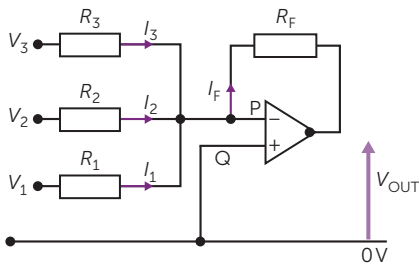


Fig. 16.28 The summing amplifier

Since P is at virtual earth, assuming the output voltage is not saturated, the current through resistor R_1 can be written as

$$I_1 = \frac{V_P - V_1}{R_1} = -\frac{V_1}{R_1}$$

the current through resistor R_2 can be written as

$$I_2 = \frac{V_P - V_2}{R_2} = -\frac{V_2}{R_2}$$

the current through resistor R_3 can be written as

$$I_3 = \frac{V_P - V_3}{R_3} = -\frac{V_3}{R_3}, \text{ etc.}$$

Hence the total current through the feedback resistor R_F can be written as

$$I_F = I_1 + I_2 + I_3 + \dots$$

$$\therefore I_F = -\frac{V_1}{R_1} - \frac{V_2}{R_2} - \frac{V_3}{R_3} - \dots$$

Since the output voltage, $V_{OUT} = I_F R_F$ then

$$V_{OUT} = -\frac{R_F V_1}{R_1} - \frac{R_F V_2}{R_2} - \frac{R_F V_3}{R_3} = \dots$$

Notes

1. The output voltage = constant $\times$ the sum of the input voltages if $R_1 = R_2 = R_3$, etc.
2. The summing amplifier may be used to convert a data byte (e.g. 1110) into an analogue voltage by choosing the resistances such that

$$V_{OUT} = -V_1 - 2V_2 - 4V_3 - 8V_4, \text{ etc.}$$
 and applying the least significant bit to V_1 , the next bit to V_2 , etc. Thus the data byte, 1110, would give an output voltage of -14 V if each bit except the least significant bit is at 1 V.

16.3 WORKED EXAMPLE

Fig. 16.29 shows an inverting operational amplifier and a graph of its output voltage against its input voltage.

- Calculate (i) its voltage gain from the graph, (ii) the feedback resistance necessary to produce this voltage gain.
- An alternating voltage is applied between the input terminal of this operational amplifier and its zero volt line. Sketch graphs to show how the output voltage varies with time if the peak-to-peak value of the input voltage is (i) 2.0 V, (ii) 4.0 V.

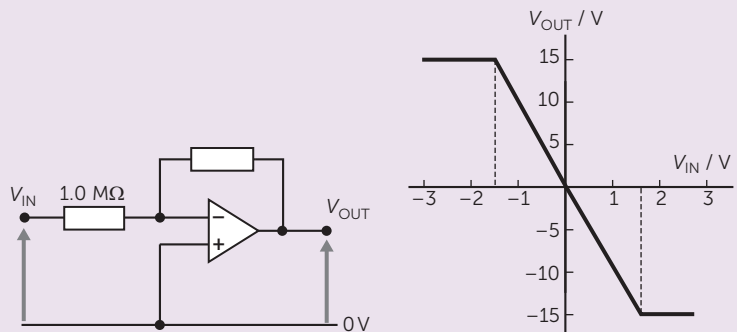


Fig. 16.29

16.3 WORKED EXAMPLE *continued***Solution**

a) (i) Voltage gain = gradient of graph

$$= -\frac{15 \text{ V}}{1.5 \text{ V}} = -10$$

(ii) Voltage gain = $-\frac{R_F}{R_1} = -10$

$$\therefore R_F = 10R_1 = 10 \text{ M}\Omega$$

b) See Fig. 16.30.

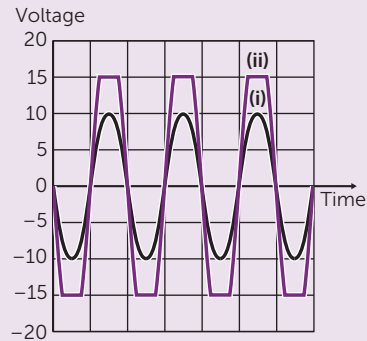


Fig. 16.30

Experiment to investigate the inverting amplifier

Adjust the null potentiometer of the op-amp so that the output voltage is zero when the input voltage is zero. This is necessary to correct internal bias voltages within the op-amp. See Fig. 16.31. Use a signal generator to apply an alternating voltage to the input of an inverting amplifier. Display the output and input waveforms using a double beam oscilloscope. Note that the output wave should be amplified and inverted compared with the input wave.

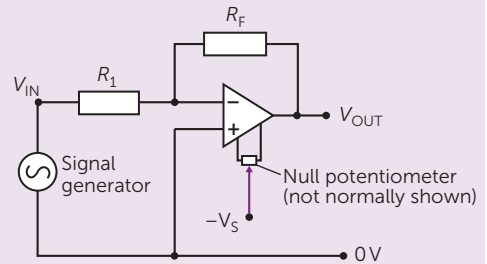


Fig. 16.31 Investigating the inverting amplifier

- 1 Measure the amplitude of the input voltage and of the output voltage and calculate the voltage gain. Compare this result with the theoretical voltage gain $-R_F \div R_1$.
- 2 Increase the amplitude of the input voltage until the output wave just saturates. Observe the effect of increasing further the amplitude of the input wave.
- 3 Measure the voltage gain for different frequencies and plot a graph of voltage gain against frequency.

16.4 QUESTIONS

- 1 For each circuit in Fig. 16.32, calculate
 - a) the voltage gain,
 - b) the maximum input voltage if the output voltage is not to exceed 15 V.

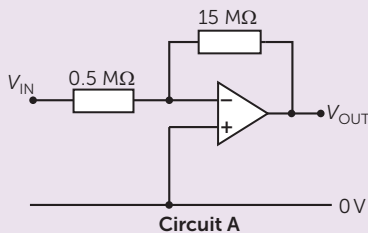
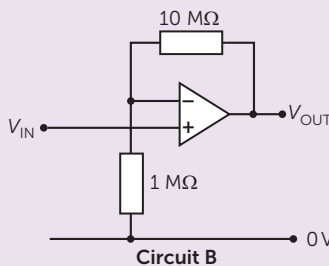


Fig. 16.32



- 2 Fig. 16.33 shows an operational amplifier on open-loop connected to a Wheatstone bridge circuit which includes a thermistor and a variable resistor. The variable resistor is increased until the output voltage of the op-amp switches from +15 V to -15 V.
 - a) Explain why the output voltage switches from +15 V to -15 V when the variable resistor reaches a certain resistance.
 - b) If the thermistor is then cooled, the output voltage of the op-amp switches from -15 V to +15 V. Explain why this happens at a certain temperature as the thermistor's temperature is lowered.

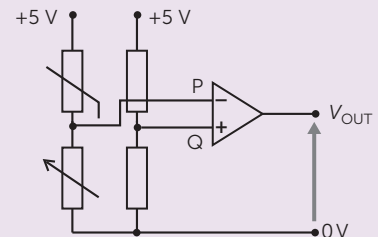


Fig. 16.33

16.5 Multivibrators

A multivibrator is a circuit in which the output voltage switches between two states, either automatically or as a result of receiving an input pulse.

See www.palgrave.com/foundations/breithaupt for an experiment to investigate a multivibrator.

Monostable multivibrator

A monostable multivibrator has one stable output state. It can be forced into its unstable state by an input pulse but returns to its stable state after a certain time. A monostable multivibrator can therefore be used in a logic circuit to provide a time delay. For example, a home security alarm system is usually fitted with a delay circuit so the system can be switched off when someone with an alarm key enters by the front door.

Fig. 16.34 shows how a monostable multivibrator works. With the switch at A, the voltage at X is zero, so the output indicator is off. When the switch is moved from A to B, the voltage at X is raised to $+V_S$ without delay so the output indicator is switched on. However, as the capacitor charges up, the voltage at X decreases and at some point can no longer supply a high enough voltage for logic 1 to the first NOT gate. The input to the second NOT gate therefore becomes '1' at this point and the indicator at its output is therefore switched off.

The time delay depends on how fast the capacitor charges up. This depends on the time constant, CR , of the capacitor, C , in series with the resistor R .

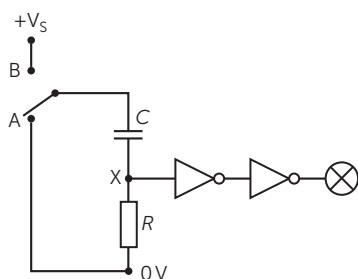


Fig. 16.34 A monostable multivibrator

Astable multivibrator

An astable multivibrator repeatedly switches automatically from one state to the other and back. Thus an astable multivibrator can be used to produce a continuous stream of pulses. Fig. 16.35 shows how an astable multivibrator works. When the output of NOT gate 1 goes high, indicator A switches on and the voltage at Y goes high at the same time, which makes the voltage at Z and W low, switching indicator

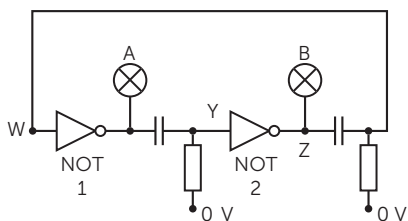


Fig. 16.35 An astable multivibrator

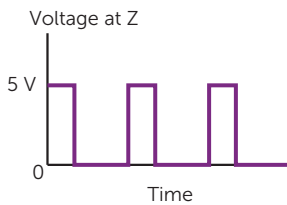


Fig. 16.36 Unequal pulses

B off. However, the capacitor at the output of NOT 1 gradually charges up and the voltage at Y therefore falls. When this voltage falls low enough, the output of NOT 2 at Z and at W goes high, which switches indicator B on and indicator A off. The capacitor at the output of NOT 2 now charges up gradually and the voltage at Z falls until it is low enough to make the output of NOT 1 high again. In this way, indicator A repeatedly switches on and off and indicator B repeatedly switches off and on.

Whenever A is on, B is off and vice versa. The 'on' time for each indicator is determined by the time constant, CR , of the capacitor and resistor it is connected to. Choosing different values of C and R at each NOT output would give pulses with unequal times for each state as shown in Fig. 16.36.

16.5 QUESTIONS

- 1 In the monostable circuit shown in Fig. 16.34, what would be the effect on the operation of the circuit of
 - a) increasing the capacitance of C ,
 - b) decreasing the resistance of R .
- 2 a) An astable multivibrator includes two $1\ \mu\text{F}$ capacitors, each in series with a $1000\ \Omega$ resistor. (i) Calculate the time constant of a $1\ \mu\text{F}$ capacitor in series with a $1000\ \Omega$ resistor.
 - (ii) Estimate the frequency of the pulses produced by this astable multivibrator.
- b) What would be the effect on the operation of the astable multivibrator in (a) of replacing one capacitor with a $4.7\ \mu\text{F}$ capacitor, and the resistor connected to the other capacitor with a $10\ \text{k}\Omega$ resistor?

SUMMARY

• Digital circuits

Logic levels:

Logic level 1 (high) is when the voltage is at or above a certain value
 Logic level 0 (low) is when the voltage is at 0 V

Logic gates:

NOT gate; Output is NOT equal to the input
 AND gate; output = 1 if **all** inputs = 1
 NAND gate; output = 0 if **all** inputs = 1
 OR gate; output = 1 if **any** input = 1
 NOR gate; output = 0 if **any** input = 1

Input sensors:

switch (i.e. position), light, temperature, pressure

Output devices:

Indicator LEDs and low-power sound alarms can be switched on or off directly by a logic gate.
 Transistor and/or relay used to switch high-current devices on or off.

• Analogue circuits

Amplifiers:

- 1 Voltage gain = output voltage $\div$ input voltage
- 2 Saturation is when the output voltage reaches its limits set by the power supply.
- 3 Bandwidth is the frequency range over which the voltage gain is constant.
- 4 Signal-to-noise ratio is the ratio of the signal amplitude to the noise.
- 5 Feedback is where a fraction of the output voltage is fed back to the input.

Operational amplifiers:

- 1 A **comparator** is an open-loop op-amp used to compare two voltages.
- 2 A **voltage follower**: $V_{OUT} = V_{IN}$; negligible current is drawn at the input.
- 3 The **inverting amplifier**: Voltage gain $= -\frac{R_F}{R_1}$
- 4 The **non-inverting amplifier**: Voltage gain $= \frac{R_F + R_1}{R_1}$
- 5 The **summing amplifier**: Output voltage, $V_{OUT} = -\frac{R_F V_1}{R_1} - \frac{R_F V_2}{R_2} - \frac{R_F V_3}{R_3} \dots$

See www.palgrave.com/foundations/breithaupt for electronic memory.

- **Multivibrators**

- 1 **Monostable**: one stable state which the output reverts to after a delay.
- 2 **Astable**: the output voltage repeatedly switches between two levels.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

16.1 Create the truth table for each combination of logic gates in Fig. 16.37.

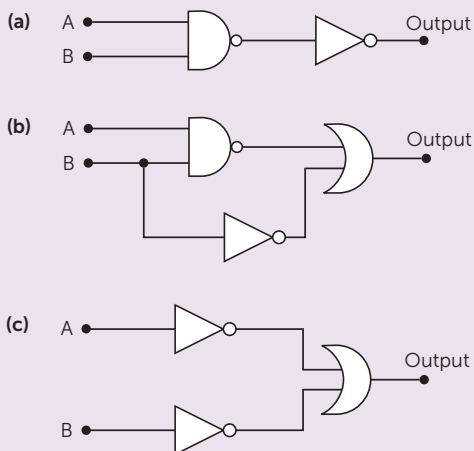


Fig. 16.37

16.2. Fig. 16.38 shows a temperature-operated electric motor used to open a vent in a greenhouse if the temperature in the greenhouse rises too much.

- a)** (i) What is the purpose of the switch marked S?
 (ii) The variable resistor can be adjusted to enable the system to switch a fan on at different temperatures.

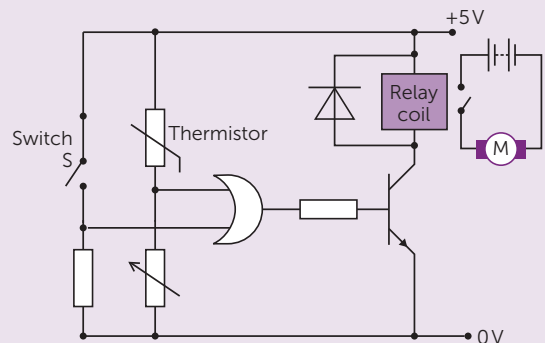


Fig. 16.38

What adjustment is needed to enable the fan to switch on at a lower temperature?

- (iii) What type of logic gate is needed in this system?
b) The electric motor in Fig. 16.38 is operated via a transistor and a relay. Explain why the motor is switched on when the output of the logic circuit goes high.

16.3

- a)** Design a logic system that switches an alarm on after a delay when either of the two main doors in a house are open and the alarm system is turned on. Include a test switch in your system.
b) What would be the effect on the delay time of increasing the capacitance of the capacitor in the delay circuit? Explain your answer.

REVISION QUESTIONS

16.4

- a) An alternating voltage of peak-to-peak voltage 2.0 V is applied to the inverting input of an op-amp on open-loop, operating from a +15 V, 0, -15 V power supply, as shown in Fig. 16.39.

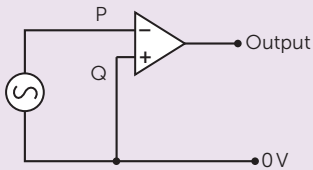


Fig. 16.39

- (i) The non-inverting input terminal, Q, is connected to earth. Sketch the output waveform produced by this circuit.
- (ii) The non-inverting input, Q, is then connected to a potential divider and a constant voltage of 0.5 V is applied to Q. Fig. 16.40 shows the output waveform produced by this arrangement. Explain the shape of this waveform.

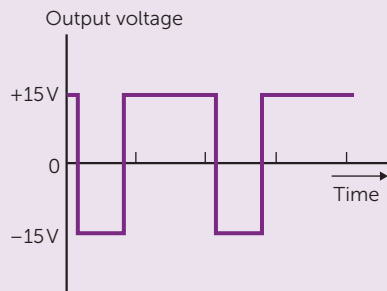
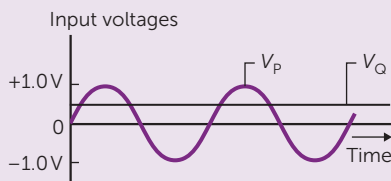


Fig. 16.40



- b) The circuit in Fig. 16.39 is modified by adding a $2.0\text{ M}\Omega$ feedback resistor and a $0.4\text{ M}\Omega$ resistor between the inverting input, P, and earth. The a.c. supply unit is disconnected from P and reconnected to Q.
- (i) What type of operational amplifier circuit is this?
- (ii) Calculate the voltage gain of this circuit.
- (iii) Calculate the maximum input voltage for no saturation at the output.

16.5

- a) Fig. 16.41 shows an operational amplifier used to add two voltages, V_1 and V_2 .
- (i) Input 2 is connected to earth and a voltage of +2.0 V is applied to input 1. Calculate the output voltage.
- (ii) Input 1 is now connected to earth and a voltage of 1.5 V is applied to input 2. Calculate the output voltage now.
- (iii) Calculate the output voltage if a voltage of 2.0 V is applied to input 1 and a voltage of 1.5 V is applied to input 2.
- b) Design a summing amplifier which can convert three input voltages, either 0 or 1 V, representing a 3-bit binary number, into a voltage between 0 and 7 V.

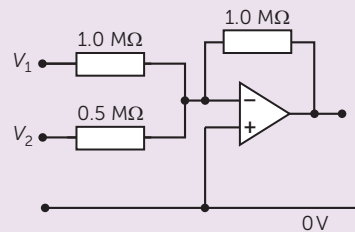


Fig. 16.41

Fields

PART

5

Part 5 concentrates on key concepts of electric and magnetic fields which underpin a wide range of appliances and devices used in the electrical industry to generate and supply electrical power. Before commencing this part, you ought to have studied **Part 4**, Electricity. The practical work in this part is closely related to the development of the principles in each unit and is intended to help you develop your understanding of the key concepts of electric and magnetic fields. Mathematical skills in algebra and graph work from earlier parts are used extensively in **Part 5**.

- 17** Electric Fields
- 18** Magnetic Fields
- 19** Electromagnetic Induction
- 20** Alternating Current

Electric Fields

OBJECTIVES

After working through this unit, you should be able to:

- use Coulomb's law to calculate the force between two point charges
- explain what is meant by a line of force and an equipotential surface
- sketch the pattern of lines of force and equipotentials in a uniform field, a radial field and near two point charges
- define electric field strength and electric potential
- calculate the electric field strength of a uniform field created by two charged parallel plates and relate it to the charge on the plates
- calculate the electric field strength and the electric potential at a given distance from a point charge

UNIT

17

CONTENTS

- 17.1 Electrostatic forces
- 17.2 Electric field patterns
- 17.3 Electric field strength
- 17.4 Electric potential
- Summary
- Revision questions

17.1 Electrostatic forces

Two charged objects exert a force on each other due to their electric charge. If the two objects carry the same type of charge, they exert equal and opposite forces of repulsion on each other. If the two objects carry opposite types of charge, they exert equal and opposite forces of attraction on each other. See Fig. 17.1.

Like charges repel; unlike charges attract.

Coulomb's investigations

The force between two point charges was investigated by Charles Coulomb in 1784. He devised a very sensitive torsion balance, as shown in Fig. 17.2, to measure the force between two charged pith balls, one on the end of a vertical rod and the other on the end of a needle suspended horizontally on a fine thread. The two balls were given the same type of charge, causing them to repel each other. By turning the torsion head at the top of the wire, the distance between the two balls could be set at any required value. The amount of turning needed to achieve that distance gave the force.

Some of Coulomb's measurements are in Table 17.1. Both variables were measured by Coulomb in degrees, as in the table, and converted to the correct units subsequently.

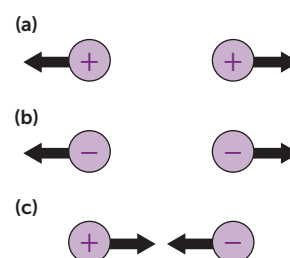


Fig. 17.1 Like charges repel; unlike charges attract (a) two positive charges repel, (b) two negative charges repel, (c) a positive and a negative charge attract

Table 17.1
Coulomb's measurements

Distance, r	36	18	8.5
Force, F	36	144	567

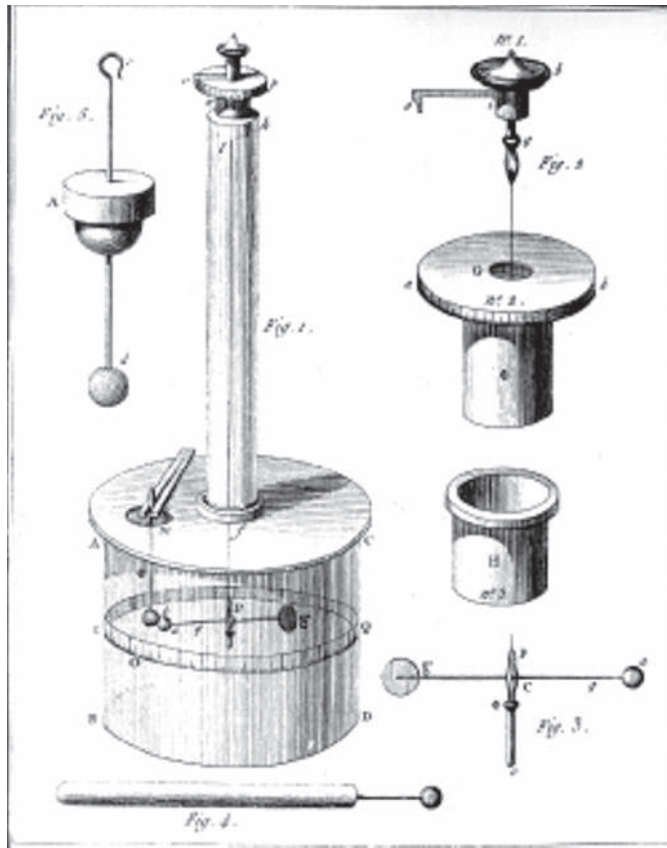


Fig. 17.2 Original drawing of Coulomb's torsion balance

The measurements in Table 17.1 show that as the distance decreases, the force increases. In fact, halving the distance from 36 to 18 causes the force to become 4 times larger and again from 18 to 8.5 (near enough 9). The measurements show that the force is inversely proportional to the square of the distance apart. In other words, doubling the distance causes the force to become 4 times weaker, trebling the distance causes the force to become 9 times weaker and so on. This relationship is known as **Coulomb's Law** and is usually expressed in the form of an equation:

$$F = \frac{kQ_1Q_2}{r^2}$$

where F is the force between two point charges, Q_1 and Q_2 , at distance, r , apart.

As explained later, the constant of proportionality

$$k = \frac{1}{4\pi\epsilon_0}$$

where ϵ_0 is the absolute permittivity of free space. This quantity was introduced on p. 224 in the section of the parallel plate capacitor and we shall see later in this unit why it appears in Coulomb's Law in the form of the constant $1/4\pi\epsilon_0$. Hence Coulomb's Law is

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{R^2}$$

Notes

$F > 0$, due to Q_1 and Q_2 carrying the same type of charge; corresponds to a repulsive force.

$F < 0$, due to Q_1 and Q_2 carrying the opposite type of charge; corresponds to an attractive force.

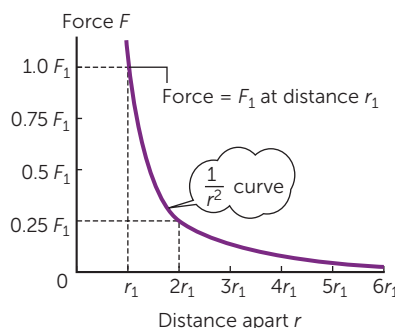


Fig. 17.3 The inverse square law of force

17.1 WORKED EXAMPLE

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$$

Calculate the force between a $+1.5 \mu\text{C}$ point charge and a $-2.0 \mu\text{C}$ point charge which are at a distance of 0.050 m apart.

Solution

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2} \\ &= \frac{+1.5 \times 10^{-6} \times -2.0 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.050)^2} \\ &= -10.8 \text{ N} \end{aligned}$$

17.1 QUESTIONS

- Calculate the force between an electron and a proton at a distance apart of 0.10 nm . The charge on a proton = $-$ the charge on an electron $= 1.6 \times 10^{-19} \text{ C}$.
- A $+2.5 \text{ nC}$ charge is placed midway between a $+1.5 \mu\text{C}$ point charge and a $-3.5 \mu\text{C}$ point charge which are at a separation of 200 mm . Calculate the magnitude and direction of the resultant force on the 2.5 nC charge.

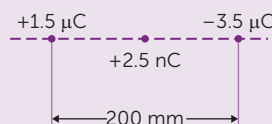


Fig. 17.4

17.2 Electric field patterns

Lines of force

Two charged bodies exert equal and opposite forces on each other without being in direct contact. This 'action at a distance' can be considered the result of a charged body creating an **electric field** around itself. Any other charged body placed in this field experiences a force due to the field.

A **line of force** in an electric field is a line along which a small positive 'test' charge would move in the field if free to do so. The 'test' charge must be small enough not to change the electric field in which it is placed.

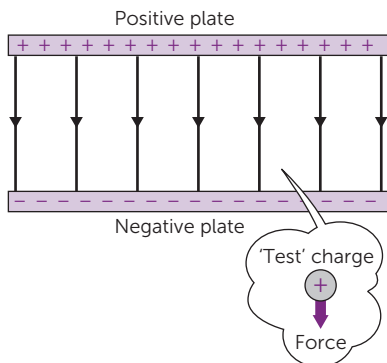


Fig. 17.5 A uniform electric field

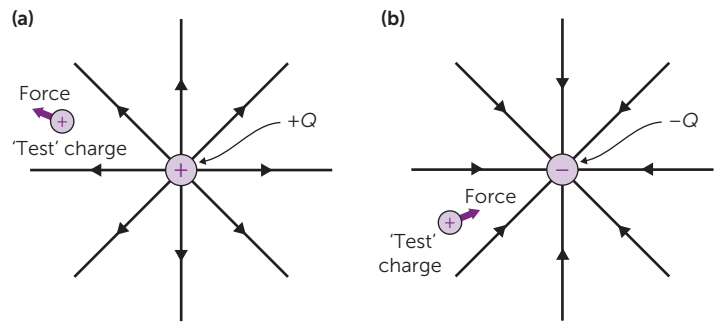


Fig. 17.6 Radial fields (a) due to a positive charge, $+Q$, (b) due to a negative charge, $-Q$

The patterns of the lines of force of two different electric fields are shown in Figs. 17.5 and 17.6.

- 1 A **uniform electric field** is created between two oppositely charged parallel plates. The lines of force always point from the positive plate to the negative plate because that is the direction of the force on a small positive 'test' charge anywhere between the plates.
- 2 A **radial electric field** exists near a point charge.
 - The lines of force created by a positive point charge, $+Q$, radiate from the charge outwards. This is because a positive test charge placed in this field would be forced away from the charge $+Q$.
 - The lines of force created by a negative point charge, $-Q$, point inwards. This is because a positive test charge placed in this field would be forced towards the charge $-Q$.

Observing electric field patterns

The pattern of the lines of force of an electric field can be seen using the arrangement in Fig. 17.7. An electric field across the surface of a viscous, clear oil is set up between two metal conductors connected to opposite poles of a high voltage supply unit. When grains of semolina are sprinkled on the oil surface, the grains align along the lines of force of the field.

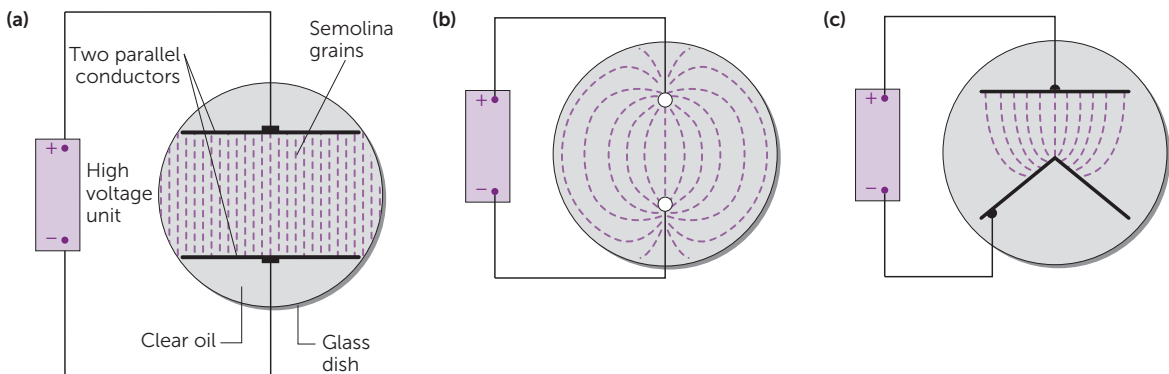


Fig. 17.7 Electric field patterns (a) top view, (b) point objects, (c) a V-shaped conductor near a straight conductor

- 1 **Between two oppositely charged parallel conductors**, the lines of force are parallel to each other and perpendicular to the conductors. The field is uniform, the same as between two oppositely charged parallel plates.
- 2 **Between two oppositely charged point objects**, the lines of force are concentrated at each point charge. A positive test charge released from an off-centre position would follow a curved path to the negative point charge.
- 3 **Between a V-shaped conductor and an oppositely charged straight conductor**, the lines of force spread out from the V onto the straight conductor. The lines are strongest where the V-shaped conductor is most curved. This is because the concentration of charge on the V-shaped conductor is greatest where the conductor is most curved.

Equipotentials

No work is needed to move a 'test' charge a small distance **at right angles** to a line of force in an electric field, even though a force acts on the test charge. This is because the direction of movement of the test charge is perpendicular to the direction of the force, so the distance moved in the direction of the force is zero. Since work done is defined as force $\times$ distance moved in the direction of the force, the work done is therefore zero. This is just the same as an object moving horizontally on a friction-free surface; its potential energy remains constant because there is no movement in the direction of the force of gravity.

The **electric potential** at a point in an electric field is the work that must be done per unit charge to move a small positive 'test' charge to that point in the field from infinity. The unit of electric potential is the volt, equal to 1 joule per coulomb. Effectively, the electric potential at a point in an electric field is the potential difference between that point and infinity. If W represents the work done to move a small 'test' charge, $+q$, from infinity to a point in an electric field, the potential, V , at that point is W/q .

To move a point charge, $+q$, from infinity to a point in an electric field where the potential is V , the work, W , that must be done is therefore qV .

$$W = qV$$

An **equipotential surface** is a surface on which the potential energy of a test charge is the same at any point on the surface. Fig. 17.8 shows the lines of force and the equipotentials of a radial electric field created by point positive charge, Q . The equipotentials are circles because the potential energy of a test charge does not change along one of these circles. The lines of force are perpendicular to the equipotentials. The equipotentials are just like contours on a map, because contours are lines of constant gravitational potential energy. The contours of a cone-shaped hill are circles like Fig. 17.8.

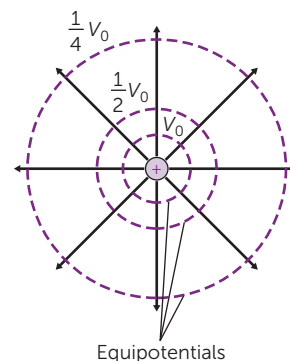


Fig. 17.8 Equipotentials in a radial field

Investigating the equipotentials in a uniform electric field

An electric field is set up along the surface of a sheet of conducting paper between two metal strips connected to a battery, as shown in Fig. 17.9a. The negative strip is earthed. The potential at different points on a sheet of conducting paper is measured relative to the negative strip, using a digital voltmeter. Points of equal potential can

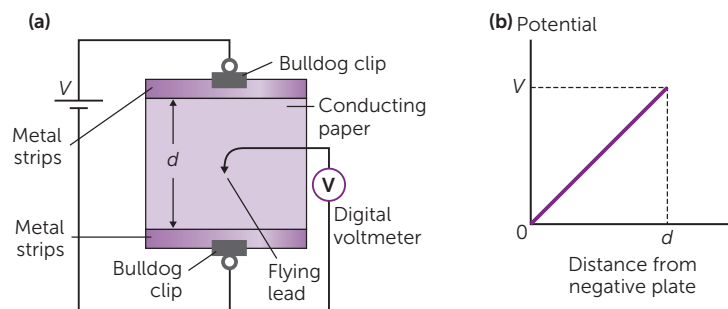


Fig. 17.9 Investigating equipotentials (a) practical arrangement, (b) potential versus distance

be traced out by moving the 'flying lead' across the surface so as to keep the voltmeter reading constant. The equipotentials between the metal strips are straight lines, parallel to the strips. The lines of force are therefore straight lines from the positive strip to the negative strip.

- The potential changes along a line of force by equal amounts in equal distances. Fig. 17.9b shows that the potential changes uniformly from the negative strip to the positive strip.
- Suppose a positive test charge, $+q$, is moved from the negative to the positive plate through a potential difference, V . The force exerted on $+q$ due to the field is towards the negative plate. To overcome this force, an equal and opposite force, F , must be applied to $+q$ to move it.

The work done by this force, $W = Fd$, where d is the distance between the two metal strips.

Since $W = qV$ from the definition of V , it follows that $Fd = qV$.

$$\therefore \text{Force, } F = \frac{qV}{d}$$

Note

This formula gives the force on a test charge, q , at any position between two oppositely charged parallel plates. You will meet it again in Unit 21 Electrons and Photons.

17.2 WORKED EXAMPLE

$$g = 9.8 \text{ m s}^{-2}$$

Two horizontal metal plates are fixed 4.0 mm apart, one above the other. A high voltage supply unit is connected to the two plates to make the top plate negative relative to the lower plate. A charged oil droplet of mass $8.8 \times 10^{-15} \text{ kg}$, carrying a charge of $+4.8 \times 10^{-19} \text{ C}$, is observed falling vertically when the plate p.d. is 500 V.

- Calculate (i) the change of potential over a distance of 1.0 mm along a vertical line in the field, (ii) the change of electrical potential energy of the droplet as a result of falling through a distance of 1.0 mm, (iii) the change of gravitational potential energy of the droplet as a result of falling through a distance of 1.0 mm.
- Calculate the potential difference necessary to hold this droplet stationary between the plates.

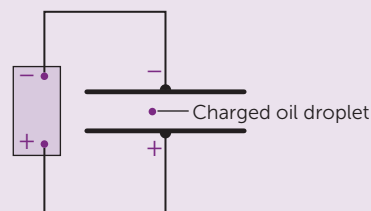


Fig. 17.10

Solution

- a) (i) The change of potential over 4.0 mm from one plate to the other is 500 V. For a distance of 1.0 mm, the change of potential is therefore $500 \div 4.0 = 125$ V.
 (ii) The electrical potential energy increases because the droplet is positive and it is falling towards the positive plate. Over 1 mm, the increase of electrical potential energy $= W = qV = 4.8 \times 10^{-19} \times 125 = 6.0 \times 10^{-17}$ J.
 (iii) The gravitational potential energy falls. Over 1 mm, the decrease of gravitational potential energy $= mgh = 8.8 \times 10^{-15} \times 9.8 \times 1.0 \times 10^{-3} = 8.6 \times 10^{-17}$ J.
- b) For the droplet to be at rest, the electric field force qV/d must be equal and opposite to its weight, mg .

$$\begin{aligned} \text{Rearranging } \frac{qV}{d} = mg \text{ gives } V &= \frac{mgd}{q} \\ &= \frac{8.8 \times 10^{-15} \times 9.8 \times 4.0 \times 10^{-3}}{4.8 \times 10^{-19}} = 720 \text{ V.} \end{aligned}$$

17.2 QUESTIONS

- 1 Fig. 17.11 shows some of the equipotentials of an electric field created by a V-shaped conductor and an oppositely charged straight conductor.
- a) Calculate the change of potential energy when a small object carrying a $+2.5$ nC charge is moved (i) from X to Y, (ii) from X to Z, (iii) from Z to Y.
- b) If the distance from X to Y is 5.0 mm, calculate the average force on the $+2.5$ nC charge when it is between X and Y.
- 2 A uniform electric field is created between two horizontal parallel plates 12 mm apart by applying a potential of $+600$ V to the upper plate and earthing the lower plate, as in Fig. 17.12. A charged oil droplet of mass 2.5×10^{-15} kg is held at rest in the field.
- a) (i) What type of charge does this droplet carry? (ii) Calculate the charge on the droplet.
- b) Sketch a graph to show how the potential between the plates changes with distance from the earthed plate to the positive plate.
- c) Calculate the electrical potential energy of the droplet if its position is (i) 2.0 mm above the earthed plate, (ii) 6.0 mm above the earthed plate, (iii) 10 mm above the earthed plate.

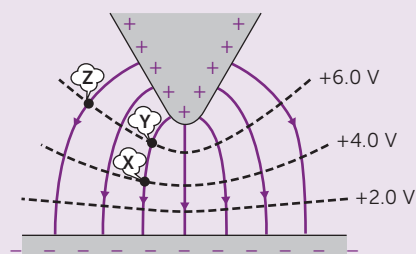


Fig. 17.11

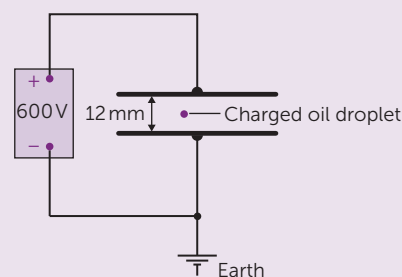


Fig. 17.12

17.3 Electric field strength

A lightning conductor discharges an overhead thundercloud because the electric field at the tip of the conductor is strong enough to ionise air molecules. See Fig. 17.13. The air becomes able to conduct and allows the charge on the cloud to discharge to the ground gradually. The lines of force concentrate at the tip because the thundercloud's charge attracts charge to the tip from the ground. An air molecule near the tip is acted upon by extremely strong equal and opposite forces

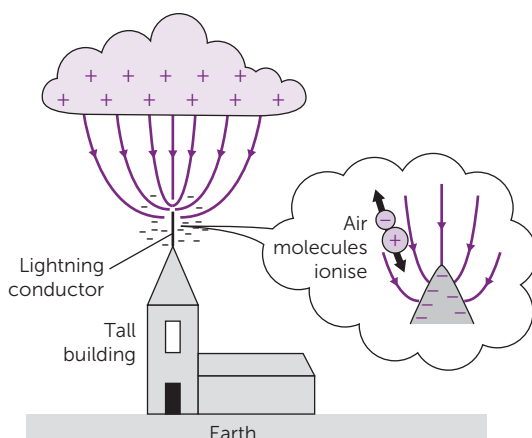


Fig. 17.13 A lightning conductor works by ionising air molecules

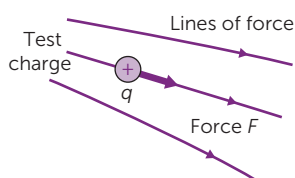


Fig. 17.14 Electric field strength

due to the electric field. The electrons of the molecule are attracted towards the positive charge and one or more electrons become detached from the molecule which is then attracted towards the negative charge.

Electric field strength, E , at a point in an electric field is defined as the force per unit charge on a positive 'test' charge at that point. See Fig. 17.14.

$$E = \frac{F}{q}$$

Notes

- 1 E is a vector quantity in the direction of the force on a positive 'test' charge.
- 2 The unit of E is the newton per coulomb (NC^{-1}) which is the same as the volt per metre (V m^{-1}). See below.

This equation shows that the unit of E can be given as the volt per metre.

The electric field between two oppositely charged parallel plates

This is a uniform field as shown in Fig. 17.15 in which the lines of force are straight, from the positive to the negative plate. As explained on p. 252, the force, F , on a small positive 'test' charge, q , is related to the plate p.d., V , and the plate spacing, d , in accordance with the equation

$$F = \frac{qV}{d}$$

The force on q is the same at all points in the field. Since the electric field strength E is defined as F/q then

$$E = \frac{V}{d}$$

For example, the electric field strength between two plates 50 mm apart at a p.d. of 100 V is 2000 V m^{-1} . Fig. 17.16 shows how the potential changes from the negative plate (which is earthed in this instance) to the positive plate.

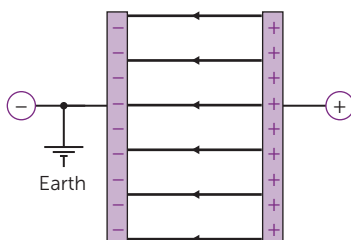


Fig. 17.15 A uniform electric field

The **potential gradient**, defined as the change of potential per unit distance, is constant, equal to

$$\frac{V}{d}$$

This is equal in magnitude and opposite in direction to the electric field strength as the potential rises from the $-$ plate to the $+$ plate, whereas the force on a positive test charge is towards the negative plate.

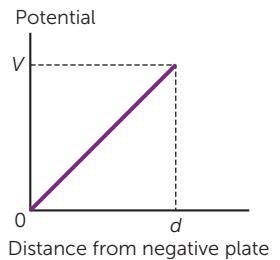


Fig. 17.16 A uniform potential gradient

The absolute permittivity of free space, ϵ_0

In **Fig. 17.15**, the charge, Q , on each plate is related to the plate p.d. in accordance with the equation

$$Q = \frac{A\epsilon_0 V}{d}$$

where ϵ_0 is the absolute permittivity of free space, and A is the plate area. This equation originated on p. 220 as a result of finding that the charge, Q , on the plates of a parallel plate capacitor is proportional to the plate area, A , and the plate p.d., V , and inversely proportional to the plate spacing, d . Provided there is nothing between the plates, the absolute permittivity of free space is introduced as the constant of proportionality.

Since the electric field strength, E , can be written as

$$E = \frac{V}{d}$$

the equation for the charge, Q , can be written in terms of the electric field strength E as $Q = A\epsilon_0 E$. Hence the surface charge density (i.e. the charge per unit area)

$$\frac{Q}{A} = \epsilon_0 E$$

$$\text{Charge per unit area, } \frac{Q}{A} = \epsilon_0 E$$

The permittivity of free space is therefore a measure of the charge per unit area on a parallel plate capacitor needed to create an electric field of strength 1 volt per metre. Since $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$, then a surface charge density of $8.85 \times 10^{-12} \text{ C m}^{-2}$ would give an electric field of strength 1 V m^{-1} .

17.3 WORKED EXAMPLE

Two parallel plates $0.10 \text{ m} \times 0.10 \text{ m}$ are at a fixed separation of 0.004 m , as shown in **Fig. 17.17**, at a p.d. of 250 V . Calculate

- the electric field strength between the plates,
- the surface charge density,
- the charge on each plate,
- the number of electrons per m^2 on the surface.

$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$; the charge of the electron, $e = 1.6 \times 10^{-19} \text{ C}$

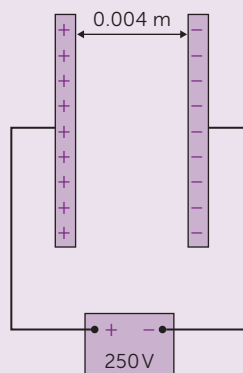


Fig. 17.17

17.3 WORKED EXAMPLE *continued***Solution**

$$\text{a) } E = \frac{V}{d} = \frac{250}{0.004} = 6.25 \times 10^4 \text{ V m}^{-1}$$

$$\text{b) } \frac{Q}{A} = \epsilon_0 E = 8.85 \times 10^{-12} \times 6.25 \times 10^4 \\ = 5.5 \times 10^{-7} \text{ C m}^{-2}$$

$$\text{c) } A = 0.1 \times 0.1 = 0.010 \text{ m}^2 \\ \therefore Q = 5.5 \times 10^{-7} \times 0.010 = 5.5 \times 10^{-9} \text{ C}$$

$$\text{d) For } A = 1.0 \text{ m}^2, Q = 5.5 \times 10^{-7} \text{ C}$$

$$\therefore \text{the number of electrons per m}^2 = \frac{5.5 \times 10^{-7}}{1.6 \times 10^{-19}} \\ = 3.5 \times 10^{12}$$

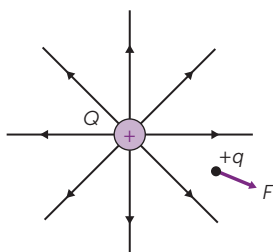


Fig. 17.18 The electric field near a point charge

Electric field strength created by a point charge

The lines of force of the electric field created by a positive point charge, $+Q$, radiate away from the charge. See Fig. 17.18. This is because a positive test charge, $+q$, placed near Q experiences a force of repulsion directly away from $+Q$. This force depends on the distance, r , between the two charges, in accordance with Coulomb's law.

$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

Because electric field strength, E , is defined as the force per unit charge on a positive test charge, then the electric field strength, E , at distance, r , from a point charge, Q , is given by the equation

$$E = \frac{F}{q} \\ = \frac{Q}{4\pi\epsilon_0 r^2}$$

This equation shows that the electric field strength due to a point charge is proportional to the inverse of the square of the distance to the charge, as shown by Fig. 17.19. For example, the electric field strength at distance, d , is four times stronger than at distance, $2d$.

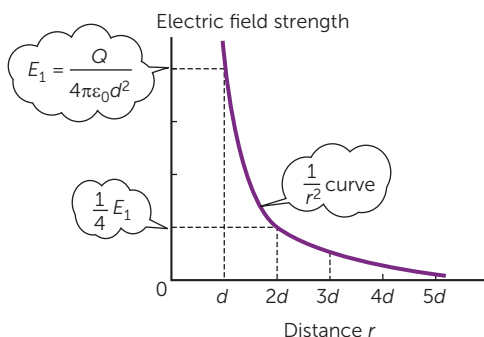


Fig. 17.19 E versus r for a point charge

Gauss's Law

Gauss was a famous nineteenth-century mathematician who recognised that there is a common link between the electric field strength near a point charge and between oppositely charged parallel plates. For two oppositely charged parallel plates, the charge on the plates, $Q = A\epsilon_0 E$. Rearranging this formula gives $E = Q/A\epsilon_0$. Gauss recognised that the field is uniform because the lines of force are parallel. At any position between the plates, the area that the lines of force pass through is unchanged, equal to the area of the plates. However, the lines of force due to a point charge spread out and the field becomes weaker with increased distance. At a certain distance, r , from a point charge, Q , the lines of force pass through a sphere of radius, r as shown in Fig. 17.20. Gauss realised that the formula for the electric field strength, E , at distance, r , from a point charge, Q

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

may be written in the form

$$E = \frac{Q}{A\epsilon_0}$$

because the surface area of a sphere of radius r is $4\pi r^2$. Gauss was able to develop his mathematical ideas much further but this limited glimpse of his ideas is sufficient to explain the underlying reason why the constant

$$\frac{1}{4\pi\epsilon_0}$$

is introduced in Coulomb's Law.

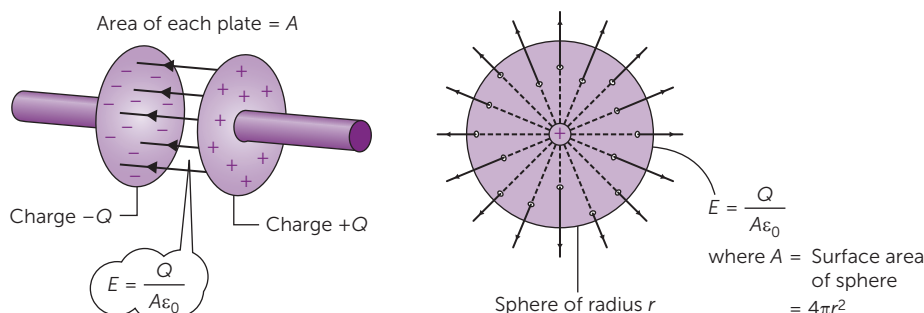


Fig. 17.20 Gauss's field ideas

17.4 WORKED EXAMPLE

Calculate the electric field strength at a distance of

a) 1.0 mm, b) 10 mm

from a small object carrying a $+4.5 \text{ pC}$ ($= 4.5 \times 10^{-12} \text{ C}$) charge. ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$)

Solution

$$\text{a) } E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{4.5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (1.0 \times 10^{-3})^2} = 4.1 \times 10^4 \text{ V m}^{-1}$$

b) At 10 mm, the electric field strength is 100 times weaker than at 1.0 mm because the distance is ten times greater. Hence $E = 410 \text{ V m}^{-1}$.

17.3 QUESTIONS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$$

- 1 A positive ion of charge of $3.2 \times 10^{-19} \text{ C}$ is in a uniform electric field of strength 120 kV m^{-1} .
 - a) Calculate the force on the ion due to the electric field.
 - b) The ion moves a distance of $5.0 \mu\text{m}$ in the direction of the lines of force of the field. Calculate the change of potential energy of the ion.
- 2 Two parallel metal plates at a separation of 10 mm are connected to a high voltage supply which maintains a constant p.d. of 300 V between the two plates.
 - a) Calculate the electric field strength in the space between the two plates.
 - b) Calculate the charge per unit area on each metal plate.
 - c) Calculate the number of electrons per mm^2 of area on each plate.

17.4 Electric potential

When a charged object in an electric field is moved, its potential energy changes unless the object is moved perpendicular to the lines of force of the field. No change of potential energy occurs if the object is moved in a direction perpendicular to the lines of force.

The electric potential at a point in an electric field is defined as the work done to move a positive 'test' charge to that point from infinity.

Consider a point charge, q , in an electric field at a point where the electric potential is V .

Potential energy of $q = qV$

This is because qV is the work done on q to move it to a point at potential V from infinity, where its potential energy is zero. For example, if a $+1 \text{ nC}$ charge is moved from infinity to a point in a field where $V = 1200 \text{ V}$, the potential energy of the charge at this point would be 1200 nJ ($= 1 \text{ nC} \times 1200 \text{ V}$).

At distance, r , from a point charge, Q ,

$$\text{the electric potential, } V = \frac{Q}{4\pi\epsilon_0 r}$$

Fig. 17.21 shows how the potential changes with distance. Note that the potential does not decrease as fast as the electric field strength does, since V is proportional to $1/r$, whereas E is proportional to $1/r^2$.

The formula for potential can be proved by considering a small positive 'test' charge, q , moved from infinity to distance, r , from charge, Q .

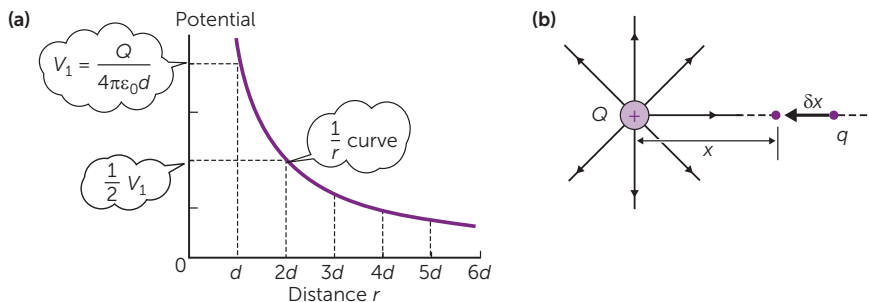


Fig. 17.21 Electric potential near a point charge (a) potential versus distance, (b) moving a test charge

The work done to move q a small distance, δx , towards Q ,

$$\delta W = F\delta x = \frac{qQ}{4\pi\epsilon_0 x^2} \delta x$$

since the force between q and Q at separation x is $\frac{qQ}{4\pi\epsilon_0 x^2}$ in accordance with Coulomb's Law

$$\therefore \text{the change of potential, } \delta V = \frac{\delta W}{q} = \frac{Q}{4\pi\epsilon_0 x^2} \delta x$$

$$\text{Since } \frac{1}{x} - \frac{1}{(x + \delta x)} = \frac{\delta x}{x^2}, \text{ provided } \delta x \ll x,$$

$$\text{then } \delta V = \frac{Q}{4\pi\epsilon_0 x} - \frac{Q}{4\pi\epsilon_0 (x + \delta x)} = \text{potential at } x - \text{potential at } x + \delta x.$$

By adding the change of potential for every step, δx , from infinity to r , the total change of potential from infinity is therefore

$$\frac{Q}{4\pi\epsilon_0 r}$$

The charged sphere

The charge on a conducting sphere is spread evenly over its surface. The lines of force therefore radiate from the surface, as shown in Fig. 17.22, giving the same pattern as if all the charge were concentrated at the centre. At and beyond the surface, the electric field strength and the electric potential at distance, r , from the centre are therefore given by the same formulae as for a point charge.

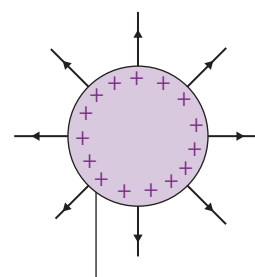
For $r \geq R_S$, the sphere radius

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Inside the sphere, the electric field strength is zero as the lines of force radiate outwards from the surface. Since there is no force on a test charge inside the sphere, the electric potential inside the sphere is therefore constant, equal to the surface potential

$$\frac{Q}{4\pi\epsilon_0 R_S}$$



Charge Q spread evenly over surface

Fig. 17.22 E and V near a charged sphere

17.4 QUESTIONS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$$

- 1 Calculate the electric field strength and the potential
 - a) at a distance of 5.0 mm from a point object carrying a charge of +8.5 pC,
 - b) midway between two parallel plates 40 mm apart, one plate being earthed and the other plate at a potential of +200 V.
- 2 A conducting sphere of radius 0.20 m is charged to a positive potential of 100 000 V. Calculate
 - a) the charge on the sphere,
 - b) the electric field strength at the surface,
 - c) the electric field strength and the potential at a distance of 2.0 m from the sphere.

SUMMARY

- **Coulomb's Law**

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

- **Electric field patterns**

- 1 A **line of force** is a line along which a positive 'test' charge would move if free to do so.
- 2 An **equipotential surface** is a surface on which the potential is constant.
- 3 A **uniform field** is a field in which the lines of force are parallel and the force on a 'test' charge is independent of its position.
- 4 A **radial field** is where the lines of force are straight lines towards or away from the point charge.

- **Electric potential, V** , is the work done per unit charge to move a positive 'test' charge from infinity. The unit of V is the volt, equal to 1 joule per coulomb.

- **Electric field strength, E** , is the force per unit charge on a positive test charge. The unit of E is the newton per coulomb or the volt per metre.

- **In a uniform field between two oppositely charged parallel plates**

- 1 The force on a charge q between the plates $= \frac{qV}{d}$
- 2 The electric field strength between the plates $= \frac{V}{d}$
- 3 The charge on each plate, $Q = \frac{A\epsilon_0 V}{d}$
- 4 The potential gradient is equal and opposite to the electric field strength.

- **In a radial field**

- 1 **Due to a point charge, Q**

$$E = \frac{Q}{4\pi\epsilon_0 r^2}, \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

- 2 **At or beyond the surface of a charged sphere, Q**

$$E = \frac{Q}{4\pi\epsilon_0 r^2}, \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

- **Inside a charged sphere**

$$E = 0, \quad V = \frac{Q}{4\pi\epsilon_0 R_S}$$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.
 $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$

17.1

- Sketch the pattern of lines of force between two equal and opposite point charges.
- A $+2.5 \text{ pC}$ point charge is placed at a distance of 20 mm from a -7.2 pC point charge. Calculate the force between these two charges and state if it is an attractive or a repulsive force.
- Calculate (i) the electric field strength, (ii) the potential at the midpoint between these two point charges.

17.2 A uniform electric field was established between two horizontal plates which were spaced 5.0 mm apart using a voltage supply unit and a potential divider as shown in Fig. 17.23. A charged oil droplet of mass $5.6 \times 10^{-15} \text{ kg}$ was held stationary between the plates when the plate p.d. was adjusted to 430 V .

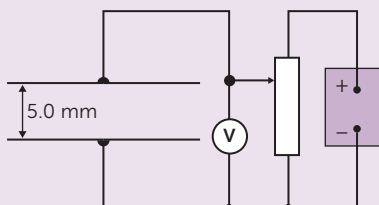


Fig. 17.23

- Calculate
 - the electric field strength between the plates at this potential difference,
 - the charge carried by this droplet.
- The droplet coalesced with an uncharged droplet and the potential difference needed to be adjusted to 620 V . Calculate the mass of the uncharged droplet before it coalesced with the charged droplet.

17.3

- Calculate the force of attraction between a proton and an electron at a separation of $1.0 \times 10^{-10} \text{ m}$.
- Calculate the work that must be done to separate a proton and an electron at a separation of $1.0 \times 10^{-10} \text{ m}$ to infinity. The charge of the electron, $e = 1.6 \times 10^{-19} \text{ C}$.

17.4 Two identical $+1.6 \text{ nC}$ point charges are placed 40 mm apart.

- Explain why the resultant electric field strength at the midpoint is zero.
- Show that the total potential at the midpoint is 1440 V .
 - Sketch a graph to show how the potential changes from infinity to the midpoint along a line that is equidistant from the two point charges.
- Calculate the work that must be done to move a particle carrying a charge of $1.5 \times 10^{-17} \text{ C}$ to the midpoint from infinity.

17.5

- A metal sphere is charged to a potential of $10\,000 \text{ V}$. Calculate the electric field strength at the surface of the sphere if its radius is
 - 100 m ,
 - 1.0 mm .
- Use your calculation in (a) to explain why a charged conductor discharges into the surrounding air if there are sharp points on its surface.
- An uncharged metal sphere of radius 50 mm is briefly brought into contact with an isolated charged sphere of radius 100 mm , which is at a potential of 5000 V . Calculate
 - the initial charge on the 100 mm sphere,
 - the charge on each sphere after they have made contact,
 - the final potential of each sphere.

UNIT

18

Magnetic Fields

CONTENTS

- 18.1 Magnetic field patterns
- 18.2 Electromagnets
- 18.3 The motor effect
- 18.4 Magnetic flux density
- 18.5 Magnetic field formulae
- 18.6 Magnetic materials
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- use a plotting compass to plot magnetic lines of force
- sketch the magnetic field patterns for different-shaped magnets and current-carrying conductors
- describe the operation of an electromagnet and its use in a relay coil
- describe the motor effect and use it to explain the operation of an electric motor, a moving coil meter and a loudspeaker
- define magnetic flux density and calculate the force on a current-carrying wire in a magnetic field.
- describe and calculate the effect of a uniform magnetic field on a beam of charged particles
- explain the Hall effect and its use in measuring magnetic flux density
- describe the factors that determine the magnetic flux density at the centre of a solenoid and near a long straight wire
- explain why two parallel current-carrying wires interact and describe how the ampere is defined
- describe the magnetic properties of iron and steel and relate these properties to uses

18.1 Magnetic field patterns**Magnetic materials**

A bar magnet is capable of attracting and holding iron filings or a steel paper clip. You can test a nail to find out if it is magnetic by seeing if it will attract iron filings or a steel paper clip. If it is magnetic, iron filings will stick to either end of the nail. The ends of a bar magnet are called **magnetic poles** because the magnetism seems to be concentrated at the ends.

The most common magnetic materials are iron and steel. An iron bar can be easily magnetised by ‘stroking’ it with a bar magnet, as shown in [Fig. 18.32](#) on

p. 279. Steel is less easy to magnetise but it retains its magnetism better than iron. Another way to magnetise an iron nail is to wrap an insulated wire around it and pass a direct current through the wire. The current produces its own magnetic field that magnetises the iron nail.

Magnetic lines of force

A magnetic compass is an essential piece of equipment for a hill-walker or a pilot on a boat as the compass needle always points north. The line along which a compass needle points is an example of a **magnetic line of force**. The exercise illustrated in Fig. 18.1a shows how to plot the lines of force of the Earth's magnetic field. On a global scale, the lines of force of the Earth's magnetic field are concentrated at the magnetic poles, as shown in Fig. 18.1b.

EXERCISE

Move the plotting compass so the tail is over a dot. Use the pencil to mark a dot where the head of the plotting compass is. Move the tail over this dot, etc. etc. . .

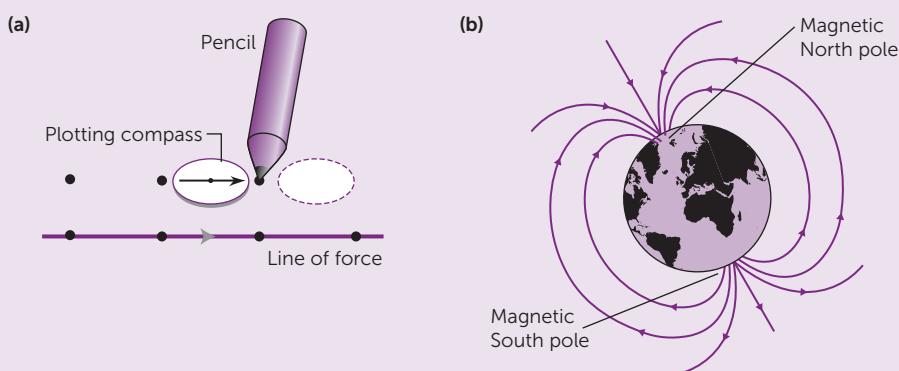


Fig. 18.1 The Earth's magnetic field (a) plotting lines of force, (b) the Earth's magnetic poles

A bar magnet suspended horizontally on a thread is a magnetic compass since it aligns itself along a line from north to south. The end that points north is called the north-seeking (N) pole and the other end the south-seeking (S) pole. If a N-pole of a second bar magnet is held near a N-pole of a suspended bar magnet, the two poles repel each other. The same happens if a S-pole is held near a S-pole. If, however, a N-pole is held near a S-pole, the two poles attract each other.

Like poles repel; unlike poles attract

The lines of force of a bar magnet form a characteristic pattern of loops from its N-pole to its S-pole, as shown in Fig. 18.2. The head of a plotting compass (i.e. its N-pole) is attracted to the S-pole of the bar magnet so the lines of force always point towards the S-pole of a bar magnet. The pattern of lines of force of a U-shaped magnet is also shown in Fig. 18.2. Note that the lines of force between the poles of the U-shaped magnet are parallel to each other, directed from the N-pole to the S-pole.

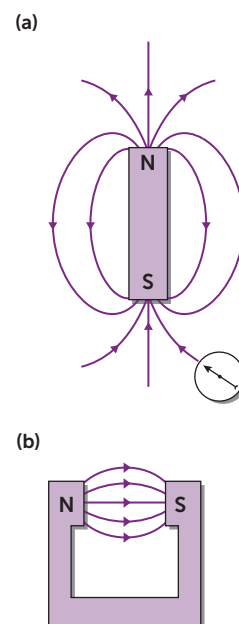


Fig. 18.2 Magnetic field patterns in (a) a bar magnet, (b) a U-shaped magnet

Investigating the magnetic field patterns between two magnets

Use a plotting compass to plot the lines of force between two bar magnets in the arrangement shown in Fig. 18.3. Note there is a **neutral point** between the two N-poles. This is where the compass needle is attracted equally by the two N-poles so the resultant magnetic field is zero at this position.

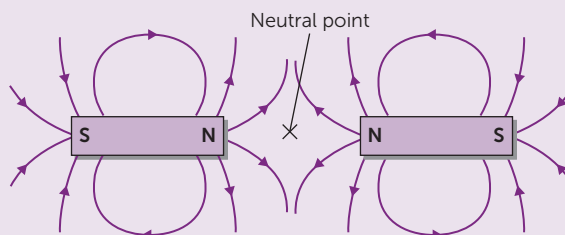


Fig. 18.3 More magnetic fields

Electromagnetism

A magnetic field is created when a current is passed along a wire. This magnetic field disappears when the current is switched off.

- 1 A **long straight wire** carrying a constant current produces lines of force which are concentric circles around the wire in a plane perpendicular to the wire. Note that the lines of force around a current-carrying wire are continuous loops and do not originate or terminate like the lines of force of a permanent magnet.

The direction of the lines of force depends on the direction of the current, as shown by Fig. 18.4. The 'corkscrew' rule is a convenient way of remembering how the direction of a line of force is related to the current direction.

- 2 A **solenoid** carrying a constant current produces lines of force that emerge from one end of the solenoid and loop around to the other end. For an empty solenoid, the lines of force pass through the solenoid, parallel to its axis, as in Fig. 18.5. The end of the solenoid where the lines of force emerge is like the N-pole of a bar magnet and the other end is like the S-pole. The 'solenoid' rule is a convenient way of remembering how the direction of the lines of force at each end relates to the current direction.

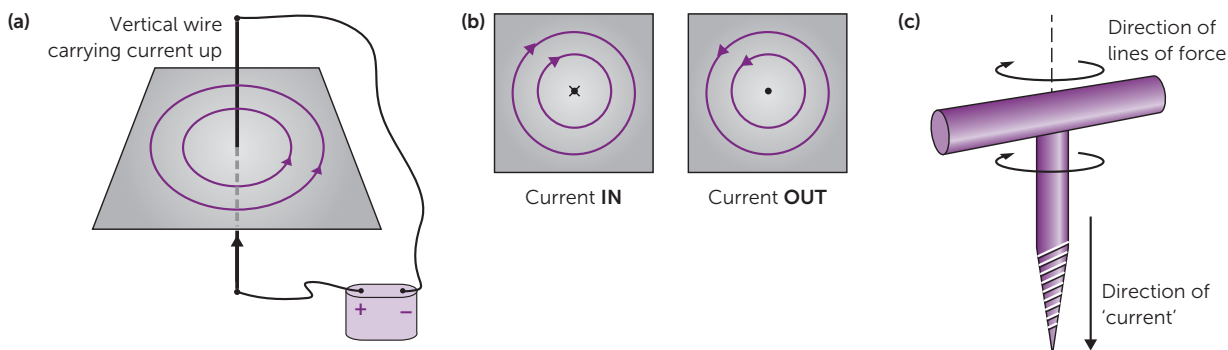


Fig. 18.4 Lines of force around a long straight wire (a) practical arrangement, (b) top view, (c) the corkscrew rule

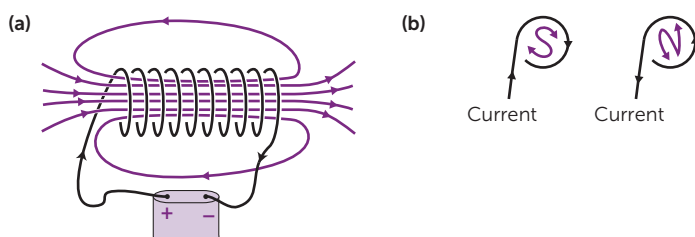


Fig. 18.5 (a) lines of force through and around an empty solenoid, (b) the solenoid rule

18.1 QUESTIONS

- Why is a permanent magnet made from steel rather than iron?
 - Fig. 18.6 shows a plotting compass and two bar magnets, X and Y, at right angles to each other.

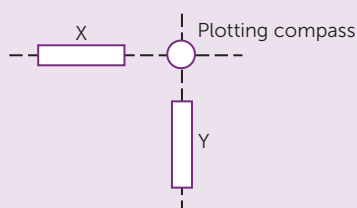


Fig. 18.6

Describe how you would use this arrangement to find out (i) which magnet is the stronger of the two, (ii) if the two poles of a magnet are the same strength.

- Sketch the pattern of lines of force of each arrangement shown in Fig. 18.7.

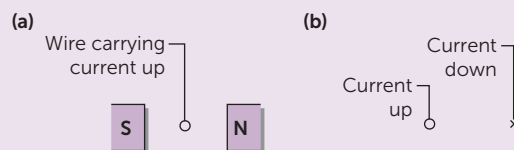


Fig. 18.7

18.2 Electromagnets

An electromagnet is a coil of insulated wire wound around an iron core. When a current passes through the coil, the iron becomes magnetised and a very strong magnetic field is created. With the iron core, the magnetic field is about 2000 times stronger than without the iron core. In addition to its use in scrapyards to lift steel car bodies, an electromagnet is found in a variety of devices including the electric bell, the electric relay and the circuit breaker.

The electric bell

Fig. 18.8 shows the construction of an electric bell. When the 'push' switch is closed, current passes through the electromagnet windings and the electromagnet core is magnetised. The core attracts the iron armature which makes the striker hit the gong. However, the movement of the armature opens the 'make and break' switch which switches the electromagnet off. The iron armature springs back to its initial position, closing the 'make and break' switch to start the cycle again. The 'make and break' switch causes the striker to hit the bell repeatedly. An electric buzzer operates on the same principle as an electric bell except there is no striker or gong and the armature is much lighter so it moves to and fro much faster.

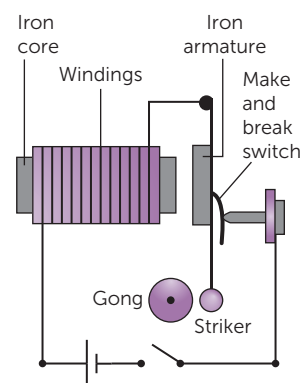


Fig. 18.8 The electric bell

The electric relay

A relay is an electromagnet which opens and closes a switch, enabling a much larger current to be turned on or off by switching on or off a much smaller current through the electromagnet windings. Fig. 16.14 shows the construction of a relay. When current is passed through the electromagnet windings, the electromagnet core becomes magnetised and attracts the iron armature. The movement of the armature opens or closes the relay switch which is in a different circuit. The relay may be 'normally open' (NO) or 'normally closed' (NC), where 'normal' refers to the switch state when there is no current through the windings of the electromagnet.

The circuit breaker

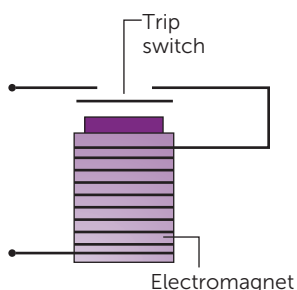


Fig. 18.9 A circuit breaker

- An **excess current circuit breaker** is a 'trip' switch opened by an electromagnet in the same circuit when the current through the windings exceeds a certain value. See Fig. 18.9. Unlike a 'make and break' switch, a 'trip' switch is designed to stay open after it has been opened by the electromagnet. The trip switch is reset manually after the cause of excessive current has been removed.
- A **residual current circuit breaker** is a trip switch opened by an electromagnet which has two coils wound in opposite directions around its iron core. It is designed to disconnect an appliance from the mains supply if the current in the neutral wire and that in the live wire differ. This could happen if the insulation of the live wire deteriorates and current passes to earth instead of to the neutral wire. A residual current circuit breaker is designed to trip if the live and neutral currents differ by more than 30 mA.

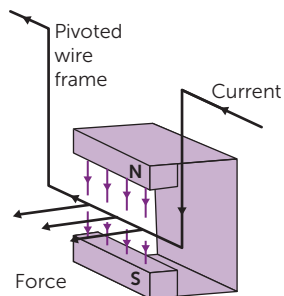


Fig. 18.10 The motor effect

18.2 QUESTIONS

- 1 In an electric bell, give a reason why
 - a) the electromagnet core is iron,
 - b) the electromagnet windings are copper,
 - c) the make and break switch contacts are platinum.
- 2 a) Explain how an excess current circuit breaker works.
b) State one advantage a circuit breaker has over a fuse.

18.3 The motor effect

When a current is passed along a wire, a magnetic field is created around the wire. If the wire is between the poles of a U-shaped magnet, equal and opposite forces act between the wire and the magnet provided the wire is not parallel to the lines of force of the magnetic field. This effect, called the **motor effect**, can be readily demonstrated using the arrangement shown in Fig. 18.10. The force is greatest when the wire is at right angles to the lines of force of the magnet's field.

The direction of the force is at right angles to both the direction of the current and the direction of the lines of force of the magnetic field. The **left-hand rule** in Fig. 18.11 is a convenient way of working out the force direction from the other two directions. Note that if the direction of the current is reversed, the force direction is reversed provided the magnetic field is unchanged.

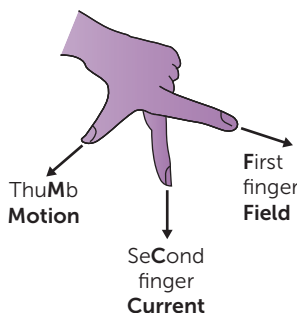


Fig. 18.11 The left hand rule

Fig. 18.12 shows the magnetic field around a current-carrying wire between the poles of a U-shaped magnet. The magnetic field due to the wire and the magnetic field of the U-shaped magnet reinforce each other on one side of the wire and cancel each other on the other side. The force on the wire is towards the neutral point, which is where the two fields cancel.

The loudspeaker

When an alternating current is passed through the coil of a loudspeaker, the coil vibrates because it is in a magnetic field. In the first half of one full cycle of the alternating current, the current through the coil is in a certain direction so the wire is forced in a particular direction according to the left hand rule. When the current reverses its direction in the second half of the cycle, the force on the coil reverses its direction. Hence the coil experiences an alternating force which makes it vibrate, forcing the diaphragm to vibrate and create sound waves in the surrounding air. The coil consists of insulated wires wound on a plastic tube. The magnet's cross-section is shown in Fig. 18.13. Its poles are a central disc and an outer ring. The coil fits between the magnetic poles, oscillating freely when an alternating current is passed through the coil windings.

The direct current electric motor

Fig. 18.14 shows the construction of an electric motor. The armature coil spins between the poles of a U-shaped magnet when a direct current is passed through the windings of the armature coil. This occurs because current enters and leaves the coil via a **split-ring commutator** so each side of the coil carries a current at right angles to the lines of force of a uniform magnetic field. Because the two sides carry current in opposite directions, the force on one side is in the opposite direction to the force on the other side. The coil is therefore forced to turn.

When the coil reaches a position with its plane perpendicular to the lines of force, its spinning motion takes it past this position and reverses the connection of the split-ring commutator to the contact brushes. The current now passes around the coil in the reverse direction, but each side is now adjacent to the other pole to which it was adjacent previously. Consequently, the forces on the two sides continue to turn the coil in the same direction of rotation.

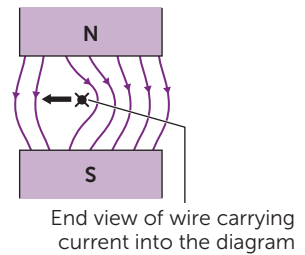


Fig. 18.12 Lines of force around a current-carrying wire between the poles of a U-shaped magnet

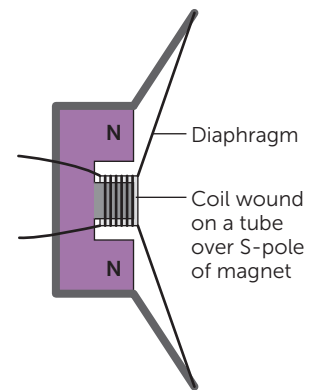


Fig. 18.13 The construction of a loudspeaker

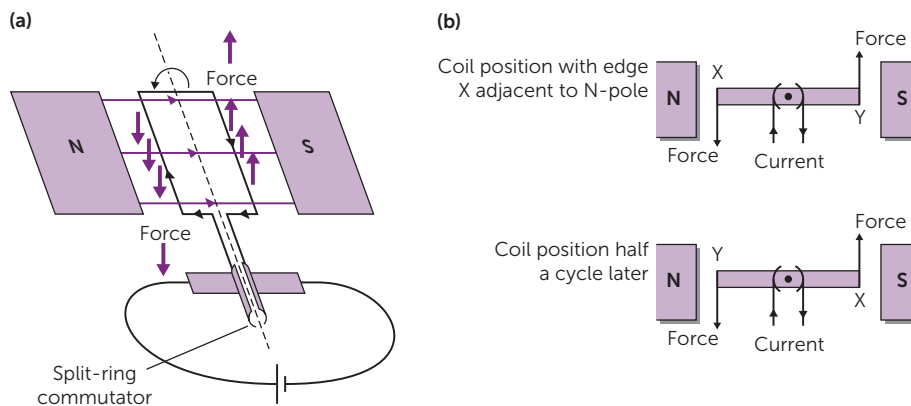


Fig. 18.14 An electric motor (a) construction and operation, (b) the action of the split-ring commutator

A practical d.c. electric motor has several identical coils wound on the same armature. The coils are evenly spaced, each coil connected to opposite sections of a multiple split-ring commutator. This design ensures that the armature spins evenly when a steady current passes through the coils.

The moving coil meter

The construction of a moving coil meter is shown in Fig. 18.15. Current enters and leaves the coil via two spiral springs. The coil turns when current is passed through it because equal and opposite forces due to the magnetic field act on the sides of the coil. As the coil turns, the spiral springs tighten until the coil cannot turn anymore. The current is measured from the position on the scale of the pointer attached to the coil. When the current is switched off, the spiral springs return the coil to its initial position.

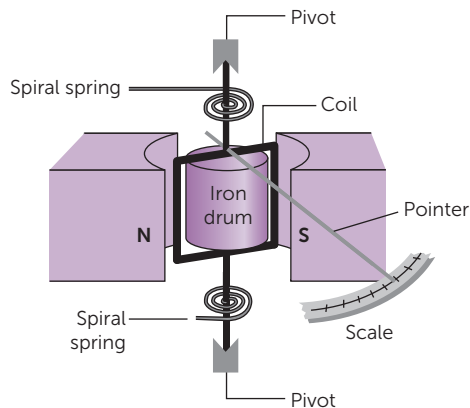


Fig. 18.15 The moving coil meter

The scale is **linear** if the coil turns by a constant amount for equal increases of current. This occurs if the magnetic field is radial. This design feature is achieved by fitting an iron drum at the centre between curved pole pieces. The coil turns in the gap between the pole pieces and the iron drum. The radial field ensures the coil plane is always perpendicular to the lines of force, ensuring equal increases of current gives equal degrees of turning.

Deflection, $\theta = kI$

where I is the current and k is the deflection per unit current.

18.3 QUESTIONS

- 1 Fig. 18.16 is an end view of the rotating coil of a d.c. electric motor.
 - a) In which direction is the coil rotating?
 - b) Explain why the contact brushes are connected to the coil via a split-ring commutator.

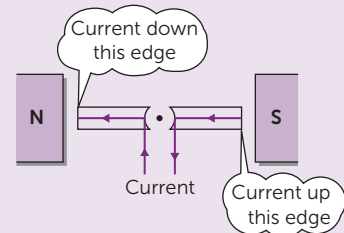


Fig. 18.16

- 2 Two moving coil meters, X and Y, are identical except X has a weaker magnet than Y. The two meters are connected in series and a current is passed through them.
- a) State and explain how the deflection of meter X compares with that of Y.
- b) If the magnets of X and Y had been the same strength but X had a weaker spiral spring than Y, how would the deflection of X have compared with that of Y?

18.4 Magnetic flux density

The force on a current-carrying conductor in a magnetic field is used to define the strength of the magnetic field. The stronger the magnetic field, the greater the force on a wire of fixed length carrying a constant current. The force can be measured directly by using a rigid wire fixed to a top pan balance, connected to the rest of the circuit via flexible leads, as in Fig. 18.17. The wire is positioned at right angles to the lines of force of the magnetic field. The force is measured from the change of the reading of the top pan balance when the current is switched on.

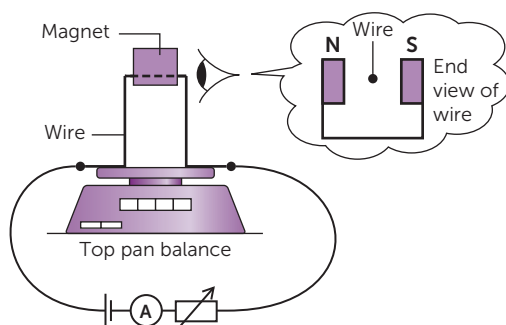


Fig. 18.17 Measuring the force on a current-carrying conductor

The measurements show that the force F is proportional to

- 1 the current, I
- 2 the length, L , of the wire in the field.

These results enable the strength of the magnetic field, the **magnetic flux density**, B , to be defined as the force per unit current per unit length on a wire placed perpendicular to the lines of force of the magnetic field.

$$\text{Magnetic flux density, } B = \frac{F}{IL}$$

The unit of B is the tesla (T), equal to 1 newton per ampere per metre. Rearranging the above equation therefore enables the force to be calculated if the current, length and magnetic flux density are known.

$$F = BIL$$

Notes

- 1 The direction of the force is given by the left-hand rule. See Fig. 18.11.
- 2 If the wire is parallel to the lines of force, no force is exerted on the wire.
- 3 If the wire is at angle θ to the lines of force, the perpendicular component of B ($= B \sin \theta$) gives the force, as shown in Fig. 18.18:

$$F = BIL \sin \theta$$

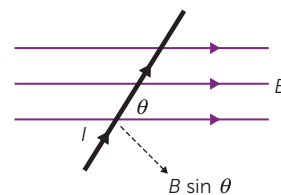


Fig. 18.18 The force on a wire at angle θ to the lines of force of a uniform magnetic field

18.1 WORKED EXAMPLE

A wire of length 0.040 m is placed perpendicular to a uniform magnetic field of magnetic flux density 0.30 T. Calculate

- the force on the wire when the current is 5.0 A,
- the current in the wire when the force is 0.050 N.

Solution

a) $F = BIL = 0.30 \times 5.0 \times 0.040 = 0.060 \text{ N}$

b) Rearrange $F = BIL$ to give $I = \frac{F}{BL} = \frac{0.050}{0.30 \times 0.040} = 4.2 \text{ A}$

The force on a moving charge in a uniform magnetic field

A charged particle moving across the lines of force of a uniform magnetic field is deflected by the field. A moving charged particle is effectively an electric current, since current is charge flow per second. The moving charged particle therefore experiences a force due to the magnetic field if its direction is not parallel to the lines of force of the magnetic field.

Consider a particle carrying charge Q , moving at constant speed, v , in a direction perpendicular to the lines of force of a uniform magnetic field as shown in Fig. 18.19.

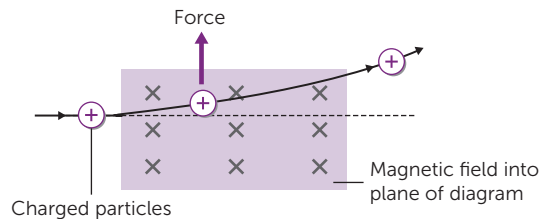


Fig. 18.19 The force on a moving charge in a magnetic field

In time t , it travels a length, $L = vt$ and is equivalent to a current $I = Q/t$ since current is charge flow per second. Hence the force due to the magnetic field, F , can be written as

$$F = BIL = B \frac{Q}{t} vt = BQv$$

More generally, if the direction of motion of the particle is at angle θ to the lines of force,

$$F = BQv \sin \theta$$

See www.palgrave.com/foundations/breithaupt for an experiment to investigate the effect of a magnetic field on an electron beam.

The Hall effect

When a current passes through a conductor or a semiconductor in a magnetic field, the charge-carrying particles are forced by the magnetic field to one face of the conductor. Consequently, a potential difference is created between this face and the opposite face as shown in Fig. 18.20. This effect is called the **Hall effect** after its discoverer. The p.d. between the two faces is called the **Hall voltage**.

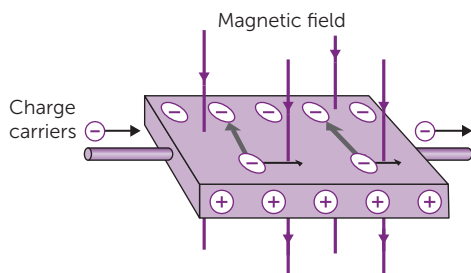


Fig. 18.20 Explanation of the Hall effect

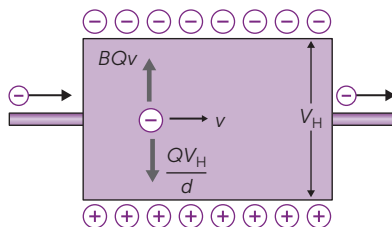


Fig. 18.21 The Hall voltage

The magnetic field forces the charge carriers to one edge, causing an electric field to build up from one edge to the other edge which acts on the charge carriers against the magnetic force. Charge carriers therefore pass through the conductor without deflection because the magnetic force is balanced by the electric field force. For a carrier of charge Q , the electric field force $= QV_H/d$, where V_H is the Hall voltage and d is the distance between the two faces (see Fig. 18.21).

Since the magnetic field force $= BQv$ where v is the speed of the charge carriers through the conductor, then because the magnetic field force is equal and opposite to the electric field force

$$\frac{QV_H}{d} = BQv$$

$$\text{Hence } V_H = Bvd$$

The Hall voltage is therefore proportional to the magnetic flux density, B . The effect is much greater in a semiconductor than in a metal because the charge carriers move through a semiconductor much faster than through a metal for the same current. A Hall probe designed to measure magnetic flux density contains a small sample of a semiconductor through which a constant current is passed. See Fig. 18.22. The Hall voltage across opposite faces is therefore a measure of the magnetic flux density when the sample is placed in a magnetic field. Because the Hall voltage is proportional to the magnetic flux density, the voltmeter used to measure the Hall voltage can be calibrated using a magnetic field of known magnetic flux density.



Fig. 18.22 A calibrated Hall probe

18.4 QUESTIONS

- 1 a) (i) Calculate the magnitude and direction of the force acting on a 5.2 cm vertical length of wire carrying a current of 3.2 A downwards in a uniform magnetic field of flux density 80 mT, acting horizontally from east to west.
(ii) Calculate the current in a horizontal wire of length 40 mm aligned at right angles to a uniform horizontal magnetic field of flux density 250 mT with lines of force from west to east if the force on the wire is 15 mN vertically upwards. State the direction of the current.
- b) A 30 mm × 40 mm rectangular coil consists of 80 turns of insulated wire wrapped on a plastic frame. The coil is placed in a uniform magnetic field of flux

density 65 mT with its plane parallel to the lines of force, as in Fig. 18.23. A current of 7.2 A is passed through the coil. Calculate (i) the force on each long edge of the coil, (ii) the couple on the coil due to the force on each long edge.

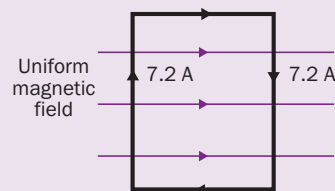


Fig. 18.23

18.4 QUESTIONS *continued*

- 2 In an experiment to measure the magnetic flux density of a U-shaped magnet midway between its poles, the magnet was placed on a top pan balance. A stiff wire was held rigidly between the poles of the magnet at right angles to the lines of force of the magnetic field. With no current in the wire, the top pan balance reading was 105.38 g. When a current of 6.5 A was passed through the wire, the top pan balance reading decreased to 104.92 g.
- a) Calculate (i) the force on the wire due to the magnet, (ii) the magnetic flux density midway between the poles of the magnet, if the length of the wire between the poles was 35 mm.
- b) The wire was then turned horizontally through 30° with the current unchanged. Calculate the new reading of the top pan balance.

18.5 Magnetic field formulae

Investigating the magnetic field in a solenoid

When a solenoid carries a constant current, the lines of force of the magnetic field inside the solenoid are parallel to the axis of the solenoid. Fig. 18.24a shows how a Hall probe may be used to investigate the factors that determine the magnetic flux density in a 'slinky' solenoid.

- 1 The probe can be used to measure the magnetic flux density across the solenoid and along its axis. The results show that the magnetic flux density is unchanged across the solenoid and along its axis inside it, away from the ends. Fig. 18.24b shows how the magnetic flux density varies with position along the axis.
- 2 The variation of magnetic flux density with current, I , can be investigated by keeping the probe at a fixed position and measuring the magnetic flux density for different currents. The results may be plotted as a graph of magnetic flux density against current as in Fig. 18.24c. The results show that the magnetic flux density, B , is proportional to the current, I .
- 3 The magnetic flux density depends on the number of turns per unit length of the solenoid. For constant current, the magnetic flux density at a fixed position is measured with the coil stretched by different amounts. For each measurement,

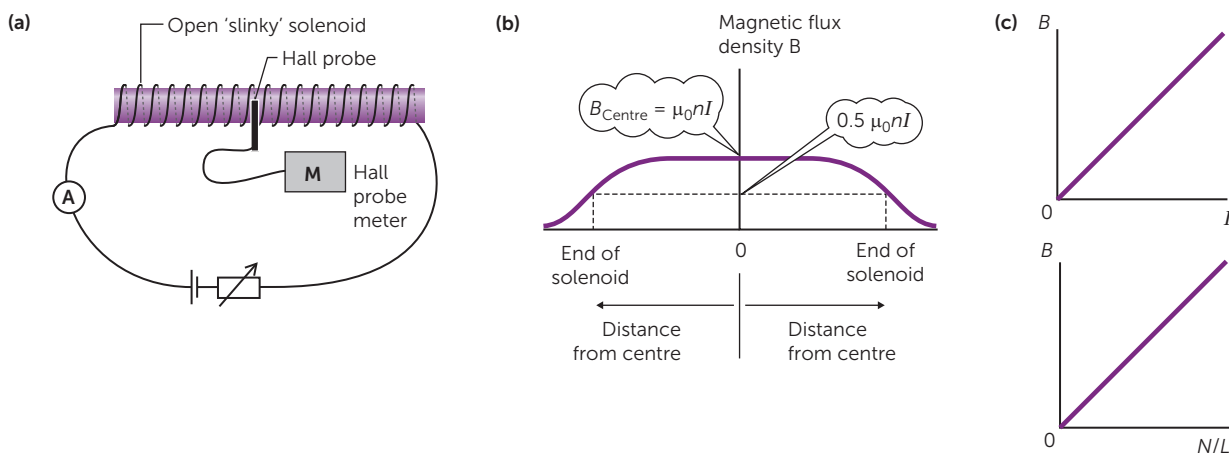


Fig. 18.24 Investigating a solenoid (a) using a Hall probe, (b) magnetic flux density at different positions, (c) factors affecting B

the number of turns, N , in a measured length, L , is counted. The results may be plotted as a graph of magnetic flux density, B , against the number of turns per unit length, N/L . The results show that the magnetic flux density is proportional to the number of turns per unit length.

The magnetic flux density, B , at and near the centre of a solenoid is proportional to

- the current, I ,
- the number of turns per unit length, N/L

$$\therefore B = \text{constant} \times \frac{N}{L} \times I$$

The constant in the equation is referred to as the **permeability of free space**, μ_0 . As explained on p. 274, the value of μ_0 is $4\pi \times 10^{-7}$ tesla metres per ampere, as a result of how the ampere is defined.

$$B = \mu_0 n I$$

where n = the number of turns per unit length, N/L .

Note

The magnetic flux density at the end of a solenoid is exactly half the value at or near the centre (i.e. $= 0.5\mu_0 n I$).

18.2 WORKED EXAMPLE

$$\mu_0 = 4 \times 10^{-7} \text{ T m A}^{-1}$$

A constant current of 4.0 A is passed through a solenoid of length 0.60 m and 1200 turns. Calculate

- the magnetic flux density at its centre,
- the current needed to produce a magnetic flux density of 250 mT at its centre.

Solution

$$\text{a) } B = \mu_0 n I = 4\pi \times 10^{-7} \times \frac{1200}{0.6} \times 4.0 = 0.010 \text{ T}$$

$$\text{b) Rearrange } B = \mu_0 n I \text{ to give } I = \frac{B}{\mu_0 n} = \frac{0.25}{4\pi \times 10^{-7} \times (1200 \div 0.60)} = 100 \text{ A}$$

Ampere's rule

The lines of force of the magnetic field created by a current-carrying wire are circles centred on the wire in a plane perpendicular to the wire. See p. 264. A circular magnetic field can also be produced by joining the ends of a long solenoid together with the axis of the solenoid as a circle. Ampere recognised that these two fields could be described by a common formula because they were the same shape as shown in Fig. 18.25.

- For the solenoid

$$B = \frac{\mu_0 N I}{L} \text{ hence } B L = \mu_0 N I$$

- For a long straight wire, the length of the line of force once round a circle is equal to $2\pi r$, where r is the distance from the line of force to the centre of the wire. Also, only one wire emerges through the 'line of force' loop so $N = 1$. Hence $B \times 2\pi r = \mu_0 I$.

Therefore, at distance r from the centre of a long straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

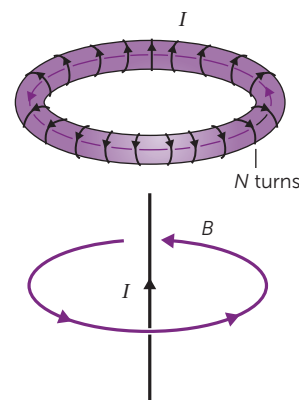


Fig. 18.25 Ampere's rule

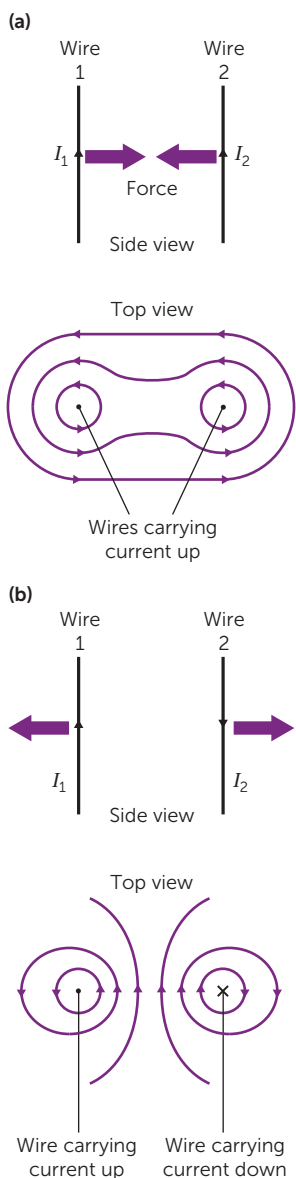


Fig. 18.26 Interaction between parallel wires (a) like currents attract, (b) unlike currents repel

The force between two parallel current-carrying wires

Two current-carrying wires interact with each other because each wire produces a magnetic field which acts on the other wire. Fig. 18.26 shows the magnetic field patterns around two wires carrying current in the same direction (in (a)) and in opposite directions (in (b)).

- (a) When the two currents are in the same direction, the two wires attract. The magnetic field of either wire at the position of the other wire creates a force towards the first wire.
- (b) When the two currents are in opposite directions, the two wires repel. The magnetic field of either wire at the position of the other wire creates a force away from the first wire.

Let I_1 and I_2 represent the currents in the two wires. Hence the magnetic flux density B_1 at distance r from the centre of the wire carrying current I_1 is given by

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

The force, F , on length L of the wire carrying current I_2 due to magnetic flux density B_1 is given by $F = B_1 I_2 L$. Therefore the force per unit length, F/L , on this wire

$$= B_1 I_2 = \frac{\mu_0 I_1 I_2}{2\pi r}$$

The same expression is obtained for the force per unit length on the other wire.

$$\therefore \text{The force per unit length on either wire} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

The definition of the ampere

One ampere is defined as that current which causes a force per unit length of $2.0 \times 10^{-7} \text{ N m}^{-1}$ in two infinitely long parallel wires of negligible thickness 1.0 m apart connected in series in a vacuum. Substituting $I_1 = I_2 = 1.0 \text{ A}$ and $r = 1.0 \text{ m}$ into the formula for the force per unit length gives

$$\frac{\mu_0}{2\pi} = 2.0 \times 10^{-7} \text{ N m}^{-1} \text{ m A}^{-2}$$

Hence

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

The value of μ_0 therefore follows from the definition of the ampere.

18.3 WORKED EXAMPLE

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

The magnetic flux density due to a current-carrying wire at a distance of 40 mm from the wire is 84 μT . Calculate

- a) the current in this wire,
- b) the force per unit length on a parallel wire carrying a current of 5.0 A at a perpendicular distance of 20 mm from the first wire.

Solution

- a) Rearrange $B = \frac{\mu_0 I}{2\pi r}$ to give $I = \frac{2\pi r B}{\mu_0} = \frac{2\pi \times 40 \times 10^{-3} \times 8.4 \times 10^{-5}}{4\pi \times 10^{-7}} = 16.8 \text{ A}$
- b) At 20 mm from the first wire, $B = \frac{\mu_0 I}{2\pi r} = \frac{2.0 \times 10^{-7} \times 16.8}{0.020} = 1.68 \times 10^{-4} \text{ T}$
- $\therefore \frac{F}{L} = BI_2 = 1.68 \times 10^{-4} \times 5.0 = 8.4 \times 10^{-4} \text{ N m}^{-1}$

18.5 QUESTIONS

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

- 1 a) Calculate the number of turns needed for a solenoid of length 800 mm to produce a magnetic field of flux density 25 mT when the current is 8.0 A.
 - b) The solenoid in (a) is placed with its internal magnetic field parallel to the lines of force of a uniform magnetic field of flux density 50 mT. (i) Calculate the magnetic flux density in the solenoid when the current is 8.0 A, (ii) Calculate the current needed to produce zero field in the solenoid and indicate its direction in relation to the external magnetic field.
- 2 a) A power cable of diameter 25 mm carries a current of 1000 A. Calculate the magnetic flux density (i) at the surface of the cable, (ii) at a distance of 10.0 m from the cable.
 - b) A second identical power cable is parallel to the first cable at a perpendicular distance of 10.0 m. Calculate the force per unit length between the two cables when they are each carrying a current of 1000 A.

18.6 Magnetic materials**Ferromagnetism**

Materials such as iron, steel, cobalt and nickel can be permanently magnetised. Such materials are described as **ferromagnetic** because they retain their magnetism. The cause of ferromagnetism is the tiny magnetic fields created by electrons orbiting the nucleus of each atom. Each electron in its orbit is equivalent to an electric current and therefore creates a magnetic field. In ferromagnetic atoms, the electrons in each atom produce a resultant magnetic field; in all other types of atoms, the electrons produce zero resultant magnetic field. Ferromagnetic atoms are therefore like tiny magnets. When a bar of iron is placed in a direct current solenoid, the atomic magnets line up along the solenoid field, all pointing in the same direction. The result is a very strong magnetic field. When the solenoid current is switched off, the atomic magnets remain partly lined up.

Investigating ferromagnetism

Fig. 18.27 shows a bar of ferromagnetic material in a solenoid which is in series with a resistor, R, and an alternating current supply unit. The X-plates of an oscilloscope are connected across the resistor so the alternating voltage across the resistor makes the oscilloscope spot oscillate horizontally. Since the current is proportional to the voltage for a resistor, the horizontal position of the spot on the screen represents current in the circuit.

The end of the bar is positioned adjacent to the screen so the magnetic field of the bar deflects the electron beam either vertically up or vertically down. The magnetic

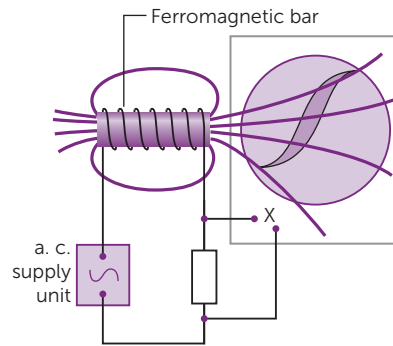


Fig. 18.27 Ferromagnetism

field alternates at the same frequency as the alternating current through the solenoid. The vertical position of the spot represents the magnetisation of the bar. The oscilloscope tube must be unscreened if the magnetic field of the bar is to affect the beam.

The beam traces a loop on the screen as shown in Fig. 18.28. This loop is called a **hysteresis** loop. The magnetisation lags behind the current, changing direction after the current has changed direction, not at the same time. All ferromagnetic materials display the following features:

- **Saturation** occurs when the magnetisation reaches a maximum value which cannot be increased for that material, no matter how large the current. The atomic magnets are all aligned when the material is magnetically saturated.
- **Remanence** occurs where the current is zero and the material remains magnetised. The atomic magnets remain partly aligned.
- The **loop area** is a measure of the energy needed to magnetise and demagnetise the same in each cycle. Materials such as steel have a large loop area and are hard to magnetise and demagnetise. They are therefore described as magnetically **hard**. Materials such as iron have a much smaller loop area because they are much easier to magnetise. Such materials are described as magnetically **soft**.

The presence of a ferromagnet in a solenoid increases the magnetic flux density considerably. The greatest effect is in a toroidal (i.e. circular) solenoid which is completely filled with the material as shown in Fig. 18.29.

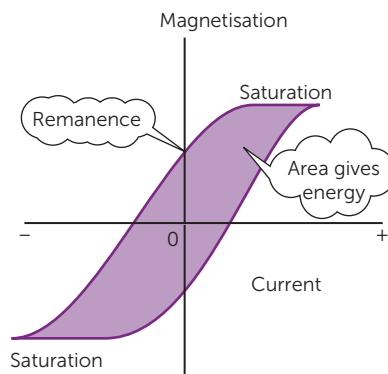


Fig. 18.28 A hysteresis loop

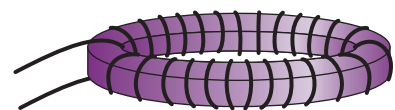


Fig. 18.29 A toroidal solenoid

This arrangement can be used to define the **relative permeability**, μ_r , of the material as

$$\frac{\text{the magnetic flux density with the material}}{\text{the magnetic flux density without the material}}$$

provided the material does not become magnetically saturated. Hence the magnetic flux density, B , in a toroidal solenoid with n turns per unit length wrapped around a ferromagnetic core

$$B = \mu_r \mu_0 n I$$

For example, suppose the magnetic flux density in a solenoid is 0.10 mT in the absence of ferromagnetic material and 0.10 T with the unsaturated material present. Hence the relative permeability of the material is 1000 ($= 0.10 \text{ T} \div 0.10 \text{ mT}$). However, if the material saturates at 0.15 T, then doubling the solenoid current would not double the field with the material present.

The domain theory of ferromagnetism

The atoms of a ferromagnetic material affect each other because they interact magnetically. When the material is unmagnetised, the atoms align in small regions called **domains** as shown in Fig. 18.30. The direction of magnetisation of the domains differs from one domain to the next in a random way. The result is that the overall magnetisation of the whole sample is zero. When the material is in an external magnetic field, the domains align and merge with each other to produce an overall magnetisation. When the external field is removed, domains are recreated. However, the material retains some magnetism because the domains remain partly aligned.

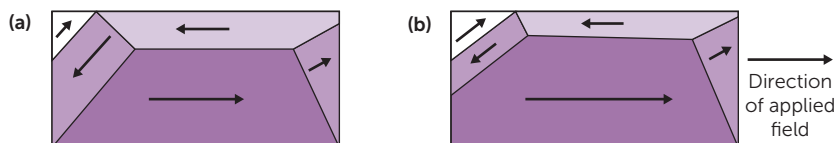


Fig. 18.30 Domains (a) unmagnetised, (b) partly magnetised

18.6 QUESTIONS

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

- 1 Use your knowledge of the domain theory to explain why a ferromagnet
 - a) becomes magnetised when it is in a direct current solenoid,
 - b) becomes demagnetised when it is slowly withdrawn from an alternating current solenoid.
- 2 a) Iron is easier to magnetise and demagnetise than steel. Which of these two ferromagnetic materials (i) has the greater remanence, (ii) has a larger hysteresis loop?
 b) A solenoid consists of 200 turns of insulated wire wound around an iron ring of diameter 40 mm. Estimate the magnetic flux density in the ring when a current of 50 mA is passed through the wire. Assume the relative permeability of iron is 2000.

SUMMARY

Rules to remember

Like poles repel, unlike poles attract.

Lines of force:

- of a permanent magnet are from the north pole to the south pole;
- around a long, straight, current-carrying wire are circles; see the corkscrew rule, Fig. 18.4;
- in a direct current solenoid are parallel to the axis, emerging from one end and looping around to the other end. See the solenoid rule Fig. 18.5.

The left hand rule:

First finger = Field; seCond finger = Current; thuMb = Motion.

Devices

Electromagnets are used in the electric bell, the relay and the circuit breaker.

The motor effect is used in the moving coil meter, the d.c. electric motor and the loudspeaker.

Magnetic field formulae

- 1 Force on a current-carrying conductor, $F = BIL \sin \theta$
- 2 Force on a moving charge, $F = BQv \sin \theta$
- 3 Hall voltage, $V_H = Bvd$
- 4 Magnetic flux density at the centre of a solenoid, $B = \mu_0 nI$, where n = number of turns per unit length
- 5 Magnetic flux density at distance r from the centre of a long straight current-carrying wire

$$B = \frac{\mu_0 I}{2\pi r}$$

- 6 Force between two parallel current-carrying wires

$$= \frac{\mu_0 I_1 I_2}{2\pi r}$$

where r = distance apart.

Ferromagnetism

- Saturation occurs when the magnetisation of a ferromagnetic material is at maximum.
- Remanence is where a ferromagnet remains magnetised after the magnetising current has been switched off.
- The area of a hysteresis loop is a measure of the energy needed to magnetise and demagnetise a ferromagnetic material.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.
 $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$

18.1

- a) Copy and complete the magnetic field patterns in Fig. 18.31.

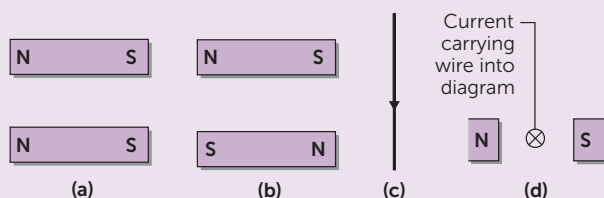


Fig. 18.31

- b) An iron nail can be magnetised by stroking it with the end of a bar magnet, as in Fig. 18.32. Use your knowledge of the domain theory of ferromagnetism to explain why (i) the iron nail becomes magnetised, (ii) the polarity of the iron nail is as shown in Fig. 18.32.

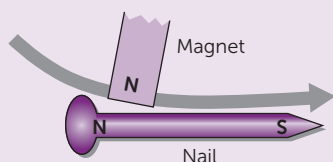


Fig. 18.32

18.2

- a) With the aid of a diagram, describe the construction and operation of a direct current electric motor.
 b) A direct current electric motor contains a rectangular armature coil 40 mm long by 30 mm wide, consisting of 200 turns of insulated wire. The coil spins in a magnetic field of flux density 120 mT about an axis parallel and midway between its long edges. Calculate the force on each long edge of the coil when the coil current is 5.0 A and the coil plane is
 (i) parallel to the lines of force of the field,
 (ii) perpendicular to the lines of force of the field.

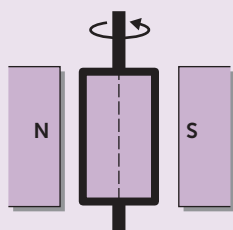


Fig. 18.33

- 18.3 A narrow beam of electrons at a speed of $2.8 \times 10^7 \text{ m s}^{-1}$ is directed into a uniform electric field between two oppositely charged parallel plates 50 mm apart. When the top plate is at a positive potential of 3500 V relative to the lower plate, the beam curves towards the top plate, as shown in Fig. 18.34. The deflection of the beam is then cancelled out as a result of a uniform magnetic field applied perpendicular to the diagram.



Fig. 18.34

- a) (i) State the direction of the magnetic field.
 (ii) By considering the forces on each electron, show that the magnetic flux density

$$B = \frac{V}{vd}$$

where v is the speed of the electrons in the beam, d is the spacing between the plates and V is the plate p.d.
 (iii) Calculate the magnetic flux density of this magnetic field.

- b) The magnetic flux density is doubled. Calculate the potential difference between the plates necessary to make the beam pass through undeflected.

18.4

- a) A current of 6.5 A was passed through a 500 turn open solenoid of length 250 mm. Calculate the magnetic flux density (i) at the centre of this solenoid, (ii) at the end of this solenoid.
 b) The solenoid was aligned north-south with its axis parallel to the lines of force of the Earth's magnetic field, as shown in Fig. 18.35, at a location where the Earth's magnetic flux density was $60 \mu\text{T}$. Calculate the current that needs to be passed through this solenoid to create zero magnetic flux density in its interior.

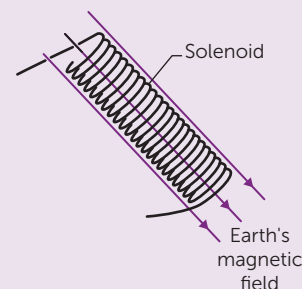


Fig. 18.35

- 18.5 A rectangular 50-turn coil of dimensions 60 mm $\times$ 40 mm is placed with its long edges parallel to a long, straight, vertical wire at a distance of 40 mm between the wire and the nearest edge of the coil. A current of 8.5 A is passed vertically up the wire. When the coil current is 2.0 A as shown in Fig. 18.36, calculate

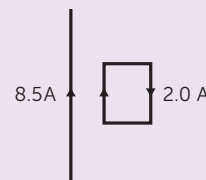


Fig. 18.36

- a) the force on each long edge of the coil,
 b) the magnitude and direction of the resultant force on the coil due to the wire.

UNIT

19

Electromagnetic Induction

CONTENTS

- 19.1 The dynamo effect
- 19.2 Magnetic flux
- 19.3 The alternating current generator
- 19.4 The transformer
- 19.5 Self-inductance
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- describe the dynamo effect
- explain the cause of an induced voltage in a conductor cutting the lines of force of a magnetic field
- state Lenz's law and relate the direction of an induced current to the change which causes it
- define magnetic flux and use Faraday's law to relate change of flux to induced voltage
- describe and explain the operation of an alternating current generator
- explain why a back e.m.f. is generated in an electric motor
- explain the cause and effect of eddy currents
- describe and explain the operation of a transformer
- use the transformer rule and relate the input power and the output power to the efficiency
- describe the main causes of inefficiency in a transformer
- explain why electrical power is transmitted using alternating currents at high voltages
- describe and explain self-inductance

19.1 The dynamo effect

A cyclist at night has to pedal harder to keep supplying electricity to the cycle lamps to light the way ahead and to be seen from behind, if the electricity for the lamps is supplied from a dynamo. The electricity from a dynamo is free, unlike that from a battery which needs to be replaced when it runs down. Some of the work done by the cyclist is used to turn the dynamo in order to generate electricity. [Fig. 19.1](#) shows the internal construction of a simple dynamo. The magnet is forced to spin

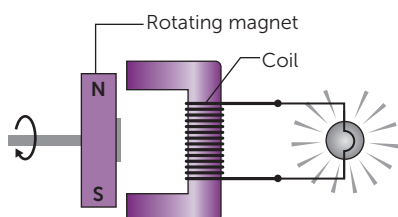


Fig. 19.1 Inside a dynamo

past the coil, causing the lines of force of the magnetic field to sweep past the coil windings. The effect is to induce an e.m.f. which forces current round the circuit through the lamp. The faster the cyclist pedals, the brighter the lamps are as more work is done each second by the cyclist.

Investigating the dynamo effect

Connect a length of insulated wire to a microammeter as shown in Fig. 19.2 and observe the deflection of the meter when a section of the wire is passed between the poles of a U-shaped magnet. No deflection is produced if the wire is stationary in the field or if the wire is moved along the lines of force. The wire must cut across the lines of force of the magnetic field to produce a deflection. It does not matter if the magnet is moving and the wire is stationary or if the wire is moving and the magnet is stationary. As long as the wire cuts the lines of force of the magnetic field, an e.m.f. is induced in the wire.

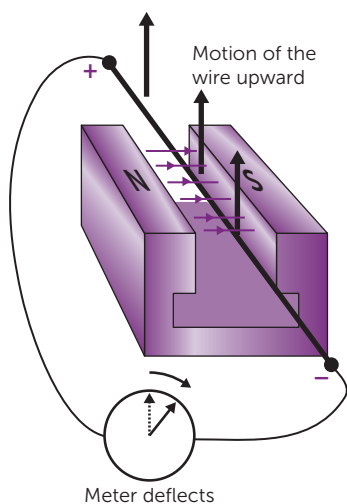


Fig. 19.2 Testing the dynamo effect

Note the difference when the wire is moved in the opposite direction or when it is moved at a different speed. You should find that the direction of the induced current depends on the direction the wire is moved through the field. Also, the faster the wire is moved through the field, the greater the deflection.

To explain why an e.m.f. is induced in a wire when it cuts the lines of force of a magnetic field, consider Fig. 19.3 in which a straight wire moves at constant speed through a magnetic field at right angles to the lines of force of the field. The conduction electrons move at the speed of the wire inside the wire across the field.

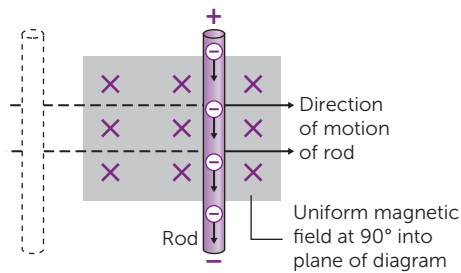


Fig. 19.3 Explaining the dynamo effect

Just like electrons in a beam, the magnetic field forces the electrons in the wire down the wire. Consequently, the top of the wire is at positive potential and the bottom is at negative potential. If the wire is part of a complete circuit, the electrons are forced around the circuit just the same as if the wire had been replaced by a battery with its positive terminal at the top and its negative terminal at the bottom. Note that an e.m.f. is induced in the wire regardless of whether or not the wire is part of a complete circuit. The induced e.m.f. produces a current only if the wire is part of a complete circuit.

Lenz's law

The direction of the induced current in a circuit is always such as to oppose the change that causes it. This is known as Lenz's law and it can be demonstrated using the arrangement in Fig. 19.4. When the north pole of a bar magnet is pushed into the coil, the meter connected to the coil deflects. The current direction in the coil is such as to generate a magnetic field that opposes the incoming north pole of the bar magnet. A north pole is therefore induced at the end of the coil where the bar magnet is inserted. When the north pole of the bar magnet is withdrawn, a south pole is induced at this end of the coil, which makes the meter deflect in the opposite direction.

Lenz's law follows from the conservation of energy. Work must be done to generate an induced current, hence a force must be applied to do the necessary work. The induced current in the conductor in the magnetic field creates a reaction force due to the motor effect, which acts against the applied force. The force created by the induced current must be in the opposite direction to the applied force otherwise it would increase the speed of the conductor, which would generate more current, which would increase the speed further, etc. thus creating electrical energy

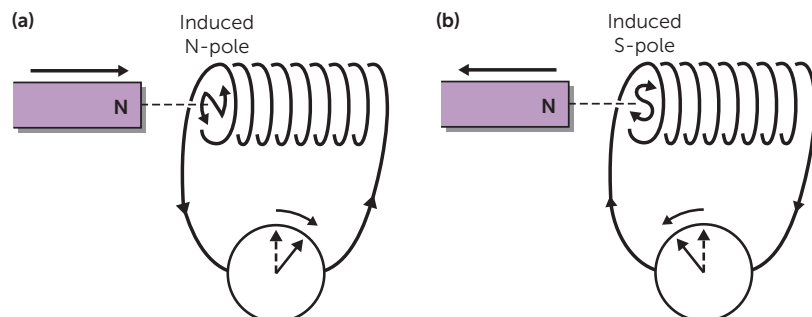


Fig. 19.4 Demonstrating Lenz's law (a) inserting a N-pole, (b) withdrawing a N-pole

and kinetic energy from nothing. Clearly, the principle of conservation of energy requires that the reaction force must be in the opposite direction to the applied force so that work done by the applied force is used to overcome the reaction force and create electrical energy.

The e.m.f. induced in a rod moving across a uniform magnetic field

Fig. 19.5 shows a conducting rod of length L moving at speed v in a direction at right angles to the lines of force of a uniform magnetic field. If the rod is part of a complete circuit, the induced voltage V drives a current I around the circuit. A reaction force F acts on the rod due to this current.

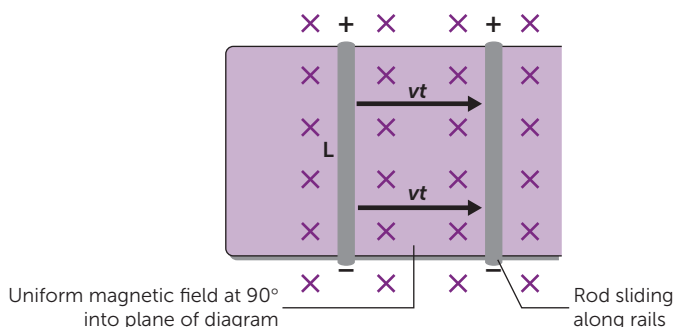


Fig. 19.5 The e.m.f. induced in a moving rod

- The electrical energy generated in time $t = IVt$
- The work done, W , to overcome the reaction force $= Fvt$, since the rod moves distance vt in time t .

Since $F = BIL$, where B is the magnetic flux density, then the work done,

$$W = BILvt$$

According to the principle of conservation of energy

$$IVt = BILvt$$

$$\therefore \text{the induced voltage, } V = BLv$$

19.1 WORKED EXAMPLE

Calculate the e.m.f. induced in a rod of length 40 mm moving at a speed of 2.8 m s^{-1} in a direction perpendicular to its length at right angles to a uniform magnetic field of flux density 85 mT.

Solution

Before using the formula, $V = BLv$, make a sketch to ensure the question is clear. See Fig. 19.3.

The induced voltage, $V = BLv$

$$= 0.085 \times 0.040 \times 2.8$$

$$= 9.5 \times 10^{-3} \text{ V}$$

19.1 QUESTIONS

1 Fig. 19.6 shows several examples of how an e.m.f. can be generated by moving a magnet relative to a wire or a coil of wire. In each case, state the polarity of the induced e.m.f. and the direction of the induced current.

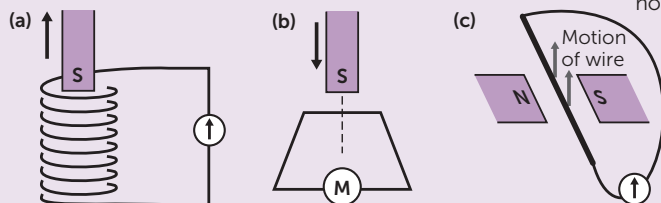


Fig. 19.6

2 An aircraft with a wing span of 22.0 m is in level flight, heading due north at a constant speed of 180 m s^{-1} . Calculate the e.m.f. induced between its wing tips due to the Earth's magnetic field, which is $60 \mu\text{T}$ in a direction due north at 70° to the horizontal at the aircraft's location.

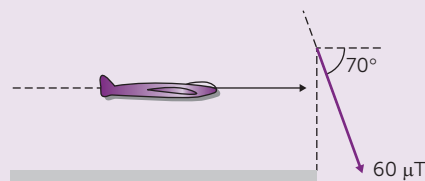
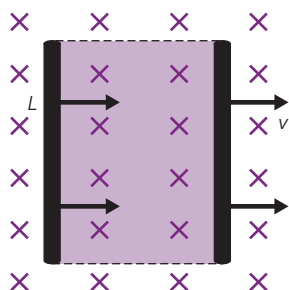


Fig. 19.7

19.2 Magnetic flux



Uniform magnetic field at 90° into plane of diagram

Fig. 19.8 Sweeping out magnetic flux

Note

The flux linkage (or total magnetic flux) through a coil of area, A and N turns, at right angles to a uniform magnetic field of flux density, B , is defined as BAN . The symbol Φ (capital phi) is used for flux linkage. Hence $\Phi = BAN = N\phi$.

The magnetic flux density of a U-shaped magnet can be increased by making the poles tapered. The magnet's field is concentrated between the poles as a result of using narrower poles. The lines of force are sometimes referred to as lines of 'flux', hence the term 'flux density' is a measure of how concentrated the lines are. Although, technically, magnetic flux density, B , is defined in terms of the force on a current-carrying wire, the term 'magnetic flux density' is used in preference to 'magnetic field strength' because it is easier to visualise lines of force. Conceptually, lines of force are no more than a useful way of picturing a magnetic field and should not be seen as the mechanism by which magnetic fields act on objects.

Consider again a moving rod of length L cutting through the lines of force of a uniform magnetic field, as in Fig. 19.8.

As explained on p. 283, the induced e.m.f. is equal to BLv . The rod sweeps out an area equal to its length, L , $\times$ its speed, v , each second because it moves distance v each second. By defining magnetic flux, ϕ (pronounced 'phi'), as the magnetic flux density, B , $\times$ the area, A , swept out, the rod can be thought of as sweeping out magnetic flux as it cuts through the lines of force.

Magnetic flux, $\phi = BA$

through an area of cross-section A at right angles to a uniform magnetic field of flux density B . The unit of magnetic flux is the weber (Wb), equal to 1 T m^{-2} .

From the definition of magnetic flux, it follows that the induced e.m.f. ($= BLv$) is equal in magnitude to the magnetic flux swept out per second. This relationship is known as Faraday's law of electromagnetic induction after Michael Faraday, who discovered how to generate electricity and established the theory of electromagnetic induction.

Faraday's law of electromagnetic induction states that the induced e.m.f. in a circuit is proportional to the rate of change of total magnetic flux through the circuit.

$$\text{Induced e.m.f., } V = -\frac{d\Phi}{dt}$$

where d/dt means rate of change and Φ = the flux linkage through the circuit.

Flux changes through a coil moved across a uniform magnetic field

An e.m.f. is induced in a conductor whenever it cuts across the lines of force of a magnetic field. In Fig. 19.9, a rectangular 1-turn coil of length L and width d is moved at speed v into and then out of a uniform magnetic field of flux density B .

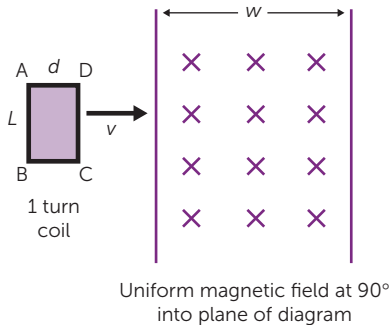


Fig. 19.9 Induced e.m.f. in a coil moved across a magnetic field

Notes

- 1 The constant of proportionality in Faraday's law is unity as a result of the definition of $\Phi = BAN$.
- 2 Lenz's law is incorporated in the equation by including the minus sign. This means that the induced e.m.f. acts against the change that causes it.

- 1 An e.m.f. equal to BLv is induced in the leading edge, CD, as it passes through the field. In time δt as CD enters the field, the flux through the coil changes by an amount $\delta\phi = BLv\delta t$ as CD moves a distance $v\delta t$ and sweeps out area $Lv\delta t$ in time δt . The flux change per second $\delta\phi/\delta t$ is therefore equal to the induced e.m.f. (BLv).
- 2 A reverse e.m.f. equal to $-BLv$ is induced in the trailing edge, AB, as it passes through the field. Therefore, a net e.m.f. is created only when one of the edges, AB or CD, is moving in the field. When both edges are in the field, the net e.m.f. is zero. A non-zero e.m.f. is produced when the coil area is partly in or out of the field but not when it is totally inside the field. To generate a non-zero e.m.f., the magnetic 'flux' through the coil must change. Fig. 19.10 shows how the flux changes as the coil is moved into, across and out of the magnetic field.

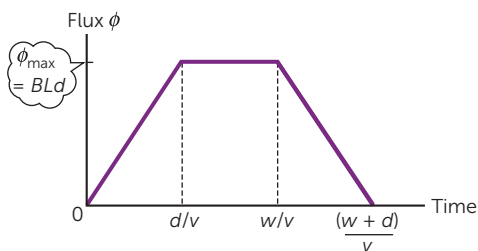


Fig. 19.10 Flux change

Investigating flux changes

- 1 Connect a flat, circular coil of known diameter to a data recorder linked to a microcomputer or an oscilloscope. Measure the induced e.m.f. when the coil is removed from between the poles of a U-shaped magnet. Display the measurements as a graph to show how the induced e.m.f. varies with time. This can usually be done automatically using the data recorder options menu. A typical graph is shown in Fig. 19.11 overleaf.

See www.palgrave.com/foundations/breithaupt for further investigations on flux changes using a data recorder.

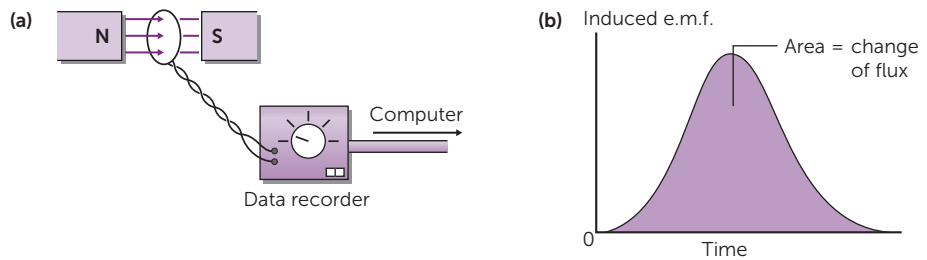


Fig. 19.11 Investigating flux changes (a) using a data recorder, (b) induced e.m.f. versus time

The area under the curve represents the total change of magnetic flux through the coil.

The idea of area under a curve representing a physical quantity was introduced through speed versus time graphs on p. 31. Just as the area under a graph of speed (= rate of change of distance) against time represents distance, so too the area under a graph of induced e.m.f. (= rate of change of magnetic flux) against time represents flux.

In formal mathematical terms,

$$\text{speed, } v = \frac{ds}{dt}$$

Hence s = area under the speed versus time graph. The induced e.m.f., V , can be written as

$$V = (-) \frac{d\Phi}{dt}$$

Hence Φ = area under induced e.m.f. versus time graph.

- 2 Measure the area under the curve in volt seconds. This is equal to the total initial flux (= BAN) through the coil, since the total final flux through it is zero. Hence calculate the magnetic flux density through the coil in its initial position.

19.2 WORKED EXAMPLE

A circular coil of 150 turns and of diameter 20 mm connected to a data recorder is placed with its face perpendicular to the lines of a uniform magnetic field. The coil is rapidly removed from the field. The data recorder is used to record and display a graph of the induced voltage with time as shown in Fig. 19.12. Calculate

- a) the total flux change,
- b) the initial magnetic flux density through the coil.

Solution

- a) Each block of the grid corresponds to 10 mV for 100 ms. Hence each block represents a flux change of 1000 mV ms which is equal to 1.0×10^{-3} Wb. There are 12 blocks under the curve (counting part blocks as 1 if over half a block and zero if under half). Hence the total magnetic flux change is 1.2×10^{-2} Wb (= $12 \times 1.0 \times 10^{-3}$ Wb)

$$\text{b) Total flux change} = BAN \quad \therefore B = \frac{\text{total flux change}}{AN} = \frac{1.2 \times 10^{-2}}{\pi(10 \times 10^{-3})^2 \times 150} = 0.25 \text{ T}$$

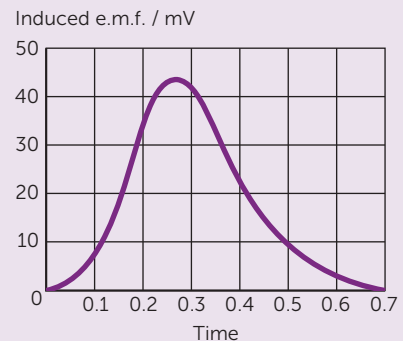
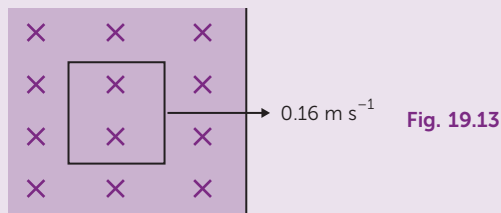


Fig. 19.12

19.3 QUESTIONS

- 1 A horizontal rod of length 750 mm aligned east-west is dropped from rest at a location where the Earth's magnetic field is $60 \mu\text{T}$ due north at an angle of 70° to the horizontal. Calculate
 - a) the speed of the rod 0.5 s after being released,
 - b) the e.m.f. induced across the ends of the rod 0.5 s after being released.
- 2 A 50 turn square coil, $40 \text{ mm} \times 40 \text{ mm}$, is placed with its plane perpendicular to the lines of force of a uniform magnetic field of flux density 90 mT.
 - a) Calculate the total magnetic flux through the coil.
 - b) The coil is moved out of the magnetic field at a steady speed of 0.16 m s^{-1} , as shown in Fig. 19.13. Calculate
 - (i) the time during which the flux decreases,
 - (ii) the e.m.f. induced as the coil leaves the field.



19.3 The alternating current generator

Mains electricity is generated in power stations and transmitted through the grid system to users up to hundreds of miles away. Each power station generates alternating current at the same frequency as any other power station. The frequency is determined by the rate at which the a.c. generator in the power station rotates.

A simple a.c. generator consists of a rectangular coil that rotates at a constant frequency in a uniform magnetic field, B . See Fig. 19.14. The coil is connected to an external circuit via two 'brushes', each in contact with its own slip ring. The magnetic flux through the coil changes as the coil position changes.

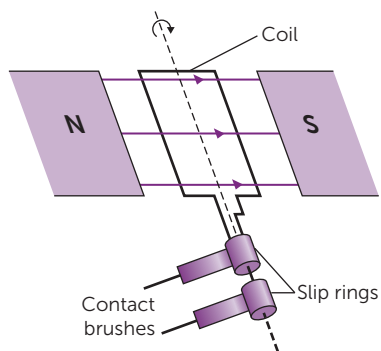


Fig. 19.14 The simple a.c. generator

- The maximum flux linkage through the coil = BAN , where A is the coil area and N is the number of turns.
- When the angle between the coil plane and its position for maximum flux is θ , the magnetic flux linkage, Φ , through the coil is $BAN \cos \theta$, since the component of the flux density perpendicular to the coil plane is $B \cos \theta$.
- Fig. 19.15 overleaf shows how the flux linkage changes with position. The flux linkage alternates between $+BAN$ and $-BAN$ every half-cycle. The minus sign for the flux linkage corresponds to the lines of force passing through the coil in the opposite direction, which is what occurs when the coil turns through half a cycle.

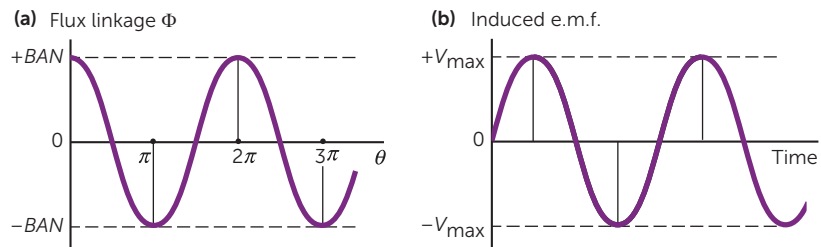


Fig. 19.15 e.m.f. and flux changes (a) alternating flux, (b) alternating voltage

- The induced e.m.f.

$$V = -\frac{d\Phi}{dt} = -\text{gradient of the flux linkage versus time graph.}$$

Since the gradient alternates between a positive maximum and a negative maximum, the induced e.m.f. therefore alternates in a similar way. This type of variation is described as **sinusoidal** because its shape is a sine wave. Fig. 19.15 also shows how the induced e.m.f. varies with time. As explained below, the waveform is described mathematically by the equation

$$V = V_{\text{MAX}} \sin(2\pi ft)$$

- The induced e.m.f. is at a peak whenever the coil is parallel to the field lines, as this is where the rate of change of flux is greatest. The sides of the coil cut directly through the lines of force in this position, each wire producing a contribution to the induced e.m.f. equal to BLv . The maximum induced e.m.f., V_{MAX} , is therefore $2 \times (BLv) \times N$, taking account of the two sides and the N turns. The speed of each side $v = \text{circumference } (\pi d) \times \text{the number of turns per second } (f) = \pi df$, where f is the frequency of rotation and d is the width of the coil.

$$\text{Hence } V_{\text{MAX}} = 2 \times (BLv) \times N = 2 \times (BL\pi fd) \times N = 2\pi fBAN$$

where $A = \text{the coil area} = Ld$.

$$V_{\text{MAX}} = 2\pi fBAN$$

- At time t after passing through a flux peak, the coil has turned through a fraction $f \times t$ of one cycle as shown in Fig. 19.16. Since the angle it turns through for a full cycle is 2π radians, the angle it turns through in time t , in radians, is given by

$$\theta = 2\pi ft$$

Each side of the coil at this instant is moving in a direction at this angle to the field lines. The component of velocity of each coil side perpendicular to the field lines is therefore $v \sin \theta$. Hence the induced voltage at this instant

$$V = V_{\text{MAX}} \sin \theta = V_{\text{MAX}} \sin(2\pi ft)$$

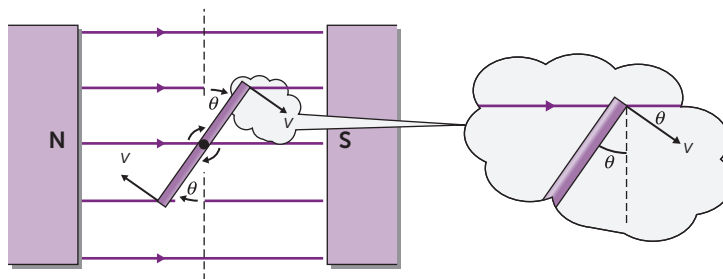


Fig. 19.16
Sinusoidal
changes

19.3 WORKED EXAMPLE

A 200 turn rectangular coil of length 50 mm and width 30 mm spins at a constant frequency of 50 Hz in a uniform magnetic field of flux density 150 mT.

- Calculate the peak induced e.m.f. at this frequency of rotation.
- Sketch a graph to show how the induced e.m.f. varies with time.
- Calculate (i) the angle in radians which the coil turns through in 4 ms after a voltage zero, (ii) the induced e.m.f. at this instant.

Solution

$$\begin{aligned} \text{a) } V_{\text{MAX}} &= 2\pi f BAN = 2\pi \times 50 \times 0.150 \times (0.050 \times 0.030) \times 200 \\ &= 14.1 \text{ V} \end{aligned}$$

b) See Fig. 19.17.

- In 4 ms, $\theta = 2\pi ft = 2\pi \times 50 \times 4.0 \times 10^{-3} = 1.26$ radians
- $V = V_{\text{MAX}} \sin(2\pi ft) = 14.1 \sin 1.26 = 13.4 \text{ V}$

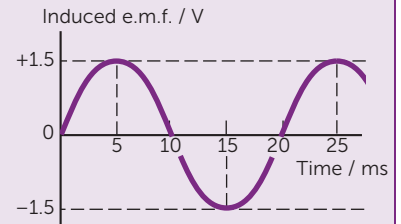


Fig. 19.17

The power-station alternator

This consists of a direct current electromagnet, the rotor, that spins at a constant frequency between three pairs of 'stator' coils. The two coils in each pair are connected in series so that an alternating voltage is induced between the terminals of each pair of coils. The three pairs of coils are aligned at 120° to each other, so that each pair produces an alternating voltage one-third of a cycle out of phase with the voltage produced by the other coils. See Fig. 19.18. The rotor is supplied with direct current from a d.c. generator driven on the same shaft.

One wire from each pair of coils, the neutral wire, is earthed at the power station. The potential of the other wire, the live wire, alternates from positive to negative and back every cycle. Mains electricity is supplied from a power station along four cables, one for each separate live wire and the fourth cable as the neutral wire for each phase. Each phase is distributed at the local substation to a different group of users. Sometimes one phase switches off due to a fault but the other phases remain on. This is why sometimes a power cut affects some users served by a local substation but not others.

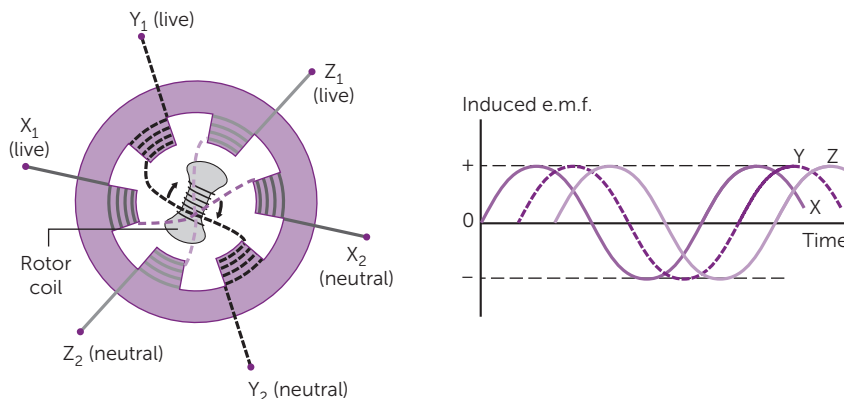


Fig. 19.18 The power-station alternator

Back e.m.f.

A simple electric motor consists of a coil rotating in a magnetic field as a result of passing a current through the coil. The rotating motion of the coil in the field generates an e.m.f. that acts against the voltage supply to the motor. This induced e.m.f. is referred to as a back e.m.f. as it opposes the voltage supply.

The voltage supply, V_S , must overcome the back e.m.f. V_B , to force current through the armature windings. Since the net e.m.f. is $V_S - V_B$, the current

$$I = \frac{V_S - V_B}{R}$$

where R is the armature resistance.

The back e.m.f. induced in a d.c. electric motor is proportional to the speed of the motor, in accordance with Faraday's law of electromagnetic induction. The faster the coil rotates, the greater the rate of change of flux through the coil and the larger the induced e.m.f. acting against the voltage supply. It follows that the current is low when the motor is running at high speed but the current increases if an increased load on the motor makes it slow down. When this happens, the back e.m.f. becomes smaller and so the net e.m.f. and the current become larger.

19.3 QUESTIONS

- 1 a) Sketch the output voltage waveform from a simple a.c. generator.
b) Indicate on your sketch at which point in a cycle the coil plane is (i) perpendicular to, (ii) parallel to the lines of force of the magnetic field.
c) On the same axes, sketch the output voltage waveform from the same a.c. generator if its speed of rotation is halved.
- 2 A 1500 turn rectangular coil of length 500 mm and width 300 mm spins at a constant frequency of 50 Hz in a uniform magnetic field of flux density 150 mT.
a) Calculate the peak induced e.m.f. at this frequency of rotation.
b) Sketch a graph to show how the induced e.m.f. varies with time.
c) The generator delivers an alternating current of peak value 10.0 A to a circuit. Calculate the couple acting on the generator coil due to the magnetic field which the drive shaft couple must overcome when the current is at its peak value.

19.4 The transformer

An e.m.f. is induced in a circuit if the magnetic flux through the circuit changes. So far we have considered changes of magnetic flux due to relative motion between the circuit and the source of the magnetic field. Another way of changing the magnetic flux through a circuit is to use an electric current to create the magnetic flux and change the magnetic flux by altering the current. The circuit that creates the magnetic flux is the **primary** circuit and the circuit in which the e.m.f. is induced is the **secondary** circuit. Transformers and induction coils are just two of a large number of devices that are based on this idea.

Changing current in the primary circuit

⇒ **changing magnetic flux through the secondary circuit**

⇒ **induced e.m.f. in the secondary circuit**

The operation of a transformer

A transformer consists of two coils of insulated wire wound around the same iron core, as shown in Fig. 19.19. When an alternating current is passed through the primary coil, an alternating magnetic field is created in the core through both coils. This alternating magnetic field induces an alternating e.m.f. in the secondary coil.

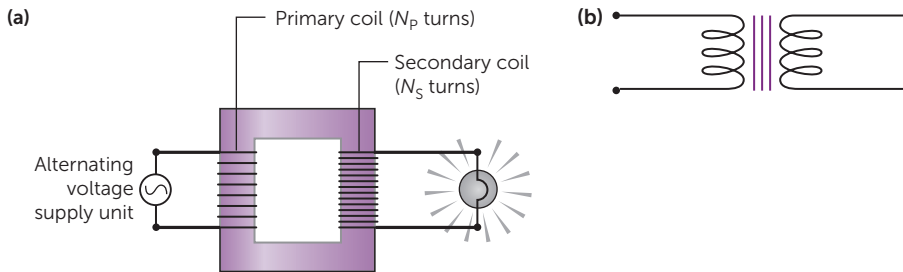


Fig. 19.19 (a) a model transformer, (b) the transformer symbol

The peak voltage across the secondary coil, V_S , depends on the turns ratio of the two coils and on the peak voltage across the primary coil, V_P , according to the transformer rule

$$\frac{V_S}{V_P} = \frac{\text{number of turns in the secondary coil, } N_S}{\text{number of turns in the primary coil, } N_P}$$

- 1 A **step-up transformer** has more turns in the secondary coil than in the primary coil (i.e. $N_S > N_P$) so $V_S > V_P$. The secondary voltage is greater than the primary voltage in a step-up transformer.
- 2 A **step-down transformer** has fewer turns in the secondary coil than in the primary coil (i.e. $N_S < N_P$) so $V_S < V_P$. The secondary voltage is less than the primary voltage in a step-down transformer.

To understand the transformer rule, let ϕ represent the flux per turn in the core at some instant.

- The total flux linkage through the secondary coil is therefore $N_S\phi$. Using Faraday's law

$$V = -\frac{d\Phi}{dt}$$

therefore gives

$$V_S = -N_S \frac{d\phi}{dt}$$

- The total flux linkage through the primary coil is $N_P\phi$, so the induced e.m.f. across the primary which the supply voltage must overcome (i.e. the primary voltage, V_P) is

$$-N_P \frac{d\phi}{dt}$$

The transformer rule therefore follows, since $d\phi/dt$ is the same for both coils provided no flux is lost from the core.

Transformer efficiency

The secondary coil of a transformer is used to deliver power to a circuit which it is connected to. When 'on load', the power delivered by the secondary coil $= I_S V_S$, where I_S is the current through the secondary coil.

The power supplied to the primary coil $= I_P V_P$, where I_P is the current through the primary coil.

The efficiency of the transformer

$$\begin{aligned} &= \frac{\text{the power delivered by the secondary coil}}{\text{the power supplied to the primary coil}} \\ &= \frac{I_S V_S}{I_P V_P} \end{aligned}$$

Note that efficiency values are quoted either as a number between 0 and 1 or, if multiplied by 100, as a percentage. The efficiency of a practical transformer is typically of the order of 98% or more. Very little power is wasted in practical transformers.

For a transformer which is 100% efficient, $I_P V_P = I_S V_S$, so

$$\frac{I_S}{I_P} = \frac{V_P}{V_S}$$

which means that if the voltage is stepped up, the current is stepped down and if the voltage is stepped down, the current is stepped up.

The main reasons for loss of efficiency are listed below.

- 1 **Resistance heating** occurs in the windings of the coils; copper wire is used to make the resistance of the windings as low as possible.
- 2 **Eddy currents** are induced in the core by the changing magnetic flux in the core. They are called 'eddy currents' because they swirl around in the core like eddy currents in water and they cause the core to heat up if they are excessive. In a practical transformer, the core is made of laminated strips of iron, separated by insulating layers which prevent eddy currents from travelling across the laminations.
- 3 **Hysteresis losses** occur because energy is used to magnetise and demagnetise the core each cycle. The core is made of iron because iron is easier than steel to magnetise and demagnetise.

Fig. 19.20 shows the construction of a practical transformer. The core is designed to ensure all the flux from the primary coil passes through the secondary coil.

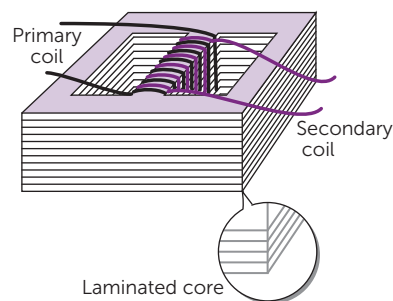


Fig. 19.20 A practical transformer

The grid system

Electricity is supplied to your home from a power station through the grid system which is a national network of cables. The voltage at which the grid system operates, typically 132 kV, is much higher than power-station voltages, typically 25 kV, which are much higher than the voltage of 230 V supplied to consumers as shown in Fig. 19.21. Transformers are used to step up the voltage from a power station onto the grid system and to step down the grid voltage at local substations to the mains voltage supplied to consumers.

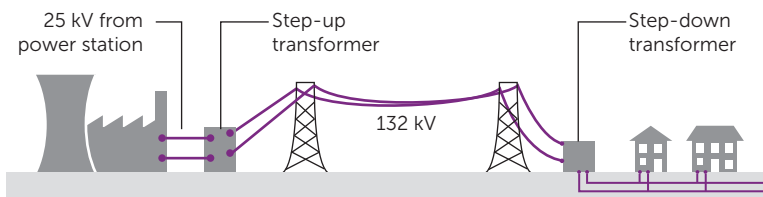


Fig. 19.21 The grid system

The reason why voltages are stepped up between a power station and the grid system is that the current needed to transfer a certain amount of power through the grid system is reduced if the voltage is increased. The lower the current through the cables, the smaller the fraction of power wasted due to resistance heating. Alternating voltages are used rather than direct voltages because alternating voltages can be stepped up or stepped down using transformers, thus enabling efficient power transfer through the grid system.

Application 8

Mobile phone chargers

A mobile phone charger contains a transformer called a 'switch-mode' transformer that is much smaller and lighter than an iron-core transformer. Instead of iron, the core is made from a ceramic material containing non-conducting metal compounds called ferrites. A ferrite-core transformer needs to operate at a much higher frequency than an iron-core transformer as ferrites become easier to magnetise and demagnetise the higher as the alternating frequency increases. Because the ferrite core is non-conducting, eddy currents are not created in the core so a switch-mode transformer is much more efficient than an iron-core transformer.

Fig. 19.22 shows the block diagram of a mobile-phone charger.

- 1 The mains input at 230 V, 50 Hz is converted by an electronic circuit to a high-frequency alternating voltage.
- 2 The high-frequency voltage is stepped down from 230 V to a much lower voltage by the switch-mode transformer.

The output voltage of the transformer is then converted to a 'direct' voltage which is then used to charge the rechargeable battery in the mobile phone.

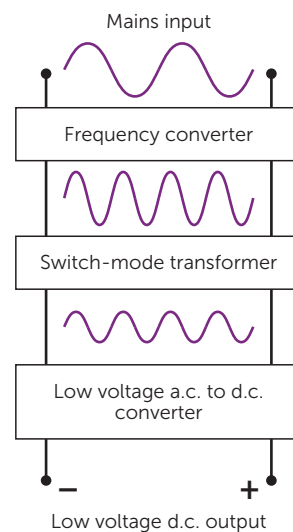


Fig. 19.22 Block diagram of a mobile phone charger

19.4 WORKED EXAMPLE

A transformer is to be used to step down an alternating voltage of 230 V to provide electrical power to a 12 V, 36 W light bulb connected to the secondary coil. Calculate

- the number of turns necessary on the secondary coil if the primary coil has 2400 turns,
- the maximum current in each circuit.

Solution

a) Rearrange $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ to give $N_S = \frac{N_P V_S}{V_P} = \frac{2400 \times 12}{230} = 125$ turns

b) Power delivered by the secondary coil $I_S V_S = 36$ W, hence $I_S = \frac{36 \text{ W}}{12 \text{ V}} = 3.0$ A

Assuming 100% efficiency, $I_P V_P = I_S V_S$

Rearranging this equation gives $I_P = \frac{I_S V_S}{V_P} = \frac{3.0 \times 12}{230} = 0.16$ A

19.4 QUESTIONS

- A transformer consists of a 60 turn primary coil and a 1200 turn secondary coil wound on the same iron core. A 230 V, 100 W light bulb is to be connected to the secondary coil. Calculate
 - the primary voltage that needs to be applied to make the light bulb light normally,
 - the current through (i) the light bulb when it lights normally, (ii) the primary coil, assuming 100% efficiency.
- An alternating current generator is used to supply electrical power in a building in the event of a mains power failure. The generator's output voltage of 110 V is stepped up to 230 V using a transformer. The generator is capable of supplying 20 kW of electrical power. Calculate
 - the turns ratio of a suitable transformer to connect the generator to the mains cable,
 - the maximum current the generator is able to supply,
 - the maximum current that could be drawn from the mains cables when a mains power failure occurs and the generator is in use.

19.5 Self-inductance

When a current through a coil changes, the change of the magnetic field produced by the coil induces an e.m.f. in the coil itself which acts against the change that causes it. Changes of the magnetic field produced by the wires in any part of the coil induce an e.m.f. in the other parts of the coil. This effect is called self-induction. In a direct current circuit, self-induction slows current growth and prolongs current decay. In an alternating current circuit, self-induction limits the current through the coil as the coil reacts against the voltage supply.

Self-induction in a direct-current circuit

Fig. 19.23a shows two light bulbs in parallel, one in series with a coil and the other in series with a variable resistor. The variable resistor is adjusted so that the light bulbs become equally bright after the battery switch is closed. However, the light bulb in series with the coil does not light up as rapidly as the other one. This is because of the effect of self-induction in the coil. As the current increases through the coil, a self-induced e.m.f. is generated in the coil, which acts against the battery

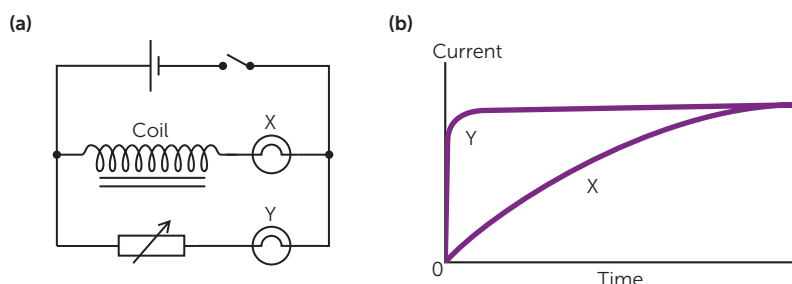


Fig. 19.23 Self-induction

and slows the growth of current through the coil and the light bulb in series with it. Fig. 19.23b shows how the current changes with time in each bulb. If the iron core of the coil is removed, both bulbs light up equally rapidly because the magnetic field in the coil is much weaker without the iron core.

Because the magnetic flux density, B , in the coil is proportional to the current, I , through the coil, as explained on p. 272, the flux linkage, Φ , through the coil is proportional to the current, I , in the coil. This relationship is used to define the **self-inductance**, L , of the coil as the flux linkage per unit current.

Flux linkage, $\phi = LI$

Using Faraday's law

$$V = -\frac{d\Phi}{dt}$$

therefore gives $-LdI/dt$ for the self-induced e.m.f. in a coil.

Self-induced e.m.f., $V = -L \frac{dI}{dt}$

The unit of self-inductance is the henry (H), equal to 1 ohm second (= 1 volt second per ampere).

Self-inductance and current growth

Consider a coil of self-inductance L connected to a battery and a switch. Suppose the total resistance of the circuit is R , as in Fig. 19.24. After the switch is closed, the current increases from zero to reach a maximum value, I_0 . At any instant after the switch is closed, the battery p.d. is used to overcome the self-induced e.m.f. to force current through the resistance of the circuit.

$$\therefore \text{the battery p.d., } V_{\text{batt}} = IR + L \frac{dI}{dt}$$

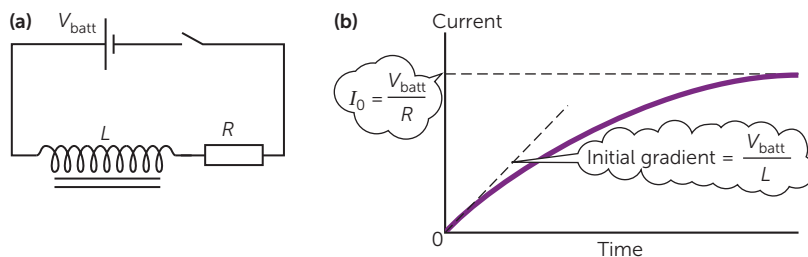


Fig. 19.24 Current growth

See www.palgrave.com/foundations/breithaupt for an experiment to investigate self-inductance.

- 1 **Initially**, the current is zero so there is no p.d. due to resistance. Hence

$$L \frac{dI}{dt} = V_{\text{batt}}$$

initially so the initial rate of growth of current

$$\left(\frac{dI}{dt}\right)_0 = \frac{V_{\text{batt}}}{L}$$

- 2 **Eventually** the current reaches a maximum value and so dI/dt becomes zero. Hence the maximum current

$$I_0 = \frac{V_{\text{batt}}}{R}$$

Energy stored in a self-inductance

When a current I passes through a coil of self-inductance L energy is 'stored' in the coil due to its self-inductance. This energy is stored in the magnetic field of the coil and is given by the equation

$$\text{Energy stored, } E = \frac{1}{2}LI^2$$

This formula can be understood by considering a small increase of the current from i to $i + \delta i$. The energy supplied for this increase, $\delta E = iV\delta t$.

However, $V\delta t = L\delta i$ for small changes since the magnitude of $V = Ldi/dt$.

$$\therefore \delta E = iL\delta i$$

Since $(i + \delta i)^2 = i^2 + 2i\delta i + \delta i^2$, then neglecting δi^2 , as it is much smaller than the other two terms, gives

$$\delta E = \frac{1}{2}L(i + \delta i)^2 - \frac{1}{2}Li^2$$

It therefore follows that the energy stored increases during this interval from $\frac{1}{2}Li^2$ to $\frac{1}{2}L(i + \delta i)^2$, so the energy stored for current I must be $\frac{1}{2}LI^2$.

19.5 QUESTIONS

- 1 In an experiment to test the effects of self-induction in a d.c. circuit, two coils were connected in series with a switch, a 0.25 A torch bulb and a 3 V battery.
 - a) When the switch was closed, the bulb lit up normally after a short delay of about a second. (i) Explain why the bulb did not light up immediately the switch was closed. (ii) Estimate the self-inductance of the two coils.
 - b) When the connections to one of the coils were reversed and the switch was closed, the torch bulb lit up immediately. Explain why there was no delay in this circuit.
- 2 A coil of resistance $2.0 \, \Omega$ and self-inductance $25 \, \text{H}$ is connected in series with a switch and a $12 \, \text{V}$ battery which has an internal resistance of $1.0 \, \Omega$. Calculate
 - a) the initial rate of growth of current in the circuit when the switch is closed,
 - b) (i) the final current in the circuit, (ii) the final energy stored in the circuit.

SUMMARY

- **Magnetic flux** = BA through an area, A , at right angles to the lines of force of a magnetic field, B .
- **Magnetic flux linkage** through a coil of N turns = BAN .
- **Laws of electromagnetic induction**
 - 1 **Lenz's law** states the induced current is always in a direction so as to oppose the change that causes it.
 - 2 **Faraday's law** states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.
- **Equations**
 - 1 **Faraday's Law** $V = -\frac{d\Phi}{dt}$
 - 2 **e.m.f. induced in a moving rod** $V = BLv$
 - 3 **Voltage from an a.c. generator** $V = V_{\text{MAX}} \sin(2\pi ft)$ where $V_{\text{MAX}} = 2\pi fBAN$
 - 4 **Transformer equation** $\frac{V_S}{V_P} = \frac{N_S}{N_P}$
 - 5 **Efficiency of a transformer** $= \frac{I_S V_S}{I_P V_P} \times 100\%$
 - 6 **Self-inductance** $L = \frac{\Phi}{I} = \frac{V}{(dI \div dt)}$

Energy stored = $\frac{1}{2}LI^2$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

19.1 The north pole of a bar magnet is pushed into one end of an open solenoid connected to a sensitive meter, as shown in Fig. 19.25.

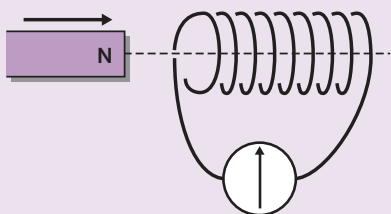


Fig. 19.25

- (i) What is the induced polarity at this end of the solenoid?
(ii) In which direction does the current pass through the meter?
- How does the deflection of the meter differ if
 - (i) the north pole is inserted more slowly,
 - (ii) a south pole is inserted more rapidly?

19.2

- An electromagnetic brake fitted to a heavy vehicle consists of a rotating magnet coupled to the drive shaft which spins near one end of a coil. The brake is applied by short-circuiting the coil terminals.
 - (i) Explain why the action of short-circuiting the coil terminals has a braking effect on the rotating magnet.
 - (ii) The braking effect reduces the kinetic energy of the vehicle. Describe the energy transformations that occur as a result of this loss of kinetic energy.
- (i) Explain why the current through an electric motor increases if the motor is slowed down by increasing the load on it.
 - (ii) Explain why the current through the primary coil of a transformer increases if a suitable load is connected across the secondary coil.

19.3

- An aircraft of wingspan 35.0 m, flying horizontally at a speed of 550 m s^{-1} , passes over a location at a height where the vertical component of the Earth's magnetic flux density is $80 \mu\text{T}$. Calculate the induced e.m.f. between the wing tips of the aircraft.

REVISION QUESTIONS

- b) A 120-turn circular coil of diameter 25 mm is positioned mid-way between the poles of a U-shaped magnet with its plane parallel to the pole faces of the magnet. The terminals of the coil are connected to a data recorder which is used to record the induced e.m.f. in the coil when it is withdrawn from the magnet. Fig. 19.26 shows how the induced e.m.f. varies with time.
- (i) Estimate the change of flux linkage through the coil from the area under the curve in Fig. 19.26.
- (ii) Hence determine the magnetic flux density between the poles of the magnet.

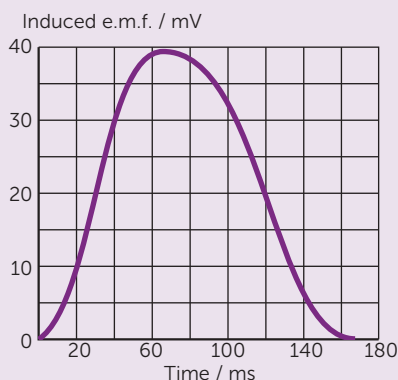


Fig. 19.26

19.4

- a) A primary coil and a secondary coil are wound on the same core. The primary coil is connected in series with a battery and a switch. The terminals of the secondary coil are connected to two rods, as in Fig. 19.27. When the switch is closed, a spark is generated between the ends of the two rods.

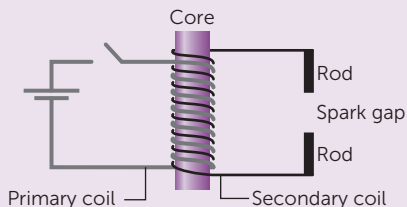


Fig. 19.27

- (i) Explain the cause of this spark.
- (ii) Explain why a spark is also produced when the switch is opened.
- (iii) Why are no sparks produced when the switch is left closed?
- b) A transformer is to be used to step down the 230 V main voltage to 9 V, to provide electrical power to a 36 W light bulb.
- (i) Calculate the number of turns needed in the secondary coil if the primary coil has 1200 turns.
- (ii) Calculate the current through the secondary coil when the light bulb lights normally.
- (iii) Assuming the transformer is 100% efficient, calculate the current in the primary coil when the light bulb lights normally.

19.5 An inductive coil consists of 80 turns of insulated wire wound tightly around a ferromagnetic ring of mean diameter 35 mm and of cross-sectional area $6.0 \times 10^{-5} \text{ m}^2$.

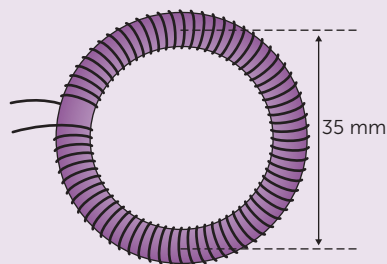


Fig. 19.28

- a) (i) The relative permeability, μ_r , of the ring is 2000. Use the formula, $B = \mu_r \mu_0 n I$ (where n is the number of turns per unit length) to calculate the magnetic flux density in the ring when the current is 0.06 A. ($\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$)
- (ii) Hence calculate the flux linkage through the coil when the current is 0.06 A, and the self-inductance of the coil.
- b) The ring is connected in series with a 1.5 V battery, a switch, an ammeter and a torch bulb.
- (i) Calculate the initial rate of growth of current through the bulb when the switch is closed.
- (ii) The ammeter reads 0.06 A when the current stops increasing after the switch is closed. Calculate the energy stored in the self-inductance.

Alternating Current

OBJECTIVES

After working through this unit, you should be able to:

- define and measure the peak value and the time period of an alternating voltage waveform
- calculate the frequency of an alternating current or voltage from its time period
- describe a sinusoidal waveform graphically and mathematically
- define the root mean square value of an alternating current or voltage and relate it to the peak value
- calculate the heating effect of an alternating current
- describe rectifier circuits to convert alternating current to direct current
- describe the effect of a capacitor or a coil in an a.c. circuit
- calculate and use reactance values for coils and capacitors
- explain how a reactive component differs from a resistive component
- describe applications of reactive components in a.c. circuits
- calculate currents and voltages for a.c. circuits containing reactive and resistive components in series

CONTENTS

- 20.1 Measurement of alternating currents and voltages
- 20.2 Rectifier circuits
- 20.3 Alternating current and power
- 20.4 Capacitors in a.c. circuits
- 20.5 Coils in a.c. circuits
- 20.6 Resonant circuits
- Summary
- Revision questions

20.1 Measurement of alternating currents and voltages

Mains appliances are supplied with alternating current via two wires, the live wire and the neutral wire. The potential of the live wire alternates repeatedly between a positive maximum and a negative maximum. The neutral wire is earthed at the local substation. The current through the appliance reverses direction repeatedly. A transformer in an appliance is used to step the mains voltage down if necessary. Fig. 20.1 overleaf shows how the voltage from a step-down transformer connected to the mains supply varies. This type of waveform is described as ‘sinusoidal’, as explained on p. 288.

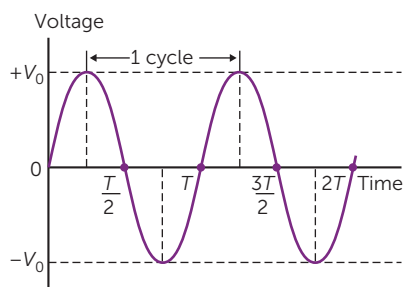


Fig. 20.1 An alternating waveform

- The **peak value** of an alternating current or voltage is the maximum value. This is represented in Fig. 20.1 by the height of the waveform from the centre.
- One **complete cycle** of an alternating current or voltage waveform is the interval from a current or voltage peak in a certain direction to the next peak in that direction.
- The **time period** of an alternating current or voltage is the time taken for one complete cycle.
- The **frequency** of an alternating current or voltage is the number of complete cycles per second. The unit of frequency is the hertz (Hz), equal to 1 cycle per second. The frequency, f , of an alternating waveform can be calculated from its time period, T , using the equation

$$f = \frac{1}{T}$$

Using an oscilloscope to measure an alternating voltage

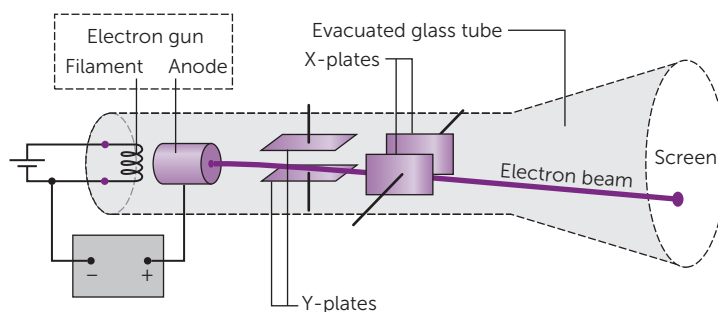


Fig. 20.2 Inside an oscilloscope tube

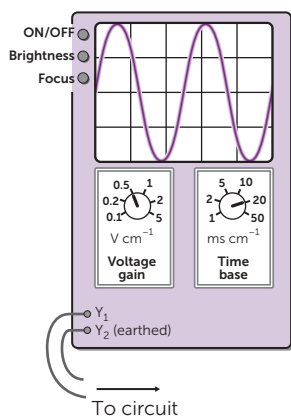


Fig. 20.3 Using an oscilloscope

An oscilloscope is designed to measure voltages. As shown in Fig. 20.2, an electron beam from an electron gun at the back of the tube hits the screen and creates a spot of light at the point of impact. The beam is deflected by two pairs of parallel plates, one pair (the X-plates) designed to deflect the beam horizontally and the other pair (the Y-plates) designed to deflect it vertically. In normal operation, a 'time base' voltage is applied to the X-plates which makes the spot move across the screen slowly from left to right, then returns it much more rapidly to start the next 'sweep'. At the same time, the waveform to be displayed on the screen is applied to the Y-plates, so the waveform is traced by the spot on the screen as it sweeps across from left to right as shown in Fig. 20.3.

The main controls for measurement purposes on an oscilloscope are:

- the **Y-gain**, which is the voltage per unit deflection in the vertical direction. For example, if the Y-gain control knob is set at 0.5 V cm^{-1} and the waveform is 40 mm from the bottom (negative) peak to the top (positive) peak, the voltage change from the negative to the positive peak is 2.0 V ($= 0.5 \text{ V cm}^{-1} \times 4.0 \text{ cm}$). The peak voltage is therefore 1.0 V .
- the **time-base**, which is the time taken by the spot to move unit distance horizontally. For example, if the time-base control knob is set at 20 ms cm^{-1} and one cycle of the waveform measures a horizontal distance of 25 mm, the time period of the waveform must be 50 ms ($= 20 \text{ ms cm}^{-1} \times 2.5 \text{ cm}$), giving a frequency of 20 Hz ($= 1/0.050 \text{ s}$).

The equation for a sinusoidal waveform

The voltage from an a.c. generator is a sinusoidal waveform like Fig. 20.1. As explained on p. 288, the voltage, V , varies with time in accordance with the equation

$$V = V_0 \sin(2\pi ft)$$

where V_0 is the peak voltage, f is the frequency of rotation of the generator and t is the time after the voltage is zero as it reverses from $-$ to $+$ polarity. The voltage at a given time can be either read off a waveform graph like Fig. 20.1 or calculated using the above equation.

Another method of finding the voltage at a given time is to represent the peak value as a rotating vector or **phasor**, as in Fig. 20.4a. When the phasor is at angle θ to the $+x$ -axis, the coordinates of its tip are $x = V_0 \cos \theta$ and $y = V_0 \sin \theta$. By considering the phasor rotating anticlockwise at constant frequency, f , at time, t , after it passes through the $+x$ -axis, the phasor has turned through a fraction, ft , of a cycle and therefore the angle it makes in radians to the x -axis is $2\pi ft$ (i.e. $\theta = 2\pi ft$). Hence its y -coordinate, $V_0 \sin \theta$, represents the voltage at time t .

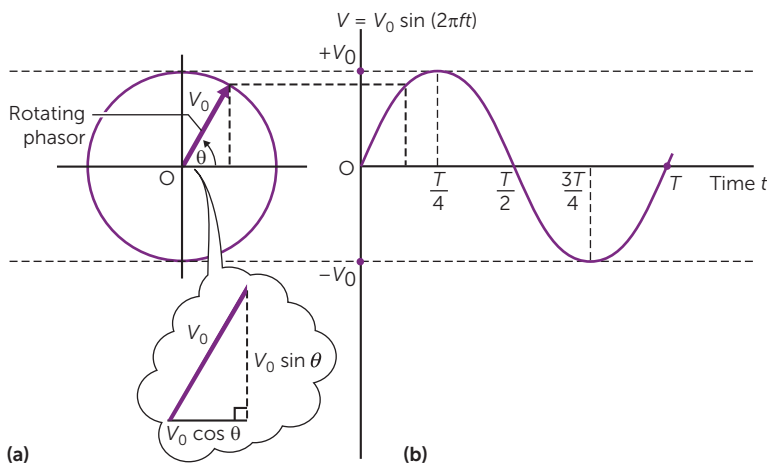


Fig. 20.4 (a) a rotating phasor, (b) a sinusoidal waveform

20.1 WORKED EXAMPLE

A sinusoidal alternating voltage with a peak value of 12 V has a frequency of 50 Hz. Calculate

- its time period,
- the voltage (i) 4 ms, (ii) 7 ms, (iii) 18 ms after the voltage was zero as it changed from negative to positive.

Solution

a) $T = \frac{1}{f} = \frac{1}{50} = 0.020 \text{ s} = 20 \text{ ms}$

- b) Fig. 20.5 shows the position of the phasor for each case. The length of the phasor represents 12 V so the y-coordinate at each position gives the voltage at that position. The values read off from the phasor diagram should be the same as the calculated values as follows:

(i) $V = V_0 \sin(2\pi ft) = 12 \sin(2\pi \times 50 \times 0.004 \text{ radians}) = 11.4 \text{ V}.$

(ii) $V = V_0 \sin(2\pi ft) = 12 \sin(2\pi \times 50 \times 0.007 \text{ radians}) = 9.7 \text{ V}.$

(iii) $V = V_0 \sin(2\pi ft) = 12 \sin(2\pi \times 50 \times 0.018 \text{ radians}) = -7.1 \text{ V}.$

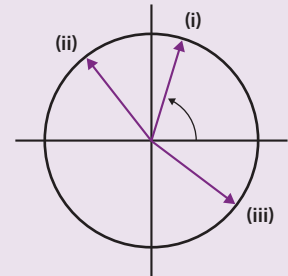


Fig. 20.5

20.1 QUESTIONS

- Fig. 20.6 shows an oscilloscope screen displaying an alternating waveform. Calculate the peak voltage and the frequency of this waveform if the control knob settings are
 - a Y-gain of 0.2 V cm^{-1} and a time base of $10 \mu\text{s cm}^{-1}$,
 - a Y-gain of 5.0 V cm^{-1} and a time base of 5 ms cm^{-1} .
- An alternating voltage has a peak voltage of 4.0 V and a frequency of 200 Hz.
 - Construct a phasor diagram and use it to determine the voltage (i) 1.0 ms, (ii) 3.0 ms after the voltage was zero and changing from $-$ to $+$.
 - Use the equation $V = V_0 \sin(2\pi ft)$ to calculate the voltage (i) 1.0 ms, (ii) 3.0 ms after the voltage was zero and changing from $-$ to $+$.

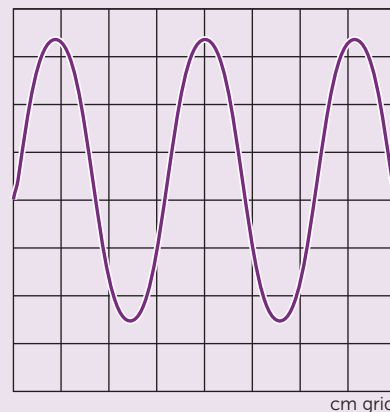


Fig. 20.6

20-2 Rectifier circuits

Electronic organs, transistor radios and portable computers are just three of many devices that can be powered from either a battery or a mains-operated, low voltage supply unit, which consists of a transformer and a rectifier circuit. The transformer steps the mains voltage down to the desired voltage and the rectifier circuit converts the alternating voltage from the transformer to a direct voltage.

Investigating rectifier circuits

- A **single diode in series with a resistor** and an a.c. supply unit allow current through every other half-cycle when the supply polarity is such that the diode is forward-biased. This is referred to as **half-wave rectification**. The current

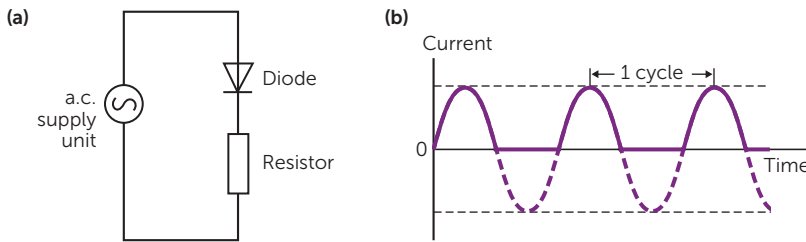


Fig. 20.7 Half-wave rectification (a) using a single diode, (b) a half-wave direct current

waveform is shown in Fig. 20.7b. Since the voltage across the resistor is proportional to the current, the voltage across the resistor also has a 'half-wave' waveform like Fig. 20.7. This can be displayed on an oscilloscope screen by connecting the Y-input of the oscilloscope across the resistor.

- 2 **Two diodes and a transformer** connected as in Fig. 20.8 give **full-wave rectification**, in which the voltage varies from zero to a peak value and back to zero every half-cycle. This can be achieved by tapping off the output voltage from the centre of the transformer's secondary coil and through a diode connected at each end of the secondary coil. In Fig. 20.8, when the alternating supply connected to the primary coil makes end X of the secondary coil positive (and the other end negative), the diode at this end, D_X , conducts so current passes through the load resistor back to the central terminal of the secondary coil. When the supply reverses polarity, the other end of the secondary coil, end Y, is positive so the diode at this end, D_Y , now conducts and current passes through the load resistor in the same direction as before, back to the central terminal of the transformer's secondary coil.

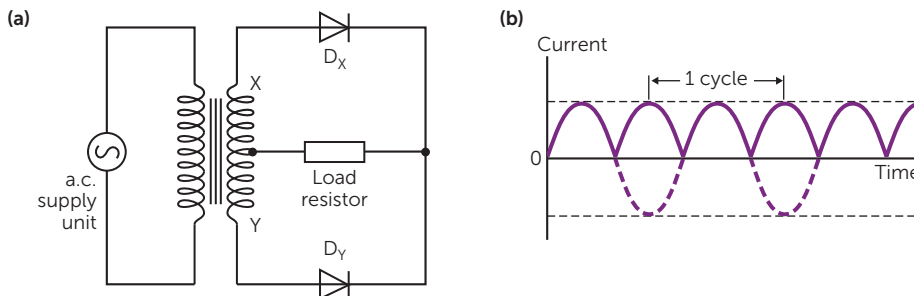


Fig. 20.8 Full-wave rectification (a) using two diodes, (b) a full-wave direct current

The current waveform and the resistor voltage waveform are shown in Fig. 20.8b. This waveform is described as a 'full-wave' rectified waveform because the current is non-zero and in the same direction in both halves of each cycle.

- 3 The **bridge rectifier** consists of four diodes as in Fig. 20.9 overleaf, connected directly to the a.c. supply to be rectified. On one half-cycle of each cycle, diodes P and R conduct, so current passes through the load resistor from left to right. On the other half-cycle, diodes Q and S conduct and current passes through the load resistor left to right again. Full-wave rectification is once again produced as the current through the resistor and the resistor voltage do not change direction.

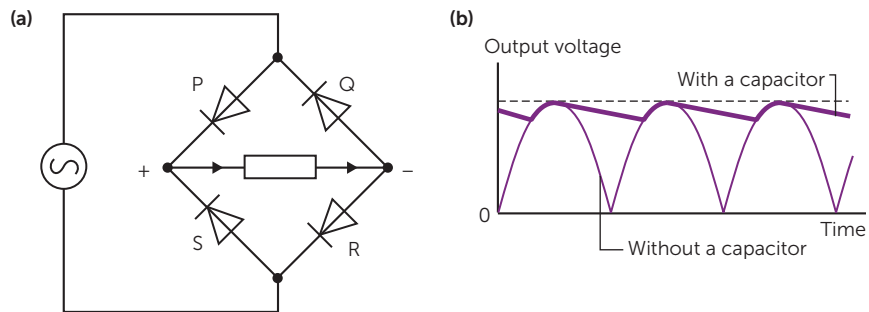


Fig. 20.9 (a) using a bridge rectifier, (b) capacitor smoothing

Capacitor smoothing

A large capacitor connected across the output terminals of a full-wave rectifier will charge up in the first half-cycle then release its charge very slowly after the supply voltage peaks, thus maintaining the output voltage from the full-wave rectifier. Fig. 20.9 shows the idea. The capacitor must be large, so the time constant RC (see p. 223) of the circuit is much longer than the time period of the alternating supply voltage.

20.2 QUESTIONS

- Fig. 20.10 shows a bridge rectifier comprising four diodes, A, B, C and D, connected to an a.c. supply unit.
 - (i) In which direction does current pass through the load resistor? (ii) When terminal X of the a.c. supply unit is negative, which diodes conduct?
 - Sketch the current waveform if diode A fails and becomes non-conducting.

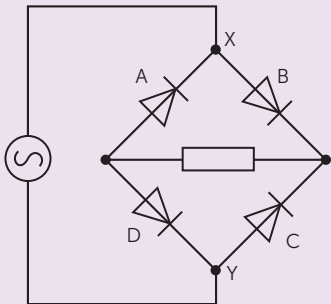


Fig. 20.10

- Fig. 20.11 shows two diodes in a circuit in opposite directions connected to an a.c. supply unit. Two unequal resistors are also connected in the circuit as shown.
 - In which direction does current pass through the meter?
 - With the aid of a waveform sketch, explain how the supply current varies with time over several cycles.

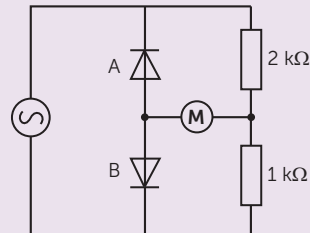


Fig. 20.11

20.3 Alternating current and power

Imagine an electric heater operating at a very low frequency. The heating element would glow at peak current and fade at zero current every half-cycle, regardless of the direction in which the current passed through. The power supplied to the heater would peak every half-cycle when the current was at its peak in either direction. The heating effect is due to the resistance of the heating element.

The power supplied to a resistor R carrying current, $I = IV = I^2R$, since the resistor p.d., $V = IR$.

For a sinusoidal current

$$I = I_0 \sin 2\pi ft$$

1 the resistor voltage

$$V = IR = I_0 R \sin 2\pi ft$$

2. the power supplied

$$P = IV = I_0^2 R \sin^2 2\pi ft$$

The power supplied varies from zero to a peak value of $I_0^2 R$ and back to zero every half-cycle. Fig. 20.12 shows how the power supplied varies with time over a few cycles. This power curve is symmetrical about $\frac{1}{2} \times$ the peak power ($= \frac{1}{2} I_0^2 R$).

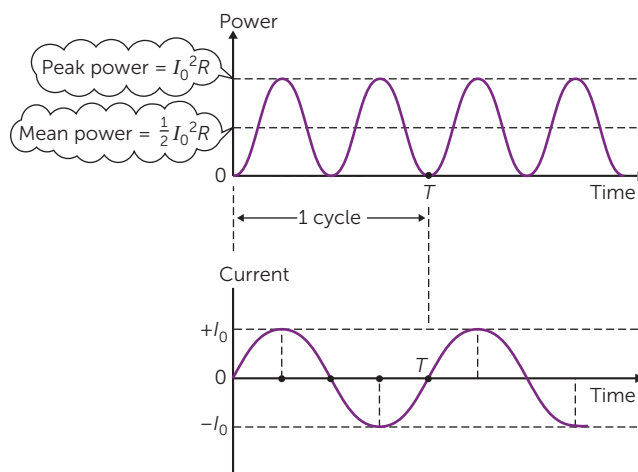


Fig. 20.12 Power versus time for a sinusoidal current through a resistor

The root mean square (r.m.s.) value of an alternating current (or voltage) is the value of the direct current (or voltage) that gives the same heating effect in a given resistor as the alternating current (or voltage).

For a sinusoidal alternating current of peak value I_0 , its r.m.s. value, I_{rms} , is therefore related to the peak current by equating the d.c. power ($I_{\text{rms}}^2 R$) to the mean a.c. power ($\frac{1}{2} I_0^2 R$).

$$\text{i.e. } I_{\text{rms}}^2 R = \frac{1}{2} I_0^2 R$$

Cancelling R therefore gives

$$I_{\text{rms}}^2 = \frac{1}{2} I_0^2$$

Taking the square root of both sides,

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_0$$

Similarly, the r.m.s. voltage V_{rms} is related to the peak voltage of a sinusoidal alternating voltage waveform by the equation

$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_0$$

See www.palgrave.com/foundations/breithaupt for more about mains electricity.

Note

$$\begin{aligned} \text{Mean power} &= I_{\text{rms}}^2 R \\ &= \frac{1}{2} I_0^2 R \\ &= I_{\text{rms}} V_{\text{rms}} \\ &= \frac{1}{2} I_0 V_0 \end{aligned}$$

20.2 WORKED EXAMPLE

A sinusoidal alternating voltage which has an r.m.s. value of 230 V is connected to a $60\ \Omega$ heating element designed to operate at an r.m.s. voltage of 230 V.

Calculate

- a) (i) the peak voltage, (ii) the voltage change from a positive peak to a negative peak.
 b) (i) the r.m.s. current, (ii) the peak current.
 c) (i) the peak power, (ii) the mean power.

Solution

a) (i) Rearrange $V_{\text{rms}} = \frac{1}{\sqrt{2}} V_0$ to give $V_0 = \sqrt{2} V_{\text{rms}} = \sqrt{2} \times 230 = 325\ \text{V}$

(ii) Peak to peak voltage $= 2V_0 = 2 \times 325 = 650\ \text{V}$

b) (i) $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{230}{60} = 3.8\ \text{A},$

(ii) $I_0 = \sqrt{2} I_{\text{rms}} = \sqrt{2} \times 3.8 = 5.4\ \text{A}$

c) (i) Peak power $= I_0^2 R = 5.4^2 \times 60 = 1750\ \text{W}$

(ii) Mean power $= \frac{1}{2} I_0^2 R = 0.5 \times 1750 = 875\ \text{W}$

Adapting a moving coil meter to measure an alternating current or voltage

If an alternating current at low frequency is passed through a moving coil meter, the pointer moves to and fro about the zero reading, changing direction as the current changes direction every half-cycle. At frequencies above about 5 Hz, the pointer vibrates about the zero reading as it is unable to keep up with the repeated changes of direction.

The pointer can be made to deflect in one direction by connecting a diode in series with the meter or by using a bridge rectifier, as in Fig. 20.13. In both cases, the pointer gives a non-zero reading which is less than the peak current. Since the reading is proportional to the peak current and hence proportional to the r.m.s. current, the meter scale is usually calibrated so the pointer position gives the r.m.s. value.

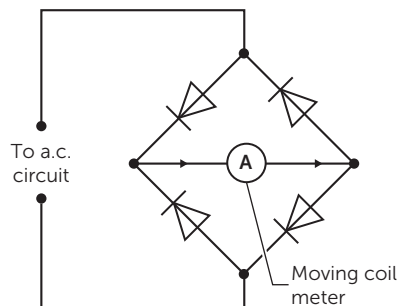


Fig. 20.13 Adapting a moving coil meter

20.3 QUESTIONS

- 1 An oscilloscope is used to display and measure an alternating voltage waveform. The waveform measures 72 mm vertically from the negative peak to the positive peak on the screen, when the Y-gain is at 0.2 V cm^{-1}
 - a) Calculate (i) the peak to peak voltage V_{pp} , (ii) the peak voltage V_o , (iii) the r.m.s. voltage.
 - b) This alternating voltage is the result of a sinusoidal current of r.m.s. value 120 mA passing through a resistor. Calculate (i) the resistance of this resistor, (ii) the mean power dissipated in this resistor.
- 2 A mains heater carries the label '230 V r.m.s., 1000 W'. Calculate
 - a) the peak voltage across the heating element,
 - b) (i) the r.m.s. current, (ii) the peak current, (iii) the resistance of the heating element.

20.4 Capacitors in a.c. circuits

Reactive and resistive components

Coils and capacitors are described as reactive components because they react against alternating current passing through without dissipating power. In comparison, a resistive component opposes an alternating current due to its resistance, which causes it to dissipate power. A resistive component is one which dissipates power when a current passes through it.

As explained in Section 20.3, in a simple a.c. circuit consisting of a resistor connected to an a.c. power supply unit, the current and the potential difference across the resistor alternate in phase, which means that they rise and fall and change direction together. The result is that power is dissipated in the resistor every half-cycle. As explained later in this section, in a simple circuit containing either a capacitor or a coil connected to an a.c. supply unit, the current and p.d. are out of phase by a quarter of one cycle. As a result, power is supplied to the component then returned to the power supply unit every half-cycle so the mean power supplied is zero. The term reactance is used for a capacitor or a coil because these components react against the power supply unit without taking power from it.

The reactance of a capacitor

Consider an a.c. supply unit connected to a single capacitor, as shown in Fig. 20.14. As the potential difference between the capacitor plates alternates, electrons flow onto one plate then back to the supply unit again, then onto the other plate and back to the supply unit again.

- The flow of electrons to and from the supply is an alternating current, therefore the capacitor allows an alternating current in the circuit, whereas it would block a direct current if the supply unit was a d.c. unit.
- The faster the frequency of the a.c. supply, the greater the flow of charge per second. The charge flows to and from the capacitor at a faster rate if the frequency is increased. Therefore, the higher the frequency is, the greater the current is, even though the peak supply voltage is constant. Conversely, the lower the frequency, the smaller the current.

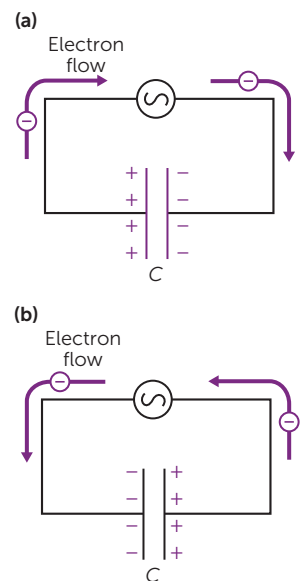


Fig. 20.14 A capacitor in an a.c. circuit (a) at a given instant and (b) half a cycle later

For any capacitor, the charge stored, $Q = CV$, where V is the p.d. across its terminals and C is its capacitance. In an alternating current circuit, the charge and the potential difference are therefore in phase with each other. In mathematical terms, if $V = V_0 \sin(2\pi ft)$, then $Q = CV_0 \sin(2\pi ft)$. Figs 20.15 and 20.16 show how the voltage and charge vary over a few cycles.

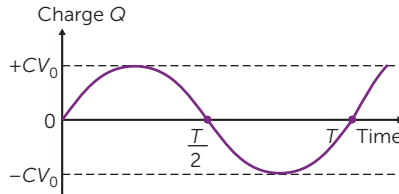


Fig. 20.15 Charge versus time

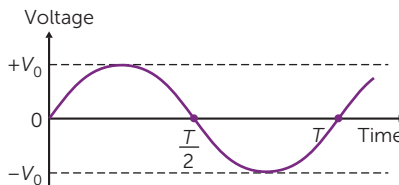


Fig. 20.16 Current versus time

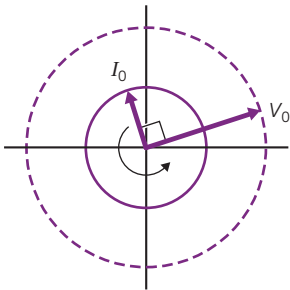
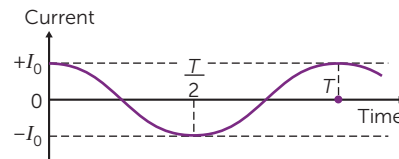


Fig. 20.17 Phasors

Since the current $I = \text{the rate of flow of charge, } dQ \div dt$, which is the gradient of the charge versus time graph, the current at any point is given by the gradient of the charge versus time graph. Fig. 20.16 shows how the current (i.e. the gradient of the charge versus time graph) varies with time when the charge varies sinusoidally as in Fig. 20.15. The current is one quarter of a cycle ahead of the potential difference and charge. Essentially, the rate of flow of charge on the capacitor is at its peak at the instant the capacitor is uncharged and about to charge up.

As explained on p. 310, if $Q = CV_0 \sin(2\pi ft)$ then

$$I = \frac{dQ}{dt} = \frac{d}{dt}(CV_0 \sin(2\pi ft)) = 2\pi f CV_0 \cos(2\pi ft)$$

- 1 The peak current $I_0 = 2\pi f CV_0$ since the peak value of $\cos(2\pi ft) = 1$.
- 2 The current is ahead of the p.d. by one quarter cycle ($= 90^\circ$). The phasor diagram for the current and voltage for a capacitor in an a.c. circuit is shown in Fig. 20.17.
- 3 The **reactance**, X_C , of a capacitor is defined as $\frac{1}{2\pi f C}$

$$\text{Since } I_0 = 2\pi f CV_0, \text{ then } \frac{1}{2\pi f C} = \frac{V_0}{I_0}$$

The unit of reactance is therefore the ohm, the same as the unit of resistance.

$$X_C = \frac{V_0}{I_0} = \frac{1}{2\pi f C}$$

Note that the reactance decreases as the frequency increases. Fig. 20.18 shows how the reactance decreases with increasing frequency.

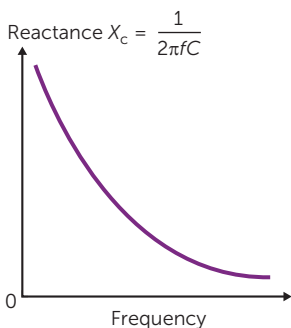


Fig. 20.18 Reactance versus frequency for a capacitor

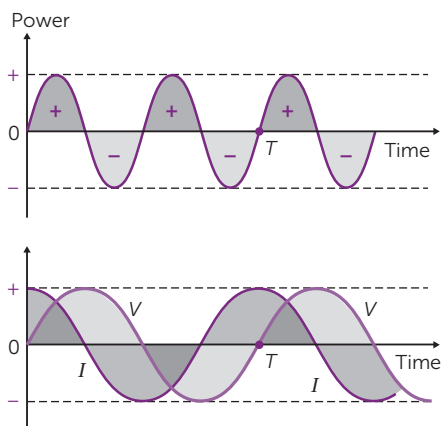


Fig. 20.19 The power curve for a capacitor

- 4 Fig. 20.19 shows how the power ($= \text{current} \times \text{potential difference}$) varies with time. Every other quarter cycle, power is returned to the supply unit as electrons are forced into the negative terminal. In these quarter cycles, the current and the potential are in opposite 'directions' so IV is negative and power returns to the supply unit. In between these quarter cycles, the current and the potential difference are in the same direction, so IV is positive, so power is delivered by the supply unit. As a result, no power is delivered by the supply over a complete cycle.

Measuring the reactance of a capacitor

- 1 Connect the capacitor in series with a digital ammeter and an a.c. supply unit. Connect an oscilloscope across the capacitor terminals, as shown in Fig. 20.20, to display the voltage waveform. Keeping the frequency constant, measure the r.m.s. current using the ammeter for different peak to peak voltages. For each current, measure the peak voltage and calculate the r.m.s. voltage. Use the oscilloscope to measure the time period of the waveform and hence calculate the frequency.
- 2 Record the measurements and use them to plot a graph of the capacitor voltage against the capacitor current. The gradient of this graph is the reactance of the capacitor. Hence determine the capacitor reactance and use its value to calculate the capacitance from the equation

$$X_C = \frac{1}{2\pi fC}$$

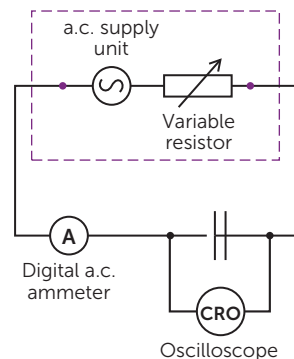


Fig. 20.20 Measuring reactance

20.3 WORKED EXAMPLE

A $2.2 \mu\text{F}$ capacitor is connected to a 50 Hz a.c. power supply unit which has an r.m.s. output p.d. of 6.0 V. Calculate

- a) the reactance of this capacitor at this frequency,
- b) the r.m.s. current in the circuit,
- c) the peak current in the circuit.

20.3 WORKED EXAMPLE *continued***Solution**

$$\text{a) } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 2.2 \times 10^{-6}} = 1.45 \times 10^3 \Omega$$

$$\text{b) Since } X_C = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}},$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{6.0}{1.45 \times 10^3} = 4.1 \times 10^{-3} \text{ A}$$

$$\text{c) } I_0 = \sqrt{2} I_{\text{rms}} = 5.9 \times 10^{-3} \text{ A}$$

Differentiation of a sine function**Proof of $\frac{d}{dt}[\sin(2\pi ft)] = 2\pi f \cos(2\pi ft)$**

In Fig. 20.21, OP is a rotating phasor of length r at angle $\theta = 2\pi ft$ to the $+x$ -axis at time t after passing through the x -axis in an anticlockwise direction. A short time, δt , later, at time $t + \delta t$, after passing through the $+x$ -axis, its angle to the x -axis has increased by $\delta\theta = 2\pi f\delta t$ and its tip moves through a distance $\delta s = r\delta\theta$.

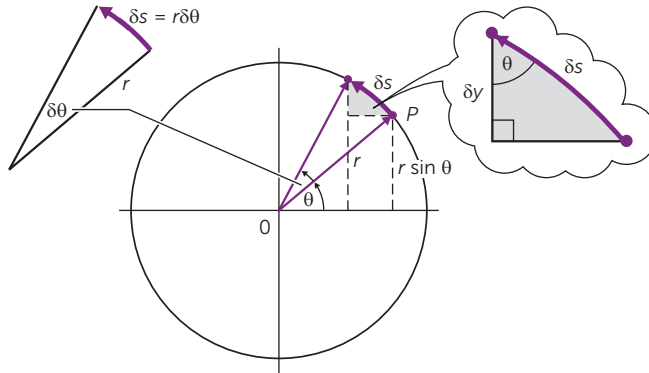


Fig. 20.21 Small changes

The y -coordinate of the tip of the vector OP is given by $y = r \sin \theta$, where $r =$ length OP. In the time interval from t to $t + \delta t$, the y -coordinate increases by δy . However, since δy and δs almost form a right-angled triangle, then $\delta y = \delta s \cos \theta$, provided δs is small.

Since $\delta s = r\delta\theta$ and $\delta\theta = 2\pi f\delta t$, then $\delta y = \delta s \cos \theta = 2\pi fr\delta t \cos \theta$. Dividing δt into both sides of $\delta y = 2\pi fr\delta t \cos \theta$ therefore gives

$$\frac{\delta y}{\delta t} = \frac{2\pi fr\delta t}{\delta t} \cos \theta$$

Hence in the limit $\delta t \rightarrow 0$,

$$\frac{dy}{dt} = 2\pi fr \cos \theta$$

$$\therefore \frac{d}{dt}[r \sin(2\pi ft)] = 2\pi fr \cos(2\pi ft)$$

Cancelling r on both sides therefore gives the required equation.

20.4 QUESTIONS

- 1 In an experiment to measure the capacitance of a capacitor, the capacitor current was measured for different p.d.s applied across the capacitor at a constant frequency of 2000 Hz. The results are in the table below.

r.m.s. voltage/V	0.0	2.0	4.0	6.0	8.0	10.0
r.m.s. current/mA	0.0	12.1	24.0	35.5	48.5	60.0

- Plot a graph of the r.m.s. voltage against the r.m.s. current.
 - Use your graph to calculate the reactance of the capacitor.
 - Hence calculate the capacitance of this capacitor.
- 2 A capacitor is connected in series with an a.c. ammeter and a variable frequency signal generator, as in Fig. 20.21. An oscilloscope is connected across the capacitor terminals. The oscilloscope trace is also shown in Fig. 20.22.

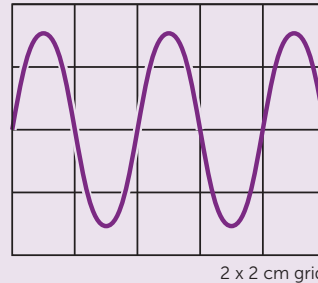


Fig. 20.22

- The voltage gain of the oscilloscope was 2.0 V cm^{-1} . Calculate (i) the peak voltage across the capacitor, (ii) the r.m.s. voltage across the capacitor.
- The time base of the oscilloscope was 0.5 ms cm^{-1} . Calculate (i) the time period, (ii) the frequency of the alternating voltage.
- The r.m.s. current was 0.48 mA , measured using the ammeter. Calculate (i) the reactance, (ii) the capacitance of the capacitor.

20.5 Coils in a.c. circuits

Consider an a.c. supply unit connected to a single coil of negligible resistance, as shown in Fig. 20.23. As the current through the coil alternates, the changing magnetic field due to the alternating current induces an e.m.f. in the coil which acts against the alternating voltage applied to the coil.

The faster the frequency of the a.c. supply, the greater the rate of change of current and hence the greater the induced e.m.f. The voltage supply must therefore be made greater to keep the current the same if the frequency is raised. Therefore, the smaller the current, the higher the frequency if the supply voltage is unaltered. Conversely, the lower the frequency, the larger the current.

In mathematical terms, if $I = I_0 \sin(2\pi ft)$, and the self-inductance of the coil is L , then the induced e.m.f. V can be written as

$$V = L \frac{dI}{dt} = L \frac{d}{dt} [I_0 \sin(2\pi ft)]$$

Because

$$\frac{d}{dt} [\sin(2\pi ft)] = 2\pi f \cos(2\pi ft)$$

as explained on p. 310, then

$$V = 2\pi f L I_0 \cos(2\pi ft)$$

As the induced e.m.f. acts against the supply voltage, the supply voltage must therefore be equal to the induced e.m.f. to force current around the circuit.

$$V = 2\pi f L I_0 \cos(2\pi ft)$$

- The peak voltage $V_0 = 2\pi f L I_0$, since the peak value of $\cos(2\pi ft) = 1$.
- The p.d. is ahead of the current by one quarter cycle ($= 90^\circ$) as shown in Fig. 20.24. The phasor diagram for the current and voltage for a coil in an a.c. circuit is also shown in Fig. 20.24.

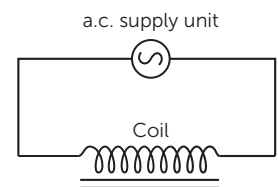


Fig. 20.23 An inductive coil in an a.c. circuit

See www.palgrave.com/foundations/breithaupt for more information about measuring reactance of a coil.

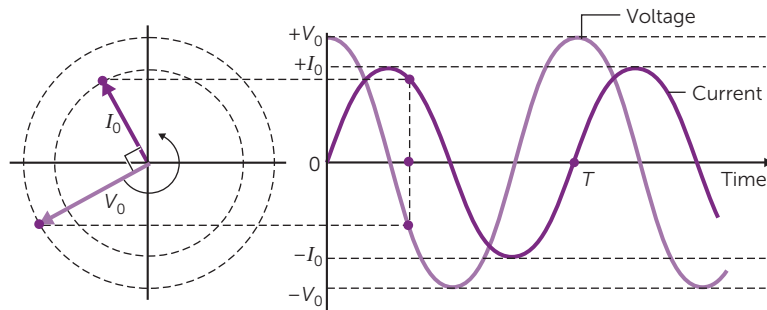


Fig. 20.24 The current and p.d. waveforms for a coil in an a.c. circuit

Reactance $X_L = 2\pi fL$

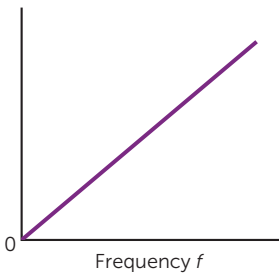


Fig. 20.25 Reactance versus frequency for an inductor

- 3 The **reactance**, X_L of a coil of inductance L is defined as $2\pi fL$.

Since $V_0 = 2\pi fLI_0$, then

$$2\pi fL = \frac{V_0}{I_0}$$

The unit of reactance is therefore the ohm, the same as the unit of resistance.

$$X_L = \frac{V_0}{I_0} = 2\pi fL$$

Note that the reactance increases as the frequency increases. **Fig. 20.25** shows how the reactance increases with increasing frequency.

- 4 **Fig. 20.26** shows how the power ($=$ current $\times$ potential difference) varies with time. Every other quarter cycle, the current and the potential difference are in the same direction, so IV is positive, so power is delivered by the supply unit. In between these quarter cycles, the current and the potential difference are in opposite 'directions' so IV is negative and power returns to the supply unit. As a result, no power is delivered from the supply unit over a complete cycle.

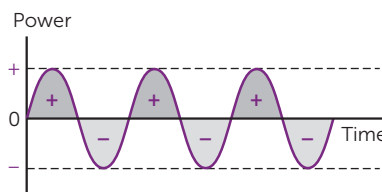


Fig. 20.26 The power curve for an inductor

20.4 WORKED EXAMPLE

A coil of self-inductance 28 mH and negligible resistance is connected across the terminals of a 2000 Hz a.c. supply unit which has an r.m.s. output voltage of 1.5 V. Calculate

- the reactance of the coil,
- the r.m.s. current.

Solution

a) $X_L = 2\pi fL = 2\pi \times 2000 \times 28 \times 10^{-3} = 352 \Omega$

b) $X_L = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} \therefore I_{rms} = \frac{V_{rms}}{X_L} = \frac{1.5}{352} = 4.26 \times 10^{-3} \text{ A}$

20.5 QUESTIONS

- 1 A coil of negligible resistance is connected in series with an ammeter and a 1500 Hz a.c. supply unit with an r.m.s. output voltage of 12.0 V. The r.m.s. current in the circuit is 110 mA.
 - a) Calculate (i) the reactance of the coil, (ii) the self-inductance of the coil.
 - b) Calculate the r.m.s. current for the same r.m.s. output voltage if the frequency is 20 000 Hz.
- 2 a) A coil of negligible resistance and self-inductance of 45 mH is connected in series with an a.c. ammeter and a variable frequency signal generator which

supplies an r.m.s. output voltage of 6.0 V. The r.m.s. current measured using the ammeter is 28 mA at a certain frequency. Calculate (i) the reactance of the coil, (ii) the frequency.

- b) If the coil is replaced by a capacitor, what capacitance would pass the same current as the coil for the same frequency and r.m.s. output voltage?

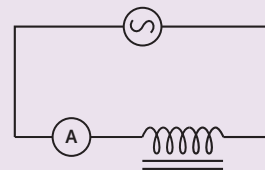


Fig. 20.27

20.6 Resonant circuits

Coils and capacitors have opposite effects in a.c. circuits.

- The reactance of a capacitor decreases with increasing frequency, whereas the reactance of a coil increases as the frequency increases.
- The current through a capacitor is 90° ahead of the voltage, whereas the current through a coil is 90° behind the voltage.

The series LCR circuit

Consider a circuit in which a coil of self-inductance L is in series with a capacitor of capacitance C and an a.c. supply unit. Let R represent the total resistance of the circuit, including resistors and internal resistance of the a.c. supply unit.

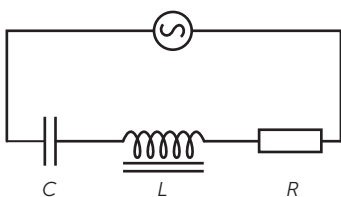


Fig. 20.28 A series LCR circuit

Suppose the current is represented by the equation $I = I_0 \sin(2\pi ft)$.

- 1 The peak resistor voltage $V_R = I_0 R$ and the resistor voltage is in phase with the current.
- 2 The peak capacitor voltage

$$V_C = I_0 X_C = I_0 \frac{1}{2\pi f C}$$

and the capacitor voltage is behind the current one-quarter of a cycle.

- 3 The peak inductor voltage $V_L = I_0 X_L = I_0 2\pi f L$ and the inductor voltage is ahead of the current by one quarter of a cycle.

Fig. 20.29 shows the phasor diagram for these voltages in relation to the current phasor I_0 . The capacitor voltage is 180° out of phase with the inductor voltage and

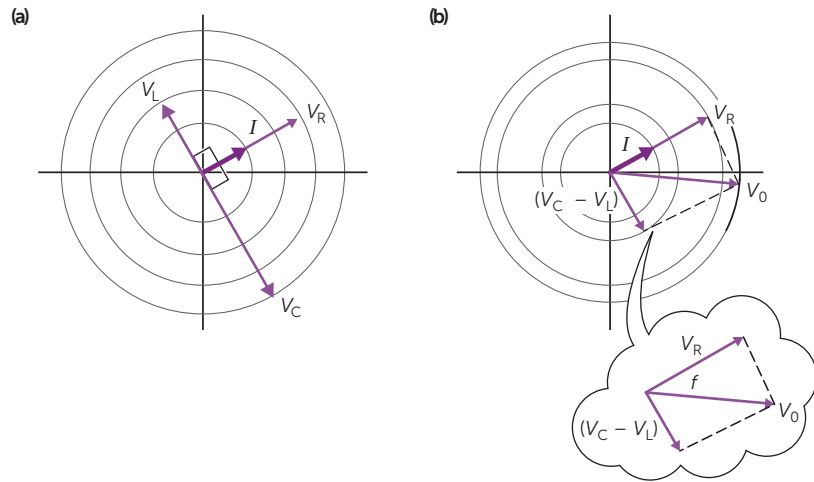


Fig. 20.29 The phasor diagram for a series LCR circuit **(a)** V_R , V_L , V_C phasors, **(b)** simplified phasor diagram

therefore V_C and V_L are in opposite directions. Their combined voltage is therefore the difference between V_C and V_L . The peak supply voltage, V_0 , is therefore the resultant of V_R and $(V_C - V_L)$ which are at 90° to each other. Using Pythagoras' theorem therefore gives

$$V_0^2 = V_R^2 + (V_C - V_L)^2$$

Since $V_R = I_0 R$, $V_C = I_0 X_C$ and $V_L = I_0 X_L$

$$V_0^2 = I_0^2 R^2 + (I_0 X_C - I_0 X_L)^2$$

Hence the circuit impedance, Z , defined as $\frac{V_0}{I_0}$ is given by

$$Z^2 = \frac{V_0^2}{I_0^2} = \frac{I_0^2 R^2 + (I_0 X_C - I_0 X_L)^2}{I_0^2} = R^2 + (X_C - X_L)^2$$

$$\therefore Z = [R^2 + (X_C - X_L)^2]^{1/2}$$

Also, the angle between the current phasor and the supply voltage phasor ϕ is given by

$$\tan \phi = \frac{(V_C - V_L)}{V_R} = \frac{(X_C - X_L)}{R}$$

The circuit impedance is a measure of the opposition of the whole circuit to the a.c. supply unit. The impedance is a minimum when $X_C = X_L$, corresponding to the capacitor voltage being exactly equal and opposite to the inductor voltage. The peak current I_0 is therefore a maximum at minimum impedance (as $I_0 = V_0/Z$). Fig. 20.3 shows how I_0 varies with frequency f . The circuit is said to be in **resonance** when $X_L = X_C$. The resonant frequency, f_0 , is given by the equation

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Rearranging this equation gives the following equation for the resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

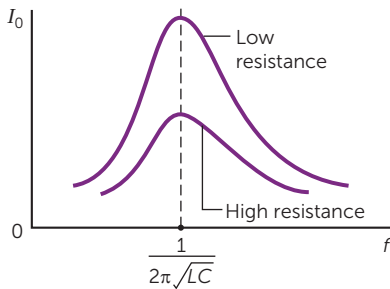


Fig. 20.30 Resonance

Notes

- 1 At resonance, the supply voltage is equal to the resistor voltage and therefore is in phase with the current.
- 2 Off resonance, the supply voltage is ahead of the current if $V_C < V_L$ and behind the current if $V_C > V_L$.
- 3 If either the capacitor or the coil is short-circuited, the impedance is given by the above equation with either X_C or X_L made zero as appropriate. Resonance is only possible if the circuit includes both a coil and a capacitor.

20.5 WORKED EXAMPLE

A $2.2\ \mu\text{F}$ capacitor was connected in series with a coil, an a.c. ammeter and a variable frequency signal generator. The frequency was increased, keeping the output voltage from the signal generator at an r.m.s. voltage of $5.0\ \text{V}$. The r.m.s. current reached a maximum of $40\ \text{mA}$ when the frequency was $1550\ \text{Hz}$. Calculate

- a) the impedance of the circuit at resonance,
- b) the capacitor reactance at this frequency,
- c) the self-inductance of the coil.

Solution

$$\text{a) } Z = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{5.0\ \text{V}}{0.040\ \text{A}} = 125\ \Omega$$

$$\text{b) } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 1550 \times 2.2 \times 10^{-6}} = 47\ \Omega$$

$$\text{c) } X_L = X_C \text{ at resonance, so } 2\pi fL = X_C \quad \therefore L = \frac{X_C}{2\pi f} = \frac{47}{2\pi \times 1550} = 4.8 \times 10^{-3}\ \text{H}$$

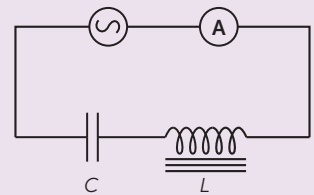


Fig. 20.31

Investigating resonance of a series LCR circuit

- 1 Connect a coil and a capacitor of known capacitance in series with a digital ammeter and a variable frequency supply unit. Use a digital voltmeter to measure the r.m.s. supply voltage. Measure the frequency from the frequency dial of the supply unit. For different measured frequencies, measure the supply voltage and the current.
- 2 Use the results to plot a graph of the impedance, Z , against the frequency. Hence determine the resonant frequency and calculate the self-inductance of the coil.

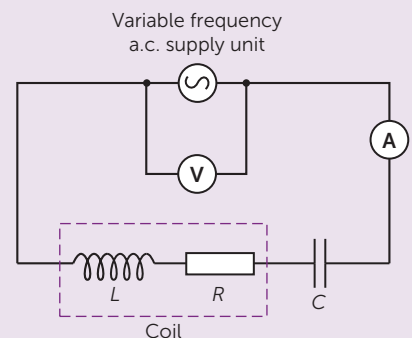


Fig. 20.32 Investigating resonance

20.6 QUESTIONS

- 1 A series LCR circuit consists of a $10\ \mu\text{F}$ capacitor and a coil of self-inductance $0.48\ \text{H}$ and resistance $55\ \Omega$ in series with a $50\ \text{Hz}$ a.c. supply unit with an r.m.s. output voltage of $6.0\ \text{V}$.

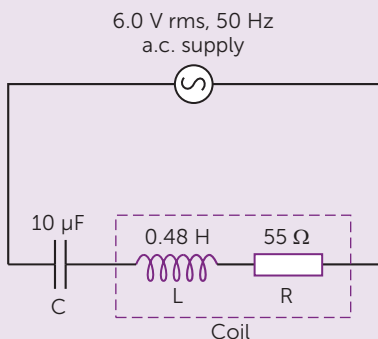


Fig. 20.33

- a) Calculate (i) the reactance of the capacitor, (ii) the reactance of the coil, (iii) the circuit impedance, (iv) the r.m.s. current.
- b) Calculate the r.m.s. voltage across (i) the resistance, (ii) the self-inductance, (iii) the capacitance.
- c) (i) Sketch the phasor diagram for this circuit. (ii) Calculate the angle between the current and the supply voltage phasors.
- 2 A circuit consists of a coil of self-inductance $0.15\ \text{mH}$ in series with a capacitor of capacitance $0.47\ \mu\text{F}$, an a.c. ammeter and a variable frequency signal generator.
- a) Calculate (i) the resonant frequency of this circuit, (ii) the reactance of the capacitor at resonance.
- b) At resonance, the r.m.s. current is $350\ \text{mA}$ when the r.m.s. supply voltage from the signal generator is $5.0\ \text{V}$. (i) Calculate the resistance of the circuit, (ii) Calculate the voltage across the capacitor at resonance, (iii) Sketch the phasor diagram for this circuit at resonance.

SUMMARY

- **Frequency** $f = \frac{1}{\text{time period } T}$
- **r.m.s. value** $= \frac{\text{peak value}}{\sqrt{2}}$ for sinusoidal a.c.
- **Power** $= I_{\text{rms}} V_{\text{rms}} = \frac{1}{2} I_0 V_0$
- **Components in a.c. circuits**

Type of circuit	V_0/I_0	Phase	Power
Resistance only	R_0	0	$\frac{1}{2} I_0^2 R$
Capacitance only	$X_C = \frac{1}{2\pi f C}$	I ahead of V by 90°	zero
Inductance only	$X_L = 2\pi f L$	I behind V by 90°	zero
Series LCR	$Z = [R^2 + (X_C - X_L)^2]^{1/2}$	$\tan \phi = \frac{(X_C - X_L)}{R}$	$\frac{1}{2} I_0^2 R$

• Resonance

- Current and supply voltage in phase as $X_L = X_C$
- Resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

20.1

- a) With the aid of a diagram, describe how a bridge rectifier is used to convert an alternating voltage into a full-wave direct voltage.
- b) Describe and explain how a large capacitor may be used to smooth a full-wave direct voltage to produce a constant direct voltage.

20.2

- a) A $47\ \Omega$ resistor in an a.c. circuit has an r.m.s. current of 0.75 A through it. Calculate
- the peak current through the resistor,
 - the peak voltage across the resistor,
 - the peak power supplied to the resistor,
 - the mean power supplied to the resistor.
- b) An a.c. supply unit has a sinusoidal output voltage of peak value 12.0 V and of frequency 200 Hz. Sketch a phasor diagram to show the voltage phasor when
- the voltage is zero and changing from negative to positive,
 - 2.0 ms after its position in (i),
 - 6.0 ms after its position in (i). State the voltage in (ii) and (iii).

20.3 In an experiment to measure the capacitance of a capacitor, an a.c. ammeter was connected in series with the capacitor and an a.c. supply unit. An oscilloscope was connected across the capacitor. Fig. 20.34 shows the circuit and the oscilloscope screen.

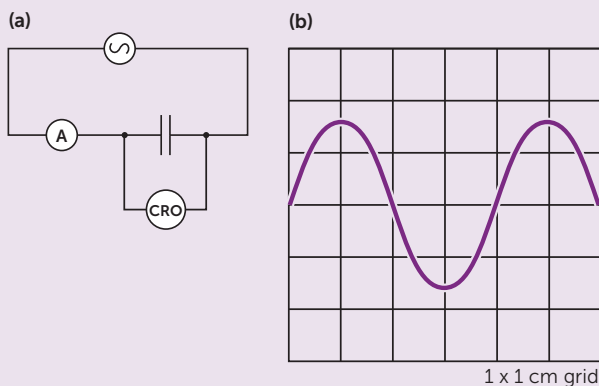


Fig. 20.34 (a) circuit diagram, (b) waveform

- a) The Y-gain of the oscilloscope was set at $2.0\ \text{V cm}^{-1}$ and the time base was set at $5\ \text{ms cm}^{-1}$. Calculate
- the frequency of the alternating current,
 - the peak voltage across the capacitor,
 - the r.m.s. voltage across the capacitor.
- b) The r.m.s. current was 2.8 mA, measured using the a.c. ammeter. Calculate
- the reactance of the capacitor,
 - the capacitance.

20.4

- a) A coil of negligible resistance was connected in series with an a.c. ammeter and a 50 Hz a.c. supply unit. The r.m.s. current was 0.35 A, measured using the a.c. ammeter, when the output voltage of the a.c. supply unit was adjusted to an r.m.s. value of 6.0 V. Calculate
- the reactance of the coil,
 - the self-inductance of the coil.
- b) Calculate the capacitance of a capacitor that would give the same ammeter reading at this frequency for the same output voltage.
- c) If the frequency of the a.c. supply unit was changed to 100 Hz, what difference would it make to the current for the same output voltage
- in (a),
 - in (b).

20.5 A $2.2\ \mu\text{F}$ capacitor and a coil of unknown resistance R and self-inductance L were connected in series with an a.c. ammeter and a variable frequency signal generator. The r.m.s. output voltage of the supply unit was maintained at 6.0 V and the frequency was adjusted until the r.m.s. current, measured using the ammeter, was at a maximum of 65 mA. This occurred at a frequency of 750 Hz.

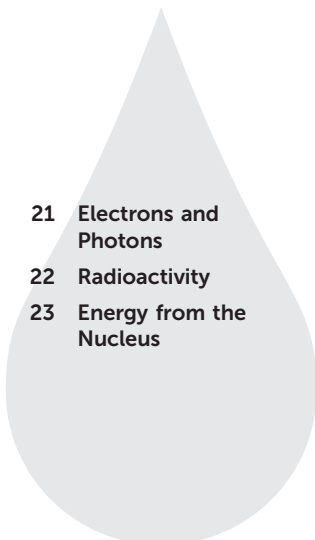
- a) Explain why the r.m.s. current is at a maximum at 750 Hz.
- b) Calculate
- the reactance of the capacitor at this frequency,
 - the self-inductance of the coil,
 - the resistance of the coil.
- c) Calculate the r.m.s. voltage at this frequency across
- the resistance,
 - the capacitance,
 - the inductance.
- d) Sketch the phasor diagram and explain why the current is in phase with the supply voltage at this frequency.

PART

6

Atomic and Nuclear Physics

In [Part 6](#), the properties of electrons in and out of the atom are studied in depth, linking up to the nature of light and why optical spectra can be used to identify elements. The principle of operation of the X-ray tube is developed before moving on to consider the properties and nature of radioactive substances. The section moves on to consider how energy is released from the nucleus and how the structure of matter at its most fundamental level has been discovered. The section includes essential practical work and makes use of mathematical skills developed earlier. Before commencing this part, it is useful to have studied [Unit 4](#), Energy and Power; [Unit 5](#), Matter and Molecules; [Unit 12](#), Electromagnetic Waves; [Unit 17](#), Electric Fields and [Unit 18](#), Magnetic Fields.

- 
- 21 Electrons and Photons
 - 22 Radioactivity
 - 23 Energy from the Nucleus

Electrons and Photons

OBJECTIVES

After working through this unit, you should be able to:

- describe and explain how a beam of electrons is produced
- calculate the speed of the electrons in a beam from the accelerating voltage
- measure the specific charge of the electron, e/m
- measure the charge of the electron, e
- describe and explain the photoelectric effect
- carry out calculations using the photon energy equation, $E = hf$
- describe and explain ionisation and excitation of atoms
- use energy level diagrams to explain line spectra
- describe the principle of operation of an X-ray tube and the intensity distribution of its output
- describe how X-rays are used in medicine
- describe and explain what is meant by wave-particle duality
- calculate the de Broglie wavelength of a particle of a given momentum
- explain in simple terms energy levels in an atom
- describe the electron microscope

CONTENTS

- 21.1 Electron beams
- 21.2 The charge of the electron
- 21.3 Photoelectricity
- 21.4 Electrons in the atom
- 21.5 X-rays
- 21.6 Wave mechanics
- Summary
- Revision questions

21.1 Electron beams

Cathode rays

An atom is the smallest particle of an element that is characteristic of the element. The atomic theory of matter was first proposed by Democritus in Ancient Greece. The theory was developed on a scientific basis by John Dalton and fellow chemists in the nineteenth century to describe how elements form compounds as a result of atoms joining together to form molecules. The molecular theory of matter successfully explains the general features of solids, liquids and gases. Dalton put forward the idea that atoms are indestructible and showed that the hydrogen atom is lighter than any other type of atom. The atomic theory was very successful because it explained many observed reactions and correctly predicted many new reactions.

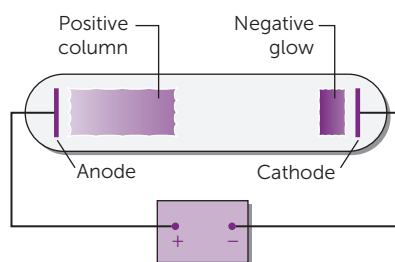


Fig. 21.1 A discharge tube

Imagine the surprise when it was discovered that atoms contain much smaller particles which carry a negative charge. These particles became known as **electrons** because they carry electric charge. They were discovered as a result of investigations into electrical conduction through gases at low pressures. Fig. 21.1 shows a discharge tube used for such an investigation.

With a very high voltage between the two electrodes, the gas is evacuated from the tube using a vacuum pump. At a sufficiently low pressure, the gas remaining inside the tube glows with light. This occurs because electrons within each atom are pulled out by the strong electrical field created by the high voltage. The atoms become positively charged ions as a result of losing electrons and are attracted to the **cathode**, the negative electrode in the tube. The electrons are attracted to the **anode**, the positive electrode. The gas glows with light because positive ions moving to the cathode recombine with electrons moving to the anode, releasing light when they combine.

When these investigations were first carried out, the cause of the glow was a mystery. It was realised that the glowing gas contained charged particles moving towards each end because a magnet held near the tube distorted the glow. This is the same effect as the force on a current-carrying wire in a magnetic field which is used to explain the operation of an electric motor. The investigations on gas discharges became possible as a result of the invention of high-voltage generators and better vacuum pumps. It was thought that radiation from the cathode, referred to as **cathode rays**, caused ionisation in the gas.

Scientists used different gases in redesigned tubes to find out more about the nature of cathode rays and the charged particles produced in the discharge tube. By using an electrode with a small hole in it, some of the charged particles attracted to the electrode passed through the hole to form a beam. Such beams could then be deflected by a magnetic field, or by an electric field created by two oppositely charged plates, as in Fig. 21.2.

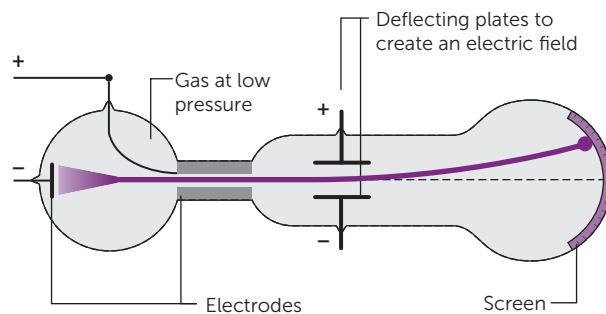


Fig. 21.2 Testing a beam of charged particles

The end of the tube forms a screen coated internally with fluorescent paint. The impact of the beam on this screen creates a spot of light at the point of impact. The effect of a magnetic or electric field on the beam can therefore be measured. The results showed the following:

- 1 The deflection of the positive ions by an electric or a magnetic field depends on the gas. For example, positive ions from chlorine gas are deflected less easily than positive ions from oxygen gas. It was realised that the heavier the mass of a molecule of a gas, the smaller the deflection. The positive ions were therefore identified as gas molecules that had become positively charged in the discharge tube.
- 2 The deflection of the cathode rays does not depend on the type of gas in the tube. The exact nature of the cathode rays was disputed and some scientists thought that these rays were waves like X-rays which ionised the gas, whereas others thought they were definitely composed of negatively charged particles. The dispute was settled in 1897 by J.J. Thomson, who proved conclusively that cathode rays are composed of identical, negatively charged particles, referred to subsequently as **electrons**. Thomson carried out a series of detailed experiments using electric and magnetic fields to deflect a beam of cathode rays and discovered that the electron's charge to mass ratio, its specific charge, e/m , is 1800 times greater than the hydrogen ion's specific charge, the largest known specific charge at that time. This was known to be $9.6 \times 10^7 \text{ C kg}^{-1}$ from electrolysis experiments (see Section 13.2) in which it was found that $9.6 \times 10^7 \text{ C}$ of charge was needed to release 1 kg of hydrogen gas. Thomson's measurements on cathode rays gave a specific charge of $1.8 \times 10^{11} \text{ C kg}^{-1}$.

The reason why the specific charge of the electron is much higher than that of the hydrogen ion could be due either to the electron having a much greater charge or a much smaller mass than the hydrogen ion. Thomson had no firm evidence on whether a much greater charge or a much smaller mass was the reason for the much greater specific charge. Nevertheless, he proved conclusively that cathode rays are sub-atomic, negative particles. In 1915 in the United States, Robert Millikan measured the charge of the electron (see p. 327) and showed that electrons and hydrogen ions carry equal and opposite charges, thus proving conclusively that the electron is much lighter than the atom.

Thermionic emission

Before flat screen televisions were invented, every television used one or more beams of electrons in a tube to build up a picture on its screen. An electron beam in a TV tube is controlled by a pair of electromagnets which are supplied with a varying electric current from a control circuit. The beam creates a spot of light where it hits the screen. The picture is built up as a result of the beam scanning the screen. The beam itself is produced by an effect known as **thermionic emission**. This was discovered by Thomas Edison in 1903, who demonstrated that electrons could be attracted onto a positive metal plate from a hot wire filament in a vacuum. This discovery led to the invention of the electron gun which produces a narrow beam of electrons in every TV tube. See [Fig. 21.3](#) overleaf.

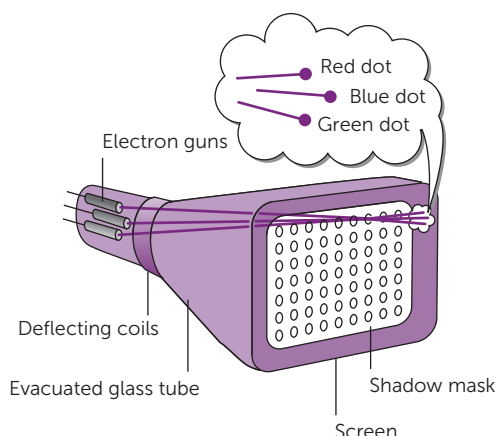


Fig. 21.3 Inside a TV tube

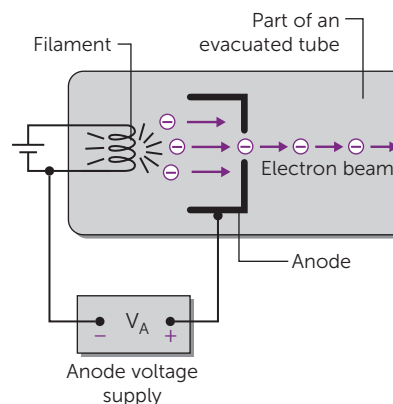


Fig. 21.4 An electron gun

Notes

- 1 One electron volt (eV) is defined as the work done on an electron when it is moved through a potential difference of 1.0 V. Since $e = 1.6 \times 10^{-19}$ C, then 1 electron volt is equal to 1.6×10^{-19} J.
- 2 The kinetic energy formula $\frac{1}{2}mv^2$ is only valid at speeds much less than the speed of light ($= 3 \times 10^8$ m s⁻¹). See p. 55.

Fig. 21.4 shows how an electron gun in an evacuated tube works. The filament glows as a result of an electric current passing through it. At this temperature, conduction electrons in the filament possess sufficient kinetic energy to leave the filament. These thermionically emitted electrons are attracted to the metal anode because it is positive, relative to the filament. The electrons gain more kinetic energy as they are attracted to the anode. Some of these electrons pass through a small hole in the anode to form a beam of electrons. The presence of gas molecules would prevent the electrons reaching the anode so the apparatus must be contained in a vacuum tube.

By definition, the potential difference between the filament and the anode is the work done per unit charge moved between the filament and the anode. The work done on each electron attracted from the filament to the anode is therefore eV_A , where V_A is the potential difference between the anode and the filament (i.e. the anode voltage) and e is the charge of the electron. As the work done on each electron increases its kinetic energy very significantly, its kinetic energy at the anode is therefore eV_A . The speed, v , of each electron in the beam is therefore given by the equation

$$\frac{1}{2}mv^2 = eV_A$$

21.1 WORKED EXAMPLE

Calculate

- a) the kinetic energy
- b) the speed of an electron after it has been accelerated through a potential difference of 2500 V in an electron gun. The specific charge of the electron, e/m , $= 1.76 \times 10^{11}$ C kg⁻¹. The charge of the electron, $e = 1.6 \times 10^{-19}$ C.

Solution

a) Kinetic energy $= eV_A = 1.6 \times 10^{-19} \times 2500 = 4.0 \times 10^{-16}$ J

b) $\frac{1}{2}mv^2 = eV_A$, hence $v^2 = \frac{2eV_A}{m} = 2 \times 1.76 \times 10^{11} \times 2500 = 8.8 \times 10^{14}$ m² s⁻²

$$\therefore v = (8.8 \times 10^{14})^{1/2} = 3.0 \times 10^7 \text{ m s}^{-1}$$

Measurement of the specific charge of the electron, e/m

Fig. 21.5 shows a vacuum tube in which a narrow beam of electrons from an electron gun is directed between two oppositely charged, parallel plates. The path of the beam between the plates is made visible by a vertical fluorescent screen in line with the beam.

- With a constant potential difference, V_p , between the parallel plates, the electron beam is attracted towards the positive plate.

$$\text{The force due to the plate voltage} = \frac{(eV_p)}{d}$$

where d is the separation between the plates. See p. 252 for a proof of this equation.

- A uniform magnetic field applied perpendicular to the plane of the screen deflects the beam down or up depending on the magnetic field direction. Two current-carrying coils in series are used to apply the magnetic field. The strength of the magnetic field and hence the deflection of the beam depend on the current in the coils.

$$\text{The force due to the magnetic field} = Bev$$

where v is the speed of the electrons and B is the magnetic flux density. See p. 270 for a proof of this equation.

The deflection due to the plate voltage can be cancelled out by the magnetic field. To do this, the beam is first deflected by a constant plate voltage and then the coil current is gradually increased so that the deflection becomes less and less until the beam passes straight through. This occurs when the force due to the plates is equal and opposite to the force due to the magnetic field. Hence the beam passes through undeflected when the magnetic flux density is such that

$$Bev = \frac{(eV_p)}{d}$$

Note

The magnetic flux density can be calculated from the coil current using the equation

$$B = 0.9 \times 10^{-6} NI \div R,$$

where N is the number of turns of each coil, R is the coil radius and I is the coil current. The unit of B is the tesla (T).

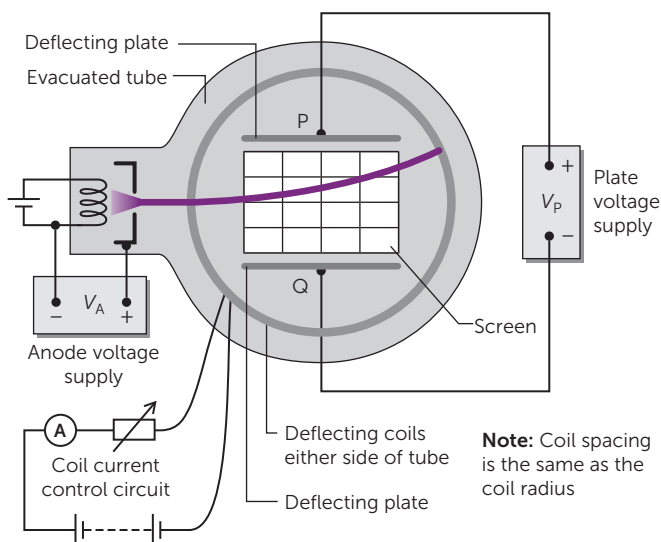


Fig. 21.5 Measuring e/m

Thus the speed of the electrons can be calculated from the equation

$$v = \frac{V_P}{Bd}$$

The electron gun equation, $\frac{1}{2}mv^2 = eV_A$, can then be used to calculate the specific charge of the electron.

21.2 WORKED EXAMPLE

In an experiment to measure the specific charge of the electron, e/m , a beam of electrons was produced from an electron gun operating at an anode voltage of 3500 V. The beam was directed between two plates at a potential difference of 2800 V and a separation of 50 mm. This deflection was cancelled by a uniform magnetic field of magnetic flux density 1.6 mT. Calculate

- the speed of the electrons in the beam,
- the specific charge of the electron, e/m .

Solution

$$\begin{aligned} \text{a) } v &= \frac{V_P}{Bd} = \frac{2800}{1.6 \times 10^{-3} \times 50 \times 10^{-3}} \\ &= 3.5 \times 10^7 \text{ m s}^{-1} \end{aligned}$$

b) Rearranging $\frac{1}{2}mv^2 = eV_A$ gives

$$\begin{aligned} e/m &= \frac{v^2}{2V_A} = \frac{(3.5 \times 10^7)^2}{2 \times 3500} \\ &= 1.75 \times 10^{11} \text{ C kg}^{-1} \end{aligned}$$

21.1 QUESTIONS

The specific charge of the electron, $e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$.

- Calculate the speed of electrons accelerated from rest in a vacuum through a potential difference of
 - 100 V,
 - 4000 V.
- A beam of electrons from an electron gun operating at an anode voltage of 3200 V is directed

between two parallel plates as in Fig. 21.5. A potential difference of 4200 V is applied between the plates which are spaced 40 mm apart. The deflection of the beam is cancelled when a uniform magnetic field is applied to the beam. Calculate

- the speed of the electrons in the beam,
- the magnetic flux density of this magnetic field.

21.2 The charge of the electron

An electric current in a wire is a flow of charge carried by electrons forced along the wire by the source of e.m.f. A wire carrying a current of 1 ampere has 6.25 million million electrons passing along its length every second. The fact that each electron carries a charge of $1.6 \times 10^{-19} \text{ C}$ was discovered by Robert Millikan, as a result of measuring the charge carried by tiny oil droplets. He also deduced that the

charge of the electron is the least possible amount of charge and that the charge of any charged body is always a multiple of the charge of the electron. The method Millikan used over 90 years ago to determine the charge of the electron is described in this section. The result of this measurement was of profound importance, as all matter is now known to be composed of quarks in combination with each other (see p. 384) and electrons.

Balancing an oil droplet

Millikan observed oil droplets from an oil spray as they fell between two parallel, horizontal metal plates, one above the other, in a beam of light, as shown in Fig. 21.6. When a potential difference was applied between the two plates, Millikan discovered that the many of the droplets were affected by the electric field created between the plates. Some droplets fell faster, whereas some reversed their motion and rose.

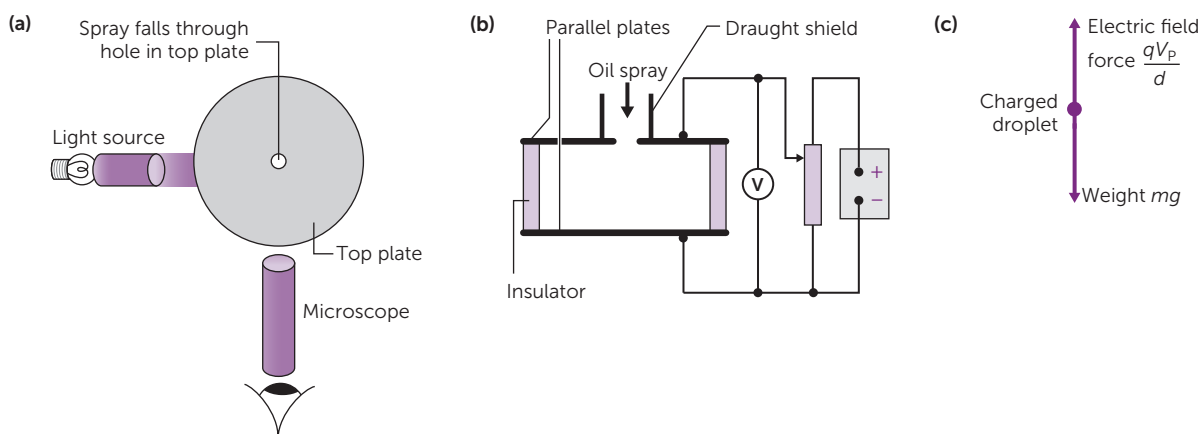


Fig. 21.6 Holding an oil droplet stationary in an electric field (a) top view, (b) cross-section, (c) a balanced droplet

By adjusting the plate p.d., Millikan was able to hold individual droplets, one at a time, in the electric field. This happens if the droplet carries an opposite charge to the top plate which causes it to be attracted by the top plate with a force equal and opposite to its weight.

As explained on p. 252, the electrostatic force on a charged droplet $= QV_p/d$, where d is the separation of the plates and Q is the charge of the droplet.

Hence $QV_p/d = mg$, where m is the mass of the droplet.

If the droplet mass is known, the droplet charge can therefore be calculated from the equation,

$$Q = \frac{mgd}{V_p}$$

Millikan discovered that the charge on an oil droplet is always a whole number $\times 1.6 \times 10^{-19}$ C, and that the least possible charge is 1.6×10^{-19} C. He concluded that the charge on the electron is 1.6×10^{-19} C and that this is the **quantum** of electric charge. Any charged body has either gained or lost a number of electrons and therefore its charge is a whole number $\times e$, the charge of the electron.

21.3 WORKED EXAMPLE

A droplet of mass 6.2×10^{-15} kg was held stationary between two parallel plates spaced 5.0 mm apart when the plate p.d. was 950 V. Calculate

- the weight of the droplet,
- the charge of the droplet.

Solution

$$\begin{aligned} \text{a) Weight} = mg &= 6.2 \times 10^{-15} \times 9.8 \\ &= 6.1 \times 10^{-14} \text{ N} \end{aligned}$$

$$\begin{aligned} \text{b) Rearranging } \frac{QV_p}{d} = mg \text{ gives} \\ Q = \frac{mgd}{V_p} &= \frac{6.1 \times 10^{-14} \times 5.0 \times 10^{-3}}{950} \\ &= 3.2 \times 10^{-19} \text{ C} \end{aligned}$$

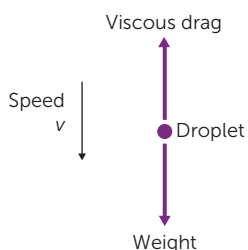
Measuring the mass of an oil droplet

Fig. 21.7 Terminal speed

With the plate p.d. switched off, each droplet falls at a constant speed, being acted upon by a drag force due to the air. When the plate p.d. is first switched off, the droplet accelerates due to gravity. As the droplet moves faster and faster, the drag force increases until it becomes equal and opposite to the force of gravity on the droplet. The resultant force is then zero and the droplet moves at its terminal speed. See Fig. 21.7. The build-up to terminal speed is almost instant so the terminal speed can be measured by timing the droplet as it falls through a measured distance. The terminal speed, v , can therefore be calculated from

$$v = \frac{s}{t}$$

where s is the measured distance and t is the time taken.

Assuming the droplet is a sphere of radius r :

- Its volume $= 4\pi r^3/3$, hence its mass $m = 4\pi r^3 \rho/3$, where ρ is the density of the oil.
- The viscous drag $F = 6\pi \eta r v$, where η is the viscosity of air, in accordance with Stokes' law.

At the terminal speed v , the droplet weight $mg =$ the viscous drag F . Hence

$$\frac{4}{3}\pi r^3 \rho g = 6\pi \eta r v$$

assuming buoyancy is negligible.

The droplet radius can therefore be calculated by rearranging this equation to give

$$r^2 = \frac{9\eta v}{2\rho g}$$

The droplet mass can then be calculated using the equation

$$m = \frac{4}{3}\pi r^3 \rho$$

To determine the charge of a charged oil droplet

- 1 The plate p.d. is adjusted and measured when the droplet is stationary.
- 2 The plate p.d. is then switched off and the droplet is timed as it falls through a measured distance.
- 3 The droplet's terminal speed is calculated (using $v = \frac{s}{t}$).
- 4 The droplet radius is then calculated (using the equation $r^2 = \frac{9\eta v}{2\rho g}$).
- 5 The droplet mass is then calculated (using the equation $m = \frac{4}{3}\pi r^3 \rho$).
- 6 The droplet charge is then calculated (using $Q = \frac{mgd}{V_p}$).

21.4 WORKED EXAMPLE

An oil droplet was observed between two horizontal plates which were at a separation of 4.2 mm. The droplet was held stationary when the top plate was at a positive potential of 235 V relative to the lower plate. When the plate p.d. was switched off, the droplet fell at a constant speed, taking 11.3 s to fall through a distance of 1.0 mm.

- a) Calculate (i) the terminal speed of the droplet, (ii) the droplet radius, (iii) the droplet mass, (iv) the droplet charge.
- b) (i) What type of charge did the droplet carry? (ii) How many electrons were responsible for the droplet's charge? The density of the oil equals 950 kg m^{-3} ; the viscosity of air $= 1.8 \times 10^{-5} \text{ N s m}^{-2}$; the acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$.

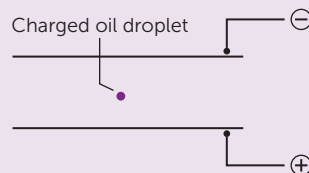
Solution

- a) (i) $v = \frac{s}{t} = \frac{1.0 \times 10^{-3}}{11.3} = 8.8 \times 10^{-5} \text{ m s}^{-1}$
- (ii) $r^2 = \frac{9\eta v}{2\rho g} = \frac{9 \times 1.8 \times 10^{-5} \times 8.8 \times 10^{-5}}{2 \times 950 \times 9.8} = 7.7 \times 10^{-13} \text{ m}^2 \quad \therefore r = 8.8 \times 10^{-7} \text{ m}$
- (iii) $m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi (8.8 \times 10^{-7})^3 \times 950 = 2.7 \times 10^{-15} \text{ kg}$
- (iv) $Q = \frac{mgd}{V_p} = \frac{2.7 \times 10^{-15} \times 9.8 \times 4.2 \times 10^{-3}}{235} = 4.7 \times 10^{-19} \text{ C}$
- b) (i) negative,
- (ii) $3 \left(= \frac{4.7 \times 10^{-19}}{1.6 \times 10^{-19}} \right)$

21.2 QUESTIONS

$$g = 9.8 \text{ m s}^{-2}$$

- 1 A charged oil droplet of mass $3.8 \times 10^{-15} \text{ kg}$ was observed between two oppositely charged parallel plates, one above the other, at a separation of 50 mm. With the upper plate negative, the p.d. between the two plates was adjusted to make the droplet stationary. The p.d. needed to do this was 595 V.

**Fig. 21.8**

- a) State the sign of the charge on the droplet.
- b) Calculate the charge carried by this droplet.

21.2 QUESTIONS *continued*

- 2 In an experiment to measure the mass and charge of an oil droplet, the following readings were taken:
 Plate p.d. to hold the droplet stationary = 375 V
 Plate separation = 40 mm
 Time taken by the droplet to fall a distance of 1.0 mm = 16.5 s for zero plate p.d.
 (The density of the oil = 960 kg m^{-3} , the viscosity of air = $1.8 \times 10^{-5} \text{ N s m}^{-2}$)
- a) Calculate (i) the terminal speed of the droplet when it fell in zero electric field, (ii) the droplet's radius, (iii) the droplet's mass, (iv) the droplet's charge.
- b) Without calculation, (i) explain why the droplet fell at steady speed when the plate p.d. was zero, (ii) describe and explain the droplet's motion if the plate p.d. was suddenly reversed when the droplet was stationary.

21.3 Photoelectricity

Electrons are emitted from a metal plate as a result of illuminating the metal plate with ultraviolet light. Fig. 21.9 shows how this can be demonstrated. The metal plate is attached to a gold leaf electroscope and charged negatively. When the plate is illuminated with ultraviolet light, the gold leaf gradually falls. If a glass sheet is inserted between the metal plate and the ultraviolet light source, the leaf stops falling because glass does not transmit ultraviolet light. Removing the glass sheet causes the leaf to continue its fall. The leaf falls because electrons are emitted from the metal plate when the plate is illuminated with ultraviolet light.

This effect, called **photoelectric emission**, was discovered over a century ago. It led to a completely new theory of light and a drastic rethink about the nature of matter and energy. The key discovery was that photoelectric emission from a metal surface does not occur unless the frequency of the incident radiation exceeds a minimum value, referred to as the **threshold frequency** that depends on the metal. For example, in Fig. 21.9, photoelectric emission does not occur with visible light even if the incident light is extremely intense, yet ultraviolet light at very low intensity is capable of producing photoelectric emission. Investigations on photoelectricity carried out at the end of the nineteenth century produced the following results:

- 1 Photoelectric emission only occurs if the frequency of the incident radiation exceeds a threshold value that depends on the metal.
- 2 Increasing the intensity of the incident radiation increases the number of photoelectrons per second emitted by the metal plate, provided the frequency exceeds the threshold frequency. If the frequency is less than the threshold frequency, no photoelectric emission occurs no matter how intense the incident radiation is.
- 3 Emission of electrons occurs instantly when the incident radiation is directed at the metal surface.

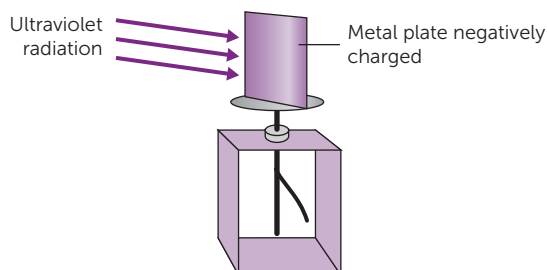


Fig. 21.9 Photoelectricity

The results could **not** be explained using the wave theory of light. According to wave theory, every wavefront arriving at the metal surface ought to share its energy with all the conduction electrons on or near the surface. The electrons would therefore gain enough energy to leave the metal plate, regardless of the frequency of the incident radiation. Photoelectric emission ought to occur at all frequencies according to the wave theory of light. The results demonstrated the existence of a threshold frequency and this could not be explained by the wave theory of light.

The photon theory of light

Einstein explained the photoelectric effect by putting forward the theory that electromagnetic radiation consists of packets of waves called **photons**, each carrying energy, E , in proportion to the frequency, f , of the radiation in accordance with the equation

$$\text{Energy of a photon, } E = hf$$

where h , known as the Planck constant, has a numerical value of 6.63×10^{-34} Js.

The idea that atoms emit and absorb radiation in discrete amounts or 'quanta' was first advanced by Planck to explain black body radiation (see p. 88 and p. 161). Einstein realised that the radiation itself is quantised and that when an atom emits electromagnetic radiation, it releases a short 'burst' of electromagnetic waves that leave the atom in one direction rather than spreading out in all directions.

According to Einstein, photoelectric emission from a metal can only occur if an electron at the surface of the metal gains sufficient energy from a single photon to leave the metal. Since each photon incident on the metal surface is absorbed by a single electron, a photoelectron (i.e. an electron that has escaped from the surface) must have gained energy equal to hf as a result of absorbing a photon. Because the electron needs to use some of this energy to leave the metal, its maximum kinetic energy on leaving the metal is equal to $hf - \phi$, where the **work function**, ϕ , of a metal is defined as the work that must be done by an electron to leave the metal completely.

$$\text{Maximum kinetic energy of an emitted electron} = hf - \phi$$

According to Einstein's theory, electrons cannot escape from the metal surface if the photon energy, hf , is less than the work function of the metal, ϕ .

$$\therefore hf > \phi$$

for photoelectric emission to occur.

The threshold frequency, f_0 , the minimum frequency of the incident radiation for photoelectric emission to occur, is therefore given by the condition $hf_0 = \phi$

$$\therefore f_0 = \frac{\phi}{h}$$

Photoelectric emission can be stopped by making the metal surface sufficiently positive, relative to the metal terminal where the photoelectrons are collected. When the metal surface is at a positive potential, V , relative to the collecting terminal, each electron must do extra work, equal to eV , to move from the metal plate to the collecting terminal. Photoelectric emission is therefore stopped if the potential difference is such that $eV_s = hf - \phi$, where V_s is referred to as the **stopping potential**.

$$eV_s = hf - \phi$$

Investigating photoelectric emission using a vacuum photocell

A vacuum photocell contains a clean metal plate, the photocathode, capable of emitting electrons when illuminated with visible radiation. A smaller metal terminal near the plate, the anode, is necessary to collect the emitted photoelectrons.

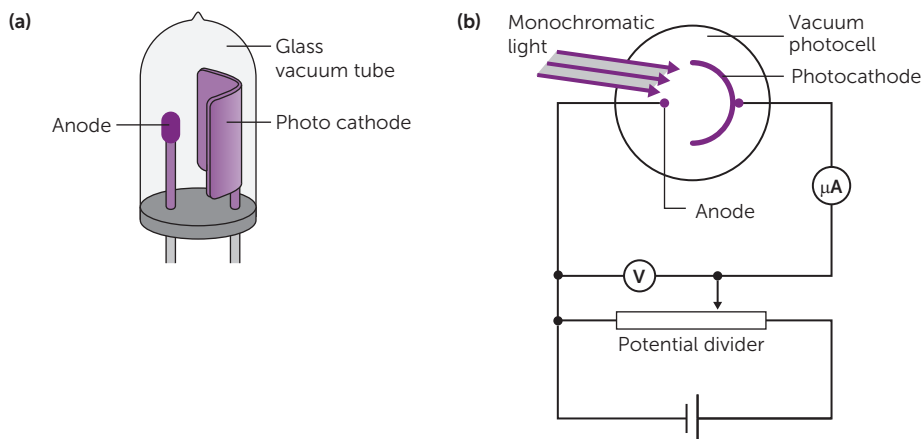


Fig. 21.10 Using a vacuum photocell (a) construction, (b) measuring stopping potential

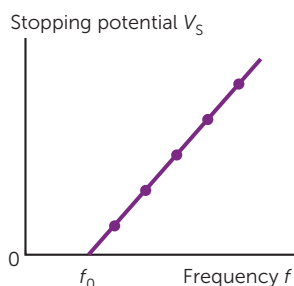


Fig. 21.11 Stopping potential against frequency

Note

See p. 147 if you have forgotten the approximate wavelength of each part of the electromagnetic spectrum. Remember that visible light covers the wavelength range from about 400 nm (violet) to about 700 nm (deep red).

Fig. 21.10 shows the photocell in series with a microammeter which registers the current due to the photoelectrons. A potential divider and a voltmeter are used to apply a variable positive potential to the photocathode, relative to the anode. When the photocathode is illuminated with light of known frequency, the potential divider is adjusted to increase the potential of the photocathode until the current is reduced to zero. The stopping potential is then measured from the voltmeter. The experiment is repeated using different light frequencies to illuminate the photocathode. Note that if the wavelength, λ , of the light is known, the frequency, f , can be calculated using the equation $f\lambda = c$, where c is the speed of light.

The results may be plotted as a graph of stopping potential, V_s , on the vertical axis against frequency on the horizontal axis, as shown in **Fig. 21.11**. The results define a straight line intercepting the $+x$ -axis.

According to Einstein's explanation of photoelectricity,

$$eV_s = hf - \phi$$

$$\therefore V_s = \frac{hf}{e} - \frac{\phi}{e}$$

In comparison with the general equation for a straight line, $y = mx + c$,

- 1 the gradient of the line, $m = \frac{h}{e}$
- 2 the y -intercept, $c = -\frac{\phi}{e}$
- 3 the x -intercept (i.e. when $V_s = 0$) is therefore the threshold frequency $f_0 = \frac{\phi}{h}$

Thus the results from the experiment confirm Einstein's theory and can be used to measure the Planck constant, h , and the work function, ϕ , of the photocathode.

21.5 WORKED EXAMPLE

The photocathode of a vacuum photocell in series with a microammeter is illuminated with light of wavelength 430 nm. The microammeter reading is reduced to zero when the potential of the photocathode is increased to 1.0 V relative to the anode. Calculate

- the energy of a photon of wavelength 430 nm,
- the work function of the photocathode,
- the maximum wavelength at which photoelectric emission can occur from the photocathode when it is at zero potential.

$$h = 6.6 \times 10^{-34} \text{ J s}, e = 1.6 \times 10^{-19} \text{ C}, c = 3.0 \times 10^8 \text{ m s}^{-1}$$

Solution

$$\text{a) } E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{430 \times 10^{-9}} = 4.6 \times 10^{-19} \text{ J}$$

$$\text{b) Rearrange } eV_s = hf - \phi \text{ to give } \phi = hf - eV_s = 4.6 \times 10^{-19} - (1.6 \times 10^{-19} \times 1.0) = 3.0 \times 10^{-19} \text{ J}$$

$$\text{c) Threshold frequency } f_0 = \frac{\phi}{h} = \frac{3.0 \times 10^{-19}}{6.6 \times 10^{-34}} = 4.5 \times 10^{14} \text{ Hz, Maximum wavelength } \lambda_0 = \frac{c}{f_0} = 6.7 \times 10^{-7} \text{ m}$$

Application 9

CCD cameras

The CCD camera has dramatically changed all our lives. It provides images that can be stored, transmitted and enhanced electronically. CCD stands for ‘charge-coupled device’. A CCD is an array of light-sensitive pixels which become charged when exposed to light. Each pixel has three small rectangular metal electrodes (labelled A, B and C in Fig 21.12) which are thin enough to allow light photons to pass through and liberate electrons in the light-sensitive material underneath.

The pixel electrodes are connected to three voltage supply ‘rails’. After the pixels have collected charge for a certain time, the charge in each pixel is shifted via the adjacent pixels towards an output electrode connected to a capacitor. This is achieved by altering the voltage level of each pixel electrode in a sequence of three-step cycles, as shown in Figure 21.12. The voltage of the output electrode is ‘read out’ electronically then the capacitor is discharged before the next pulse of charge is received. In this way, the output electrode produces a stream of voltage pulses, each one of amplitude in proportion to the light energy received by an individual pixel. A CCD is much more sensitive than a photographic film. Also, it is sensitive to ultraviolet and infra-red radiation as well as to visible light.

Repeat voltage sequence:

A	10 V	2 V	2 V
B	2 V	10 V	2 V
C	2 V	2 V	10 V

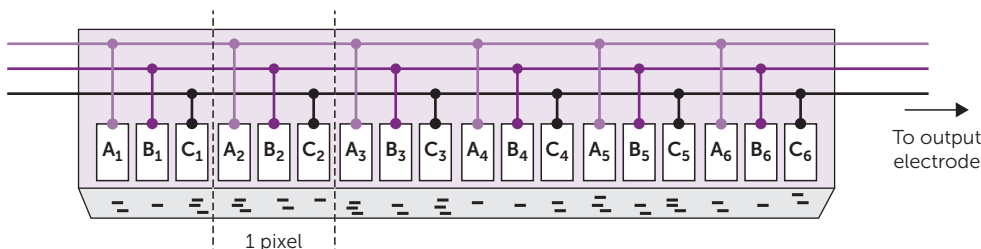


Fig. 21.12 Inside a CCD

21.3 QUESTIONS

$$h = 6.6 \times 10^{-34} \text{ Js}, e = 1.6 \times 10^{-19} \text{ C}, c = 3.0 \times 10^8 \text{ m s}^{-1}$$

- 1 a) Calculate the energy of a photon of wavelength
(i) 600 nm, (ii) 100 nm.
- b) A certain metal surface has a work function of 0.64 eV. When this metal surface is at zero potential, calculate the maximum kinetic energy of a photoelectron when the surface is illuminated with photons of wavelength (i) 600 nm, (ii) 100 nm.
- 2 A metal surface is illuminated with light of wavelength 550 nm causing it to emit photoelectrons. The emission

of photoelectrons is reduced to zero as a result of applying a positive potential of 0.85V to the surface.

- a) Calculate (i) the energy of a photon of wavelength 550 nm, (ii) the work done by an electron that moves through a potential difference of 0.58 V, (iii) the work function of the metal.
- b) Calculate the maximum wavelength capable of producing photoelectrons from this surface at zero potential.

21.4 Electrons in the atom

The electrons in an atom are arranged in shells, as explained on p. 76, each shell capable of holding a certain number of electrons at fixed energies. To escape from an atom, an electron must be given energy to enable it to overcome the attraction of the positively charged nucleus of the atom. If an electron is removed from an uncharged atom, the atom becomes a positively charged ion.

- **Ionisation**, the process of creating ions, occurs when radioactive particles travel through matter. A single radioactive particle passing through air ionises millions of gas atoms. A more controlled way to ionise gas atoms is to direct a beam of electrons into the gas. If the electrons in the beam have sufficient kinetic energy, they will knock electrons out of the gas atoms.

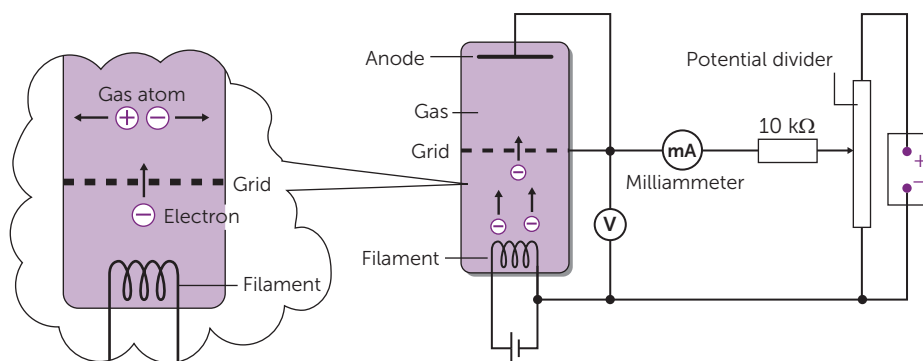


Fig. 21.13 Ionisation by collision

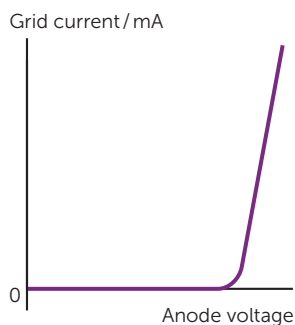


Fig. 21.14 Ionisation results for a tube filled with xenon

Fig. 21.13 shows a tube used to demonstrate ionisation by collision. Electrons are emitted from a hot-wire filament by the process of thermionic emission. As the anode voltage is increased from zero, little current is registered on the milliammeter until the voltage reaches a certain value. At this voltage, some of the electrons from the filament arrive at the grid and pass through the grid wires with just sufficient kinetic energy to ionise the gas atoms. The beam electrons knock electrons out of the gas atoms at and above a certain anode voltage. The current increases because the conductivity of the gas increases due to ions being created. Fig. 21.14 shows the results for a tube filled with xenon, an inert gas, which ionises at an anode voltage

of 12.1 V. The **ionisation energy of xenon** (i.e. work done to ionise a xenon atom) is therefore 12.1 eV.

- **Excitation** of an atom is possible without ionisation. In other words, atoms can absorb energy without losing electrons. Fig. 21.15 shows the tube in Fig. 21.13 in a modified circuit in which a microammeter is used to measure the number of electrons per second reaching the anode.

As the anode voltage is increased from zero, more and more electrons from the filament shoot through the grid to reach the anode, causing the anode current to rise. However, at certain voltages, the anode current drops sharply, as shown in Fig. 21.16 because the electrons from the filament have just enough kinetic energy to 'excite' the gas atoms. No excitation occurs if the electrons from the filament have too much or too little energy. An atom absorbs energy when excitation occurs. This happens at certain energies less than the ionisation energy.

Note

1 eV = 1.6×10^{-19} J. See p. 324 if necessary.

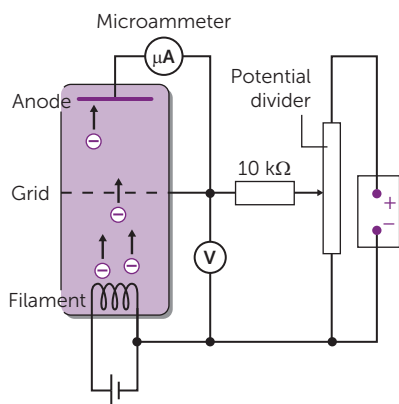


Fig. 21.15 Excitation by collision

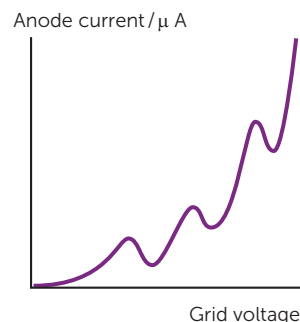


Fig. 21.16 Excitation results

When these experiments were first carried out, it was found that atoms absorb energy at different values which depend on the type of atom. For example, mercury atoms absorb energy at 4.9 eV, 7.6 eV and 8.9 eV, and they ionise at 10.4 eV. Such measurements are used to make an energy level diagram for each type of atom, as shown in Fig. 21.17.

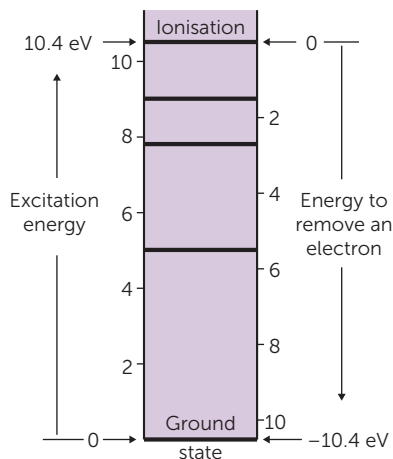


Fig. 21.17 The energy levels of a mercury atom

- 1 The ground state, the lowest possible energy level, is the normal state of the atom.
- 2 The ionisation level is the minimum energy needed to ionise the atom in its ground state.
- 3 Excitation occurs when an atom absorbs energy, raising its energy from one level to a higher level.
- 4 An atom in an excited state is unstable and it returns sooner or later to its ground state. When this occurs, the atom loses energy by emitting a photon. The energy carried away by the photon is equal to the loss of energy of the atom. For example, if a mercury atom in an excited state at 4.9 eV returns to its ground state directly, it releases a photon of energy 4.9 eV.

When excitation experiments were first carried out on mercury, it was discovered that ultraviolet radiation of wavelength 250 nm was emitted from the mercury atoms. Prove for yourself that the energy of a photon of wavelength 250 nm is 4.9 eV. Other wavelengths are emitted by mercury atoms, corresponding to photons of different energies released when the atoms move down the energy level ladder in other ways. The energy level picture of each type of atom can therefore be worked out from the energies of the photons released by that type of atom.

Electron shells and energy levels

The electrons in an atom move about the nucleus, prevented from leaving the atom by the attractive force of the nucleus. Each electron has a fixed amount of energy which depends on the shell it occupies.

- When an atom absorbs energy, it moves to a higher energy level because an electron in one of its shells has moved to a shell of higher energy. This can occur as a result of collisions between atoms or as a result of an incoming radioactive particle or electron colliding with an electron in the atom.
- When an atom releases energy, an electron in the atom moves to a shell of lower energy and releases a photon as it does so, as shown in Fig. 21.18. Because the electrons in an atom occupy shells of well-defined energies, the photons released by an atom when it loses energy are of well-defined energies.

$$\text{Photon energy, } hf = E_1 - E_2$$

where E_1 = initial energy level of electron and E_2 = final energy level.

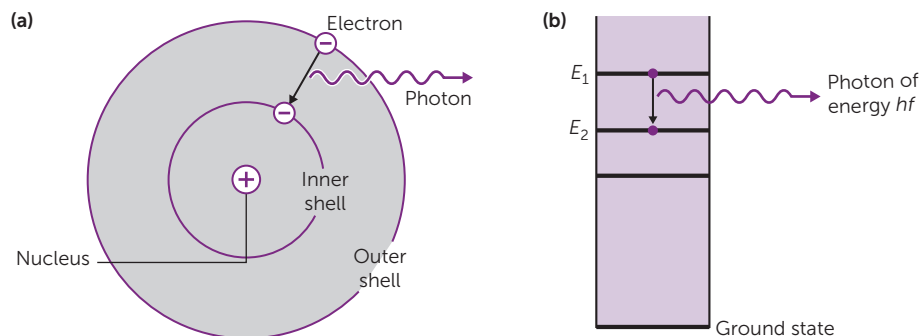


Fig. 21.18 Photon energies (a) electron shells, (b) energy levels

The emission spectrum of light from a discharge tube or a vapour lamp consists of discrete coloured lines, each line corresponding to a definite wavelength. The pattern of spectral lines from the atoms of a given element is characteristic of that element and can be used to identify the element. See p. 165 for details of how the wavelengths of light in a line spectrum can be measured.

See www.palgrave.com/foundations/breithaupt for an experiment to investigate the spectrum of hydrogen.

21.6 WORKED EXAMPLE

Fig. 21.19 shows some of the energy levels of a certain type of atom.

- a) (i) Calculate the energy and the wavelength of the photon released when an electron moves from a shell at an energy of 7.2 eV to a shell at 2.6 eV. (ii) Identify to which part of the electromagnetic spectrum this photon belongs.
b) Identify the electron transition responsible for photons of wavelength 476 nm emitted by this type of atom.

Solution

$$\begin{aligned}\text{a) (i) Photon energy} &= E_1 - E_2 = 7.2 - 2.6 = 4.6 \text{ eV} = 4.6 \times 1.6 \times 10^{-19} \\ &= 7.4 \times 10^{-19} \text{ J}\end{aligned}$$

Rearrange photon energy $E = hf = \frac{hc}{\lambda}$ to give

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{7.4 \times 10^{-19}} = 2.7 \times 10^{-7} \text{ m}$$

(ii) Ultraviolet.

$$\text{b) Photon energy, } E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{476 \times 10^{-9}} = 4.2 \times 10^{-19} \text{ J} = 2.6 \text{ eV}$$

The transition is therefore from the energy level at 2.6 eV to the ground state.

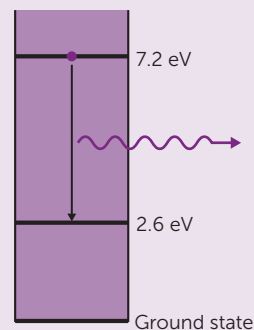


Fig. 21.19

An atomic model

The hydrogen atom is the simplest type of atom. It consists of a single electron and a proton. The energy levels of the hydrogen atom can be calculated correctly as outlined in Section 21.6 'Wave mechanics'. In addition, using the more advanced theory of quantum mechanics which is beyond the scope of this book, the shape of each shell can be worked out. The energy of each shell is given by the formula

$$E = -\frac{13.6}{n^2} \text{ eV}$$

where n is a positive integer referred to as the principal quantum number.

Application 10

Lasers

A laser works by causing atoms to emit light which cause more atoms to emit light. This is called 'Light Amplification by Stimulated Emission of Radiation' hence the acronym 'laser'. The light from a laser is almost perfectly parallel and monochromatic so can be focused by a convex lens to a tiny spot. See p. 164 to find out why you should never look along a laser beam.

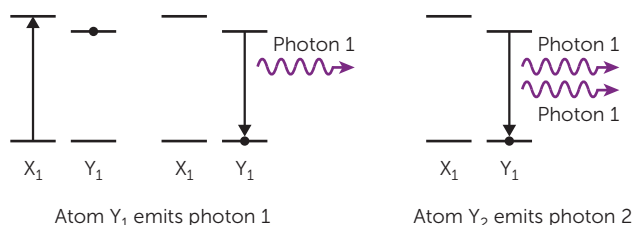


Fig. 21.20 Stimulated emission

The first lasers contained two different gases in a sealed tube. Energy is supplied to the gas atoms, for example by passing an electric current through the gas or by using a flash light, causing excitation by collision of the gas atoms. The gases are chosen because the atoms of one gas (X) have a slightly higher excited energy level than the atoms of the other gas (Y), as shown in [Figure 21.20](#).

Collisions between the atoms of X and Y then result in Y having more excited than non-excited atoms. When an atom of Y de-excites, it releases a photon which causes other excited Y atoms to de-excite and release photons. With a mirror outside each end of the tube, the photons emitted by the atoms of Y travel back and forth through the gas causing more photons to be emitted.

In addition to gas lasers, other types of lasers include solid-state lasers, semiconductor lasers, metal-vapour lasers and dye lasers. Lasers are used in a range of applications including bar code scanners, communications, DVD players, laser printers, laser surgery, industrial cutting and welding and surveying.

21.4 QUESTIONS

- 1 a) Calculate the energy and wavelength of the photon released as a result of an electron moving from a shell at 7.2 eV to the ground state.
b) Calculate the wavelength of the photon released when a mercury atom moves from an energy level of 8.9 eV to an energy level of (i) 7.6 eV, (ii) 4.9 eV.
- 2 a) Use the energy level formula $E = -13.6/n^2$ eV to calculate the energy levels in eV of the $n = 2, 3, 4$ and 5 shells of the hydrogen atom.
b) (i) Construct an energy level diagram for the hydrogen atom. (ii) Determine the wavelength emitted when an electron in a hydrogen atom moves from $n = 5$ to $n = 4$. (iii) Identify to which part of the electromagnetic spectrum this photon belongs.

21.5 X-rays

X-rays are used to take photographs of bones in the body because X-rays pass through tissue but not through bone. X-rays are photons of electromagnetic radiation of wavelength about 1 nm or less. They are produced when a beam of high-energy electrons is stopped by a metal target. The kinetic energy of the electrons in the beam is converted to X-ray energy.

[Fig. 21.21](#) shows the construction of an X-ray tube. The electrons are emitted from a hot wire filament and attracted directly to the metal anode which is at a high positive potential relative to the filament. X-rays are produced at the point of impact on the anode surface. The glass tube is evacuated and the anode surface is at 45° to the beam direction so that the X-rays emerge through the side of the tube. A step-up

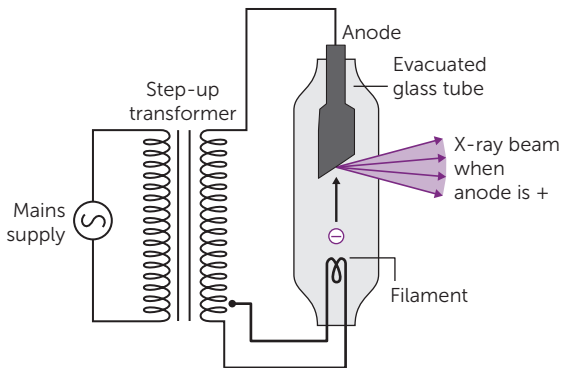


Fig. 21.21 An X-ray tube

transformer is used to apply a large alternating voltage between the anode and the filament. Electrons are attracted to the anode every other half-cycle of the alternating supply when the anode is positive relative to the filament.

- The **intensity distribution** of the X-ray beam changes continuously with frequency up to a maximum, as shown by Fig. 21.22. This is because the maximum energy of an X-ray photon is equal to the kinetic energy of an electron in the beam, corresponding to all the energy of a single electron being used to produce one photon.

For a beam of electrons produced by an electron gun operating at an anode potential V_A , the kinetic energy of each electron in the beam is eV_A . Hence the maximum frequency, $f_{\max}$, of the X-rays produced when the beam is stopped is given by equating the photon energy $hf_{\max}$ to the kinetic energy of a single electron.

$$hf_{\max} = eV_A$$

$$\therefore f_{\max} = \frac{eV_A}{h}$$

The X-rays produced as a result of stopping a beam of electrons have a spread of frequencies up to $f_{\max}$. Consequently, the spread of wavelengths is continuous down to a minimum value

$$\lambda_{\min} = \frac{c}{f_{\max}} = \frac{hc}{eV_A}$$

Fig. 21.22 shows the intensity distribution for two different tube voltages. The higher the voltage, the higher the maximum frequency and the greater the intensity of the X-ray beam.

- **Intensity spikes** are produced at certain frequencies that are characteristic of the metal target. Each spike occurs because beam electrons knock electrons out of the inner shells of the target atoms. These inner shells are at very large negative energies because they are deep within each atom. The vacancies created in these shells are filled by electrons from the outer shells or by conduction electrons. The photons released are therefore high energy photons in the X-ray part of the electromagnetic spectrum. See Fig. 21.23. The spikes are characteristic of the target atoms because they correspond to the possible electron transitions between the energy levels of the target atoms.

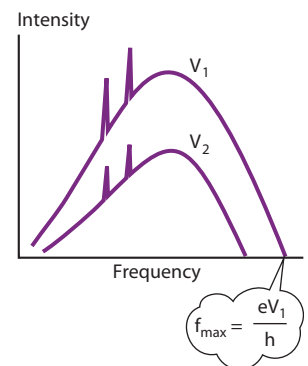


Fig. 21.22 Intensity distribution

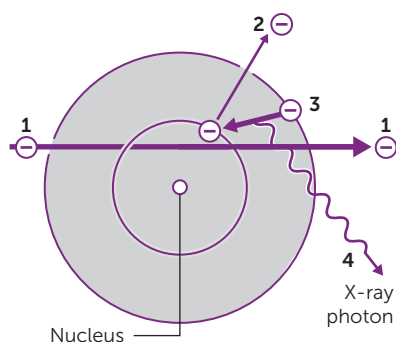


Fig. 21.23 Characteristic X-rays

- 1 Incoming electron knocks an atomic electron out
- 2 Vacancy created in shell
- 3 Outer electron fills vacancy and emits X-ray photon 4

- The **power** used by an X-ray tube can be calculated by multiplying the beam current by the anode potential. Only a small proportion of the beam energy is converted into X-ray energy. Typically, a tube operating at an anode potential of 20 000 V and a beam current of 40 mA would use 800 W of electrical power (= beam current $\times$ anode potential), perhaps producing about 20 W of X-ray energy per second. The rest of the power delivered to the tube would be converted to internal energy at the anode which therefore becomes very hot. For this reason, the anode is usually a tungsten plate (tungsten has a very high melting point) set in a copper block (copper is a good heat conductor).

21.7 WORKED EXAMPLE

An X-ray tube operates at an anode potential of 40 kV and a beam current of 5 mA. Calculate

- a) the minimum wavelength of the X-rays produced,
- b) (i) the number of electrons striking the anode per second, (ii) the X-ray energy produced per second if the tube efficiency is 1%.

Solution

$$\text{a) } \lambda_{\min} = \frac{hc}{eV_A} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-19} \times 40\,000} = 3.0 \times 10^{-11} \text{ m}$$

$$\text{b) (i) Beam current, } I = \text{no. of electrons per second } n \times e, \text{ hence } n = \frac{I}{e} = 3.1 \times 10^{16}$$

$$\text{(ii) Electrical power used by the tube} = IV_A = 5 \times 10^{-3} \times 40\,000 = 200 \text{ W}$$

$$\therefore \text{X-ray energy per second} = 1\% \times 200 \text{ W} = 2 \text{ W}$$

X-rays in medicine

Fig. 21.24 shows an X-ray tube used to take an X-ray picture of a broken limb. A shadow of the limb is formed on the photographic film (or on a flat-panel X-ray detector connected to a separate LCD screen) because bone absorbs X-rays whereas flesh does not. To produce a clear picture, the beam must originate from as small an area of the target as possible. However, if this target area is too small, the beam might melt the target at the point of impact. As X-rays are harmful due to their ionising effect, the tube is surrounded by lead shielding to prevent X-rays reaching people in the vicinity of the tube.

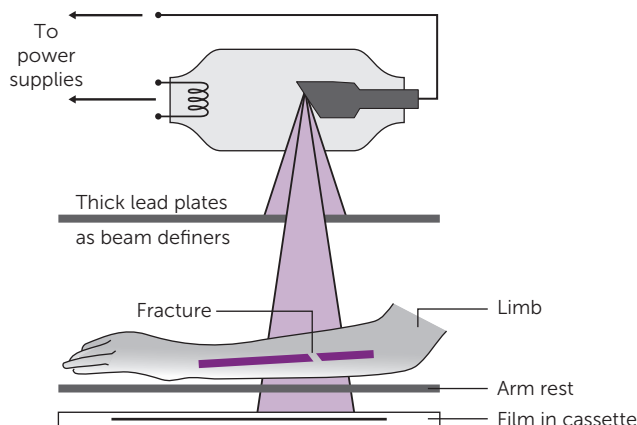


Fig. 21.24 X-rays in use

- **Beam definers** are used to limit the area of the patient exposed to the beam. The lead plates absorb X-rays which would otherwise reach other parts of the patient.
- An **aluminium metal plate** is placed in the path of the beam between the patient and the tube. This absorbs low-energy X-rays that would otherwise be absorbed by the soft tissue of the patient.
- A **metal grid** between the patient and the photographic film prevents X-rays scattered by the patient from reaching the film. Only those X-rays that pass through the patient without scattering travel through the narrow tubular holes in the grid block. This device therefore improves the contrast of the image. See Fig. 21.25.
- A **contrast medium** is used to obtain X-ray photographs of organs or blood vessels. For example, before taking an X-ray photograph of the stomach, the patient is given a barium drink which lines the stomach. Barium is used because it absorbs X-rays and eventually passes out of the patient without any harmful effects.
- A **film badge** must be worn by all personnel in an X-ray unit. The film is sealed in a light-proof container covered in different parts by different materials. When the film is developed, the exposure of the wearer to different types of ionising radiation can be measured. Strict limits are imposed by law on the maximum permitted exposure of radiation workers to ionising radiation. This is because ionising radiation is harmful, either killing living cells or causing mutation.

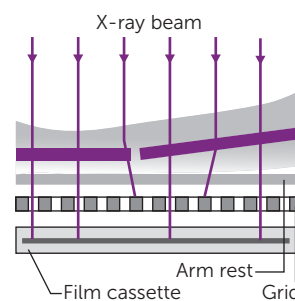


Fig. 21.25 Improving contrast

21.5 QUESTIONS

- 1 An X-ray tube operates at a potential of 25 kV and a beam current of 30 mA with an efficiency of 2%. Calculate
 - a) the minimum wavelength of the X-ray photons from this tube,
 - b) the electrical energy used by the tube each second,
 - c) the heat produced per second at the target.
- 2 Describe how an X-ray picture is made of a broken limb, explaining the steps taken to ensure good contrast between the bones and the surrounding soft tissues.

21.6 Wave mechanics

Wave-particle duality

In the preceding topics, we have seen that electromagnetic radiation consists of photons which are packets of electromagnetic waves. A photon can knock an electron out of an individual atom (or a metal) or promote it to a higher energy level. Such ‘particle-like’ effects of light are in contrast with the wave-like properties of light such as diffraction. Light (and all other electromagnetic radiation) therefore has a dual nature in that it can display wave-like properties and particle-like properties. In 1923, the French physicist Louis de Broglie put forward the idea that all matter particles (e.g. protons, neutrons, electrons) have a **dual wave-particle nature**. In his hypothesis, de Broglie proposed that the wavelength of a matter particle, its **de Broglie wavelength**, depends on the particle’s momentum in accordance with the equation

$$\text{wavelength} \times \text{momentum} = h,$$

where h is the Planck constant.

In other words, since a particle of mass m moving at velocity v has momentum mv , its de Broglie wavelength is given by

$$\lambda = \frac{h}{mv}$$

Evidence for de Broglie’s hypothesis was discovered a few years later when it was observed that a beam of electrons passing through a thin metal foil was diffracted in certain directions only. Fig. 21.26 shows an outline of the arrangement.

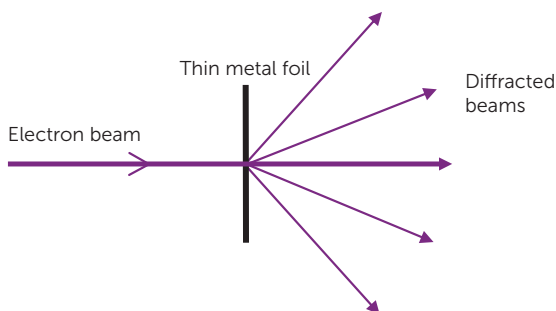


Fig. 21.26 Electron diffraction

Notes

- 1 In the electron diffraction experiment outlined above, the apparatus must be in an evacuated container so the electron beam is not stopped by air molecules. The electrons are directed at the metal foil at a known speed that is calculated from the anode voltage as explained in topic 21.1. Increasing the anode voltage causes less diffraction because the speed of the electrons is increased so they have more momentum and therefore a smaller wavelength.
- 2 The spacing between the rows of the metal atoms is measured from X-ray diffraction experiments. The angle of diffraction in the electron diffraction experiment is measured for each diffraction order and used with the spacing of the rows to calculate the de Broglie wavelength of the electrons. This value agrees with the wavelength calculated using the de Broglie equation after calculating the momentum of the electrons from the mass and known velocity of each electron in the beam.

As explained in topic 4.2, the same effect happens when a beam of light passes through a diffraction grating. The atoms in each grain of a thin metal foil are arranged in a regular pattern of rows like the slits of a diffraction grating. The effect of the rows of atoms on the electron waves is similar to the effect of the regularly spaced slits of the diffraction grating on the light waves. The diffracted waves are in certain directions only.

21.8 WORKED EXAMPLE

Calculate the de Broglie wavelength of an electron moving at a velocity of $3.8 \times 10^6 \text{ m s}^{-1}$.

Mass of the electron = $9.1 \times 10^{-31} \text{ kg}$, $h = 6.6 \times 10^{-34} \text{ J s}$

Solution

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 3.8 \times 10^6} \\ &= 1.9 \times 10^{-10} \text{ m}\end{aligned}$$

Energy levels

An electron in an atom is held in the atom by the force of electrostatic attraction between it and the nucleus of the atom. The electron is trapped in the atom so its potential energy is negative. In effect, it is trapped in a 'potential well' created by the nucleus. In this situation, its wave-like nature restricts its energy to certain values. In other words, its energy is said to be 'quantised'. This applies to any situation where a particle is trapped in a potential well.

The simplest atom is the hydrogen atom as it has one electron that moves around the nucleus (consisting of one proton only). If the electron is moving round the nucleus in a circular orbit of radius r , its de Broglie wavelength λ is such as to enable it to form a stationary wave on the circumference of the orbit ($= 2\pi r$). From topic 10.4, a stationary wave of wavelength λ forms on a string of length L if $L = n\lambda$ where n is a whole number.

Applying this condition to the electron on a circular orbit gives $2\pi r = n\lambda$. In other words, the circumference must be such that a whole number of wavelengths must fit on it. Since the momentum of the electron depends on its de Broglie wavelength, it therefore follows that the momentum of the electron and hence its energy can only take certain values.

A detailed analysis using the above ideas and the formulae for kinetic energy ($= \frac{1}{2}mv^2$) and electrostatic potential energy (see topic 17.4) gives

- the radius of each allowed orbit, $r = r_0 n^2$
- the allowed energy levels of the electron, $E = -E_0/n^2$, where $n = 1, 2, 3, \dots$ and $E_0 = 13.6 \text{ eV}$.

Measurements of the hydrogen spectrum give photon energies in agreement with transitions between energy levels at the above values. See topic 21.4.

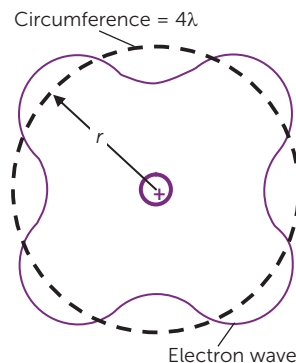


Fig. 21.27 Electron stationary waves in an atom (for $4\lambda = 2\pi r$)

The electron microscope

Electron microscopes depend on the dual nature of electrons. In an electron microscope, the particle nature of electrons enables them to be directed by electric and/or magnetic fields at the atoms in a sample. The wave nature of electrons allows the electrons to form a high-resolution image of the sample.

The **transmission electron microscope (TEM)** uses a magnetic ‘lens’ to direct electrons from a hot wire filament at a thin sample via a high voltage anode and the magnetic lens, as shown in Fig. 21.28. The lens creates a magnetic field which deflects the electrons when they pass through the coil. Further magnetic lenses are used to focus the electrons that pass through the sample onto a fluorescent screen. A magnified image of the sample is formed on the screen.

The speed of the electrons can be increased by increasing the anode voltage. This has the effect of reducing the de Broglie wavelength of the electrons (in accordance with the de Broglie equation). As a result, the electrons are diffracted less as they pass through the lenses so more detail can be seen in the image. The same general effect occurs if an object is observed in blue light using an optical microscope. Blue light has a smaller wavelength than the average wavelength of white light so it diffracts less when it passes through the microscope lenses and the image details are therefore clearer.

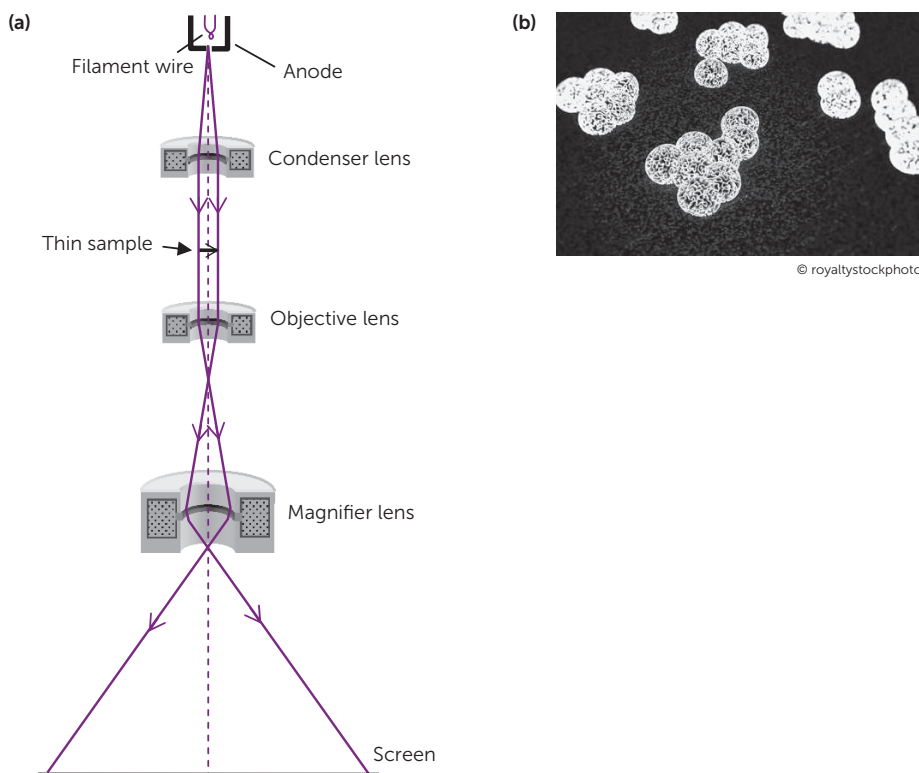


Fig. 21.28 The transmission electron microscope (a) construction (b) a TEM image of MRSA particles

In practice, because of limitations mostly due to the magnetic lenses, the TEM cannot be used to see objects smaller than about 10^{-9} m ($= 0.001$ μm). Individual atoms are too small to see with a TEM.

The **scanning tunnelling microscope** gives images of atoms on the surface of an object. It uses a wave property known as **tunnelling**. This is a property of a particle that enables it to 'tunnel' or pass through a barrier. For example, the work function of a metal is a measure of the potential barrier that prevents electrons from leaving the surface of a metal. However, the wave nature of the electron means there is a small probability that electrons can cross the gap between the surface and a nearby object if the gap is sufficiently small. This is because the electron waves can extend across such a narrow gap.

Fig. 21.29 shows the arrangement in a scanning tunnelling microscope where electrons tunnel across the gap between the surface and the tip of a metal probe above the surface.

The probability of an electron crossing the gap decreases if the gap is made wider. In effect, electrons cross the gap if

- the gap is narrow enough (i.e. of the order of 10^{-9} m)
- there is a constant potential difference between the tip and the surface.

The p.d. is necessary to ensure that the electrons cross the gap in one direction only from the negative to the positive side.

In practice, the metal probe is moved repeatedly across the surface at a fixed horizontal level. The unevenness of the surface causes the gap width to change. As a result the current due to electrons crossing the gap changes. The changing current is used to recreate a 3-dimensional computer image of the gap surface. The change of current is extremely sensitive to changes of the gap width which is why individual atoms on the surface can be seen in the image.

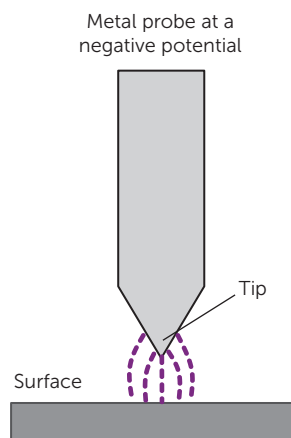


Fig. 21.29 The scanning tunnelling microscope

Application 11

Flash memory

Memory cards and USB flash drives (memory sticks) used in digital cameras, in MP3 players and in or with computers have revolutionised electronic storage. Without any moving parts, they can store much more data than the storage devices they replaced such as the floppy discs.

Flash memory consists of rows of individual memory cells, each one capable of storing electrons as an electronic bit (i.e. a '0' or '1'). Each cell is a type of transistor that contains an insulated 'floating gate', as shown in Figure 21.30 overleaf, which stores a single data bit. The 3 terminals of the cell are referred to as the source, the drain and the control gate. When a potential difference is applied between the source and the drain, a current passes through the semiconducting material between them. With the control gate at $+12$ V, a bit is stored on the floating gate by applying $+7$ V to the drain for logic 0 or close to zero volts for logic 1.

When a suitable potential is applied to the control gate, electrons transfer to or from the floating gate by tunnelling through the insulating layer between the floating gate and the material between the source and the drain. Table 21.1 shows the potential applied at each terminal in order to erase or write in or read out a data bit.

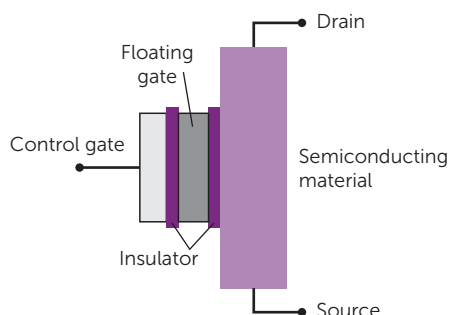


Fig. 21.30 A flash memory cell

Table 21.1

	Erase	Write	Read
Source	+6 V	0	0
Drain	open	+7 V for logic 0, ~ 0 V for logic 1	+1 V
Control gate	−9 V	+12 V	+5 V
Effect on the floating gate (FG)	Loses electrons to source and becomes 0 V	Gains electrons from source-drain current and becomes negative	If there are electrons on FG, source-drain current is low (logic 0) If there are no electrons on FG, current is 'high' (logic 1)

21.6 QUESTIONS

$h = 6.6 \times 10^{-34}$ J s, $e = 1.6 \times 10^{-19}$ C, mass of the electron = 9.1×10^{-31} kg, mass of proton = 1.7×10^{-27} kg

- 1 a) State one observation that demonstrates that matter particles have a wave-like nature.
b) Calculate the velocity of a proton for it to have a de Broglie wavelength of 1.5×10^{-9} m.
- 2 a) With the aid of a diagram, describe the essential features of a scanning tunnelling microscope and explain why its operation depends on the wave nature of the electron.
b) Calculate the de Broglie wavelength of an electron moving at a velocity of 9.2×10^6 m s^{−1}.

SUMMARY

- The electron gun equation

$$\frac{1}{2}mv^2 = eV_A$$

- Electrons in fields

- 1 Force, $F = \frac{eV_P}{d}$ for an electron between two oppositely charged parallel plates.
- 2 Force, $F = Bev$ for an electron moving at speed v perpendicular to a magnetic field, B .

- Millikan's experiment

- 1 $\frac{QV_P}{d} = mg$ for a charged droplet at rest.
- 2 $F = 6\pi\eta rv$ gives the viscous force on a droplet of radius r moving at speed v .
- 3 $Q = ne$, where n is a whole number.

● **Photons**

- 1 Energy of a photon, $E = hf = \frac{hc}{\lambda}$
- 2 Maximum k.e. of a photoelectron $= hf - \phi$, where ϕ is the work function.
- 3 $eV_S S = hf - \phi$ gives the stopping potential V_S .

● **Energy level equation**

$$hf = E_1 - E_2$$

● **X-ray intensity distribution**

$$f_{\max} = \frac{eV_A}{h}$$

● **de Broglie's equation**

$$\lambda = \frac{h}{mv}$$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.
 $h = 6.6 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C, $c = 3.0 \times 10^8$ m s $^{-1}$

21.1

- a) A beam of electrons is produced by an electron gun operating at an anode potential of 4200 V. Calculate
- (i) the kinetic energy in joules,
 - (ii) the speed of the electrons in the beam.
- b) The beam in (a) is directed between two oppositely charged parallel plates 50 mm apart, along a line midway between the plates. When the top plate is at a positive potential of 5000 V relative to the lower plate, the beam curves as shown in Fig. 21.31.

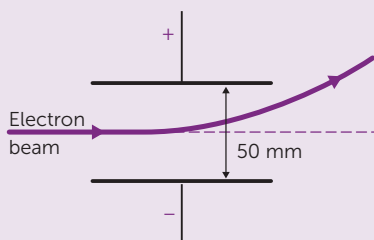


Fig. 21.31

- (i) Calculate the force on an electron in the beam when it is between the plates.
- (ii) The force due to the electric field is cancelled out if a suitable magnetic field is applied at 90° to the plane of the diagram. State whether this magnetic field should be directed into or out of the plane of the diagram and calculate the magnetic flux density necessary for the beam to be undeflected.

21.2 In an experiment to measure the charge on an oil droplet, an oil droplet was held stationary between two parallel plates 4.0 mm apart at a potential difference of 590 V. When the plate p.d. was switched off, the droplet fell at steady speed, moving through a distance of 1.2 mm in 14.6 s.

- a) (i) The top plate was positive relative to the lower plate. What type of charge was carried by the droplet?
 (ii) Why did the droplet fall at constant speed when the plate p.d. was switched off?
- b) Calculate
- (i) the terminal speed of the droplet,
 - (ii) the radius of the droplet,
 - (iii) the droplet's mass,
 - (iv) the charge on the droplet.

The viscosity of air $= 1.8 \times 10^{-5}$ N s m $^{-2}$, the density of the oil $= 960$ kg m $^{-3}$.

21.3

- a) Calculate the frequency and the energy of a photon of wavelength
- (i) 500 nm,
 - (ii) 50 nm.
- b) In a photoelectricity experiment, light of wavelength 500 nm is directed at the surface of the photocathode in a vacuum photocell. The photoelectric current is reduced to zero as a result of applying a positive potential of 0.36 V to the photocathode, relative to the anode. Calculate
- (i) the work done by an electron moved through a potential difference of 0.36 V,
 - (ii) the work function of the photocathode surface in electron volts,
 - (iii) the maximum wavelength of incident radiation capable of causing photoelectric emission from this surface when it is at zero potential.

REVISION QUESTIONS

21.4

- a) The energy levels of the hydrogen atom are given by the equation

$$E = \frac{-13.6}{n^2} \text{ eV.}$$

Calculate the wavelength of the photon emitted by an electron in a hydrogen atom when it transfers from

- (i) $n = 4$ to $n = 3$,
 (ii) $n = 2$ to $n = 1$.
 b) Fig. 21.32 shows the energy levels of a certain type of atom known to emit photons of wavelengths 565 nm and 430 nm. Calculate the energy of a photon of each wavelength and identify the transition responsible for each photon.

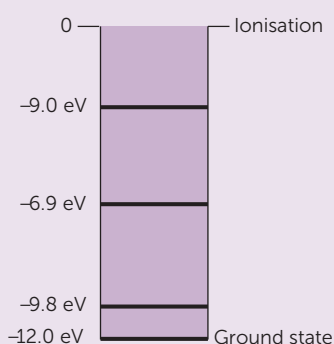


Fig. 21.32

21.5

- a) (i) Sketch curves showing how the intensity of the X-rays in an X-ray beam varies with frequency for two different tube voltages.
 (ii) Explain why the intensity is very high at certain wavelengths that depend on the target element.
 b) Calculate the minimum wavelength of the X-rays produced from an X-ray tube operating at
 (i) 25 kV,
 (ii) 100 kV.

21.6 In a transmission electron microscope, provided the speed v of the electrons is much less than the speed of light, the kinetic energy of the electrons in the beam depends on the potential difference V between the filament and the anode in accordance with the equation $\frac{1}{2}mv^2 = eV$, where e is the charge of the electron and m is the mass of the electron.

- a) Use this equation and the de Broglie equation to show that the electrons in the beam have a de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{(2meV)}}$$

Mass of the electron = 9.1×10^{-31} kg.

- b) (i) Calculate the de Broglie wavelength for an anode p.d. of 10 000 V.
 (ii) Explain why the resolution (i.e. detail) of a TEM image is increased if the anode p.d. is increased.

Radioactivity

OBJECTIVES

After working through this unit, you should be able to:

- describe the structure of the atom in terms of electrons, protons and neutrons
- explain what the term 'isotope' means
- explain what a radioactive substance is, and describe the main properties of the radiation from radioactive substances
- describe the changes that occur when an unstable nucleus emits radiation
- carry out experiments to measure background radiation and to investigate the absorption of radiation from radioactive substances
- describe and explain the operation of a Geiger-Müller tube and a cloud chamber
- outline Rutherford's alpha-scattering experiment and how the results led Rutherford to deduce the nuclear model of the atom
- explain what is meant by radioactive decay and the term 'half-life'
- carry out half-life calculations

CONTENTS

- 22.1 Inside the atom
- 22.2 Radiation from radioactive substances
- 22.3 The range and penetrating power of alpha, beta and gamma radiation
- 22.4 Radioactive decay
- 22.5 The mathematics of radioactive decay
- Summary
- Revision questions

22.1 Inside the atom

An atom is the smallest particle characteristic of an element. Until near the end of the nineteenth century, the atom was thought to be indivisible and indestructible. Joseph John Thomson's experiments on the electron proved that atoms contain these much lighter particles. In this unit, we will look at how the discovery of radioactivity during the same period led to the conclusion that some atoms are unstable and that every atom consists of electrons surrounding a nucleus composed of protons and neutrons. See [Fig. 22.1](#) overleaf. Before considering radioactivity and the properties of radioactive substances, here are some reminders about the structure of the atom. Electrons, protons and neutrons form the building blocks of every atom. Electrons are negatively charged particles of mass about $1/2000$ th of the mass of a hydrogen atom. The protons and neutrons in an atom are concentrated in the nucleus which has a diameter about 10 millionths of the diameter of the atom.

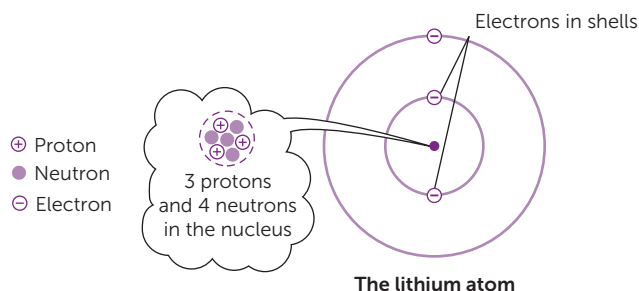


Fig. 22.1 Inside the atom

Almost all the mass of an atom is located in its nucleus. Some further key points about the structure of the atom are listed below.

- The electrons occupy shells at fixed energy which surround the nucleus of the atom.
- The proton carries a positive charge, equal and opposite to the charge of the electron, and has a mass almost equal to that of a hydrogen atom.
- The neutron is uncharged and has a mass about the same as that of the proton.
- Every atom of a given element contains the same number of protons.
- The number of neutrons in the nucleus of an atom is not the same for all the atoms of an element.

An isotope of an element consists of atoms of that element that have the same number of neutrons.

For example, the three isotopes of hydrogen are shown in Fig. 22.2. Most hydrogen atoms contain a nucleus consisting of a proton only. Deuterium is a rare isotope of hydrogen in which the nucleus of every atom consists of a proton and a neutron. Tritium, the third isotope of hydrogen, is even rarer than deuterium and consists of atoms which each have a nucleus consisting of a proton and two neutrons.

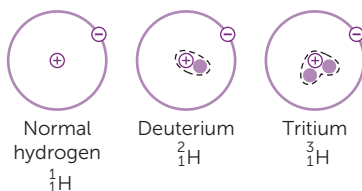


Fig. 22.2 The three isotopes of hydrogen

Numbers and symbols

The proton number, Z , of an atom is the number of protons in the nucleus of the atom.

Each atom of a given element contains Z protons. The proton number of an atom is also referred to as the atomic number of the element as this is the order number of the element in the Periodic Table (see p. 76).

The mass number, A , of an atom is the number of protons and neutrons in its nucleus.

The lightest known atom is the hydrogen atom, which has a single proton as its nucleus. The neutron has about the same mass as the proton. Electrons are much lighter than protons or neutrons. Therefore, the mass of an atom relative to the mass of a proton is approximately equal to its mass number, A .

An isotope is represented by the symbol ${}_Z^AX$.

where X is the symbol for the chemical element. An isotope is sometimes referred to by the name of the element followed by the mass number. For example, uranium 235 (or more simply U 235) is the isotope of uranium which has a mass number of 235.

- There are Z protons in the nucleus of an atom of this isotope.
- The nucleus also contains $(A - Z)$ neutrons.
- An uncharged atom has Z electrons in shells surrounding the nucleus.

The unit of atomic mass, 1 u, is defined as $\frac{1}{12}$ th of the mass of an atom of ${}_{6}^{12}\text{C}$ (i.e. carbon 12).

Since there are 12 neutrons and protons in the nucleus of carbon 12, the mass of a proton or a neutron is approximately equal to 1 u.

The Avogadro constant, N_A , is defined as the number of atoms in exactly 12 g of carbon 12.

The accepted value of N_A is 6.02×10^{23} .

- A carbon 12 atom has a mass of $\frac{12}{N_A}$ grams or $\frac{0.012}{N_A}$ kilograms. Since this is equal to 12 u by definition, then it follows that

$$1 \text{ u} = \frac{1}{N_A} \text{ grams} = \frac{0.001}{N_A} \text{ kilograms} = 1.67 \times 10^{-27} \text{ kg}.$$

- The mass of 1 atom of mass number A is approximately equal to A in atomic mass units. For example, the mass of a uranium 235 atom is approximately equal to 235 u.

A mole is the quantity of matter in N_A particles of a substance.

The molar mass of a substance is the mass of 1 mole of the substance. The unit symbol for the mole is mol.

22.1 QUESTIONS

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

- 1** Calculate the number of protons and neutrons in one atom of each of the following isotopes:

a) ${}_{92}^{235}\text{U}$ **b)** ${}_6^{14}\text{C}$ **c)** ${}_{17}^{37}\text{Cl}$

- 2** Calculate the number of atoms in
a) 1.0 kg of uranium 235,
b) 1 g of helium 4.

22.2 Radiation from radioactive substances

Becquerel's discovery

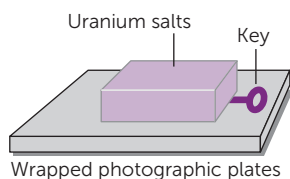


Fig. 22.3 Becquerel's discovery

Radioactivity was discovered by Henri Becquerel in 1896, when he was investigating materials that glow in X-ray beams and ultraviolet light. He wanted to find out if uranium salts glow in strong sunlight so he prepared a sample and placed it in a drawer ready for a sunny day. The sample was placed on a key on some wrapped photographic plates, as shown in Fig. 22.3, which he was using to test the penetrating power of X-rays. When he developed the plates, he saw an image of the key and realised this was due to radiation emitted by the uranium salts. The uranium salts were referred to as 'radioactive'. As he was more interested in X-rays than this curious radiation from uranium salts, he passed the task of investigating radioactivity onto Marie Curie, one of his research students. She discovered other natural substances which also emit radiation, including two new elements, polonium and radium. Radium is over a million times more radioactive than uranium.

The nature of the radiation from radioactive substances

Ernest Rutherford showed that there were two types of radiation from radioactive substances. One type, which he called **alpha** (α) radiation, is easily absorbed and another, more penetrating component, he called **beta** (β) radiation. A third type, called **gamma** (γ) radiation, which is even more penetrating than beta radiation, was discovered later. Fig. 22.4 shows the effect of different materials on each type of radiation.

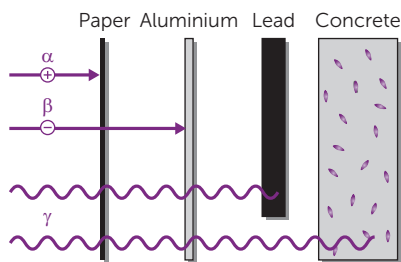


Fig. 22.4 Absorption of α , β and γ radiation

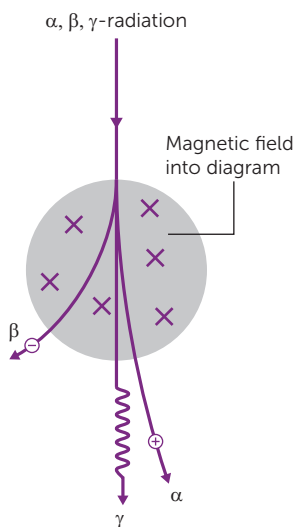


Fig. 22.5 The separation of α , β and γ radiation

Further investigations on the nature and properties of radioactive substances were carried out as outlined below.

Deflection by a magnetic field

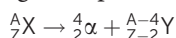
A magnetic field deflects moving charged particles. Investigations showed that alpha radiation consists of positively charged particles since a beam of alpha radiation is deflected as shown in Fig. 22.5. This type of investigation was also used to show that beta radiation consists of negative particles which are more easily deflected than alpha particles. Gamma radiation is unaffected by a magnetic field hence gamma radiation does not consist of charged particles.

The nature of each type of radiation

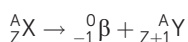
A radioactive substance is an isotope composed of atoms with unstable nuclei. Alpha, beta or gamma radiation is emitted when an unstable nucleus becomes stable.

- **Alpha particles** were shown to be identical to the nuclei of helium atoms, each composed of two protons and two neutrons. An alpha particle therefore has a mass of $4u$ and a charge of $+2e$. This was proved by showing that the spectrum of light from a discharge tube containing alpha particles is the same as that from a helium discharge tube.

A nucleus that emits an alpha particle loses two protons and two neutrons so its proton number decreases by 2 and its mass number decreases by 4. This change is represented by:



- **Beta particles** are fast-moving electrons, emitted by unstable nuclei which have too many neutrons. The identity of beta particles as electrons was made by measuring the path of the particles in a uniform magnetic field. A beta particle is created and emitted from a nucleus as a result of a neutron in the nucleus changing into a proton. The proton number of the nucleus increases by 1 and the mass number of the nucleus is unchanged. This change is represented by:



- **Gamma radiation** is electromagnetic radiation of wavelength 10^{-12} m or less. An unstable nucleus of a gamma-emitting isotope emits a high energy photon when it becomes stable. The mass number and the proton number of an unstable nucleus are unchanged when a gamma photon is emitted by the nucleus.

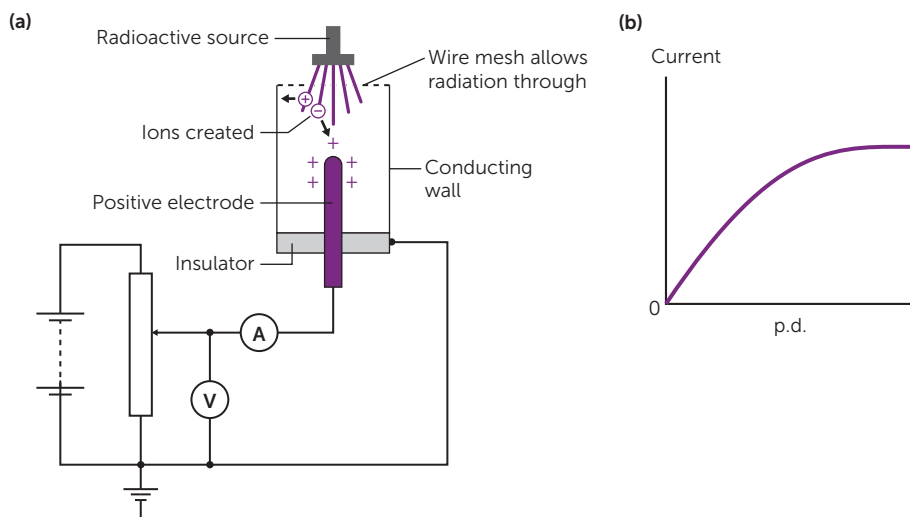
Ionisation

An ion is a charged atom, formed as a result of the atom gaining or losing one or more electrons. All three types of radiation emitted by radioactive substances are capable of creating ions. Fig. 22.6 overleaf shows how the ionising effect of each type of radiation can be measured.

The radiation ionises air molecules in the ionisation chamber so the positive ions created are attracted to the negative surface of the chamber and the negative ions to the positive electrode. The positive ions gain electrons and become neutral at the negative surface. The negative ions lose electrons and become neutral at the positive electrode. As a result, an ionisation current passes around the circuit and this is measured using a microammeter. The current is proportional to the number of ions created each second in the chamber.

- **Alpha radiation** passing through matter is capable of producing much more ionisation than beta or gamma radiation. This is because an alpha particle is much more likely to knock electrons out of the shells of an atom than are gamma radiation or a beta particle. An alpha particle on passing through matter therefore loses its kinetic energy more rapidly than a beta particle or gamma radiation.
- **Beta radiation** is more strongly ionising than gamma radiation but less strongly ionising than alpha radiation. A beta particle is a fast-moving electron created in and emitted from the nucleus. To ionise an atom, a beta particle must knock an

Fig. 22.6 An ionisation chamber (a) in a circuit, (b) characteristics



electron out of one of the shells of an atom. A beta particle is much lighter than an alpha particle so it has less effect on an atom with which it collides than does an alpha particle.

- **Gamma radiation** is much less ionising than alpha or beta radiation. Gamma radiation is uncharged and therefore has less effect on the electrons in an atom than either alpha or beta radiation.

Application 12

Radiation risks

Ionising radiation is harmful to the human body. The effect of ionising radiation on the human body is measured in **millisieverts (mSv)** which takes into account the type of tissue exposed, the type of radiation that is used and the energy it delivers per kilogram of tissue. Everyone is exposed to 'background' ionising radiation from a variety of sources including radioactive isotopes in the air, in the ground and in food and drink, and from cosmic rays and medical X-rays. The radiation dose a person receives from all such background sources depends on location and environment. The average annual dose in the UK is 2.7 mSv and 6.2 mSv in the USA.

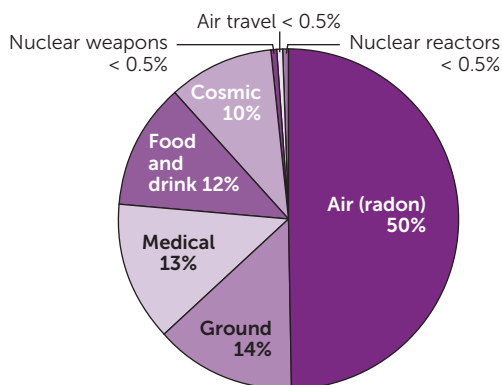


Fig. 22.7 The origins of background radiation

Radiation workers are monitored carefully as they are exposed to additional ionising radiation because of the nature of their work. The current regulatory dose limit is 20 mSv per year for employees aged 18 years or over, 6 mSv per year for trainees and 1 mSv per year for members of the public and employees under 18. These figures are based on epidemiological studies of groups of people exposed to ionising radiation and what are considered to be acceptable risks compared with everyday activities. For example, estimates based on the survivors of the atomic bombs dropped on Hiroshima and Nagasaki indicate 3 fatal cancers per 100 000 survivors for each millisievert of radiation. On that basis, an average annual exposure of 2. mSv per year would cause $8 (= 3 \times 2.7)$ fatal cancers per year per 100 000 people which is considered acceptable when compared to road accidents causing about 10 deaths per 100 000 people.

Detectors of ionising radiation

The ionisation chamber

The operation of an ionisation chamber is explained above. The ionisation current increases and reaches a maximum value as the potential difference between the chamber wall and the central electrode is increased. The maximum current is reached when all the ions produced reach the electrode or chamber wall instead of recombining.

The Geiger–Müller tube

A Geiger–Müller (GM) tube consists of a hollow metal tube which is sealed and filled with an inert gas at low pressure. See Fig. 22.8. The tube is earthed and it contains a metal rod along its axis, insulated from the tube. The rod is maintained at a high positive potential so a strong electric field exists between the rod and the tube walls.

When an alpha particle, a beta particle or a gamma photon enters the tube, ion pairs are created and the ions are attracted to the appropriate oppositely charged electrode. The ions gain kinetic energy due to this attraction and create further ion pairs as a result of collisions with uncharged gas molecules. These ions then produce further ionisation and an ‘avalanche’ of ions is therefore triggered from a single radiation particle. When the ions reach the electrodes, they are discharged and a pulse of charge passes round the circuit through the resistor in series with the GM

Note

As explained later on p. 359, radioactive emission is a random process and the number of counts in a given time from a source at constant distance will fluctuate. For this reason, the count rate is more accurate, the longer the time taken. Also, to ensure timing errors do not occur, the number of counts in a given time is repeated several times to give an average number. The count rate is then calculated from the average number of counts divided by the time taken.

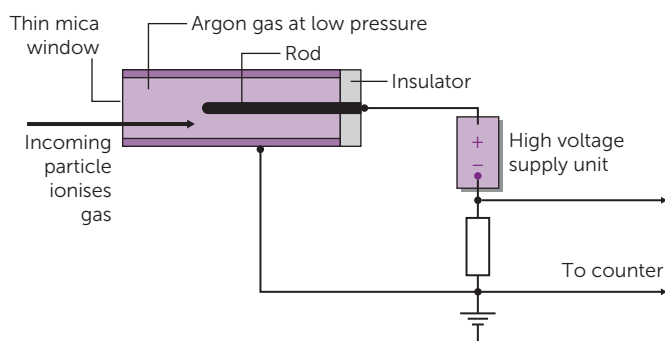


Fig. 22.8 The Geiger–Müller tube

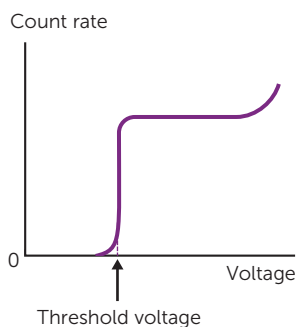


Fig. 22.9 The characteristics of a GM tube

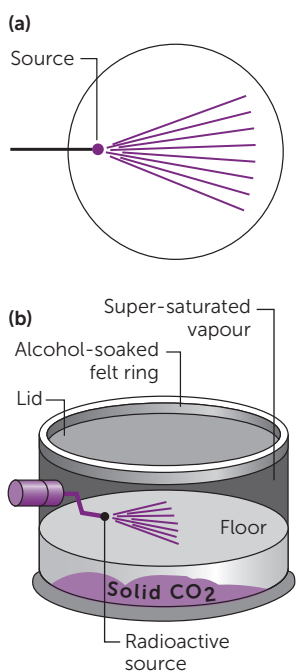


Fig. 22.10 The cloud chamber
(a) alpha tracks,
(b) construction

tube. The voltage pulse created across the resistor is then registered as a single count by the electronic counter. The gas in the tube returns to its non-conducting state less than a millisecond after the particle enters the tube. The GM tube and the counter can therefore be used to measure the number of ionising particles entering the tube in a certain time. The **count rate**, the number of counts per unit time, can then be calculated by dividing the number of counts by the time taken.

The count rate depends on the tube voltage, as shown by Fig. 22.9. Below a certain voltage, the threshold voltage, no counts are detected because the tube voltage is not high enough to trigger further ionisation after an ionising particle enters the tube. As the voltage is increased above the threshold voltage, the count rate increases sharply to a plateau. The tube voltage is usually set so the tube is operating at the 'plateau' away from the steep part of this curve.

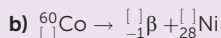
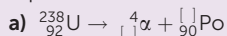
The cloud chamber

The tracks of alpha particles can easily be seen in a cloud chamber because the air space in the cloud chamber, as shown in Fig. 22.10, contains a supersaturated vapour. When an ionising particle passes through this vapour, tiny droplets form from the vapour along the track of the particle because the ions formed along the track initiate condensation.

- **Alpha tracks** are clear, straight and of the same length for a given isotope.
- **Beta tracks** are irregular and poorly defined. This is because beta particles are less ionising and more easily deflected by air molecules than alpha particles.
- **Gamma particles** do not produce tracks as they produce insufficient ionisation.

22.2 QUESTIONS

1 Copy and complete the following equations:



2 a) State one similarity and two differences between alpha and beta radiation.

b) Describe the change that occurs in an unstable nucleus as a result of emitting
(i) an alpha particle, (ii) beta particle.

22.3 The range and penetrating power of alpha, beta and gamma radiation

Fig. 22.11 shows how to investigate the range in air or the penetrating power of alpha or beta or gamma radiation. A GM tube is used to detect the radiation from a radioactive isotope in a small container. A different radioactive source is used to investigate each type of radiation. The background count rate (i.e. the count rate with no source present) must first be measured. This is subtracted from each count rate measured with the source present to give the corrected count rate.

- 1 **To investigate the range**, the count rate is measured with the tube at different distances from the source. Because the particles spread out as they move away from the source, the count rate decreases with increasing distance from the source.

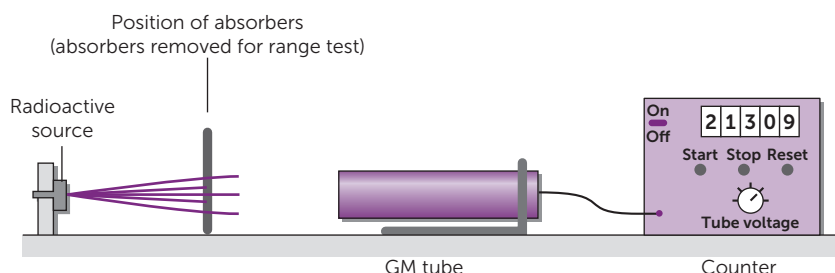


Fig. 22.11 Investigating range and penetrating power

- **Alpha particles** penetrate only a certain distance in air so the count rate suddenly falls to the background level when the tube is moved beyond the range of the alpha particles.
- **Beta particles** have a range in air up to no more than about 0.5 m, depending on their initial energy. The count rate decreases as the tube is moved away from the source because the particles are absorbed by air as well as spreading out.
- **Gamma radiation** is scarcely absorbed by air and therefore the corrected count rate, C , decreases with distance, r , from the source in accordance with the inverse square law

$$C = \frac{k}{r^2} \text{ where } k \text{ is a constant.}$$

This equation assumes that no gamma radiation is absorbed by air. As shown in Fig. 22.12, at distance, r , from the source, all the radiation passes through the surface of a sphere of radius r and surface area equal to $4\pi r^2$. If the source emits N gamma photons per second, the number of gamma photons entering the tube is therefore

$$\frac{NA}{4\pi r^2}$$

where A is the area of the tube facing the source. Hence the corrected count rate is given by the above equation with $k = NA \div 4\pi$.

- 2 **To investigate the penetrating power**, the distance between the source and the tube is kept constant. The count rate is measured with different thicknesses of each material to be tested.

- **Alpha radiation** is absorbed by paper and metal foil.
- **Beta radiation** penetrates paper and metal foil. The count rate decreases with increasing thickness of the absorber. A metal plate more than a few millimetres thick will absorb the beta radiation completely.
- **Gamma radiation** is even more penetrating than beta radiation. A lead plate of several centimetres thickness or a thick concrete block will stop gamma radiation. The count rate decreases exponentially with increasing thickness. This means that every extra millimetre thickness reduces the intensity of radiation penetrating the absorber by the same percentage. For example, if the count rate with no absorber is 1600 counts per minute and this is reduced to 90% of 1600 counts per minute (= 1440 per minute) by an absorber, then adding a further identical absorber would decrease the count rate to 90% of

See www.palgrave.com/foundations/breithaupt for an experiment to test the inverse square law for gamma radiation from a point source.

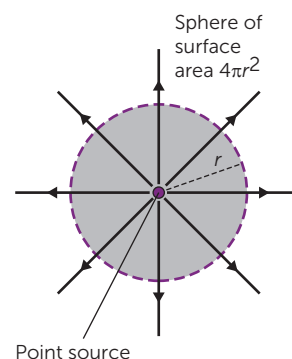


Fig. 22.12 The inverse square law for gamma radiation

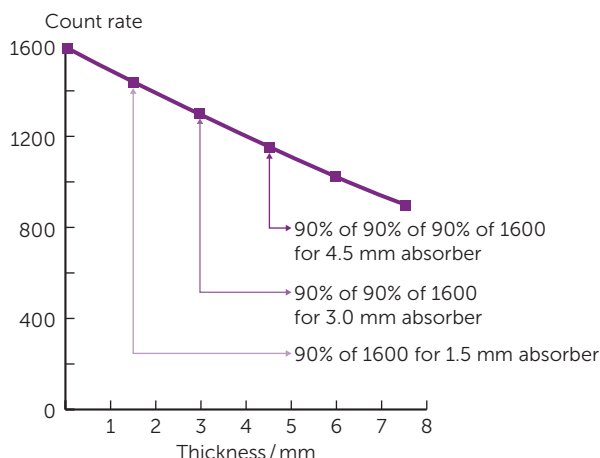


Fig. 22.13 Count rate versus thickness

1440 counts per minute (= 1296 counts per minute). Fig. 22.13 shows how the count rate decreases with absorber thickness.

- The **half-value thickness** of an absorber is the thickness needed to reduce the count rate to 50%. For example, if the half-value thickness of a certain material is 2.0 mm, then an absorber of this material of thickness 4.0 mm would reduce the count rate to 25% (= 50% of 50%).
- For **beta radiation**, the count rate also decreases with increasing thickness in a similar way to gamma radiation, except that beta radiation is totally absorbed beyond a certain thickness whereas gamma radiation continues to decrease exponentially.

22.1 WORKED EXAMPLE

A GM tube is used to measure the count rate due to a certain radioactive source which emits gamma radiation. Without the source present, the count rate due to background radioactivity is 28 counts per minute.

- When the GM tube is 100 mm from the source, the count rate is 950 per minute. Estimate the count rate if the distance between the tube and the source is increased to 200 mm.
- With the tube at 100 mm from the source, an aluminium plate is placed between the tube and the source, reducing the count rate to 655 counts per minute. Estimate the count rate if another identical absorber is placed between the source and the tube.

Solution

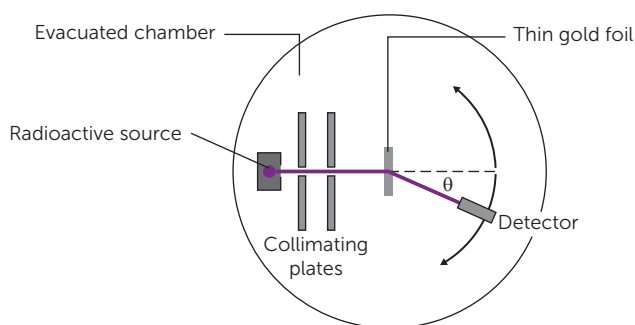
- Corrected count rate at 100 mm = $950 - 28 = 922$ counts per minute. According to the inverse square law, if the distance is doubled, the count rate is reduced to one quarter.
Corrected count rate at 200 mm = $0.25 \times 922 = 231$ counts per minute.
The observed count rate at 200 mm would therefore be about 259 counts per minute (approx. equal to $231 + 28$ counts per minute).
- Corrected count rate with single absorber = $655 - 28 = 627$ counts per minute. This is 68% (= $627 \div 922 \times 100\%$) of the corrected count rate without the absorber.
 $\therefore$ the corrected count rate with two identical absorbers = 68% of 627 = 426 counts per minute.
The observed count rate would therefore be 454 counts per minute (= $426 + 28$ counts per minute).

Table 22.1 Summary of the nature and properties of alpha, beta and gamma radiation

Property	alpha (α)	beta (β)	gamma (γ)
Nature	helium nucleus	electron	photon
Charge	+2e	e	0
Ionisation	strong	weak	very weak
Range in air	several cm	50 cm or more	infinite
Absorption by matter	paper	several mm of metal	thick lead plate

Rutherford's alpha scattering experiments

The nuclear model of the atom was proved by Rutherford. He devised an experiment in which a narrow beam of alpha particles was directed at a thin gold foil in a vacuum as shown in Fig. 22.14. Rutherford measured the number of alpha particles per second scattered in different directions by the foil.

**Fig. 22.14** Rutherford's alpha scattering experiment

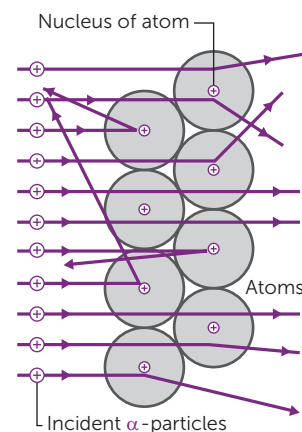
He made the following observations:

- 1 Most alpha particles passed through the foil with little or no deflection,
- 2 A very small proportion of the alpha particles rebounded from the foil.

These results are akin to finding that bullets aimed at cardboard occasionally bounce back from the cardboard. If that happened, you would conclude that there must be some small but heavy objects in the cardboard which a bullet might hit. See Fig. 22.15.

Rutherford deduced from his measurements that:

- 1 the mass of every atom is concentrated in a comparatively small nucleus. This explained why most alpha particles passed through the foil with little or no scattering.
- 2 the nucleus of the atom is positively charged. This explains why an alpha particle can rebound from an atom. Such an alpha particle must have collided directly with the nucleus. Since the nucleus and the alpha particle are both positively charged, the alpha particle is repelled by the nucleus.

**Fig. 22.15** The paths of scattered alpha particles

Rutherford used the laws of mechanics and Coulomb's law of force to show that the number of alpha particles per second scattered by the foil through an angle, θ , from the original direction should be proportional to

$$\frac{1}{\sin^4 \theta/2}$$

His experimental results fitted this formula, thus proving conclusively the nuclear model of the atom.

22.3 QUESTIONS

- 1 In an experiment to determine the type of radiation from a radioactive source, the following measurements were made with different absorbers between a GM tube and the source:
 - count rate with no absorber and no source present = 26 counts per minute;
 - count rate with source present and no absorber = 628 counts per minute;
 - count rate with paper between the tube and the source = 620 counts per minute;
 - count rate with a 4.0 mm aluminium plate between the tube and source = 30 counts per minute.
 - a) (i) What type of radiation was emitted from the source? Give a reason for your answer. (ii) What further test could be carried out to confirm your answer to (i)?
 - b) In the above experiment, the count rate with a 0.5 mm metal plate between the same source and the tube at the same distance was 482 counts per minute. Calculate (i) the corrected count rate due to the source, with and without the 0.5 mm metal plate present, (ii) the percentage of the incident radiation transmitted by this metal plate, (iii) the expected count rate if an identical metal plate had also been placed in the path of the beam.
- 2 A GM tube was used to measure the count rate due to gamma radiation from a point source. The following readings were made:
 - (1) Number of counts in 5 minutes without the source present = 143, 124, 136.
 - (2) Number of counts in 3 minutes with the source at a distance of 0.40 m from the tube = 725, 746, 738.
 - a) Calculate the corrected count rate due to the source.
 - b) Estimate the count rate if the distance between the tube and the source was reduced to (i) 0.20 m, (ii) 0.15 m.

22.4 Radioactive decay

Randomness

In a cloud chamber, the tracks created by alpha particles from a small radioactive source can easily be seen. The tracks occur at random with no indication of the direction of the next one. This is because the decay of an unstable nucleus is unpredictable. This is also why the number of counts by a Geiger counter in a given time, from a source at constant distance, fluctuates randomly.

The number of unstable nuclei in a sample of a radioactive isotope decreases gradually with time. This is because each decay of an unstable isotope means that there is one less unstable nucleus waiting to decay. Because the decay process is unpredictable, the number of unstable nuclei that decay each second (i.e. the rate of decay) is proportional to the number of unstable nuclei present. The rate of decay therefore slows down because the number of unstable nuclei decreases with time. Fig. 22.16 shows how the count rate from a radioactive isotope decreases with time.

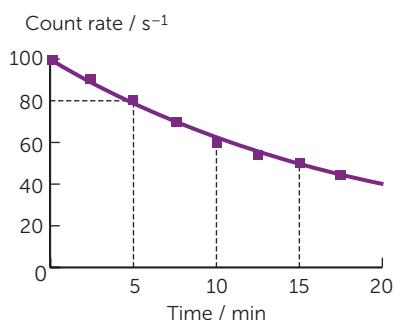


Fig. 22.16 An exponential decay curve

The curve is called an **exponential decay** curve because the rate of decay is proportional to the number of unstable nuclei remaining. For example, suppose 10% of the unstable nuclei of a certain radioactive isotope decay every hour. For every 1000 unstable nuclei of this isotope, there would be 900 unstable nuclei (= 90% of 1000) present 1 hour later and 810 unstable nuclei (= 90% of 900 of 1000) present a further hour later. Table 22.2 shows how the number of unstable nuclei continues to decrease in this example. Fig. 22.17 shows the number of unstable nuclei plotted on the y -axis with time plotted on the x -axis. The shape of the curve is the same as Fig. 22.16. Prove for yourself that the number of unstable nuclei halves every 5 hours approximately.

Table 22.2 Numbers of unstable nuclei

Time at the start of each interval/hours	0	1	2	3	4	5	6
Number of unstable nuclei present initially	1000	900	810	729	629	567	510
Number of unstable nuclei that decay	100	90	81	73	63	57	51
Number of unstable nuclei remaining	900	810	729	656	566	510	459

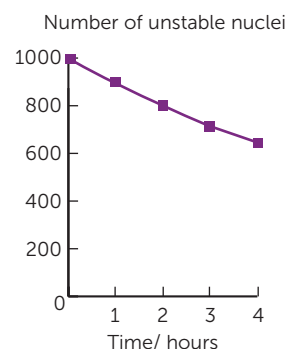


Fig. 22.17 Number of nuclei versus time

A model of radioactive decay

- 1 Suppose 1000 dice are thrown and all the dice with a six face-up are removed. Since the probability of a six is the same as any other number, the number of sixes should be 167, equal to $1/6$ th of the total thrown. The number of dice remaining would be 833 (= $1000 - 167$)
- 2 If the throw is repeated again and again with the dice not showing a six after each throw, the number of dice decreases by $1/6$ th each time so the number remaining decreases exponentially, as shown in Table 22.3 and Fig. 22.18.

Table 22.3 Numbers of sixes thrown

Throw number	1	2	3	4	5	6	7
Initial number of dice	1000	833	694	578	482	402	335
Number of sixes	167	139	116	96	80	67	56
Number of dice remaining	833	694	578	482	402	335	279

See www.palgrave.com/foundations/breithaupt to model radioactive decay using a spreadsheet.

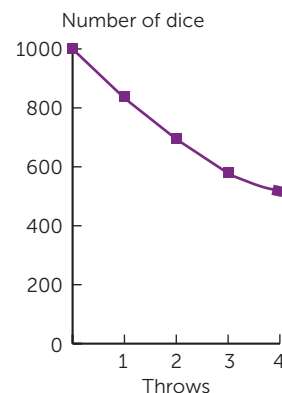


Fig. 22.18 A game of chance

Activity

The activity of a sample of a radioactive isotope is the number of particles or photons emitted per unit time from the sample.

The unit of activity is the becquerel (Bq), equal to 1 disintegration per second.

Since each unstable nucleus emits either an alpha particle, a beta particle or a gamma photon when its nucleus becomes more stable, the activity is also the number of unstable nuclei that decay (i.e. become more stable) per unit time.

Because the number of unstable nuclei decreases with time, the activity also decreases with time. As explained earlier, the decrease is exponential because the number of nuclei that decay per unit time depends only on the number of nuclei present at that time. The time taken for the activity of a radioactive isotope to halve is defined as the **half-life**.

The half-life of a radioactive isotope is the time taken for the activity of the isotope to decrease to 50% of its initial activity.

This is the same as the time taken for the number of unstable nuclei to decrease to 50% of the initial number.

For example, the half-life of strontium 90 is 28 years. Thus a sample of strontium 90 with an initial activity of 80 kBq would have an activity of 40 kBq after 28 years and an activity of 20 kBq after a further 28 years.

Fig. 22.19 shows how the activity of such a sample would decrease. The number of nuclei of strontium 90 and also the mass of strontium 90 would decrease at a similar rate because the activity is proportional to the number of strontium 90 nuclei present.

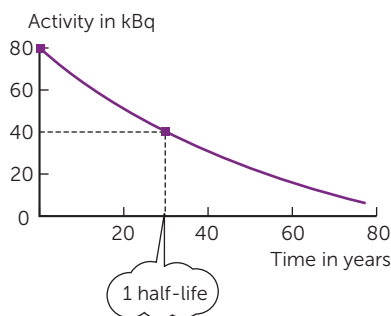


Fig. 22.19 Activity versus time for strontium 90

Measurement of the half-life of protoactinium 234

Protoactinium 234 is a beta emitter produced by radioactive decay from thorium 234, which has a much longer half-life. A sealed bottle containing an organic solvent is used to separate protoactinium from thorium to enable the activity of the protoactinium to be measured at intervals. The bottle is first shaken to mix the solvent layer and the aqueous layer, then placed at rest to allow the solvent layer to form above the aqueous layer. The end of a GM tube is held adjacent to the solvent layer. The activity of the protoactinium isotope in the solvent layer is determined at intervals by measuring the count rate at 30 second intervals. The half-life can be determined from a graph of count rate versus time as in Fig. 22.19.

22.4 QUESTIONS

- 1 A solvent contains a fixed quantity of a radioactive isotope with a half-life of 68 seconds. The isotope decays to form a stable product. The activity of the isotope when the solvent layer was formed was measured using a GM tube, which gave a corrected count rate of 420 counts per minute.
 - a) Calculate the corrected count rate (i) two half-lives (i.e. 136 s), (ii) three half-lives (i.e. 204s) after the solvent layer formed.
 - b) Estimate how long it takes for the activity to decrease so the corrected count rate falls below 25 counts per minute.
- 2 Strontium 90 is a beta-emitting isotope with a half-life of 28 years. A certain sample of strontium 90 has an activity of 160 kBq.
 - a) Calculate the activity (i) 28 years later, (ii) 56 years later.
 - b) Estimate the activity of the sample 100 years later.

22.5 The mathematics of radioactive decay

Consider a pure sample of a radioactive isotope that decays to form a stable isotope. Let N_0 represent the number of unstable nuclei initially present and suppose there are N unstable nuclei present at time, t , later. Since radioactive decay is a random process, the number, ΔN , of unstable nuclei that decay in a certain time interval, Δt , is proportional to

- 1 the number of unstable nuclei present, N ;
- 2 the time interval, Δt .

Hence $\Delta N = -\lambda N \Delta t$, where λ is called the **decay constant** of the isotope. The minus sign means that the change of N is a decrease.

Since $\Delta N/N = -\lambda \Delta t$, and $\Delta N/N$ is the probability of decay of a single nucleus, it follows that λ is the probability of decay per unit time. Rearranging this equation gives $\Delta N/\Delta t = -\lambda N$.

This is usually written in the form

$$\frac{dN}{dt} = -\lambda N$$

where dN/dt , the rate of change of the number of nuclei, is the activity of the isotope.

The mathematical solution to this equation is written

$$N = N_0 \exp(-\lambda t)$$

where $\exp$ is referred to as the **exponential function**. The function is usually written as $e^{-\lambda t}$. The special feature about this function is that its rate of change is proportional to the function itself.

Half-life and the decay constant

The half-life, $T_{1/2}$, is the time taken for the number of nuclei of a radioactive isotope to decrease by 50%

Consider a radioactive isotope in which there are initially N_0 unstable nuclei. The number of unstable nuclei at time, t , later, $N = N_0 e^{-\lambda t}$.

At $t = T_{1/2}$, $N = \frac{1}{2} N_0$, hence $\frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}$. Cancelling N_0 from both sides gives $\frac{1}{2} = e^{-\lambda T_{1/2}}$.

$$\therefore e^{\lambda T_{1/2}} = 2$$

$$\lambda T_{1/2} = \log_e 2$$

Mathematical notes

- 1 For any function of the form $y = x^n$, $\frac{dy}{dx} = nx^{n-1}$. See section 15.5, p. 223 if necessary.
- 2 The exponential function $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3 \times 2} + \frac{x^4}{4 \times 3 \times 2} + \dots$
- 3 Hence $\frac{d}{dx}(e^x) = 0 + 1 + x + \frac{x^2}{2} + \frac{x^3}{3 \times 2} + \dots = e^x$
Thus the rate of change (with respect to x) of e^x is e^x .
- 4 If $y = e^x$, then $x = \log_e y$, since $\log_e$ is the inverse function of the exponential function. Note that most calculator keys denote $\log_e$ by the symbol $\ln$.
- 5 To prove that $N = N_0 e^{-\lambda t}$ is the solution of $\frac{dN}{dt} = -\lambda N$:
$$\frac{d}{dt}N = N_0 \frac{d}{dt}(e^{-\lambda t}) = \lambda N_0 \frac{d}{dx}(e^{-x}) \text{ where } x = \lambda t$$

Since $\frac{d}{dx}(e^{-x}) = -e^{-x}$, then $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$.

Activity and the decay constant**Note**

The key for e on some calculators is $\ln$.

The activity of a radioactive isotope, $A =$ the magnitude of dN/dt , the number of disintegrations per second of the nuclei of the isotope. Since

$$\frac{dN}{dt} = -\lambda N,$$

then the activity, A , can be written as

$$A = \lambda N$$

Also, since $N = N_0 e^{-\lambda t}$, then the activity, $A = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$, where A_0 , the initial activity, $= \lambda N_0$.

22.2 WORKED EXAMPLE

Strontium 90 is a radioactive isotope with a half-life of 28 years. Calculate

- a) the decay constant of this isotope,
- b) the activity of a 1.0 milligram sample of strontium 90,
- c) the activity of a 1.0 milligram sample in 100 years time.

The Avogadro constant, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$.

Solution

$$\begin{aligned} \text{a) } AT_{1/2} &= \log_e 2, \text{ hence } \lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{28 \text{ years}} = 0.0248 \text{ yr}^{-1} \\ &= \frac{0.0248}{365.25 \times 24 \times 3600 \text{ s}} = 7.8 \times 10^{-10} \text{ s}^{-1} \end{aligned}$$

- b) 90 g of strontium 90 contains 6.02×10^{23} atoms of the isotope.

$$\text{Hence 1.0 mg contains } \frac{6.02 \times 10^{23}}{90 \times 1000} = 6.7 \times 10^{18} \text{ atoms.}$$

$$\begin{aligned} \text{Hence the activity of 1.0 mg, } A_0 &= \lambda N_0 = 7.8 \times 10^{-10} \times 6.7 \times 10^{18} \\ &= 5.2 \times 10^{-9} \text{ Bq.} \end{aligned}$$

- c) Using $A = A_0 e^{-\lambda t}$ gives $A = 5.2 \times 10^{-9} e^{-(0.0248 \times 100)} = 4.4 \times 10^{-8} \text{ Bq}$

22.5 QUESTIONS

The Avogadro constant = $6.02 \times 10^{23} \text{ mol}^{-1}$

- 1 Polonium 210 is a radioactive isotope with a half-life of 140 days. Calculate
 - a) the decay constant of this isotope,
 - b) the number of atoms in 1.0 mg of pure polonium 210,
 - c) the activity of 1.0 mg of this isotope,
 - d) the activity of this isotope one year later.
- 2 Cobalt 60 is a radioactive isotope with a half-life of 1940 days. A sample of this isotope has an activity of 2.0 MBq. Calculate
 - a) the decay constant,
 - b) the number of atoms of the isotope,
 - c) the time taken for the activity to decrease to 0.1 MBq.

SUMMARY

- **Alpha (α) radiation**

- consists of helium 4 nuclei, each containing two protons and two neutrons. An α -particle is emitted from an unstable nucleus with too many protons and neutrons
- is highly ionising, absorbed by paper and has a well-defined range in air of a few centimetres.

- **Beta (β) radiation**

- consists of fast-moving electrons. A β -particle is emitted from an unstable nucleus with too many neutrons. A neutron in the nucleus changes into a proton and a β -particle is created and emitted at the instant of change
- is less ionising than α -radiation, absorbed by metal plates of a few millimetres thickness and has a range in air of more than 50 cm.

- **Gamma (γ) radiation**

- consists of high energy photons, emitted by an unstable nucleus with excess energy
- is less ionising than beta radiation, absorbed by lead plates of several centimetres thickness and has an unlimited range in air. The intensity from a point source follows the inverse square law.

- **Definitions**

- An isotope comprises atoms with the same number of protons and neutrons.
- The activity of a radioactive isotope is the number of disintegrations per second.
- The half-life, $T_{1/2}$, of a radioactive isotope is the time for half the atoms of the isotope to disintegrate.

- **Equations**

Decay constant, $\lambda = \frac{\log_e 2}{T_{1/2}}$

Activity, $A = \text{magnitude of } \frac{dN}{dt} = \lambda N$

Radioactive decay equation, $\frac{dN}{dt} = -\lambda N$

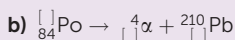
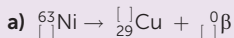
Solution of the radioactive decay equation, $N = N_0 e^{-\lambda t}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

The Avogadro constant = $6.02 \times 10^{23} \text{ mol}^{-1}$

22.1 Each of the following equations represents the decay of an unstable nucleus. Copy and complete each equation.



22.2 In an experiment to determine the type of radiation emitted by a radioactive source, the following measurements were made:

Background count in 5 minutes = 128, 136, 138;

Counts in 3 minutes with the source at 10 cm from the GM tube

- with no absorber between the tube and the source = 565, 572, 552;
- with a 0.8 mm thick metal plate between the tube and the source as shown in Fig. 22.20 = 384, 368, 372.

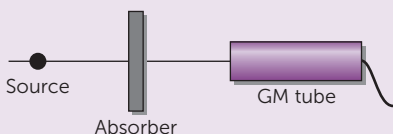


Fig. 22.20
Stimulated emission

- Calculate the average background count rate in counts per minute.
- The source is known to emit one type of radiation only. What type of radiation does it emit? Give a reason for your answer.
- What percentage of the radiation incident on the plate is transmitted by the plate?
- Calculate the expected count rate if a further identical plate was placed in the path of the beam.

22.3

- a) A GM tube was used to measure the count rate due to gamma radiation from a point source inside a sealed container, as shown in Fig. 22.21. When the distance between the tube and the container was 90 mm, the corrected count rate was 1284 counts per minute. When this distance was increased to 190 mm, the corrected count rate was 330 counts per minute.

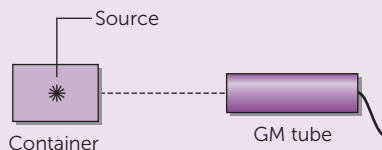


Fig. 22.21 An X-ray tube

- Use these results to show that the source was 10 mm inside the container.
 - Calculate the corrected count rate if the distance between the tube and the source had been increased to 240 mm.
- b) The source in (a) was replaced by a different source. Describe how you would identify the type of radiation emitted by the source, assuming it emits only one type.

22.4 Sodium 24 is a radioactive isotope with a half-life of 14.8 hours. Calculate

- the decay constant for this isotope,
- the number of atoms in a 1.0 mg sample of this isotope,
- the activity of a 1.0 mg pure sample of this isotope,
- the activity of such a sample exactly 24 hours later.

22.5 Plutonium 239 is a radioactive isotope with a half-life of 100 years.

- Calculate
 - the decay constant of this isotope,
 - the number of atoms in 1.0 g of this isotope.
- A pure sample of this isotope has an activity of 1000 MBq. Calculate
 - its mass,
 - its activity in 10 years time.

Energy from the Nucleus

OBJECTIVES

After working through this unit, you should be able to:

- describe the characteristics of the strong nuclear force
- explain what is meant by binding energy and mass defect
- calculate the binding energy of a nucleus and describe the shape of the binding energy curve
- use the binding energy curve to explain why energy is released as a result of fission and fusion
- describe the main features of a nuclear reactor
- discuss the advantages and disadvantages of nuclear power
- describe the use of high-energy accelerators to probe the nucleus
- describe the quark model of the nucleus

CONTENTS

- 23.1 The nature of force
- 23.2 Binding energy
- 23.3 Nuclear power
- 23.4 Probing the nucleus
- Summary
- Revision questions

23.1 The nature of force

Fundamental forces

The force of gravity holds us on to the Earth and keeps the Earth on its orbit round the Sun. The force between charged objects, the electrostatic force, holds the atoms together in the human body and prevents us from disappearing into the Earth. Most of the forces acting on objects we can see are either due to gravity or due to the electrostatic force. For example, the tension in a cable supporting a lift is due to the electron bonds between the atoms of the cable. The force of gravity and the electrostatic force, more generally known as the electromagnetic force, are two of the fundamental forces of nature. See [Fig. 23.1](#) overleaf.

- The **force of gravity** acts between any two material objects. The force of gravity between any two objects is always attractive and varies with distance apart according to the inverse square law. See p. 441.
- The **electromagnetic force** acts between any two charged objects. It was shown in the nineteenth century that electrostatic forces and magnetic forces are essentially the same, differing only because the electrostatic force acts when charges are at rest and the magnetic force acts when they are in relative motion. The electromagnetic force varies with distance according to the inverse square law. See p. 249.

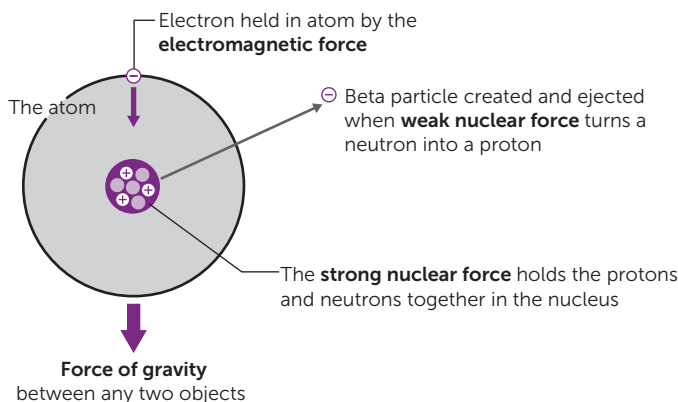


Fig. 23.1
Fundamental forces

- The **strong nuclear force** is an attractive force acting between protons or neutrons over a short range of no more than a few femtometres. (1 femtometre = $1 \text{ fm} = 10^{-15} \text{ m}$).
- The **weak nuclear force** is the force responsible for beta decay, which occurs when a neutron in a nucleus changes into a proton. The phenomenon of beta decay cannot be explained in terms of gravity or the electromagnetic force or the strong nuclear force. Recent research has led to the conclusion that the weak nuclear force and the electromagnetic force are different aspects of the same fundamental force, referred to as the **electroweak** force.

Force carriers

When two objects interact, they exert equal and opposite forces on each other. Since force is defined as change of momentum per second, an interaction causes transfer of momentum if the objects are free to move. Each fundamental type of force is thought to act through exchange of quanta between the two objects, as shown in Fig. 23.2. The quantum of each type of force may be thought of as the carrier of the force. The quanta may be described as ‘virtual’ because they cannot be detected without altering the interaction. Nevertheless, the idea of virtual quanta is supported by firm experimental evidence for all the fundamental forces except gravity (up to when this text was written in 2014).

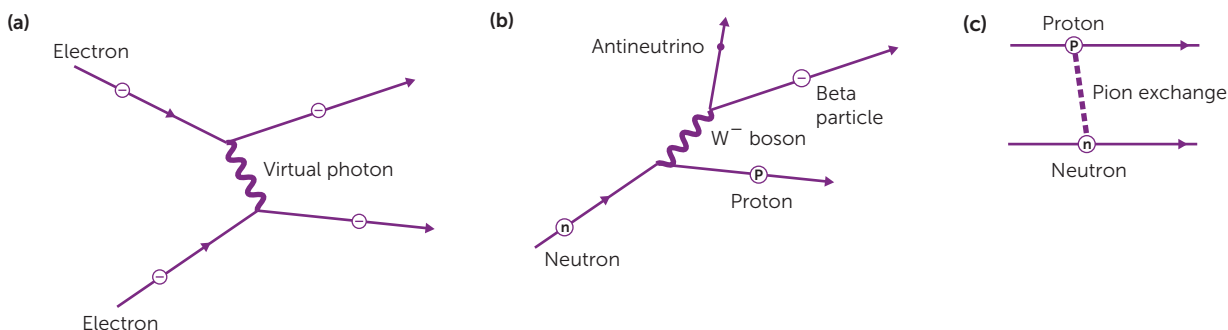


Fig. 23.2 Force carriers (a) coulomb repulsion, (b) the weak interaction, (c) the strong nuclear force

- The **electroweak force** is thought to be due to the exchange of:
 - 1 **Virtual photons** in the case of electromagnetic interactions. Photons are massless, have an infinite range and carry energy and momentum. The idea that electrons absorb photons was established in the photoelectric effect where photons are absorbed by electrons; the idea that electrons emit photons is used to explain the origin of optical and X-ray spectra. The electromagnetic interaction between two charged objects is therefore thought to be due to virtual photons exchanged between the two charged objects.
 - 2 **W bosons** in the case of the weak interaction. The weak interaction is thought to be due to virtual particles called **weak (W) bosons** which carry mass, momentum and energy. W bosons are thought to be unstable, with a range of no more than a fraction of a femtometre. A beta particle is emitted by a nucleus when a neutron in the nucleus changes into a proton. The neutron emits a W boson at the instant of change and the W boson disintegrates into a beta particle and another particle called an antineutrino.
- The **strong nuclear force** is thought to be due to the exchange of particles called **pions** between protons and neutrons in the nucleus. Pions were discovered as a result of exposing photographic plates to cosmic radiation at high altitude. Cosmic radiation consists of fast-moving particles, such as protons from the Sun. When such a high-energy particle hits a nucleus, it can knock pions as well as protons or neutrons out of the nucleus. Pions are unstable and therefore have a limited range.
- **Gravity** could be the result of the exchange of massless particles called **gravitons**. Experimental evidence for gravitational radiation was first obtained in 1975 from two neutron stars in orbit about each other. However, there is no firm evidence at present for the graviton or any other quantised form of gravity as the exchange particle of the gravitational force.

The strong nuclear force

The nucleus of an atom consists of protons and neutrons packed together at the centre of the atom. The diameter of the nucleus is of the order of a few femtometres, approximately 10 millionths of the diameter of the atom. The protons carry like charges, so they tend to repel each other due to the electrostatic force of repulsion between like charges. The fact that most nuclei are stable must mean that the protons and neutrons in a nucleus are held together by an attractive force which acts against the electrostatic force of repulsion. This attractive force is called the **strong nuclear force**.

- The **strength** of the strong nuclear force can be estimated from the fact that it must be sufficiently strong to overcome the electrostatic nuclear force between two protons at a distance of about 1 fm apart. Using Coulomb's Law (see p. 248) for the electrostatic force therefore gives

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{4\pi\epsilon_0 \times (10^{-15})^2} = 230 \text{ N}$$

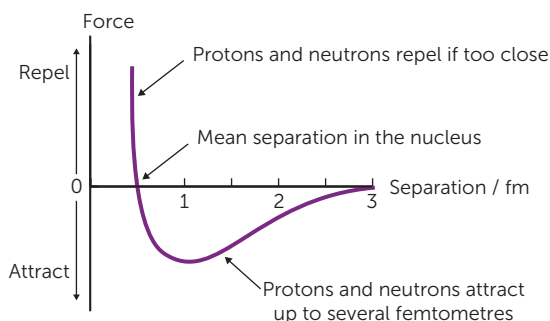


Fig. 23.3 The strong nuclear force

The strong nuclear force must be of the order of 100 N or more to overcome the electrostatic force of repulsion. The mass of a proton is of the order of 10^{-27} kg, so its weight ($= mg$) is about 10^{-26} N. The strong nuclear force is more than 10^{28} times the weight of a proton.

- The range of the strong nuclear force is no more than a few femto- metres, unlike the electrostatic force which stretches to infinity. The largest nuclei, those of the heaviest elements, are of diameters of the order of no more than 10 fm. If the strong nuclear force extended much beyond a few femtometres, nuclei would be much larger than even the largest known nuclei. See Fig. 23.3.

23.1 QUESTIONS

- State the fundamental force responsible for
 - the path of a satellite round the Earth,
 - release of energy in (i) alpha decay, (ii) beta decay,
 - preventing you from falling through the floor.
- When alpha particles from a radioactive source pass through air, some of the particles collide directly with the nuclei of the atoms.
 - Calculate the electrostatic force between an alpha particle and a nitrogen $^{14}_7\text{N}$ nucleus at a separation of (i) 10 fm, (ii) 2 fm. ($e = 8.85 \times 10^{-12} \text{ F m}^{-1}$).
 - Use your knowledge of the strong nuclear force to explain why an alpha particle directed at a nitrogen nucleus is repelled by the nucleus unless it has sufficient kinetic energy.

23.2 Binding energy

Energy from reactions

Chemical energy is released when a fuel such as oil burns in air. The fuel molecules react with oxygen molecules to release energy. The atoms of each fuel molecule break away from each other and combine with oxygen atoms to form carbon monoxide, carbon dioxide and water vapour. Sufficient energy is released when the atoms from a fuel molecule combine with oxygen to enable more fuel molecules to break up. The energy is released when the bonds between the fuel atoms and oxygen atoms are formed. Such bonds involve electrons in the atoms becoming trapped, releasing energy in the process. Burning fuel releases about 50 MJ of heat per kilogram of fuel burned, which is equivalent to about 1019 J per atomic mass unit. This is about the same as the work done to move a single electron through a potential difference of 1 volt.

The **electron volt (eV)** is the work done to move an electron through a potential difference of exactly 1 volt. Since the charge of an electron is $1.6 \times 10^{-19} \text{ C}$ and

1 volt is defined as work done per unit charge, then $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. Note that $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.

Nuclear reactions involve energy changes of the order of millions of electron volts (MeV). Imagine pulling a nucleus apart, pulling off the protons and neutrons one at a time. Work needs to be done on a proton or a neutron to pull it out of a nucleus. This is necessary to overcome the strong nuclear force. Even though this force acts over an exceedingly short distance of no more than a few femtometres, its strength (of the order of 100 N or more) is so great that the work done is of the order of 10^{-13} J ($= 100 \text{ N} \times 1 \text{ fm}$). Thus the energy changes in nuclear reactions are of the order of a million times greater than the energy changes in chemical reactions.

The binding energy of a nucleus is defined as the work done to separate a nucleus into its constituent neutrons and protons.

Mass and energy

Whenever energy is gained by a body, the body's mass increases. When a body loses energy, its mass decreases. Transfer of energy without transfer of matter to or from a body causes its mass to change. The exact mechanism of how this process works is not yet known. However, the scale of the changes was determined by Einstein who proved that the total energy, E , of a body is related to its mass, m , in accordance with the now-famous equation

$$E = mc^2$$

where c is the speed of light in free space ($= 3.0 \times 10^8 \text{ m s}^{-1}$).

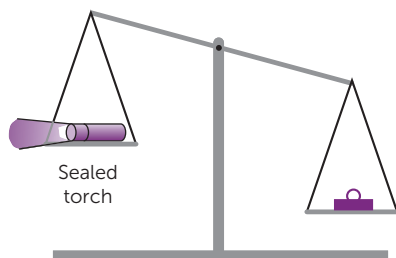


Fig. 23.4 Energy and mass – the mass of a sealed torch is reduced as it loses light energy

For example, the mass of a sealed torch emitting light becomes smaller as it loses light energy, as shown in Fig. 23.4. The scale of the change is insignificant where energy is released due to chemical changes. Prove for yourself that the mass loss when a 10 W torch is left on for 10 hours is about $4 \times 10^{-12} \text{ kg}$ ($= 10 \times 36000 \text{ J} \div c^2$).

However, in nuclear changes, the mass changes are significant as the energy changes are much greater than for chemical reactions. Prove for yourself that 10^{-13} J of energy released by a nucleus corresponds to a loss of mass of about 0.001 u ($= 10^{-30} \text{ kg}$ approximately) which is about 0.1% of the mass of a proton.

Note

1 u is equivalent to 931.3 MeV. This can be proved using $E = mc^2$, with $m = 1 \text{ u} = 1.660 \times 10^{-27} \text{ kg}$ and $c = 2.998 \times 10^8 \text{ m s}^{-1}$, which gives $E = 1.492 \times 10^{-10} \text{ J}$ or 931.3 MeV, since $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$.

When a nucleus is formed from separate neutrons and protons, energy is released equal to the binding energy of the nucleus. The mass of the nucleus is less than the mass of its constituent protons and neutrons because energy is released. This mass difference can be measured and is known as the mass defect of the nucleus.

The mass defect of a nucleus is the difference between the mass of a nucleus and the mass of its constituent protons and neutrons.

A nucleus of an isotope ${}_Z^AX$ consists of Z protons and $(A - Z)$ neutrons. Its mass defect, Δm , is therefore given by the equation

$$\Delta m = Zm_p + (A - Z)m_n - M$$

where M is the mass of the nucleus, m_p is the mass of a proton ($= 1.00728$ u) and m_n is the mass of a neutron ($= 1.00867$ u).

The binding energy of a nucleus can therefore be calculated from the mass defect, Δm , using the equation $E = mc^2$ in the form

$$\text{Binding energy, BE} = \Delta mc^2$$

23.1 WORKED EXAMPLE

The mass of a nucleus of ${}_2^4\text{He}$ is 4.00150 u. Calculate

a) its mass defect,

b) its binding energy.

The mass of a proton = 1.00728 u and the mass of a neutron = 1.00867 u.

Solution

a) $Z = 2$, $A = 4$

$$\begin{aligned} \therefore \Delta m &= Zm_p + (A - Z)m_n - M \\ &= 2 \times 1.00728 + 2 \times 1.00867 - 4.00150 = 0.0304 \text{ u} \end{aligned}$$

b) $\text{BE} = \Delta mc^2 = 0.0304 \times 931 = 28.3 \text{ MeV}$

The binding energy curve

The binding energy of a nucleus is the work needed to separate the nucleus into its constituent protons and neutrons. The protons and neutrons in a nucleus are collectively referred to as **nucleons**. The binding energy of a nucleus depends on the number of nucleons in a nucleus. The stability of different types of nuclei can be compared by calculating the binding energy per nucleon of each type of nucleus. This is the total binding energy of the nucleus divided by the number of nucleons in the nucleus. For example, if the binding energy per nucleon of nucleus, X, is 8.0 MeV per nucleon and the binding energy per nucleon of a nucleus, Y, is 8.5 MeV per nucleon, then Y is more stable than X because more work needs to be done to remove a nucleon from Y than from X.

$$\text{Binding energy per nucleon of a nucleus} = \frac{\text{binding energy of the nucleus}}{\text{total number of nucleons}}$$

The binding energy per nucleon of a nucleus ${}^A_Z\text{X}$ can be calculated as follows, if the mass of the nucleus is known.

Step 1 Calculate the mass defect of the nucleus using the equation

$$\Delta m = Zm_p + (A - Z)m_n - M$$

Step 2 Calculate the binding energy of the nucleus using the equation

$$\text{BE} = \Delta mc^2$$

Step 3 Calculate the binding energy per nucleon using the equation

$$\text{BE per nucleon} = \frac{\text{BE}}{A}$$

Note

If the atomic mass of X is given, the mass, M , of the **nucleus** is calculated by subtracting the mass of Z electrons from the atomic mass.

Fig. 23.5 shows how the binding energy per nucleon for every known type of nucleus varies with nucleon number. The graph shows that the binding energy per nucleon increases as A increases, up to about $A = 60$, then decreases gradually.

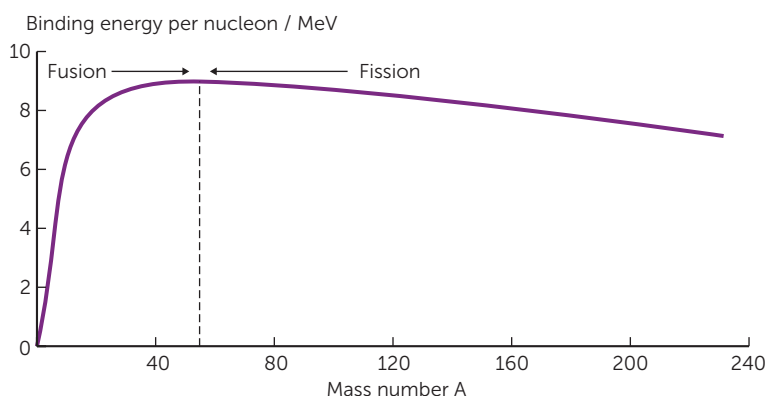


Fig. 23.5 The binding energy curve

- The most stable nuclei occur at about $A = 60$.
- The helium nucleus (i.e. the alpha particle) is very stable compared to other light nuclei.
- Certain heavy nuclei, such as U 235, can become more stable by splitting into two approximately equal nuclei. This process is known as **nuclear fission**. When a heavy nucleus fissions, the binding energy per nucleon rises by about 1 MeV per nucleon. The energy released from a single fission event is therefore of the order of 200 MeV ($= 1 \text{ MeV per nucleon} \times \text{approximately 200 nucleons in a heavy nucleus}$).
- Light nuclei can be forced to fuse together if at very high temperatures. This process is known as **nuclear fusion**. For example, this occurs in the Sun's core where hydrogen is converted into helium. When nuclear fusion occurs, the binding energy per nucleon rises, so energy is released as a result.

23.2 WORKED EXAMPLE

Calculate the binding energy per nucleon, in MeV, of

a) a $^{12}_6\text{C}$ nucleus,

b) a $^{56}_{26}\text{Fe}$ nucleus.

(Atomic masses: Carbon 12 = 12.00000 u, Iron (Fe) 56 = 55.93493 u, electron mass = 0.00055 u, proton mass = 1.00728 u, neutron mass = 1.00867 u) 1 u = 931 MeV.

Solution

a) Mass defect, $\Delta m = Zm_p + (A - Z)m_n - M$

$$= 6 \times 1.00728 + 6 \times 1.00867 - (12.00000 - 6 \times 0.00055)$$

$$= 0.099 \text{ u}$$

$$\therefore \text{BE of C 12 nucleus} = 0.099 \text{ u} \times \frac{931 \text{ MeV}}{1 \text{ u}} = 92.1 \text{ MeV}$$

$$\therefore \text{BE per nucleon} = \frac{\text{BE}}{A} = \frac{92.1}{12} = 7.7 \text{ MeV per nucleon}$$

b) Mass defect, $\Delta m = Zm_p + (A - Z)m_n - M$

$$= 26 \times 1.00728 + 30 \times 1.00867 - (55.93493 - 26 \times 0.00055)$$

$$= 0.529 \text{ u}$$

$$\therefore \text{BE of Fe 56 nucleus} = 0.529 \text{ u} \times \frac{931 \text{ MeV}}{1 \text{ u}} = 492 \text{ MeV}$$

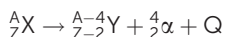
$$\therefore \text{BE per nucleon} = \frac{\text{BE}}{A} = \frac{492}{56} = 8.8 \text{ MeV per nucleon}$$

Q values

When a nuclear change occurs, binding energy changes cause the total mass of the products to differ from the total initial mass.

- If the total initial mass exceeds the total final mass, energy is released as a result of the change.
- If the total initial mass is less than the total final mass, energy is needed to make the change occur.

The **Q value** of a nuclear process is the energy released by the process ($Q > 0$) or needed for the process to occur ($Q < 0$).

Alpha decay

$$\therefore Q = \text{Mass of nucleus of } {}^A_Z\text{X} - [\text{mass of nucleus of } {}^{A-4}_{Z-2}\text{X} + \text{mass of } \alpha].$$

Since an atom of X has Z electrons, an atom of Y has $Z - 2$ electrons and a helium atom is an alpha particle + 2 electrons,

$$Q = \text{atomic mass of } {}^A_Z\text{X} - [\text{atomic mass of } {}^{A-4}_{Z-2}\text{Y} + \text{atomic mass of helium 4}].$$

Beta decay



$$\therefore Q = \text{mass of nucleus of } {}_Z^AX - [\text{mass of nucleus of } {}_{Z+1}^AY + \text{mass of } \beta \text{ particle}].$$

Since an atom of X has Z electrons, an atom of Y has Z + 1 electrons and a beta particle is an electron,

$$Q = \text{atomic mass of } {}_Z^AX - \text{atomic mass of } {}_{Z+1}^AY.$$

23.2 QUESTIONS

Data: electron mass = 0.00055 u, proton mass = 1.00728 u, neutron mass = 1.00867 u, 1 u = 931 MeV.

- 1 a) Calculate the binding energy per nucleon of
(i) an ${}^{16}_8\text{O}$ nucleus, (ii) an ${}^{90}_{38}\text{Sr}$ nucleus.
b) Calculate the change of mass of (i) an atom when it releases a light photon of wavelength 600 nm,
(ii) a nucleus when it releases a gamma photon of wavelength 100 fm.
(Atomic masses: oxygen 16 = 15.99492, strontium 90 = 89.90730)
- 2 a) The uranium nucleus ${}^{238}_{90}\text{U}$ decays by α -particle emission to form a nucleus of an isotope of thorium (Th).
(i) Copy and complete the equation for this decay.
(ii) Calculate the Q value of this decay.
(Atomic masses: U 238 = 238.05076, Th 232 = 232.03821, Th 233 = 233.04143, Th 234 = 234.04357, He 4 = 4.00260)
b) The cobalt nucleus ${}^{60}_{27}\text{Co}$ decays by beta emission to form a nucleus of an isotope of nickel (Ni). (i) Copy and complete the equation for this decay.
(ii) Calculate the Q value of this decay.
(Atomic masses: Co 60 = 59.93381, Ni 59 = 58.93434, Ni 60 = 59.93078, Ni 61 = 60.93105)

23.3 Nuclear power

Induced fission

Uranium and other heavy nuclei are unstable and usually decay by emitting alpha particles. However, these very heavy nuclei can be made to fission (i.e. split into two approximately equal fragments) as a result of bombardment by neutrons. This process is known as **induced fission**. Fast neutrons at kinetic energies of the order of 1 MeV are needed to fission most fissionable nuclei, although U 235 can also be fissioned by slow neutrons at kinetic energies of the order of 0.1 eV or less.

In a fission process, two or three neutrons are released, which may then go on to produce further fission. Energy is released in the fission process as kinetic energy of the fission fragments and the released neutrons, as well as in the form of gamma radiation. The fission fragments themselves are neutron-rich isotopes and therefore emit beta particles.

Fig. 23.6 overleaf shows how the **liquid drop model** of the nucleus is used to describe induced fission. The oscillating nucleus is similar to a vibrating liquid drop. At its most distorted, the strong nuclear force between the nucleons at the neck of the distorted nucleus is sufficient to prevent the two halves of the nucleus breaking away due to their electrostatic repulsion. If the nucleus is struck by a neutron at the point where it is most distorted, the added energy from the incident neutron is just enough to split the nucleus.

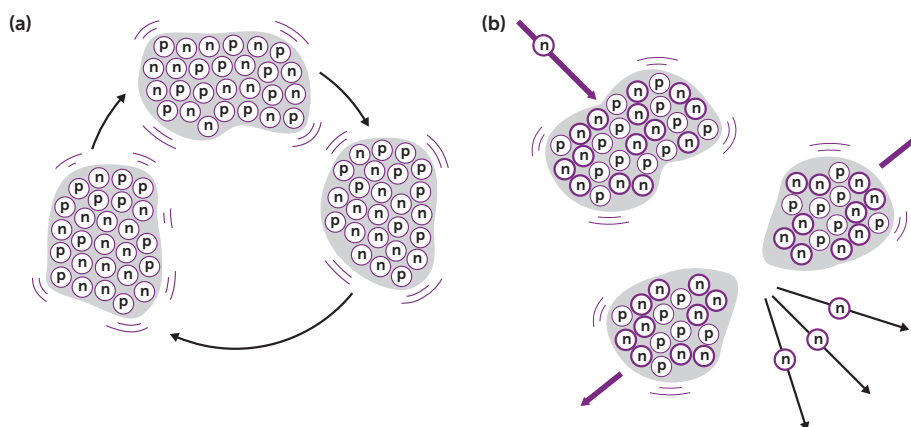


Fig. 23.6 (a) the liquid drop model of the nucleus, (b) induced fission

The Q value of a fission reaction can be estimated from the binding energy curve. Each nucleon has a binding energy of about 7.5 MeV per nucleon in the parent nucleus and about 8.5 MeV per nucleon in the fission fragments. Hence the binding energy per nucleon rises by about 1 MeV per nucleon, and about 1 MeV per nucleon is released. Thus the energy released by a single fission event is about 200 MeV ($= 200 \text{ nucleons} \times 1 \text{ MeV per nucleon}$). Most of this energy is carried away by the fission fragments as recoil kinetic energy.

Further fission

The neutrons released when a heavy nucleus is fissioned are capable of producing further fission in the same material, if the material is either uranium 235 or plutonium 239. Most other fissionable isotopes absorb fast neutrons without fission instead of undergoing further fission.

Uranium 235

Uranium 235 is one of the two isotopes in natural uranium. Natural uranium is more than 99% U 238 and less than 1% U 235.

- In pure U 235, when a nucleus fissions, the released neutrons will produce further fission if they do not escape from the material first. Further fission would result in more neutrons being released and the number of fission events would escalate in an uncontrollable **chain reaction**, releasing all the energy in a very short time and thus causing a massive explosion. See Fig. 23.7.
- Enriched uranium containing 2% or 3% of U 235 is used in a nuclear reactor. Without enrichment, the fission neutrons are more likely to be absorbed by U 238 than to produce further fission of U 235. Even so, the fission neutrons need to be slowed down to produce further fission of U 235. This is achieved in the **thermal nuclear reactor** by the presence of a moderator, as explained on p. 377. In addition, rods made of boron are used to control the chain reaction. The control rods absorb neutrons to ensure a steady rate of fission events. The control rods are inserted into the nuclear reactor core at just sufficient depth to ensure an average of exactly one neutron per fission goes on to produce further fission.

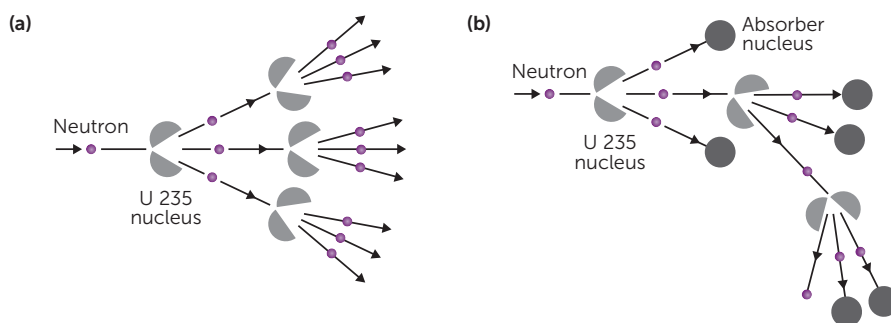


Fig. 23.7 Chain reactions (a) uncontrolled (b) controlled

Plutonium 239

Plutonium 239 is an artificial isotope produced in a thermal nuclear reactor as a result of neutron absorption by U 238 to form U 239, which decays by beta emission to form the isotope neptunium 239. This change occurs rapidly as the half-life of uranium 239 is 24 minutes. Neptunium 239 decays by beta emission with a half-life of 56 hours to form plutonium 239, which has a half-life of 100 years.

Fast neutrons cause plutonium 239 to fission, resulting in the release of further fast neutrons which then produce further fission if the total mass of plutonium 239 present is sufficiently large. The **fast breeder** reactor is designed to use plutonium 239 as its fuel. The plutonium core is surrounded by a blanket of U 238 which absorbs neutrons that escape from the core. The U 238 nuclei in the blanket are converted into plutonium 239 nuclei as a result of absorbing neutrons from the core. See Fig. 23.8. In this way, the reactor ‘breeds’ its own fuel from the surrounding uranium. No moderator is needed in a fast breeder reactor. The chain reaction is controlled by control rods in the same way as in the thermal nuclear reactor.

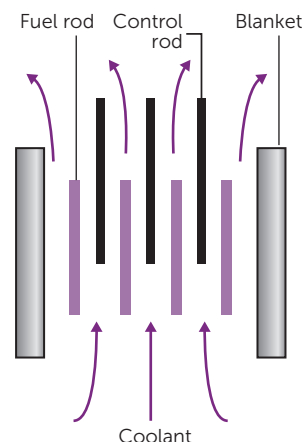


Fig. 23.8 The fast breeder reactor

The thermal nuclear reactor

Enriched uranium, used as the fuel in a thermal nuclear reactor, contains about 2% U 235. As explained on p. 375, about 200 MeV of energy is released when a U 235 nucleus fissions. Thus the energy released by 1 mole (ie. 0.235 kg) of U 235 is $2 \times 10^{13} \text{ J} (= 200 \times 1.6 \times 10^{-13} \text{ J} \times 6 \times 10^{23})$. The total energy released as a result of 1 kg of U 235 fissioning is therefore about $8 \times 10^{13} \text{ J} (= 2 \times 10^{13} \text{ J} \div 0.235 \text{ kg approx.})$. Since 50 kg of enriched uranium contains about 1 kg of U 235, the energy released as a result of fissioning all the U 235 nuclei in 1 kg of enriched uranium is about $1.6 \times 10^{12} \text{ J} (= 8 \times 10^{13} \text{ J} \div 50 \text{ kg})$. In comparison, the energy released from burning 1 kg of oil is about 30 MJ. In other words, 1 kg of enriched uranium releases the same amount of energy as 50 000 kg of oil.

The fuel in a thermal nuclear reactor is contained in cylindrical metal tubes called fuel rods. The fuel rods are inserted into the moderator which may be water, as in the pressurised water reactor (PWR), or graphite, as in the advanced gas cooled reactor (AGR). The moderator is enclosed in a thick-walled steel vessel, the reactor core, inside a concrete chamber. Fig. 23.9 shows a cross-section of a thermal nuclear reactor.

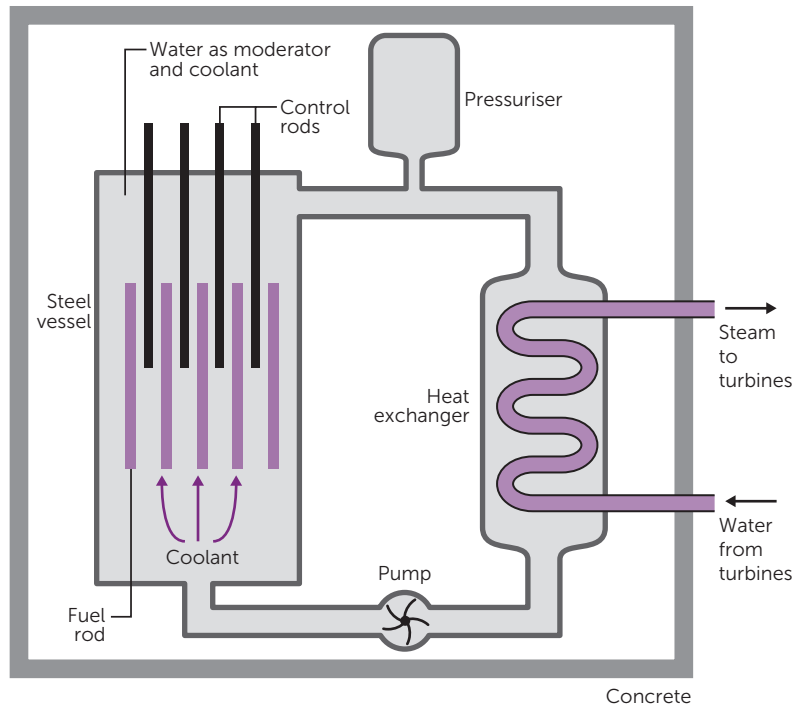


Fig. 23.9 The PWR thermal nuclear reactor

- Each fission neutron enters the moderator at high speed and loses kinetic energy as a result of repeated collisions with the moderator atoms. Eventually, one fission neutron per fission re-enters a fuel rod at sufficiently slow speed to cause a further fission. The moderator atoms must be light enough to absorb kinetic energy readily from the fission neutrons. For this reason, water or graphite is used.
- The moderator heats up as a result of gaining thermal energy from the fuel pins, which become very hot as the fission fragments in the fuel pins transfer their kinetic energy to the surrounding atoms. The moderator also gains thermal energy as a result of absorbing the kinetic energy of neutrons from the fuel pins and absorbing gamma radiation released from the fuel pins.
- A coolant is pumped through the steel vessel over the fuel rods, then through pipes to a heat exchanger, before re-entering the steel chamber. The coolant must flow easily and be non-corrosive. Water under pressure is used in the PWR as both the coolant and the moderator. In the AGR, the coolant is carbon dioxide gas, which is pumped through channels which contain the fuel pins and the control rods in the graphite moderator block. The hot coolant from the reactor core is pumped through a heat exchanger where it transfers its thermal energy before returning to the reactor core.
- Boron control rods are inserted into the moderator to ensure the neutron density remains constant. Boron nuclei easily absorb neutrons without fissioning. The depth of the rods in the moderator is controlled to ensure a steady rate of fissioning occurs.

Advantages and disadvantages of nuclear power

Advantages

- 1 The energy released from 1 kg of uranium fuel is about the same as from 50 000 kg of fossil fuel.
- 2 World oil and gas reserves are unlikely to last much beyond 2050 at the present rate of usage. Uranium reserves are sufficient to keep the present generation of thermal reactors running for 100 years or more. Extracting plutonium from the used fuel from thermal nuclear reactors could effectively extend world reserves of nuclear fuel from 100 years to many centuries.
- 3 Nuclear power stations do not produce so-called greenhouse gases such as carbon dioxide, which are thought to be responsible for atmospheric warming. New nuclear reactors are being constructed in many countries to meet the increasing worldwide demand for electricity without burning fossil fuels. See topic 4.4.

Disadvantages

- 1 The radiation from the core of a nuclear reactor is harmful. For this reason, the core of a reactor is enclosed in concrete which is intended to absorb the radiation. All operations in the core, such as adjusting the control rods or replacing spent fuel rods, must be carried out by remote control from a control room outside the concrete casing. Operating personnel must wear film badges to record exposure to radiation. If exposure exceeds the maximum permitted level, the operator is not allowed to continue.
- 2 The core is extremely hot and the coolant must remove the thermal energy of the core, otherwise meltdown could occur. Thermal energy could build up if the coolant pumps break down or the coolant leaks from the core or heat exchanger. When the reactor is shut down by inserting the control rods fully into the core, thermal energy continues to be produced for some time afterwards. If the cooling system ceases to function, the temperature of the core would rise and meltdown would occur. The Chernobyl disaster in 1986 is thought to have been caused through failure of the cooling system. See Fig. 23.10.



Fig. 23.10 The remains of the Chernobyl nuclear power station. The site will remain a prohibited area for many years

- 3 The spent fuel from a nuclear reactor is highly radioactive. This is because spent fuel contains highly radioactive fission fragments, as well as uranium 238 and plutonium 239. The metal containers are also radioactive as a result of exposure to neutrons. Each nucleus of a metal atom that absorbs a neutron becomes unstable and subsequently emits a beta particle. Some of these radioactive processes have long half-lives, so the spent fuel rods remain hot because thermal energy continues to be generated as a result of radioactivity in the fuel rods.

Fukushima

In 2011, a powerful earthquake in Japan created a tsunami that killed thousands of people and caused a meltdown at the Fukushima nuclear power station. When the earthquake struck, planned safety procedures were put into effect and the nuclear reactors at the site were shut down. When the tsunami hit the site, it flooded the generators which supplied electricity to the reactor cooling pumps and the pumps stopped working. The result was that three reactors overheated and radioactive material leaked from broken fuel rods. Traces of plutonium have since been found in soil nearby and contaminated water leaked into the sea. Over 100 000 people living near the Fukushima power station were evacuated and, four years later, many evacuees had still not been allowed to return home.

The spent fuel rods are moved by remote control as they are too radioactive and too hot to handle. After removal from the reactor core, the rods are stored in water-filled cooling ponds for some years. When sufficiently cool, the rods are cut open and the spent fuel is removed and reprocessed. The unused uranium 238 and plutonium 239, both highly radioactive isotopes, are separated and stored for possible use in fast breeder reactors. The remaining waste contains isotopes that stay radioactive for many years. This **high level waste** and the reprocessed isotopes need to be stored for many years underground in sealed containers, at sites which are secure and geologically stable.

- 4 Materials used in the reactor, such as remote handling equipment and the coolant, also become radioactive. This **medium level waste** is stored in sealed containers at the same sites as the high level waste. **Low level radioactive waste**, such as clothing, is disposed of and stored in sealed containers which are buried at restricted sites.
- 5 The first generation of thermal reactors were designed to last about 30 years. At the end of its designated life, a thermal reactor must be decommissioned and then monitored for many years, since its core continues to be radioactive and hot for some time. The process of dismantling a nuclear reactor is likely to be very costly, hazardous and a possible danger to the environment. By 2025, most first generation reactors will have been shut down and decommissioned with dismantling not likely to commence for many years after shut-down. The cost of decommissioning, monitoring and dismantling nuclear reactors is likely to prove a burden for future generations and is the main reason why the nuclear power programme in some countries has been wound down.

23.3 QUESTIONS

- 1 a) Explain what is meant by (i) induced fission, (ii) a chain reaction of fission events.
b) (i) Why is it not possible for a chain reaction to occur if the mass of fissionable material is less than a critical amount? (ii) Describe how the rate of fission events in a thermal nuclear reactor is controlled to ensure energy is released at a steady rate.
c) (i) What is the function of a moderator in a thermal nuclear reactor? (ii) When a neutron collides elastically with a moderator atom, the percentage of the neutron's kinetic energy transferred to the moderator atom depends on the atomic mass of the moderator. For graphite, up to 25% of the kinetic energy of a neutron can be transferred in a single collision. How many collisions are necessary to reduce the kinetic energy of a neutron from 1 MeV to 0.01 eV?
- 2 a) A kilogram of uranium releases as much thermal energy as 50 000 kg of oil. Calculate the mass of fuel used per day in a 1000 MW power station which is 25% efficient and which uses (i) oil, (ii) uranium. Assume 30 MJ of energy is released when 1 kg of oil is burned.
b) (i) Explain why spent fuel rods are highly radioactive. (ii) Why is spent fuel reprocessed? (iii) Why is a moderator not necessary in a fast breeder reactor?

23.4 Probing the nucleus

The discovery of the neutron

Rutherford established the nuclear model of the atom using alpha particles to probe atoms of thin metal foils. He used his measurements to show that the nucleus carries a charge of $+Ze$, where Z is the atomic number of the element (i.e. its order number

in the Periodic Table of the Elements). He deduced that nuclei are composed of two types of particles: protons and neutrons. The proton is the nucleus of a hydrogen atom. Rutherford put forward the idea that nuclei also contained uncharged particles, neutrons, of about the same mass as protons. However, he had no direct experimental evidence for the existence of neutrons.

The existence of the neutron was proved about 20 years later by Chadwick, one of Rutherford's former students. He knew that alpha particles from polonium directed at beryllium foil caused radiation to be emitted from the foil. Some scientists thought the radiation was electromagnetic in nature, but Chadwick discovered that the radiation knocked protons out of a wax plate placed in the path of the beam. He made further measurements and proved from his measurements that the radiation consisted of neutral particles of approximately the same mass as the proton.

Note

The rest mass of an electron, in atomic mass units, is 0.00055 u. The energy produced when an electron and a positron annihilate each other is therefore at least 1.02 MeV ($= 2 \times 0.00055 \times 931$). The rest mass of a proton, equal to 1.00728 u, is equivalent to about 1000 MeV or 1 GeV.

Antimatter

Chadwick's discovery provided direct evidence that the nucleus of the atom is composed of protons and neutrons. To explain the structure of any type of atom, no more than three types of particles are needed, namely the electron, the proton and the neutron. About the same time as Chadwick discovered the neutron, the first evidence for antimatter was obtained from cloud chamber studies by Carl Anderson in America. He photographed cloud chamber tracks produced by cosmic rays. These rays consist of high-energy particles from the Sun and other stars. When such a high energy particle crashes into the Earth's atmosphere, it creates a cascade of particles as a result of nuclear collisions. Anderson discovered how to photograph the cloud chamber at the instant each cascade reached the ground. By applying a strong magnetic field to the cloud chamber, charged particles passing through the chamber created curved tracks. Anderson found some tracks that were like beta particle tracks but curved in the opposite direction. He concluded that these were due to positively charged electrons known as positrons, predicted by Dirac some years earlier.

Dirac put forward the theory that for every particle, there is an antimatter particle with the opposite charge which annihilates itself and the particle when they collide with each other. The discovery of the positron was the first experimental evidence that antimatter exists. Dirac's predictions about antimatter, all of which have been confirmed, included the following key ideas:

- 1 **Annihilation:** radiation is produced when a particle meets its antiparticle. Matter is converted into radiation energy in this process which is known as annihilation. The scale of conversion is in accordance with Einstein's equation $E = mc^2$.
- 2 **Pair production:** radiation of sufficiently high energy can produce particle and antiparticle pairs. In this process, known as pair production, radiation energy is converted into mass in accordance with $E = mc^2$. For example, a gamma photon can produce an electron and a positron if the energy of the gamma photon exceeds $2m_0c^2$, where m_0 is the rest mass of the electron.

Gamma photon energy, $hf \geq 2m_0c^2$

Accelerators

Charged particles can be accelerated to speeds approaching the speed of light by using electric or magnetic fields to accelerate them. No particle or antiparticle can reach the speed of light. When a particle of charge, q , is accelerated through a potential difference, V , in a vacuum, the particle gains kinetic energy equal to qV and its mass increases in accordance with $E = mc^2$. If the particle is repeatedly accelerated, its mass becomes ever larger and larger as its speed, v , approaches c in accordance with Einstein's relativistic mass formula

$$m = m_0 \left[1 - \left(\frac{v}{c} \right)^2 \right]^{-1/2}$$

where m_0 is the rest mass of the particle.

More and more energy is needed to make the speed of a particle approach closer and closer to c . Fig. 23.11 shows how the mass of a particle rises as it approaches the speed of light. The speed of light is impossible to reach because the energy needed would be infinite.

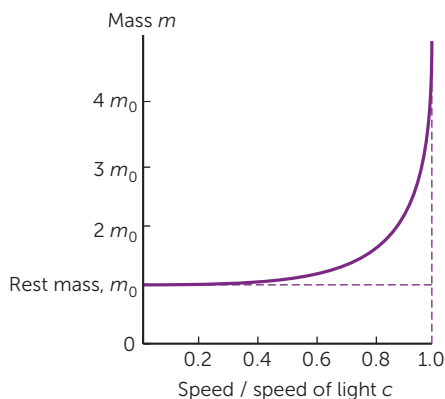


Fig. 23.11 Relationship between mass and speed

Since the 1950s, fundamental research into the nature of matter has been conducted using larger and larger accelerators to hurl fast moving particles into each other and then study the debris from the collisions. From such experiments, scientists now know that all matter is composed of

- **quarks**, which make up neutrons and protons, and
- **leptons**, which include electrons and positrons.

Quarks

Cosmic ray studies and accelerator experiments up to about 1960 resulted in the discovery of a large number of very short-lived sub-atomic particles. The properties of these new particles were measured from the tracks the particles created in photographic emulsions, cloud chambers and bubble chambers. Bubble chambers contain liquid under high pressure which is released at the instant charged particles pass through it. This process creates tiny bubbles along the particle track, thus making the track visible.

For example, pions were discovered from tiny tracks created in the emulsion of photographic plates exposed to cosmic rays at high altitude. **Strange** particles were

discovered in photographs of cloud chambers containing absorber plates. These particles were strange because they were created in short-lived impacts with nuclei but they decayed so slowly that they left tracks much longer than expected. In addition, they were created in pairs so they were assigned a **strangeness** number, S , on the basis that strangeness is conserved in an interaction involving the strong nuclear force.

The properties of all the new and existing particles were used to classify the particles in groups.

- Particles heavier than the proton (and the neutron) were called **baryons**.
- Particles lighter than the proton and heavier than the electron were called **mesons**.
- Particles lighter than the electron (including the electron) were called **leptons**.

Table 23.1 is a summary of the properties of baryons and mesons discovered by 1960.

Patterns emerged as a result of plotting each group on a grid of charge against strangeness. Fig. 23.12 overleaf shows the pattern for very short-lived baryons. This particular group forms a pattern which appears incomplete. Its completion required the existence of a negatively charged particle with a strangeness of -3 and a rest mass of 1.79 u. This particle was called the **omega minus** (Ω^-) and its discovery in 1963 led to the quark model to explain the patterns.

Table 23.1 Baryons and mesons

Name + symbol		Charge/e	Mass/u	Strangeness	Lifetime/s
Pion (π -meson)	$\pi^+ \pi^-$	± 1	0.14	0	10^{-8}
	π^0	0	0.14	0	10^{-6}
K-meson	$K^\pm$	± 1	0.5	± 1	10^{-8}
	K^0	0	0.5	+1	10^{-8}
	$\bar{K}_0$	0	0.5	-1	10^{-8}
Proton	p	+1	1.0	0	infinite
Neutron	n	0	1.0	0	15 min
Sigma	$\Sigma^\pm$	± 1	1.2	-1	10^{-10}
	Σ^0	0	1.2	-1	10^{-20}
Lambda	Λ^0	0	1.1	-1	10^{-10}
Xi	Ξ^0	0	1.3	-2	10^{-10}
	Ξ^-	-1	1.3	-2	10^{-10}
Delta	Δ^{++}	+2	1.33	0	10^{-23}
	Δ^+	+1	1.33	0	10^{-23}
	Δ^0	0	1.33	0	10^{-23}
Sigma star	$\Sigma^{\pm*}$	± 1	1.49	-1	10^{-23}
	Σ^{0*}	0	1.49	-1	10^{-23}
Xi-star	Ξ^{0*}	0	1.64	-2	10^{-23}
	Ξ^{-*}	-1	1.64	-2	10^{-23}

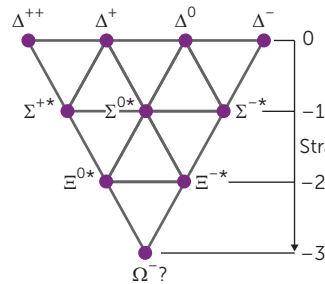


Fig. 23.12 Short-lived baryons

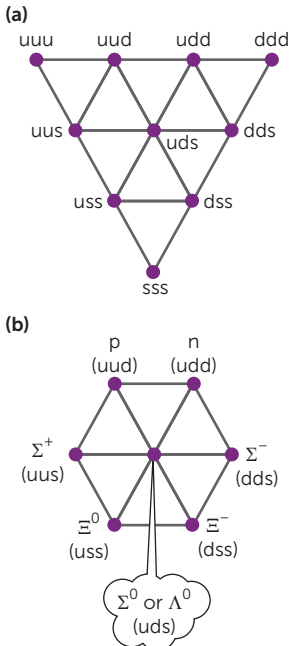


Fig. 23.13 The baryon family
(a) short- life baryons,
(b) long-life baryons

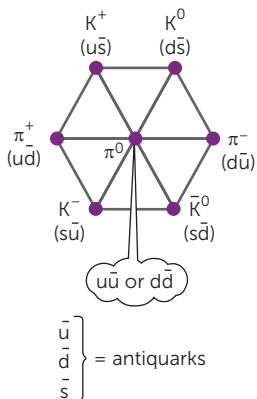


Fig. 23.14 The meson family

The quark model was originally based on the existence of three types of quarks, referred to as the up quark, the down quark and the strange quark.

The rules of the model are simple:

- 1 The up quark has a charge of $+\frac{2}{3}e$ and strangeness 0. The down quark has a charge of $-1e$ and strangeness 0. The strange quark has a charge of $-1e$ and strangeness -1 .
- 2 A baryon consists of three quarks and an antibaryon consists of three antiquarks. There are 10 possible combinations of three quarks. These are shown in Fig. 23.13. These 10 combinations explain the 10 particles of the short-lived baryon group. The quark composition of the longer lived baryons, including the neutron and the proton, is also shown in Fig. 23.13.
- 3 A meson consists of a quark and an antiquark. The quark composition of each meson in Table 23.1 is shown in Fig. 23.14.

Further experimental work and successful predictions have led to the discovery of three more members of the quark family. The six known quarks are thought to constitute the complete quark family. Whether there is a fundamental link between the quark family and the lepton family remains to be seen.

23.4 QUESTIONS

- 1 a) An alpha particle is capable of knocking a neutron out of a beryllium ${}^9_4\text{Be}$ nucleus. Write down the equation for this process.
b) The up quark has a charge of $+2e$ and strangeness, 0, and the down quark has a charge of $-1e$ and a strangeness, 0. The strange quark has a charge of $-1e$ and a strangeness of -1 . (i) A proton consists of two up quarks and a down quark. A neutron consists of two down quarks and an up quark. Show that the charge of a proton is $+1e$ and the charge of a neutron is 0. (ii) What is the quark composition of a baryon that has a charge of $+1e$ and a strangeness of -1 ?
- 2 a) Fig. 23.15 represents a photograph of a pair production event in a bubble chamber. The particle and antiparticle curve around because a magnetic field was applied. (i) Why are there two tracks in opposite directions? (ii) Why do the tracks spiral?
b) When a proton annihilates an antiproton, two gamma photons are released in opposite directions. Calculate the energy of the gamma photons produced when a proton of kinetic energy 2 GeV collides with an antiproton of the same energy moving in the opposite direction. The rest mass of a proton = 1 GeV.



Fig. 23.15

SUMMARY

• Forces

- The strong nuclear force: acts between protons and neutrons, attractive, ranges up to 2 or 3 fm.
- The electromagnetic force: acts between any two charged objects, attractive or repulsive, extends to infinity.
- The weak nuclear force: responsible for turning neutrons into protons, ranges no more than about 10^{-18} m.
- The force of gravity: acts between any two masses, attractive, extends to infinity.

• Energy

- The electron volt (1 eV) is the work done when an electron moves through 1 volt. $1\text{ eV} = 1.6 \times 10^{-19}$ J.
- Binding energy is the work done to separate a nucleus into its constituent neutrons and protons.
- Mass defect is the difference between the mass of a nucleus and its constituent neutrons and protons.
- The Q value of a nuclear process is the energy released or needed.

• Fission

- U 235 is fissioned by slow neutrons; U 238 absorbs neutrons without fission.
- One fission neutron per fission goes on to produce further fission in a steady chain reaction.
- The moderator in a thermal reactor reduces the kinetic energy of the fission neutrons to enable them to produce further fission of U 235.

• Quarks

- A baryon consists of three quarks. An antibaryon consists of three antiquarks. A meson consists of a quark and an antiquark.
- The up quark has a charge of $+\frac{2}{3}e$ and a strangeness of 0. The down quark has a charge of $-\frac{1}{3}e$ and a strangeness of 0. The strange quark has a charge of $-\frac{1}{3}e$ and a strangeness of -1 .

• Equation relating energy and mass, $E = mc^2$.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

$e = 1.6 \times 10^{-19}$ C, $\epsilon_0 = 8.85 \times 10^{-12}$ F m $^{-1}$, $1\text{ u} = 931\text{ MeV}$,
the mass of a proton = 1.00728 u , the mass of a
neutron = 1.00867 u , the mass of an electron = 0.00055 u .

23.1

- a) The electrostatic potential energy of two charged particles at distance r apart is

$$\frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Calculate the closest distance that an alpha particle of kinetic energy 5 MeV can approach a beryllium ${}^9_4\text{Be}$ nucleus, assuming it is only affected by the electrostatic force.

- b) The strong nuclear force has a range of a few femtometres. Discuss whether or not a 5 MeV alpha particle is capable of causing a nuclear reaction when it hits a beryllium 9 nucleus.

23.2

- a) Calculate the binding energy per nucleon of
(i) a ${}^{12}_6\text{C}$ nucleus,
(ii) a ${}^{206}_{82}\text{Pb}$ nucleus.
(Atomic masses: C 12 = 12.0000 u , Pb 206 = 205.97446 u)
b) (i) Sketch a graph to show how the binding energy per nucleon varies with the mass number of the nucleus.
(ii) Use your graph to explain why energy is released when a heavy nucleus undergoes fission.

23.3

- a) Polonium 210 is an unstable alpha-emitting nucleus that contains 84 protons. It decays to form an isotope of lead (Pb).
(i) Write down the equation representing the decay of polonium 210.
(ii) Calculate the Q value of the reaction. The atomic mass of polonium 210 is 209.98287 u and the atomic mass of Pb 206 is 205.97446 u .

REVISION QUESTIONS

- b) Sodium 24 is an unstable beta-emitting nucleus that contains 11 protons. It decays to form an isotope of magnesium (Mg).
 (i) Write down the equation that represents this decay.
 (ii) Calculate the Q value of this decay. The atomic mass of sodium 24 is 23.99097 u and the atomic mass of magnesium 24 is 23.98505 u.

23.4

- a) (i) What is meant by a chain reaction in a nuclear reactor?
 (ii) Describe how a chain reaction is controlled in a nuclear reactor.
- b) (i) What is the purpose of the moderator in a thermal nuclear reactor?
 (ii) What physical properties are necessary for a moderator?
- c) (i) Why is spent fuel from a nuclear reactor dangerous?
 (ii) Plutonium 239 has a half life of 100 years. Calculate the activity of 1 kg of this isotope. The Avogadro constant $= 6.02 \times 10^{23} \text{ mol}^{-1}$.

23.5

- a) In an accelerator called a synchrotron, charged particles are accelerated periodically as they travel on a circular path around the synchrotron.
 (i) How can charged particles be forced to go around in a circle?
 (ii) Charged particles radiate electromagnetic waves when they are forced around a circular path. The energy per second radiated by a particle increases with its speed. Explain why this process limits the kinetic energy of a charged particle in a synchrotron.
- b) An electron is accelerated until its kinetic energy is 20 GeV.
 (i) Calculate its mass at this energy.
 (ii) Explain why its speed is limited, no matter how much kinetic energy it acquires.
- c) Determine the charge and strangeness of
 (i) a uds baryon,
 (ii) a $u\bar{s}$ meson.
- d) Determine the quark composition of (i) a baryon of charge $+2e$ and strangeness 0, (ii) a meson of charge 0 and strangeness $+1$.

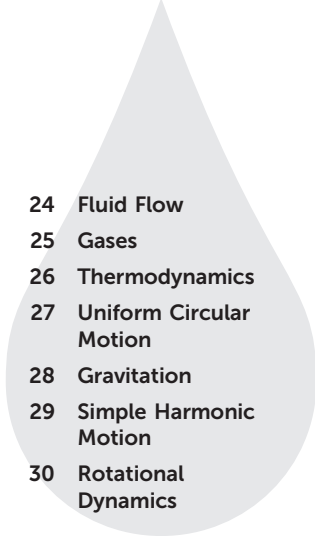
PART

7

Further Physics

Part 7 provides opportunities to develop your grasp of physics principles to give you confidence in your ability to use the principles of the subject. The unit on fluid flow builds on your studies on pressure in [unit 8](#) and on mechanics in [units 3](#) and [4](#) to enable you to understand key properties such as viscosity and important effects such as the Bernoulli effect. The unit on gases enables you to develop your understanding of thermal physics from [unit 6](#) by considering temperature in depth and its link to energy at a molecular theory. The unit on thermodynamics develops simple ideas on energy transfer from [unit 4](#) into a sophisticated appreciation of the limits of efficiency of heat engines. The final units on circular motion, gravitation, simple harmonic motion and rotational dynamics build on the earlier units on mechanics, enabling key concepts and mathematical skills to be developed to a high level.

The units in [part 7](#) offer in-depth selected studies which you can choose from in order to prepare for the specialist studies which you will undertake at the next stage of your academic carrier. In this way, [part 7](#) completes the pathway provided by Foundation Physics to the next stage of your studies.

- 
- 24 Fluid Flow
 - 25 Gases
 - 26 Thermodynamics
 - 27 Uniform Circular Motion
 - 28 Gravitation
 - 29 Simple Harmonic Motion
 - 30 Rotational Dynamics

Fluid Flow

OBJECTIVES

After working through this Unit, you should be able to:

- describe streamlined flow and turbulent flow and give examples of each type of flow
- state and use the continuity equation
- relate the volume of fluid per second flowing through a tube to the fluid speed and the area of cross-section of the tube for an incompressible fluid
- explain the nature of viscosity in terms of molecular motion
- describe what is meant by a velocity gradient in a fluid and the coefficient of viscosity of a fluid
- carry out calculations using Poiseuille's Law and Stokes' Law
- describe and give examples of the Bernoulli effect
- state the Bernoulli equation and relate it to the energy of a fluid
- apply the Bernoulli equation to streamlined flow of an incompressible fluid
- describe the use of a pitot-static tube to measure flow speed

CONTENTS

- 24.1 Flow patterns
- 24.2 Flow rates
- 24.3 Viscous flow
- 24.4 Non-viscous flow
- Summary
- Revision questions

24.1 Flow patterns

Oil, water and air are all fluids because they can flow but their flow properties are very different. Water flows more readily than oil and the flow of air easily becomes turbulent. The flow pattern of a fluid in motion depends on the forces acting in the fluid and the shape of any solid surfaces the fluid is in contact with.

Flow patterns can be seen in many everyday situations. Such examples show that flow patterns can change with time so they might be steady or unsteady and they can change with position along or across the flow. For example,

- smoke from a chimney in still air rises smoothly through the air in a stream before it breaks up and mixes into the air,
- oil poured carefully from a container flows out smoothly without splashing or spilling,
- water flows smoothly as it moves past the pillars of a bridge but often becomes turbulent immediately behind each pillar.

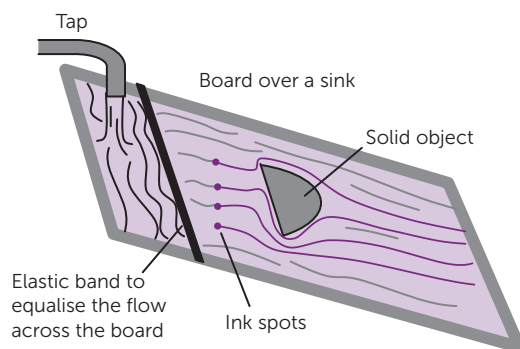


Fig. 24.1 A simple flow tank

A flow tank can be used to observe the flow pattern of a fluid in a controlled way. A simple flow tank using water is shown in Fig. 24.1. Ink can be used to show the pattern of the **flow lines** as the water drains down the board. Each flow line shows the path of a small amount of fluid.

Flat objects can be placed in the flow to see how the flow pattern is affected by their shape. The flow rate can be increased by tilting the board more. Some flow patterns are shown in Fig. 24.2.

- If the flow lines do not move about and do not become mixed up, the flow is stable and the flow lines are described as **streamlines**. Fluid does not flow across streamlines. If the lines are parallel to each other, the flow is said to be **laminar**. In streamlined flow, the velocity of the fluid may change along a streamline but will always be the same at any point on the streamline. For example, the flow lines round the sphere and the streamlined shape shown in Figs 24.2a and b are stable and do not move about. The velocity of the fluid at any point in the fluid does not vary in time but it does change with position.
- If the flow lines move about, the flow is mixed up and unstable and is described as **turbulent**. The fluid mixes itself up in turbulent flow because the path of the fluid changes unpredictably. Various factors such as the speed of the fluid, the shape and size of solid surfaces, the fluid density and fluid friction (i.e. viscous forces) determine whether or not the flow of a fluid is turbulent. People in an aircraft passing through 'turbulence' experience the effects of turbulence as the aircraft is pushed about erratically.

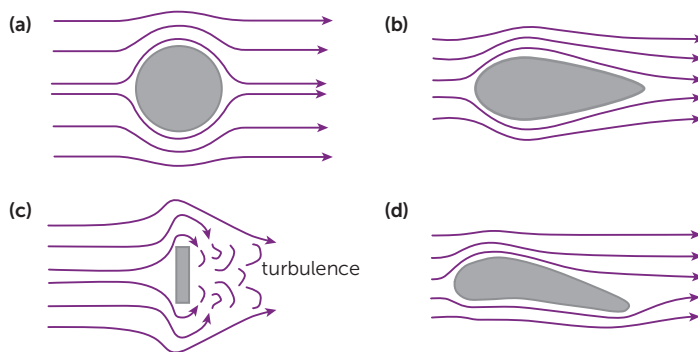


Fig. 24.2 Flow patterns around various shapes (a) Sphere, (b) Streamlined shape, (c) Disc (seen side on), (d) Aerofoil

The flow of a fluid can change from streamlined flow to turbulent flow if the fluid passes round a sharp edge of a solid object, as in Fig. 24.2c) The effect can cause an aircraft in flight to stall. Fig. 24.2d) shows an aerofoil which represents the shape of the cross-section of an aircraft wing. As explained in 29.4, the upward force on an aircraft wing, referred to as the 'lift force', occurs because air flows faster across the top surface of the wing than it does underneath. However, if the wing is inclined at an increasing angle to a horizontal plane, the aircraft stalls because the air flow over the trailing edge of the wing and behind it becomes turbulent and the lift force is drastically reduced.

Applications of streamlined flow and of turbulent flow

There are many examples of streamlined or turbulent flow that we make use of. The importance of fluid flow in two applications already covered in this book are summarised below.

1 Drag factors (Topic 4.2)

Vehicles and aircraft experience drag forces when they move. The drag force on a vehicle or aircraft is reduced if the object has a streamlined shape. The air flows past the object in streamlines and turbulence is therefore reduced. As explained in topic 4.2, if the drag force is reduced, less energy is wasted at any given vehicle speed. Fuel consumption is therefore reduced by streamlining the shape of a vehicle or aircraft.

2 The sphygmomanometer (Topic 8.4)

This instrument is used with a stethoscope to measure blood pressure. As explained in topic 8.4, as the cuff pressure on the artery in the lower arm is reduced, the flow of blood through the artery resumes at the systolic pressure. This is detected by the noise it makes. At the lower diastolic pressure, the flow changes from turbulent to laminar flow. This is detected because laminar flow in the artery is quieter than turbulent flow.

24.1 QUESTIONS

- 1 Explain what is meant by streamlined flow.
- 2 Describe the following flow situations in terms of streamlined and turbulent flow.
 - a) The flow of water at a slow rate from a tap into a bucket of water,
 - b) Oil poured from a bottle into a pan.

24.2 Flow rates

In general, when a fluid flows through a pipe or a tube, pressure must be applied to it to make it flow through the pipe or tube. The pressure might be from a pump or it might be from a supply of fluid at a higher pressure. In general, gases put under pressure are compressed to a higher density whereas liquids cannot be compressed and its density is uniform (ie. the same throughout). A fluid that cannot be compressed is said to be **incompressible**.

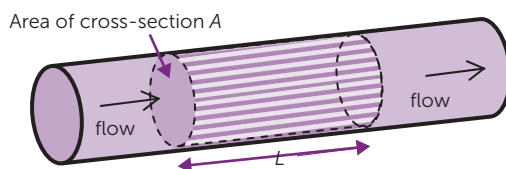


Fig. 24.3 Flow rate in a pipe

For an **incompressible fluid**, the volume of fluid flowing each second through a pipe or a channel depends on the speed of the flow and the area of cross-section of the pipe or the channel. Consider Fig. 24.3 which shows a fluid flowing through a pipe of cross-sectional area A at constant speed v .

For a length L of the pipe,

- the volume, V , of fluid in this section = length $L \times$ area of cross-section A (where $A = \pi d^2/4$)
- the time taken, t , for fluid to flow through it = length/speed = L/v

Therefore, the volume of fluid per second flowing through the pipe

$$= \frac{V}{t} = \frac{L \times A}{L \div v} = vA$$

For an incompressible fluid flowing at speed v through a uniform pipe (or tube or channel) of cross-sectional area A ,

the volume per second flowing through the tube, $V/t = vA$

Note

For a pipe of circular cross-section and internal diameter d ; its area of cross-section $A = \pi d^2/4$.

24.1 WORKED EXAMPLE

Oil flows steadily through a pipe of internal diameter 2.4 mm at a speed of 0.31 m s^{-1} .

- Show that the area of cross-section of the pipe is $4.5 \times 10^{-6} \text{ m}^2$
- Calculate the volume of oil that flows through the pipe each second.

Solution

- Area of cross-section $A = \pi d^2/4 = \pi(2.4 \times 10^{-3})^2/4 = 4.5 \times 10^{-6} \text{ m}^2$
- Volume per second = $vA = 0.31 \times 4.5 \times 10^{-6} = 1.4 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$

Continuity

In general, when a fluid flows continuously through a pipe (or a tube or a channel), the mass of fluid leaving the pipe each second is the same as the mass of fluid entering the pipe each second provided there are no leaks! The mass that flows past any position along the pipe every second is the same for all positions. In other words, for continuous flow

the mass flow per second, $m/t = \text{constant}$

Using the equation 'mass = density $\times$ volume', we can therefore write the mass flow equation as

$$\begin{aligned} \text{mass flow per second, } m/t &= \text{density } (\rho) \times \text{volume flow per second } (V/t) \\ &= \text{constant} \end{aligned}$$

Combining this equation with the equation for volume, $V/t = vA$, therefore gives

the mass flow per second, $m/t = \rho vA = \text{constant}$

This equation is referred to as **the continuity equation**. It tells us that at any position in a pipe or tube or channel through which a fluid flows continuously, the product of the fluid density, the fluid speed and the area of cross-section is constant.

Notes

- 1 **The continuity equation also applies to streamlined flow.** This is because streamlines cannot cross each other. In effect, the fluid between a 'tube' of streamlines always flows between the streamlines as shown in Fig. 24.4. So the continuity equation applies to the stream tube where A is the cross-sectional area at any position along the tube.

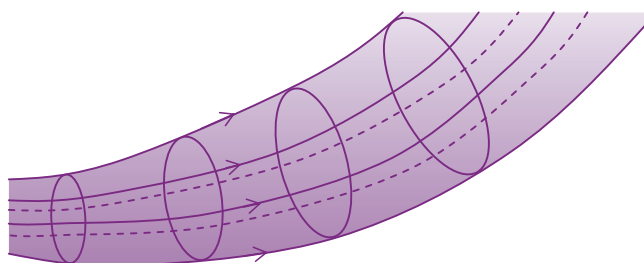


Fig. 24.4 A flow tube

- 2 **If the density is constant** (i.e. the fluid is incompressible), we can write the continuity equation as $vA = \text{constant}$. This tells us that if the area of cross-section of a pipe or tube or channel changes along its length, the flow speed v increases where the pipe becomes narrower and decreases where the pipe becomes wider. For example, water flowing through a pipe with a wide section between two narrow sections as in Fig. 24.5 moves slower in the wide section than in the narrow section.

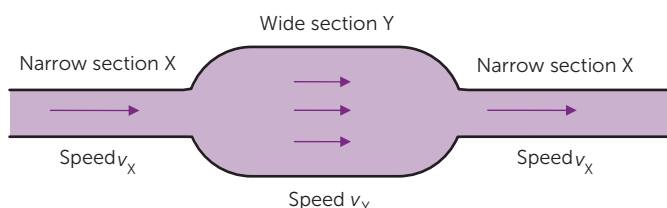


Fig. 24.5 Flow through a pipe with a wide section between two narrow sections

Using the continuity equation in the form $vA = \text{constant}$,

$$v_X A_X = v_Y A_Y$$

where v_X and v_Y are the fluid speeds in the narrow and wide sections, respectively, and A_X and A_Y are the respective areas of cross-section.

Rearranging this equation shows that

$$\frac{v_X}{v_Y} = \frac{A_Y}{A_X}$$

For example, if the cross-sectional area of the narrow section is $0.5 \times$ that of the wide section, the speed in the narrow section is twice as large as the speed in the wide section.

24.2 WORKED EXAMPLE

Density of water = 1000 kg m^{-3}

A water hose with an internal diameter of 12 mm at the outlet discharges 20 kg of water in 25 s. Calculate the water speed at the outlet.

Solution

$$\text{Mass flow per second, } m/t = \frac{20 \text{ kg}}{25 \text{ s}} = 0.80 \text{ kg s}^{-1}$$

$$\begin{aligned} \text{Cross-sectional area, } A &= \pi d^2/4 = \pi(12.0 \times 10^{-3})^2/4 \\ &= 1.13 \times 10^{-4} \text{ m}^2 \end{aligned}$$

Rearranging the continuity equation $m/t = \rho Av$, where ρ is the density of water and v is the speed to be calculated, gives

$$v = \frac{m/t}{\rho A} = \frac{0.80}{1000 \times 1.13 \times 10^{-4}} = 7.1 \text{ m s}^{-1}$$

24.2 QUESTIONS

- 1 A river flows faster under a road bridge than it does after the bridge where the river banks are further apart. Explain why the flow of the river is faster under the bridge than where the river widens out.
- 2 Oil flows steadily through a pipe of internal diameter 11 cm at a speed of 0.65 m/s.
 - a) Show that the area of cross-section of the pipe is $9.5 \times 10^{-3} \text{ m}^2$
 - b) Calculate the mass of oil that flows through the pipe each second. Density of oil = 960 kg m^{-3}

24.3 Viscous flow

Some liquids flow much more easily than other. Gases flow much more easily than liquids. Fluid flow generally involves different parts of a fluid moving at different velocities, as if there are layers in the fluid sliding past each other at different velocities. In a fluid that flows easily, the layers slide easily past each other with little friction. For example, if a cup of tea is stirred briefly, the tea moves round inside the cup and gradually slows to a standstill. Try the same with oil and you will find the oil stops moving much sooner. This is because internal friction in oil is much more significant than in water so the faster-moving layers of oil are soon stopped by the slower-moving layers. Internal friction in a fluid is referred to as **viscosity**. Oil is much more viscous than water. In general, liquids are much more viscous than gases. We use the term 'non-viscous' to refer to fluids in which viscosity is negligible.

To understand the nature of viscosity, consider the flow of a viscous fluid through a pipe, as in Fig. 24.6. A pressure difference between the ends of the pipe must be applied to make a viscous fluid flow through the pipe. Friction between the fluid and the surface of the pipe makes the fluid near the surface move slower than fluid further away from the surface. The flow is fastest along the centre of the pipe. We can think of the flow in terms of different layers of fluid moving at velocities

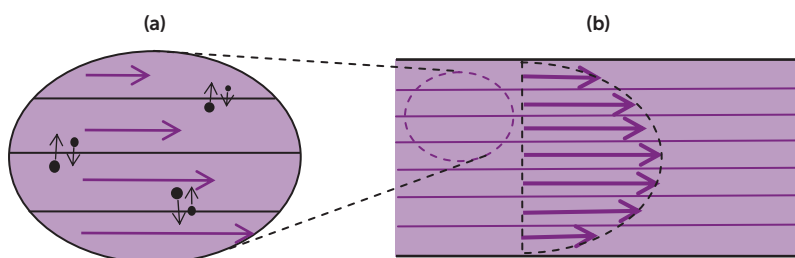


Fig. 24.6 The nature of viscosity (a) molecules moving between layers (b) velocity profile across a pipe

that decrease the nearer a layer is to the pipe surface, as shown by the velocity profile across the pipe in Fig. 24.6.

Viscosity or ‘internal friction’ is caused by molecules transferring between adjacent layers. In addition to their motion along the pipe, the fluid molecules move about erratically because they repeatedly collide with each other at random. As a result, molecules transfer between adjacent fluid layers. For two such layers moving at different velocities, as shown in Fig. 24.6,

- the molecules transfer in opposite directions between layers at the same rate on average,
- the faster layer loses momentum to the slower layer because, on average, it loses faster-moving molecules and gains slower-moving molecules.

Newton’s second law of motion tells us that loss of momentum is due to a force opposing the motion. So a faster layer moving past a slower layer experiences a force due to the slower layer opposing its motion. This internal friction is the cause of the fluid’s viscosity.

The velocity profile in Fig. 24.6 shows how the velocity of the fluid changes across the pipe. The change of velocity from one layer to the next is greatest near the pipe surface. This is because the internal friction is greatest near the pipe surface.

The velocity gradient across the flow is the change of velocity per unit distance across the flow. For example, in Fig. 24.6, if the distance from centre to centre between two adjacent layers is 1 mm (= 0.001 m) and their velocities differ by 2.0 m s^{-1} , the velocity gradient across this distance is given by calculating $2.0 \text{ m s}^{-1} \div 0.001 \text{ m}$ which equals 2000 s^{-1} . Note the correct expression for the unit ‘m s⁻¹ per metre’ is s⁻¹.

The coefficient of viscosity, η (pronounced ‘eta’), of a fluid is the internal friction per unit area of the layers in a fluid where the velocity gradient is 1 s^{-1} (i.e. the change of velocity per metre across the flow is 1 m/s per metre across the flow).

- The unit of viscosity is the newton second per square metre (N s m^{-2}). This is because the unit of friction per unit area is the newton per square metre (N m^{-2}) and the unit of the velocity gradient is s^{-1} . So the unit of viscosity is $\text{N m}^{-2} \div \text{s}^{-1}$ which is the newton second per square metre (N s m^{-2}).
- To appreciate the physical meaning of η , consider the fact that castor oil has a viscosity 10 000 times greater than that of water. Therefore, the internal friction per unit area between the layers of castor oil when it flows is 10 000 times greater than between the layers of water in identical conditions.

Notes

- 1 The viscosity of a fluid depends on its temperature. For example, oil and water both become less viscous (ie. more runny) if they become warmer. In winter, cars can be more difficult to start from cold if the engine oil is much more viscous when cold than when warm. **Viscostatic** oil used in vehicles is specially made so its viscosity is unchanged when its temperature changes.
- 2 Some liquid paints become less viscous the more they are stirred. Other liquids like wallpaper paste in water become more viscous the more they are stirred. Such liquids are examples of **non-newtonian fluids** which have a viscosity that depends on how much the fluid is stirred.

Poiseuille's equation for pipe flow

The flow of a viscous fluid through a pipe or a tube is a feature of a wide range of devices and applications ranging from blood flow through blood vessels to oil flow through pipe lines.

Consider steady laminar flow of a viscous fluid through a uniform horizontal pipe. To make the fluid flow through the pipe, the pressure at the inlet to the pipe must be greater than the pressure at the outlet as shown in Fig. 24.7. If there were no pressure difference between the inlet and the outlet, the viscous forces in the fluid would prevent the flow. The more viscous the fluid is, the greater the pressure difference that is needed.

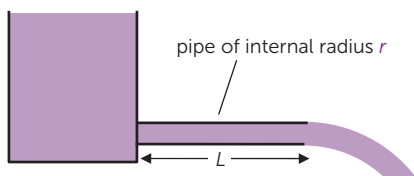


Fig. 24.7 Pipe flow

If volume V flows through the pipe in time t , the volume per second of fluid flowing through the pipe is V/t . This depends on

- the pressure difference Δp between the inlet and the outlet,
- the pipe length L ,
- the pipe radius r ,
- the fluid viscosity η .

The volume per second flowing through the pipe is increased if the pressure difference is increased or the pipe is shortened. The pressure gradient $\Delta p/L$ is the important factor here. So the volume per second depends on $\Delta p/L$, r and η . Assuming the viscosity is constant, it can be shown that the volume per second, V/t , flowing through a **horizontal** pipe is given by the following equation, known as **Poiseuille's equation** after its discoverer.

$$V/t = \frac{\pi r^4 \Delta p}{8\eta L}$$

24.3 WORKED EXAMPLE

The following measurements were made on the flow of water at 20°C through a narrow horizontal tube when the pressure at the inlet was 1.5×10^3 Pa greater than the pressure at the outlet. Use these measurements to determine the viscosity of water.

Volume of water collected in 150 s = $85 \text{ cm}^3 = 8.5 \times 10^{-5} \text{ m}^3$

Length of capillary tube = 0.135 m

Mean diameter of the tube = 1.20 mm = $1.20 \times 10^{-3} \text{ m}$

Density of water = 1000 kg m^{-3} ,

$g = 9.8 \text{ m s}^{-2}$

Solution

Volume of water per second flowing through the tube,

$$V/t = \frac{8.5 \times 10^{-5} \text{ m}^3}{150 \text{ s}} = 5.7 \times 10^{-7} \text{ m}^3$$

Pressure difference $\Delta p = 1.5 \times 10^3 \text{ Pa}$

Tube radius, $r = 0.5 \times \text{diameter} = 6.0 \times 10^{-4} \text{ m}$

Substituting the above values and the value of length L into Poiseuille's equation

$$V/t = \frac{\pi r^4 \Delta p}{8 \eta L}$$

$$\text{gives } 5.67 \times 10^{-7} = \frac{\pi \times (6.0 \times 10^{-4})^4 \times 1.5 \times 10^3}{8 \eta \times 0.135}$$

$$5.67 \times 10^{-7} = \frac{6.1 \times 10^{-10}}{8 \eta \times 0.135}$$

Rearranging this equation gives

$$\eta = \frac{6.1 \times 10^{-10}}{8 \times 0.135 \times 5.7 \times 10^{-7}} = 9.9 \times 10^{-4} \text{ N s m}^{-2}$$

Note

Poiseuille's equation is not unlike the equation for electrical resistance in the form $I = V/R$ (see section 13.4) if we compare the flow rate V/t for a pressure difference Δp with current I for a certain pd V . Just as an electric current requires a pd, a flow rate requires a pressure difference.

$$I = \frac{1}{R} \times V \quad V/t = \frac{\pi r^4}{8 \eta L} \times \Delta p$$

This comparison shows that the pipe 'resistance' is $\frac{8 \eta L}{\pi r^4}$

The expression for the pipe 'resistance' shows that the resistance to flow increases

- the more viscous the flow is,
- the longer the pipe is,
- the narrower the pipe is.

The last point is particularly significant as the resistance is inversely proportional to r^4 . This 'fourth power inverse' variation means that a reduction of the radius to half cuts the flow rate to a sixteenth ($= 1/24$). A consequence of this variation is within each of us – arteries are known to narrow due to deposits on the inside walls. The narrower the arteries become, the greater their resistance to flow. As a result, the heart has to work much harder to keep the blood flowing through them.

Stokes' Law

When an object moves through a viscous fluid, the fluid exerts a drag force on the object. For a sphere of radius, r , moving at constant velocity, v , through such a fluid, it can be shown that the drag force, F , is given by the following equation, known as

Stokes' Law

$$F = 6\pi\eta r v$$

Note

As explained in section 2.6, an object released in a viscous fluid reaches a maximum velocity, referred to as its **terminal velocity**. At this velocity, the drag force is equal and opposite to the weight of the object. Hence, for a sphere of mass m and radius r falling at terminal velocity v in a fluid of viscosity η

$$mg = 6\pi\eta r v$$

24.4 WORKED EXAMPLE

A metal sphere of mass 7.1×10^{-3} kg and radius 6.0 mm falls in a viscous liquid through a vertical distance of 0.30 m in 5.9 s. Calculate

- the terminal velocity of the sphere,
- the viscosity of the liquid ($g = 9.8 \text{ m s}^{-2}$).

Solution

a) Terminal velocity $v = 0.40 \text{ m} / 5.9 \text{ s} = 6.8 \times 10^{-2} \text{ m s}^{-1}$

b) Substituting the value of v and other data into Stokes' Law gives

$$mg = 6\pi\eta rv, \text{ i.e. } 7.1 \times 10^{-3} \times 9.8 = 6\pi\eta \times 6.0 \times 10^{-3} \times 6.8 \times 10^{-2}$$

$$\text{Rearranging this equation gives } \eta = \frac{7.1 \times 10^{-3} \times 9.8}{6\pi \times 6.0 \times 10^{-3} \times 6.8 \times 10^{-2}} = 9.0 \text{ N s m}^{-2}$$

24.3 QUESTIONS

- The flow rate of a viscous fluid through a straight tube of radius r is proportional to $1/r^4$.
 - What is meant by 'a viscous fluid'?
 - The flow of a liquid through a tube of internal radius 5.0 mm is reduced by accumulated deposits along the pipe to 4 cm^3 per second compared with 10 cm^3 per second when it was first used. Calculate the effective radius of the tube.
- The following measurements were made on the flow of oil at 20°C through a horizontal pipe of internal diameter 1.2 cm when the pressure at the inlet was $6.7 \times 10^{-3} \text{ Pa}$ greater than the pressure at the outlet. Use these measurements to determine the volume of oil flowing through the pipe each second.

Length of pipe = 3.8 m
Viscosity of the oil = 7.5 N s m^{-2}

24.4

Non-viscous flow

Pressure differences in a fluid make it flow. If the fluid along a flow line moves faster, it gains kinetic energy. If it gains height, its potential energy increases. The energy of a moving fluid is in the form of kinetic energy, potential energy and internal energy. If the fluid is non-viscous, internal friction is absent so cannot change its internal energy. Work is done by the pressure in a fluid when it gains potential energy or kinetic energy. The pressure in the fluid therefore drops where the fluid gains speed or height. Fig. 24.8 shows a demonstration of this effect in which water flows through a horizontal tube which has a narrow section between two wide sections. The water level in each of the three vertical tubes is a measure of the pressure of the water in each part of the horizontal tube.

As explained in section 24.2, the flow speed is greater in the narrow section Y than in the two wider sections, X and Z. The water level in the vertical tubes show that the pressure in Y is less than in X or Z. The demonstration therefore shows that, for horizontal flow, the pressure is low where the speed is high. This effect is referred to as **the Bernoulli effect**.

In more general terms, for steady flow of an incompressible non-viscous fluid, the pressure p and the speed v at any point along a streamline are related by

$$p + \frac{1}{2}\rho v^2 + p\rho gh = \text{constant}$$

where h is the height of that point above a horizontal 'reference' line. The equation is known as **Bernoulli's equation**.

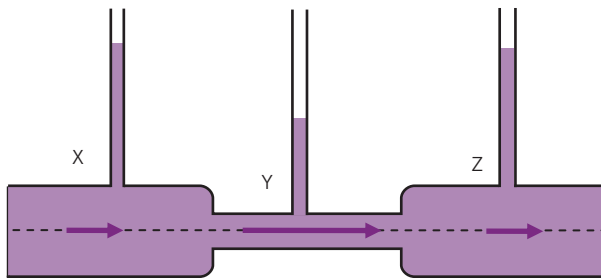


Fig. 24.8 The Bernoulli effect

Notes

1 To understand Bernoulli's equation, consider a small amount or 'element' of an incompressible fluid of volume ΔV and mass Δm moving at speed v .

- Its kinetic energy per unit volume $= \frac{1}{2} \Delta m v^2 / \Delta V = \frac{1}{2} \rho v^2$ where ρ is the fluid density ($= \Delta m / \Delta V$)
- Its potential energy per unit volume at height h above a base line $= \Delta m g h / \Delta V = \rho g h$
- The work done per unit volume by the fluid element $= p \Delta V / \Delta V = p$ (since work done = force $\times$ distance moved = (pressure $p \times$ area of cross-section A) $\times$ distance moved $\Delta x = p \Delta V$ as $\Delta V = A \times \Delta x$)

Applying the principle of conservation of energy therefore gives the work done per unit volume by the fluid element + its kinetic energy per unit volume + its potential energy per unit volume = constant. Hence

$$p + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

2 We can apply Bernoulli's equation to any two points X and Y along a streamline, as shown in Fig. 24.9, to give

$$p_X + \frac{1}{2} \rho v_X^2 + \rho g h_X = p_Y + \frac{1}{2} \rho v_Y^2 + \rho g h_Y$$

where the pressures at X and Y are p_X and p_Y , respectively, the speeds at X and Y are v_X and v_Y , respectively and the heights of X and Y above a fixed horizontal line are h_X and h_Y , respectively. Therefore, if the pressure and height at X and Y are known and the speed at X is measured, the speed at Y can be calculated.

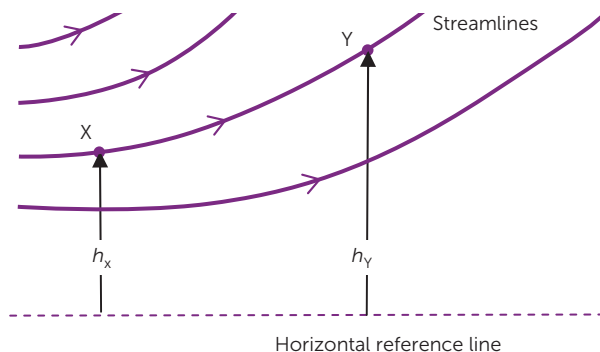


Fig. 24.9 Bernoulli's equation

The pitot-static tube

A pitot-static tube can be used to measure the flow speed at different depths in a river or the speed of a boat or an aircraft. Fig. 24.10 shows a diagram of a simple pitot-static tube.

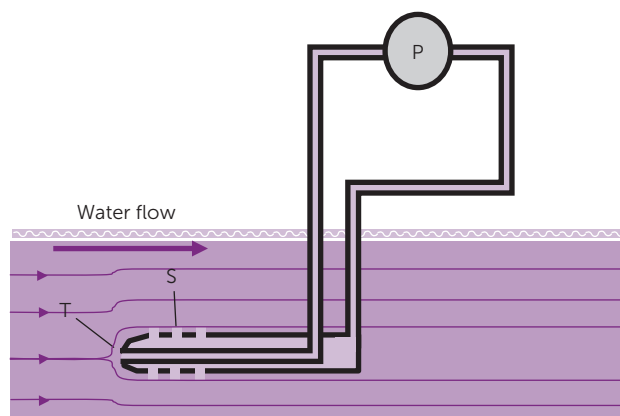


Fig. 24.10
The pitot-static tube

The pressure gauge measures the difference Δp between the pressure on the tube at its tip T and the pressure on the surface S of the tube which is parallel to the flow. This is referred to as the **dynamic pressure** of the fluid as it is the pressure due to its motion and is given by the term $\frac{1}{2}\rho v^2$ in Bernoulli's equation. In other words, the dynamic pressure

$$\Delta p = \frac{1}{2}\rho v^2$$

Rearranging this equation gives

$$v = \sqrt{\frac{2\Delta p}{\rho}}$$

The equation can therefore be used to calculate the flow speed, v , if the dynamic pressure, Δp , is measured and the density of the fluid, ρ , is known.

24.4 QUESTIONS

- Fig. 24.11 shows a cross-section of an aircraft wing in an airflow. The airflow over the top of the wing is faster than underneath it.
 - Explain why the wing experiences an upward force due to the airflow past it.
 - Explain why an aircraft taking off needs to attain a certain speed before it lifts off the ground.

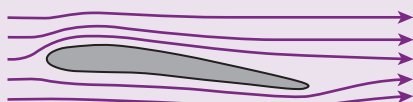


Fig. 24.11

- In an injection tube, a gas flows through a narrow section as shown in Fig. 24.12 and draws air in through a small pipe inserted in the narrow section. Explain why air is drawn into the flow.
 - A pitot-static tube used to measure the speed of river water gives a reading of 210 Pa. Calculate the speed of the water. Density of water = 1000 kg m^{-3}

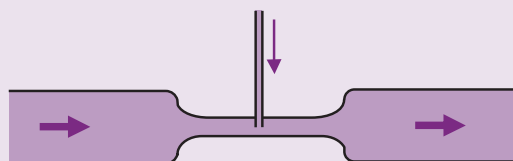


Fig. 24.12

SUMMARY

- **Continuity equation** mass flow per second, $m/t = \rho vA$
- **Velocity gradient in a fluid** change of velocity per unit distance across the flow of the fluid.
- **Viscosity** the coefficient of viscosity of a fluid is the internal friction per unit area of layers of the fluid where the velocity gradient is 1 m s^{-1} per metre across the flow.
- **Poiseuille's equation** $V/t = \frac{\pi r^4 \Delta p}{8\eta L}$
- **Stokes' Law** $F = 6\pi\eta r v$
- **Bernoulli's equation** $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$
- **Pitot-static tube** $v = \sqrt{\frac{2\Delta p}{\rho}}$
- **Dynamic pressure** the pressure of a fluid due to its motion

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.
 $g = 9.8 \text{ m s}^{-2}$

24.1 The cab roof of a lorry is fitted with a wind deflector, as shown in Fig. 24.13. The streamline over the lorry is shown with the deflector fitted. If the deflector is not fitted, the flow over the cab is turbulent instead of streamlined.



Fig. 24.13

- Describe the differences between streamlined flow and turbulent flow.
- Explain why the fuel consumption of the lorry is reduced by fitting the wind deflector.

24.2 In Fig. 24.8, the two wide sections X and Z have the same cross-sectional area.

- State why the water flows in X and Z at the same speed.
- Explain why the water level in the vertical tube at Z is lower than the water level in vertical tube at X.

24.3 A viscous fluid flows through a pipe.

- Sketch the velocity profile across the pipe of the liquid flowing through a pipe. Indicate on your diagram where the velocity gradient in the pipe is greatest.
- For a fluid of viscosity 2.4 N s m^{-2} flowing through a pipe of internal diameter 16 mm and length 160 mm, calculate the pressure difference between the ends of the pipe if the volume of fluid per second flowing through the pipe is $1.1 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$.

24.4 A steel ball of diameter 5.4 mm is released in oil and falls through a distance of 0.50 m in 38 s at constant speed.

Calculate

- (i) the speed of the ball,
(ii) the weight of the ball.
- the viscosity of the oil

Density of steel = 7800 kg m^{-3}

24.5 Water flows from an open tank through an open tap near the base of the tank. A streamline passing through the tap from the water surface in the tank is shown in Figure 29.14.

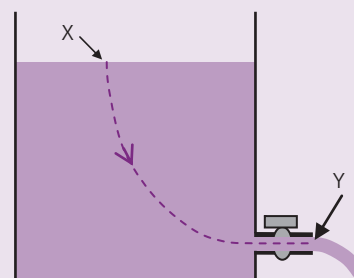


Fig. 24.14

- By applying Bernoulli's equation to points X and Y on the streamline at the water surface in the tank and at the tap, show that the flow speed v out of the tap is equal to $(2gh)^{1/2}$ where h is the vertical height of the water level in the tank above the tap.
- For $h = 0.52 \text{ m}$, calculate
 - the speed of the water from the tap,
 - the mass flow per second from the tap if the tap outlet has a circular cross-section of diameter 12 mm.

Density of water = 1000 kg m^{-3}

UNIT

25

Gases

CONTENTS

- 25.1 The gas laws
- 25.2 The ideal gas equation
- 25.3 The kinetic theory of gases
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- state and use Boyle's law and Charles' law
- explain what is meant by an ideal gas
- state and use the ideal gas equation and the molar gas constant, R
- explain gas pressure using the kinetic theory of matter
- state and use the kinetic theory equation for gas pressure
- explain what is meant by the root mean square speed of the molecules of a gas.
- use the kinetic theory equation to prove Avogadro's hypothesis

25.1 The gas laws**Boyle's law**

If a fixed amount of gas at constant temperature is compressed into a smaller volume, its pressure increases. The relationship between the pressure of a gas and its volume was first investigated in the seventeenth century by Robert Boyle. He discovered that the pressure multiplied by the volume of a gas is always the same, provided the temperature of the gas is unchanged. This relationship is known as

Boyle's law.

$$pV = \text{constant}$$

where p is the pressure and V is the volume of a fixed amount of gas at constant temperature.

Fig. 25.1 shows how Boyle's law can be investigated, using a pump to compress some trapped air in a thick-walled glass tube. Each time the volume of the trapped air is altered, its pressure is measured using the pressure gauge and its volume is measured from the length of the trapped air column. Typical results are shown in Fig. 25.2. The product, pV , is unchanged provided the gas temperature stays the same. Any pair of measurements, p_1 and V_1 , is related to any other pair of measurements, p_2 and V_2 , by Boyle's law equation in the form

$$p_1V_1 = p_2V_2$$

The measurements may be plotted on a graph of y = pressure against x = volume, as shown in Fig. 25.2. If the measurements are repeated for different fixed

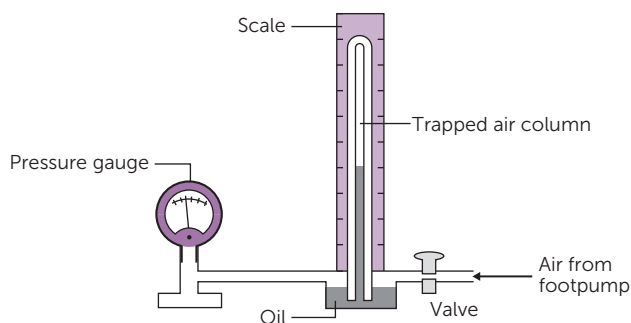


Fig. 25.1 Investigating Boyle's Law

temperatures, each set of measurements gives a different curve according to the temperature at which those measurements were obtained. These curves are referred to as **isothermals** because each curve is a constant temperature curve.

The measurements may also be plotted on a graph of $y = \text{pressure}$ against $x = 1 \div \text{volume}$, as shown in Fig. 25.3. Because the pressure $\times$ the volume ($p \times V$) is constant, the points for each set of measurements define a straight line in accordance with the equation

$$p = \text{constant} \times \frac{1}{V}$$

if Boyle's law is obeyed. A gas that obeys Boyle's law is said to be an **ideal** gas.

Note

An isothermal change is a change at constant temperature.

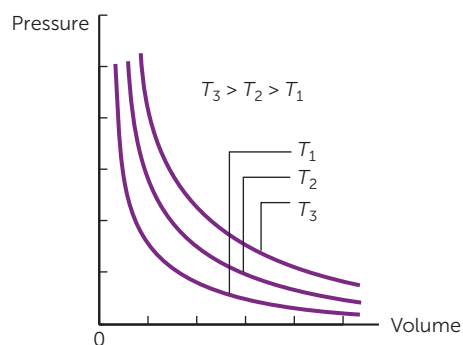


Fig. 25.2
Pressure
versus
volume

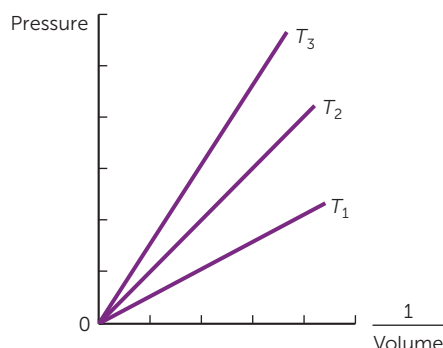


Fig. 25.3
Pressure
versus
 $1 \div \text{volume}$

25.1 WORKED EXAMPLE

A cylinder contained some trapped air which could be compressed using a piston. The volume of the trapped air could be reduced from a maximum volume of 0.0100 m^3 , when its pressure was 100 kPa , to 0.0020 m^3 . Assuming no change of temperature, calculate the pressure in the cylinder when the volume was 0.0020 m^3 .

Solution

$$pV = \text{constant}$$

$$\therefore p \times 0.0020 = 100 \times 10^3 \times 0.0100$$

$$\text{hence } p = \frac{100 \times 10^3 \times 0.0100}{0.0020} = 500 \times 10^3 \text{ Pa} = 500 \text{ kPa}$$

Using Boyle's law to measure the volume of a powder

The air in a sealed flask of known volume, V_F , connected to a pressure gauge is compressed using a pump, as in Fig. 25.4.

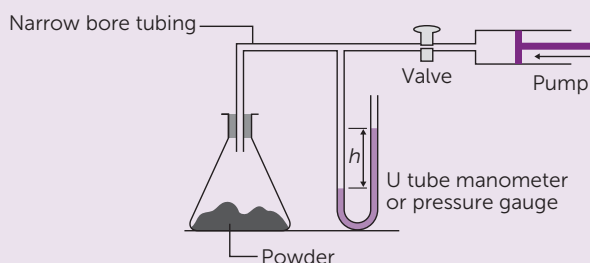


Fig. 25.4 Measuring the volume of a powder

- 1 With the flask empty, the pressure is measured before compression (p_0) and after compression (p_1).

Using Boyle's law gives $p_1 V_F = p_0 (V_F + V_P)$ where V_P is the volume of the pump. Rearranging this equation gives $p_0 V_P = (p_1 - p_0) V_F$.

- 2 With the flask containing powder of unknown volume, v , the pressure is measured before compression (p_0 , which is the same as in 1) and after compression (p_2).

Using Boyle's law therefore gives $p_2 (V_F - v) = p_0 (V_F - v + V_P)$ since the volume of the air was $(V_F - v)$ before compression and $(V_F - v + V_P)$ after compression.

Rearranging this equation gives $p_0 V_P = (p_2 - p_0) (V_F - v)$. Combining the two rearranged equations therefore gives $(p_2 - p_0) (V_F - v) = (p_1 - p_0) V_F$.

Rearranging this equation then gives the following expression from which the powder volume, v , is calculated.

$$v = V_F - V_F \frac{(p_1 - p_0)}{(p_2 - p_0)} = \frac{(p_2 - p_1)}{(p_2 - p_0)} V_F$$

25.1A QUESTIONS

- 1 A cycle pump of volume $4.5 \times 10^{-5} \text{ m}^3$ ($= 45 \text{ cm}^3$) was used to inflate a tyre of volume $1.50 \times 10^{-3} \text{ m}^3$ ($= 1500 \text{ cm}^3$), initially at a pressure of 110 kPa.
 - a) Calculate the pressure of the air in the tyre after one stroke.
 - b) Show that the pressure after each stroke was 3% higher than before the stroke.
 - c) Calculate the pressure after 50 strokes of the pump.
- 2 In an experiment to measure the volume of powder in a flask of volume $2.50 \times 10^{-4} \text{ m}^3$ ($= 250 \text{ cm}^3$), the pressure of the air in the flask was raised using a hand pump with, then without, powder in the flask.
 - a) Without powder present, the pump raised the pressure in the flask from 100 kPa to 118 kPa. Calculate the volume of air in the pump initially.
 - b) With the flask approximately half full of powder, the test was repeated, raising the pressure in the flask from 100 kPa to 140 kPa. Calculate the volume of the powder.

Charles' law

When a gas is heated, its volume increases if it is free to expand. Fig. 25.5 shows how this can be investigated. The gas under test is air trapped in the capillary tube between the sealed end and a thread of liquid. The length of the trapped air column is a measure of the volume of air and it increases if the trapped air is heated. This can be achieved using a water bath.

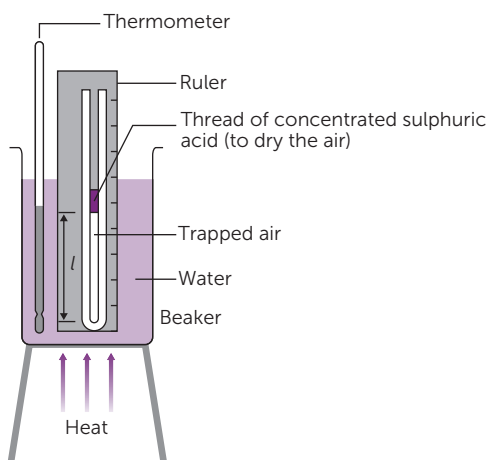


Fig. 25.5 Investigating the expansion of a gas

The volume of the gas is measured at 0°C , the temperature of pure melting ice, and at 100°C , the temperature of steam at atmospheric pressure. These two measurements may be plotted on a graph to find the temperature at which the volume of the gas would become zero if it was cooled. Provided the gas under test is an ideal gas (i.e. obeys Boyle's law over the temperature range $0^{\circ}\text{--}100^{\circ}\text{C}$), this temperature is always -273°C , regardless of which gas is used or how much of the gas is present, and is referred to as the **absolute zero of temperature**, as shown in Fig. 25.6. This is the lowest possible temperature.

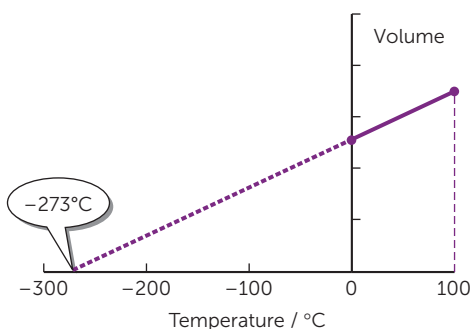


Fig. 25.6 Absolute zero

The **absolute scale of temperature**, T , in kelvins, is related to the Celsius scale, t , in $^{\circ}\text{C}$, by the equation

$$T = t + 273$$

The volume, V , of an ideal gas is therefore proportional to the absolute temperature, T , of the gas, in accordance with the equation

$$V = \frac{V_0 T}{T_0}$$

where V_0 is the volume of the gas at ice point, $T_0 = 273\text{ K}$. In the following form, this equation is known as Charles' law:

$$\frac{V}{T} = \text{constant}$$

Investigating the variation of pressure with temperature for a gas at constant volume

The pressure of a fixed mass of gas at constant volume increases if the gas is heated. This can be investigated using a U-tube mercury manometer to measure the pressure of air in the sealed flask. A water bath is used to change the temperature of the air in the flask. Each pressure measurement is made at constant temperature which is measured using a thermometer in the water bath. As explained on p. 110, the pressure of the air in the flask, p , is given by the equation

$$p = h\rho g + p_0$$

where h is the height difference between the mercury levels in the manometer, ρ is the density of mercury and p_0 is atmospheric pressure (measured separately using a barometer; see p. 111).

The results may be plotted on a graph of y = pressure against x = absolute temperature. For an ideal gas, the points define a straight line that passes through absolute zero at zero pressure. The relationship between the pressure, p , and the absolute temperature, T , of the air in the flask is therefore given by the following equation

$$\frac{p}{T} = \text{constant}$$

provided the volume and mass of gas stays fixed.

The combined gas law

The three separate gas laws can be combined into a single law which links together the pressure, p , volume V , and the absolute temperature, T , of any ideal gas, provided the mass of gas is constant.

$$\frac{pV}{T} = \text{constant}$$

For example, if the pressure, volume and temperature of a gas are p_1 , V_1 and T_1 under one set of conditions and p_2 , V_2 and T_2 under a second set of conditions, then

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$$

25.2 WORKED EXAMPLE

In an electrolysis experiment, 25 cm^3 ($2.5 \times 10^{-5} \text{ m}^3$) of a gas is collected at a pressure of 103 kPa and a temperature of 20°C . Calculate the volume of this quantity of gas at 0°C and a pressure of 102 kPa.

Solution

$$p_1 = 103 \text{ kPa}, \quad V_1 = 25 \text{ cm}^3, \quad T_1 = 20 + 273 = 293 \text{ K}$$

$$p_2 = 102 \text{ kPa}, \quad V_2 = ?, \quad T_2 = 273 \text{ K}$$

$$\text{Using } \frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \text{ gives } \frac{102 \times 10^3 \times V_2}{273} = \frac{103 \times 10^3 \times 2.5 \times 10^{-5}}{293}$$

$$\text{Hence } V_2 = \frac{103 \times 10^3 \times 2.5 \times 10^{-5} \times 273}{293 \times 102 \times 10^3} = 2.35 \times 10^{-5} \text{ m}^3 = 23.5 \text{ cm}^3$$

25.1B QUESTIONS

- 1 Use the combined gas law to complete the missing data in each row of the table.

p_1 / kPa	V_1 / m^3	T_1 / K	p_2 / kPa	V_2 / m^3	T_2 / K
100	0.05	300	110	(a)	350
105	0.24	400	101	0.12	(b)
0.35	0.85	350	(c)	0.58	250
101	0.42	(d)	101	0.38	300
110	(e)	290	101	0.16	273

- 2 a) In a chemistry experiment, 20 cm^3 of gas released from a reaction is collected in a syringe at a pressure of 105 kPa and a temperature of 288 K. Calculate the volume of this amount of gas at 0°C and 101 kPa pressure.
- b) A sealed can contains 30 cm^3 of air at 20°C and 101 kPa pressure. Calculate the pressure in the can at 100°C if its volume increases to 31 cm^3 .

25.2 The ideal gas equation

Molar mass

- The **Avogadro constant**, N_A , is defined as the number of atoms present in exactly 12 g of carbon 12, and is equal to 6.02×10^{23} .
- One mole** of a substance is defined as N_A particles of the substance. The number of moles in a certain quantity of a substance is its **molarity**. The unit of molarity is the mol.
- The **molar mass**, M , of a substance is the mass of N_A particles of the substance. This is the same as the mass of 1 mole of the substance. The unit of molar mass is kg mol^{-1} .

The molar gas constant, R

Equal volumes of ideal gases at the same temperature and pressure contain equal numbers of moles.

This rule was established as a result of measuring the volume of gas released or used in a chemical reaction. Further measurements showed that one mole of any ideal gas at 0°C and a pressure of 101 kPa has a volume of 0.0224 m^3 , regardless of the gas. In other words, the molar volume of any gas at standard temperature and pressure (0°C and 101 kPa) is always 0.0224 m^3 . The value of $PV \div T$ for one mole is known as the **molar gas constant**, R , also referred to as the universal gas constant. Hence

$$R = \frac{101 \times 10^3 \text{ Pa} \times 0.0224 \text{ m}^3}{273}$$

$$= 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

The combined gas law may then be written in the form

$$\frac{pV_m}{T} = R$$

where V_m is the volume of 1 mole. Rearranging this equation gives the **ideal gas equation**:

$$pV_m = RT \text{ for 1 mole}$$

$$\text{or } pV = nRT \text{ for } n \text{ moles}$$

Notes

- The unit of R is the same as the unit of PV/T . Since $1 \text{ pascal} = 1 \text{ N m}^{-2}$, then the unit of pV is the newton metre (Nm) or the joule (J). Hence the unit of R is $\text{J mol}^{-1} \text{ K}^{-1}$.
- The equation can also be written $pV = nMR_gT$ where M is the molar mass and $R_g = R \div M$ (which depends on the gas).

25.3 WORKED EXAMPLE

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}, R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

A sealed flask of volume 60 cm^3 contains a gas at a pressure of 10 kPa and a temperature of 27°C . Calculate

- the number of moles of the gas in the bulb,
- the number of molecules per cm^3 in the bulb.

Solution

$$\begin{aligned} \text{a) Rearrange } pV = nRT \text{ to give } n &= \frac{pV}{RT} = \frac{10 \times 10^3 \times 60 \times 10^{-6}}{8.31 \times (273 + 27)} \\ &= 2.4 \times 10^{-4} \text{ mol} \end{aligned}$$

$$\begin{aligned} \text{b) Number of molecules present} &= nN_A = 2.4 \times 10^{-4} \times 6.02 \times 10^{23} \\ &= 1.45 \times 10^{20} \end{aligned}$$

$$\text{number of molecules per cm}^3 = \frac{1.45 \times 10^{20}}{60} = 2.4 \times 10^{18}$$

25.2 QUESTIONS

$$N_A = 6.02 \times 10^{23}, R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

- A sealed gas container fitted with a safety valve contains 200 cm^3 of gas at a pressure of 120 kPa and a temperature of 15°C .
 - Calculate the number of moles of the gas in the container.
 - The safety valve releases gas if the pressure rises above 150 kPa . Calculate the lowest temperature at which the valve would open.
- If the container is heated above the lowest temperature at which the valve opens, the valve releases gas to keep the pressure at 150 kPa . If the container temperature is raised to 100°C , calculate
 - the number of moles that would be lost from the cylinder,
 - the fraction of gas that would be lost.
- The molar mass of nitrogen is 0.028 kg . Calculate the molar volume and density of nitrogen gas at
 - 0°C and 101 kPa pressure,
 - 100°C and 101 kPa pressure.

25.3 The kinetic theory of gases

Brownian motion

Smoke particles in a light beam observed using a microscope appear as quivering specks of light, moving about unpredictably and erratically. The motion of the particles is called Brownian motion after its discoverer, the botanist Robert Brown, who observed pollen grains in water moving in the manner described above. Fig. 25.7 shows an arrangement to observe the motion of smoke particles.

Brownian motion is caused by repeated and continual impacts on each smoke particle by fast-moving air molecules, too small to see but nevertheless able to make the smoke particles move about noticeably. The erratic motion of each smoke particle occurs because the molecules bombard the smoke particle unevenly and at random. Consequently, the overall force of the impacts on each smoke particle continually changes direction at random. Brownian motion provides direct evidence that a gas consists of very small, fast-moving molecules moving about at random.

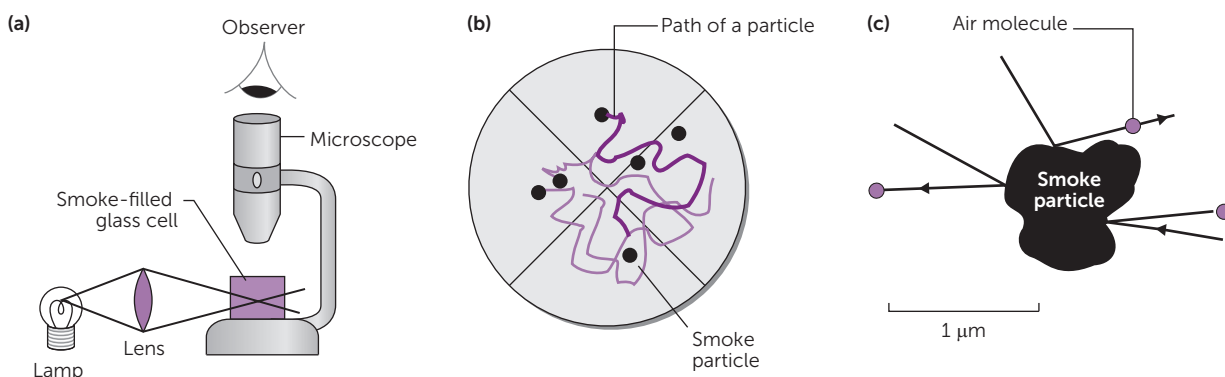


Fig. 25.7 Brownian motion (a) using a microscope, (b) observation, (c) explanation

The cause of gas pressure

The pressure that a gas exerts on any surface exposed to the gas is due to the impacts of the gas molecules on the surface. Each impact exerts a tiny force on the surface. The pressure is measurable because the number of impacts per second per unit area is very large, even though the force of each impact is very small.

- The pressure of a gas rises when its volume is reduced because more impacts take place each second. This is because the size of the gas container is reduced when the gas volume is reduced. Therefore, the molecules hit the surface of the container more frequently, thus raising the pressure of the gas.
- The pressure of the gas rises when its temperature is raised. This is because each impact of a molecule on the surface of the container is more forceful because the molecules move faster at higher temperature. In addition, the number of impacts per second increases because the molecules are moving faster and therefore take less time to move across the container. The pressure is therefore greater because there are more impacts per second and each impact is more forceful.

The kinetic theory model

The following assumptions form the basis of the kinetic theory model of a gas:

- 1 The gas consists of identical point molecules (i.e. the molecules themselves have negligible volume).
- 2 The molecules are in continual random motion.
- 3 Collisions between the molecules and the container surface or between molecules are elastic (i.e. there is no loss of kinetic energy in a collision).
- 4 The molecules do not attract each other.
- 5 When a molecule collides, the duration of the collision is much less than the time between successive collisions.
- 6 The mean kinetic energy of the molecules is proportional to the absolute temperature of the gas.

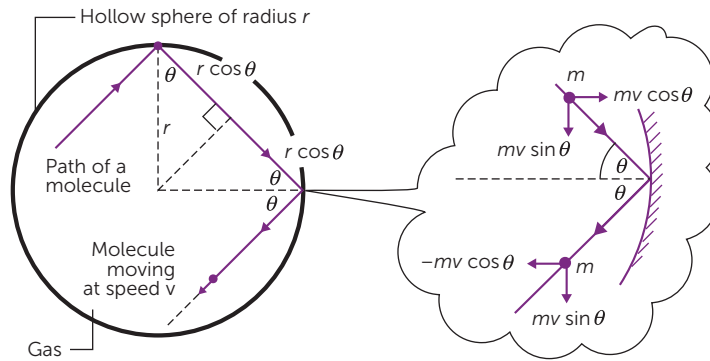


Fig. 25.8 The kinetic theory model of a gas

Consider a single gas molecule of mass, m , moving at speed, v , in a spherical container of internal radius, r , as shown in Fig. 25.8.

- The molecule repeatedly collides with the surface at the same angle each time. For each collision, its momentum component perpendicular to the surface reverses from $mv \cos \theta$ to $-mv \cos \theta$. Its change of momentum = momentum after impact – momentum before impact = $2mv \cos \theta$.

- The time between successive impacts = $\frac{\text{distance}}{\text{speed } v} = \frac{2r \cos \theta}{v}$

- Impact force = $\frac{\text{change of momentum}}{\text{time taken}} = \frac{2mv \cos \theta}{\left(\frac{2r \cos \theta}{v}\right)} = \frac{mv^2}{r}$

- Area of internal surface of the sphere = $4\pi r^2$

$$\therefore \text{pressure, } p = \frac{\text{force}}{\text{area}} = \frac{\left(\frac{mv^2}{r}\right)}{4\pi r^2} = \frac{mv^2}{4\pi r^3}$$

Since the volume of the container $V = \frac{4\pi r^3}{3}$, then $p = \frac{mv^2}{3V}$

Now consider N identical molecules in the container:

- the pressure of molecule 1, p_1 , can be written as

$$p_1 = \frac{mv_1^2}{3V}$$

where v_1 is the speed of the molecule

- the pressure of molecule 2, p_2 , can be written as

$$p_2 = \frac{mv_2^2}{3V}$$

where v_2 is the speed of the molecule

- the pressure of molecule 3, p_3 , can be written as

$$p_3 = \frac{mv_3^2}{3V}$$

where v_3 is the speed of the molecule

- the pressure of molecule N , p_N , can be written as

$$p_N = \frac{mv_N^2}{3V}$$

where v_N is the speed of the molecule

- $\therefore$ the total pressure, $p = p_1 + p_2 + p_3 + \dots + p_N$

$$p = \frac{mv_1^2}{3V} + \frac{mv_2^2}{3V} + \frac{mv_3^2}{3V} + \dots + \frac{mv_N^2}{3V}$$

$$\therefore p = \frac{Nmv_{\text{RMS}}^2}{3V}$$

$$\text{where } v_{\text{RMS}}^2 = \frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2}{N}$$

This equation is known as the **kinetic theory equation**. Multiplying each side by V gives the equation in the form

$$pV = \frac{Nmv_{\text{RMS}}^2}{3}$$

Notes

- v_{RMS} is referred to as the root mean square (rms) speed of the molecules of the gas.

$$v_{\text{RMS}} = \left(\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2}{N} \right)^{1/2}$$

- The density, $\rho = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V}$

$$\therefore p = \frac{Nmv_{\text{RMS}}^2}{3V} = \frac{1}{3}\rho v_{\text{RMS}}^2$$

25.4 WORKED EXAMPLE

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

The molar mass of oxygen is 0.032 kg. Calculate

- the density of oxygen at a temperature of 20°C and a pressure of 101 kPa,
- the rms speed of oxygen molecules at this density and pressure.

Solution

- Use $pV_m = RT$ to calculate the volume of 1 mole at 20°C and 101 kPa pressure

$$\text{Hence } V_m = \frac{RT}{p} = \frac{8.31 \times (273 + 20)}{101 \times 10^3} = 2.41 \times 10^{-2} \text{ m}^3$$

$$\therefore \text{density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{molar mass}}{\text{molar volume}} = \frac{0.032 \text{ kg}}{2.41 \times 10^{-2} \text{ m}^3} = 1.33 \text{ kg m}^{-3}$$

- Rearrange $p = \frac{1}{3}\rho v_{\text{RMS}}^2$ to give $v_{\text{RMS}}^2 = \frac{3p}{\rho} = \frac{3 \times 101 \times 10^3}{1.33} = 2.28 \times 10^5 \text{ m}^2 \text{ s}^{-2}$

$$\therefore v_{\text{RMS}} = 478 \text{ m s}^{-1}$$

Kinetic energy and temperature

The total kinetic energy of all the molecules of gas

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \dots + \frac{1}{2}mv_N^2$$

$$= \frac{1}{2}Nm \frac{(v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2)}{N}$$

$$= \frac{1}{2}Nmv_{\text{RMS}}^2$$

By assuming the total kinetic energy of the molecules of a gas, $\frac{1}{2}Nmv_{\text{RMS}}^2 = \frac{3}{2}nRT$, the kinetic theory equation $pV = \frac{1}{3}Nmv_{\text{RMS}}^2$ becomes $pV = nRT$. The ideal gas equation, $pV = nRT$, can therefore be derived by making certain assumptions (listed on p. 409) about the molecules of an ideal gas.

The mean kinetic energy of an ideal gas molecule

$$= \frac{\text{total kinetic energy}}{\text{number of molecules, } N}$$

$$= \frac{\frac{1}{2} N m v_{\text{RMS}}^2}{N} = \frac{1}{2} m v_{\text{RMS}}^2$$

Also, since the total kinetic energy of all the molecules

$$= \frac{1}{2} N m v_{\text{RMS}}^2 = \frac{3}{2} n R T$$

the mean kinetic energy of an ideal gas molecule

$$\frac{1}{2} m v_{\text{RMS}}^2 = \frac{3 n R T}{2 N} = \frac{3 k T}{2}$$

where k , the Boltzmann constant

$$= \frac{R}{N_A} = \frac{n R}{N} = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}.$$

Mean kinetic energy of a gas molecule = $\frac{3}{2} k T$

25.5 WORKED EXAMPLE

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}, k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}$$

Calculate the mean kinetic energy of a gas molecule at 20°C in

a) J, b) electron volts.

Solution

a) Mean kinetic energy of a gas molecule = $\frac{3}{2} k T$

$$= 1.5 \times 1.38 \times 10^{-23} \times (273 + 20) = 6.1 \times 10^{-21} \text{ J}$$

b) $6.1 \times 10^{-21} \text{ J} = \frac{6.1 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 0.038 \text{ eV}$

The distribution of molecular speeds in a gas

The molecules in an ideal gas possess a continuous range of speeds from zero upwards. The distribution of the number of molecules at different speeds is shown in Fig. 25.9. The peak of this distribution curve is the most probable speed of a molecule. The rms speed is not the same as the most probable speed. The distribution curve depends on temperature. At a higher temperature, more molecules move at higher speeds so the curve is broader and flatter.

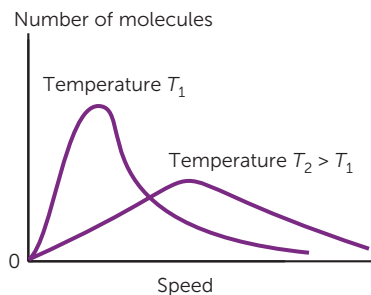


Fig. 25.9 Distribution of molecular speeds

Avogadro's hypothesis

This hypothesis states that equal volumes of gas at the same temperature and pressure contain equal numbers of molecules.

Consider equal volumes, V of two gases X and Y at the same temperature, T and pressure, p . According to the kinetic theory equation

$$pV = \frac{Nm v_{\text{RMS}}^2}{3} = NkT \text{ since } \frac{1}{2} m v_{\text{RMS}}^2 = \frac{3}{2} kT$$

For gas X, $pV = N_X kT$ and for gas Y, $pV = N_Y kT$

$$\therefore N_X kT = N_Y kT$$

$$\text{so } N_X = N_Y$$

in accordance with Avogadro's hypothesis.

25.3 QUESTIONS

$$R = 8.31 \text{ J K}^{-1}, N_A = 6.02 \times 10^{23} \text{ mol}^{-1}, k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

- Calculate (i) the density, (ii) the rms speed of molecules of nitrogen gas at 100°C and a pressure of 150 kPa.
 - Calculate the mean kinetic energy of an ideal gas molecule at 100°C .
 - Calculate the temperature at which the mean kinetic energy of an ideal gas molecule is double its mean kinetic energy at 100°C .

Molar mass of nitrogen = 0.028 kg.
- Calculate (i) the mean kinetic energy, (ii) the rms speed of molecules of the gas for hydrogen gas at
 - 0°C ,
 - 100°C .

Molar mass of hydrogen = 0.002 kg.

SUMMARY

- Absolute zero of temperature** is the lowest possible temperature, equal to -273°C .
- Absolute temperature in Kelvins, T = temperature in $^\circ\text{C}$ + 273**
- The gas laws**
 - Boyle's law $pV = \text{constant}$ for constant mass and temperature
 - Charles' law $\frac{V}{T} = \text{constant}$ for constant mass and pressure
 - Pressure law $\frac{p}{T} = \text{constant}$ for constant mass and volume
 - Combined gas law $\frac{pV}{T} = \text{constant}$ for constant mass
- The ideal gas equation** $pV = nRT$
- The kinetic theory equation** $pV = \frac{1}{3} N m v_{\text{RMS}}^2$ or $p = \frac{1}{3} \rho v_{\text{RMS}}^2$
- The root mean square speed** $v_{\text{RMS}} = \left(\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_N^2}{N} \right)^{1/2}$
- Mean kinetic energy of a gas molecule** $= \frac{3}{2} kT$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

$$R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}, k = 1.38 \times 10^{-23} \text{ J K}^{-1}, \\ N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

25.1

- a) In an electrolysis experiment, 40 cm^3 of oxygen gas is collected at 15°C and a pressure of 103 kPa . Calculate
- the volume this gas would occupy at 0°C and 101 kPa pressure,
 - the number of moles of this gas and its mass.
- The molar mass of oxygen is 0.032 kg .
- b) An air bubble trapped in a brake pipe has a volume of 1.2 cm^3 at a pressure of 101 kPa and a temperature of 10°C .
- Calculate the volume of this air bubble at a pressure of 200 kPa and a temperature of 20°C .
 - Calculate the mass of this air bubble, assuming the molar mass of air is 0.029 kg .

25.2

- a) A tyre contains 1600 cm^3 of air at a pressure of 150 kPa and a temperature of 25°C . Calculate
- the pressure of this amount of air at 0°C and a volume of 1500 cm^3 ,
 - the mass of air in the tyre, assuming the molar mass of air is 0.029 kg .
- b) A capillary tube sealed at one end contained a column of air trapped by a thread of mercury of length 150 mm , as shown in Fig. 25.10. When the tube was vertical, the length of the air column was 120 mm when the sealed end was below the open end. When the tube was inverted with the sealed end above the open end, the air column length at the same temperature was 180 mm . Calculate the atmospheric pressure in millimetres of mercury.

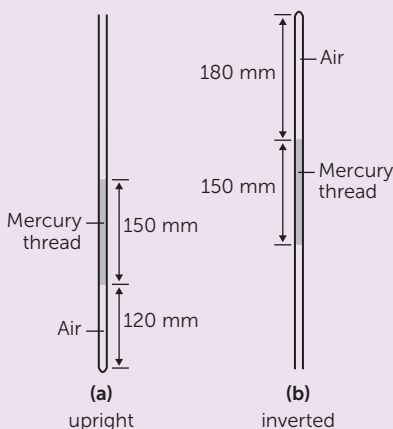


Fig. 25.10

25.3

- a) Sketch graphs to show how the product of pressure and volume, pV , for 1 mole of an ideal gas varies with the temperature of the gas in
- kelvins,
 - $^\circ\text{C}$.
- b) Show on each graph how pV varies with temperature for 0.5 moles.

25.4

- a) A gas cylinder contains 500 cm^3 of nitrogen at a pressure of 140 kPa and a temperature of 20°C . Calculate (i) the mass of gas in the cylinder, (ii) the number of molecules of gas in the cylinder, (iii) the rms speed of the molecules. The molar mass of nitrogen = 0.028 kg .
- b) The cylinder is fitted with a safety valve that operates at a pressure of 170 kPa . Calculate
- the maximum temperature the gas can be heated to without the safety valve opening,
 - the mass of gas lost if the gas temperature is raised to 100°C ,
 - the rms speed of the molecules at 100°C .

25.5

- a) Explain why the pressure of a gas falls if
- its volume is increased at constant temperature,
 - its temperature is reduced at constant volume.
- b) (i) Calculate the rms speed of hydrogen molecules at 20°C .
- (ii) Calculate the rms speed of oxygen molecules at 20°C .
- (iii) By comparing the two calculations in (i) and (ii), explain why the Earth's atmosphere retains oxygen but not hydrogen.
- The molar mass of hydrogen = 0.002 kg , the molar mass of oxygen = 0.032 kg .

Thermodynamics

OBJECTIVES

After working through this unit, you should be able to:

- describe the use of different types of thermometers, including calibration on the Celsius and absolute scales
- state and use the first law of thermodynamics
- explain what is meant by an isothermal change and an adiabatic change
- calculate the work done when the volume of a gas changes
- calculate the change of internal energy of an ideal gas when its temperature changes
- define the molar heat capacities of a gas at constant volume, C_V , and at constant pressure, C_P
- relate C_P and C_V to each other and to the atomicity of the molecules of a gas
- explain what is meant by enthalpy
- state and explain the second law of thermodynamics
- explain what a heat engine is and calculate its maximum possible efficiency
- define the thermodynamic scale of absolute temperature and relate it to the ideal gas scale

CONTENTS

- 26.1 Temperature
- 26.2 Heat and work
- 26.3 The thermodynamics of ideal gases
- 26.4 The Second Law of Thermodynamics
- Summary
- Revision questions

26.1 Temperature

As explained in [Unit 6](#), a scale of temperature is defined by assigning numerical values to ‘fixed points’ which are standard degrees of hotness that are easily reproducible. Any thermometer works on the basis of a physical property, its thermometric property, that varies smoothly with change of temperature. [Figs 26.1](#), [26.2](#) and [26.3](#) show three different types of thermometers.

- The **liquid-in-glass thermometer** contains a liquid that expands when the thermometer bulb warms up. The liquid expands up the stem and the temperature is measured from the position of the end of the liquid thread in the stem against a calibrated scale. The thermometric property is the thermal expansion of the liquid.

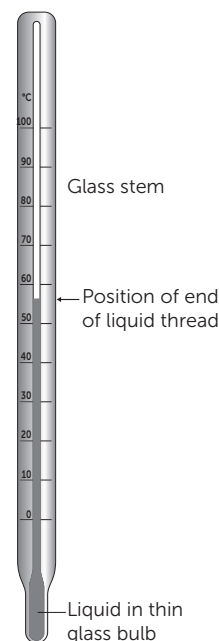


Fig. 26.1 The liquid-in-glass thermometer

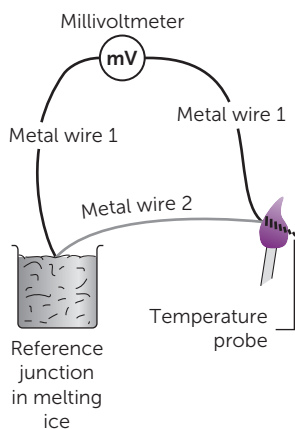


Fig. 26.2 The thermocouple thermometer

- The **thermocouple thermometer** makes use of the fact that a potential difference (p.d.) exists between two dissimilar metals in contact. This p.d. depends on the temperature of the contact junction. In practice, two wires of identical materials are joined to opposite ends of a dissimilar wire to give two junctions in series in opposition to each other, as in Fig. 26.2. A net p.d. is thus only produced when the two junctions are at different temperatures. One junction is maintained at ice point and the other is the 'temperature probe'. The temperature can then be measured from a suitably calibrated meter.
- The **constant volume gas thermometer** contains a bulb of dry air connected to a mercury manometer. The pressure of the air in the bulb is measured using the manometer. When the bulb temperature is changed, the open arm of the manometer is adjusted to keep the mercury level in the closed arm at the same position throughout. The height difference, h , between the mercury level in the closed arm and the open arm is measured and used to calculate the temperature, t , in $^{\circ}\text{C}$ from the formula

$$t = \frac{h - h_0}{H_{100} - h_0} \times 100^{\circ}\text{C}$$

where h_0 is the height difference at ice point and h_{100} is the height difference at steam point.

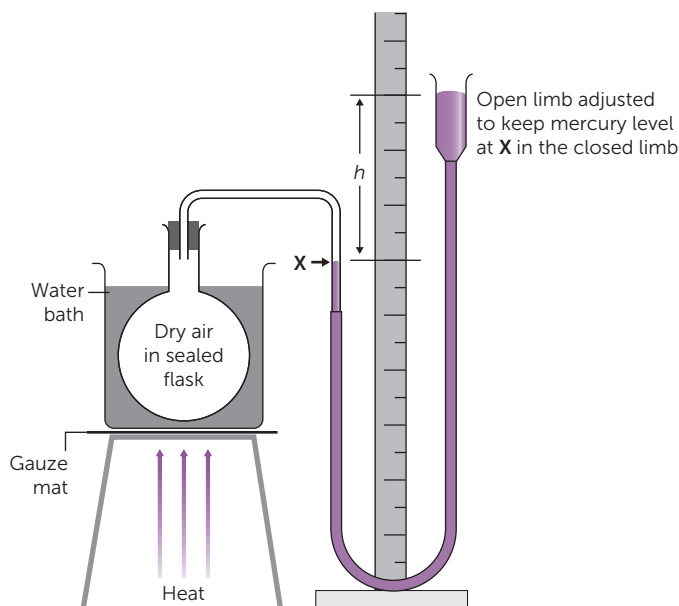


Fig. 26.3 The gas thermometer

Calibrating a thermometer

At temperatures away from the fixed points, different thermometers give different readings for the same temperature. The gas thermometer is chosen as the standard against which other types of thermometers are calibrated between the fixed points. Temperatures according to the gas thermometer are referred to as **ideal gas temperatures**.

The **Celsius scale** is achieved in practice by calibrating a gas thermometer at ice point and at steam point then using the equation on p. 416. The absolute zero of temperature on this scale is -273.15°C .

The **absolute scale**, described on p. 80, is achieved in practice by calibrating a gas thermometer at the **triple point** of water which is the point at which water, water vapour and ice coexist in thermal equilibrium. See Fig. 26.4.

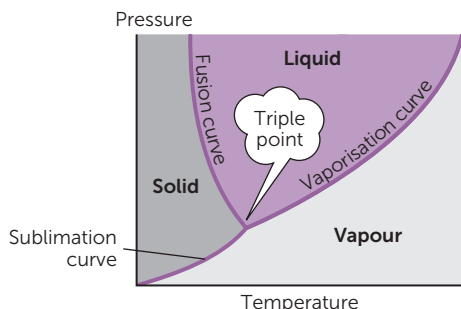


Fig. 26.4 The triple point of water

Since this temperature is 0.01°C higher than ice point, the triple point temperature on the absolute scale is defined as 273.16 K. Temperatures on the absolute scale may therefore be measured from the readings on a gas thermometer in accordance with the equation

$$T = \frac{(pV)_{p \rightarrow 0}}{(pV_{\text{Tr}})_{p \rightarrow 0}} \times 273.16 \text{ K}$$

where $(pV)_{p \rightarrow 0}$ is the limit of pressure $\times$ volume of the gas at zero pressure at temperature T and $(pV_{\text{Tr}})_{p \rightarrow 0}$ is the same quantity at the triple point of water.

By defining the absolute scale of temperatures in this way, the link between the two scales is given by the equation:

$$T \text{ (in K)} = t \text{ (in } ^{\circ}\text{C)} + 273.15$$

Thus the reading of any type of thermometer, which has been calibrated in $^{\circ}\text{C}$ against a gas thermometer, can be converted into kelvins by adding 273.15 to its reading.

Note

For any thermometer (other than a gas thermometer) which is calibrated at ice point and steam point, if the interval between the two fixed points is divided into 100 equal degrees, the scale is then a **centigrade** scale, not the Celsius scale. The unit $^{\circ}\text{C}$ should not be used for readings taken from a centigrade scale as $^{\circ}\text{C}$ means 'degree Celsius'.

26.1 QUESTIONS

- 1 a) A constant volume gas thermometer was calibrated at ice point and steam point, then used to measure the temperature of a beaker of hot water. The readings for the height difference, h , between the mercury level in the closed limb and the open limb were -50 mm for ice point, $+220 \text{ mm}$ for steam point and $+105 \text{ mm}$ for the temperature of the water. Calculate the temperature of the hot water in $^{\circ}\text{C}$.
 b) When the position of the mercury in a mercury-in-glass thermometer in some hot water was exactly midway between ice point and steam point, the height difference between the mercury levels of the constant volume gas thermometer in (a) was $+80 \text{ mm}$. Calculate the temperature (i) on the centigrade scale of the mercury-in-glass thermometer, (ii) in $^{\circ}\text{C}$.
- 2 The electromotive force (e.m.f.), E , of a certain thermocouple varies with the temperature in $^{\circ}\text{C}$ in accordance with the equation $E = at - bt^2$, where $a = 40.5 \text{ } \mu\text{V } ^{\circ}\text{C}^{-1}$ and $b = 0.065 \text{ } \mu\text{V } ^{\circ}\text{C}^{-2}$.
 a) Calculate the e.m.f. at $t = 0, 20, 40, 60, 80$ and 100°C .
 b) (i) Plot a graph to show how the e.m.f. varies with temperature. (ii) Use your graph to determine the temperature in $^{\circ}\text{C}$ when the e.m.f. is midway between its value at ice point and its value at steam point.

26.2 Heat and work

Work is done by a force when it moves its point of application in the direction of the force. Energy is the capacity of a body to do work. Energy can be transferred from one body to another by two methods:

- 1 Work is energy transferred by means of a force moving its point of application.
- 2 Heat is energy transferred by means other than a force. A temperature difference is said to exist between two bodies if heat transfer between the two bodies could occur.

If no heat transfer occurs between two bodies in thermal contact, the two bodies must be at the same temperature. This is known as the **zeroth law of thermodynamics**.

The **internal energy** of a body is the energy of the body (i.e. its capacity to do work) regardless of its position (i.e. its potential energy) or its motion (i.e. its kinetic energy). In effect, the internal energy of a body is the potential energy due to the bonds between its atoms and molecules and the kinetic energy due to the random motion of its atoms and molecules. When energy is transferred overall to a fixed body, its molecules gain either kinetic energy or potential energy or both. Increasing the kinetic energy of the molecules of a body raises its temperature. Increasing the potential energy of the molecules changes its state.

For example, suppose a sealed can of air is placed in a beaker of hot water. The air gains internal energy and the water loses internal energy due to heat transfer through the can. Thus the capacity of the air to do work has increased because its internal energy has increased. For example, if the can is pierced suddenly, air would be forced out due to the high pressure inside. The outflow of air could be used to do work. Two further examples of the use of heat to do work are shown in Figs 26.5 and 26.6. A device that uses heat to do work is called a **heat engine**.

- The four-stroke petrol engine uses the heat from the combustion of petrol and oxygen in air to do work. The sequence of strokes is shown in Fig. 26.5.

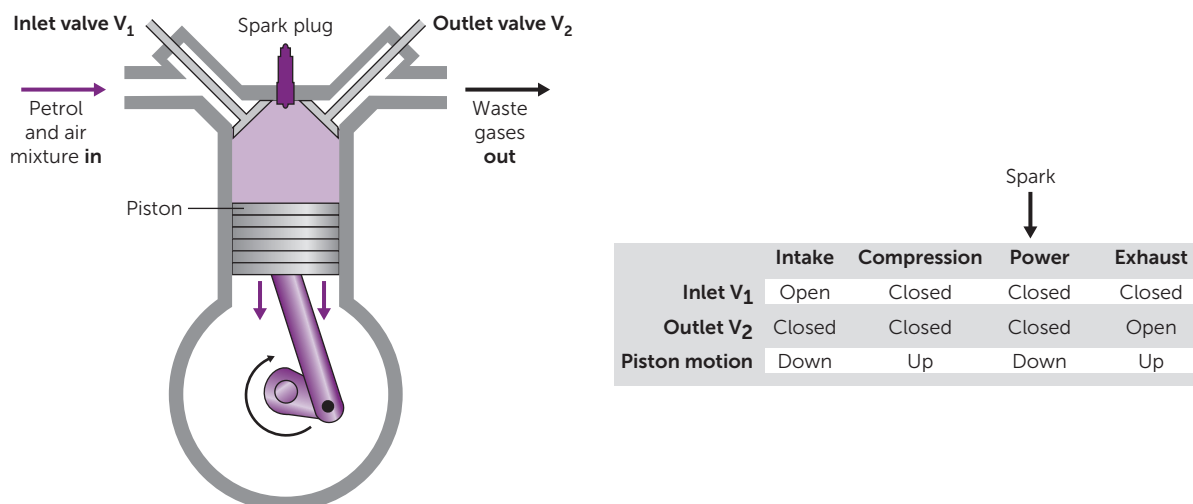


Fig. 26.5 The four-stroke petrol engine

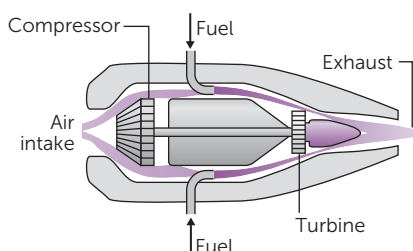


Fig. 26.6 The jet engine

- The jet engine in Fig. 26.6 burns fuel in air to raise the temperature of air drawn into the engine by a compressor. The hot air is expelled at high speed from the exhaust, creating the thrust that drives the engine forward.

The First Law of Thermodynamics

The internal energy of a body can be changed, either by heat transfer in or out of the body, or by work done on or by the body. For example, suppose a body does 1000 J of work and its internal energy decreases by 800 J. From the principle of conservation of energy, it therefore follows that the body must have gained 200 J of heat. Using conservation of energy to equate the overall transfer of energy to the change of internal energy therefore gives rise to the following statement, known as the **First Law of Thermodynamics**.

As shown in Fig. 26.7, for a body of internal energy, U , the heat entering the body, ΔQ = its change of internal energy, ΔU + the work done by the body, ΔW .

$$\Delta Q = \Delta U + \Delta W$$

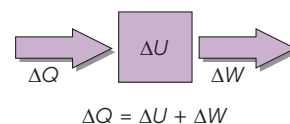


Fig. 26.7 The First Law of Thermodynamics

Notes

A sign convention is necessary to take account of the direction of energy transfer and change of internal energy.

- 1 A positive value for ΔU means an increase of internal energy and a negative value means a decrease.
- 2 A positive value for ΔQ means heat entering and a negative value means heat leaving.

- 3 A positive value for ΔW means work done by the gas (i.e. expansion) and a negative value means work done on the gas (i.e. compression).
- 4 An **adiabatic change** is defined as a change in which there is no heat transfer, that is, $\Delta Q = 0$. For an adiabatic change $\Delta U = -\Delta W$.

26.1 WORKED EXAMPLE

Calculate the change of internal energy of a gas as a result of 100 J of work done on the gas and 80 J of heat

- a) entering the gas,
- b) leaving the gas.

Solution

$$\Delta Q = \Delta U + \Delta W$$

$$\text{a) } \Delta Q = +80 \text{ J, } \Delta W = -100 \text{ J}$$

$$\therefore \Delta U = \Delta Q - \Delta W = 80 - (-100) = 180 \text{ J increase.}$$

$$\text{b) } \Delta Q = -80 \text{ J, } \Delta W = -100 \text{ J}$$

$$\therefore \Delta U = \Delta Q - \Delta W = -80 - (-100) = 20 \text{ J increase.}$$

26.2 QUESTIONS

1 Calculate the heat transfer to or from a body when

- a) its internal energy increases by 1000 J and (i) the body does 300 J of work, (ii) 300 J of work is done on the body,
 b) its internal energy decreases by 500 J and (i) the body does 200 J of work, (ii) 200 J of work is done on the body.

2 Complete the following table:

$\Delta U / \text{J}$	$\Delta Q / \text{J}$	$\Delta W / \text{J}$
2000	400	(a)
-2000	(b)	2400
(c)	-800	-2400
-1 000	-400	(d)

26.3 The thermodynamics of ideal gases

Work done by a gas

Consider a gas in a cylinder at pressure, p , held in by a piston of cross-sectional area, A , as in Fig. 26.8. The force, F , of the gas on the cylinder is therefore given by the equation $F = pA$.

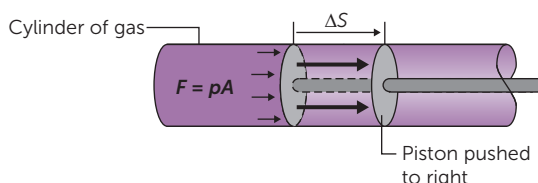


Fig. 26.8 Work done by a gas

Suppose the gas pushes the piston outwards by a distance Δs at constant pressure. The work done by the gas, $\Delta W = F\Delta s = pA\Delta s$.

Since the increase of volume of the gas $\Delta V = A\Delta s$, it therefore follows that $\Delta W = p\Delta V$.

Work done by a gas to expand at constant pressure

$$\Delta W = p\Delta V$$

Notes

For an ideal gas at constant pressure,

- before expansion, $pV = nRT$,
- after expansion $p(V + \Delta V) = nR(T + \Delta T)$. Therefore subtracting the first equation from the second equation gives $p\Delta V = nR\Delta T$. Hence the work done by an ideal gas when it expands at constant pressure is given by $\Delta W = nR\Delta T$.

Molar heat capacities

When heat is supplied to a gas, its temperature change depends on whether or not it is allowed to expand. The First Law of Thermodynamics for an ideal gas may be expressed in the form

$$\Delta Q = \Delta U + p\Delta V$$

- At **constant volume**, no work is done by or on a gas as $\Delta V = 0$, hence the heat supplied to an ideal gas at constant volume to raise its temperature is given by $\Delta Q = \Delta U$. See Fig. 26.9.

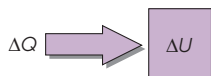


Fig. 26.9 Heating a gas at constant volume

The molar heat capacity at constant volume of a gas, C_v , is defined as the heat supplied at constant volume to raise the temperature of 1 mole by 1 K.

The heat supplied, ΔQ , to raise the temperature of n moles by ΔT at constant volume is therefore given by

$$\Delta Q = nC_v\Delta T$$

Hence the change of internal energy of a gas when its temperature changes by ΔT is given by

$$\Delta U = nC_v\Delta T$$

- At **constant pressure**, work is done by or on a gas as ΔV is not zero. See Fig. 26.10.

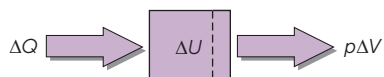


Fig. 26.10 Heating a gas at constant pressure

The molar heat capacity at constant pressure, C_p , of a gas is defined as the heat supplied at constant pressure to raise the temperature of 1 mole by 1 K.

The heat supplied, Q , to raise the temperature of n moles by ΔT at constant pressure is therefore given by

$$\Delta Q = nC_p\Delta T$$

Since $\Delta Q = \Delta U + p\Delta V$, and $p\Delta V = nR\Delta T$, then $nC_p\Delta T = nC_v\Delta T + nR\Delta T$. By considering $n = 1$ mole and $\Delta T = 1$ K, it follows that $C_p = C_v + R$.

$$C_p - C_v = R$$

Atomicity and degrees of freedom

A **monoatomic molecule** has three degrees of freedom, each degree corresponding to motion which is independent of the motion associated with any other degree of freedom, as shown in Fig. 26.11a. Thus a monoatomic molecule can move in any one of three perpendicular directions without altering its motion in the other two directions. Since the mean kinetic energy of a point molecule is $\frac{3}{2}kT$, then the mean kinetic energy associated with each degree of freedom is $\frac{1}{2}kT$.

The total kinetic energy of n moles of monoatomic molecules is $\frac{3}{2}nRT$, since n moles contain $nR \div k$ molecules (as $k = R \div N_A$, then 1 mole contains $R \div k$ molecules).

Hence the internal energy, U , of n moles of an ideal gas of monoatomic molecules is $\frac{3}{2}nRT$.

$$U = \frac{3}{2}nRT$$

If the temperature of an ideal gas changes by ΔT , the change of its internal energy, ΔU , is therefore equal to $\frac{3}{2}nR\Delta T$. Therefore, to raise the temperature of 1 mole by 1 K, the internal energy must increase by $\frac{3}{2}R$. Since this is equal to the heat supplied at constant volume, it follows that

$$C_v = \frac{3}{2}R$$

Also, since $C_p = C_v + R$, then

$$C_p = \frac{5}{2}R$$

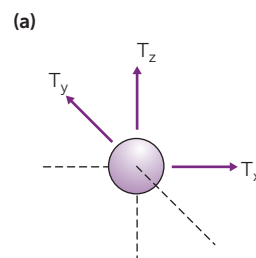
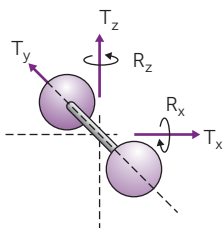


Fig. 26.11 Degrees of freedom
(a) a monoatomic molecule

(b)



(c)

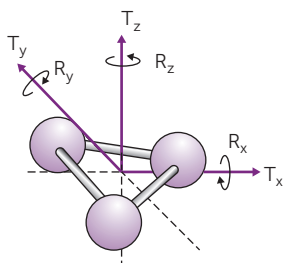


Fig. 26.11 Degrees of freedom
(b) a diatomic molecule, (c) a polyatomic molecule

Therefore, the ratio of molar heat capacities, γ , can be written as

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.67$$

for a monoatomic gas.

A **diatomic molecule** has five degrees of freedom, three of which are due to its motion in any of three perpendicular directions. The other two are due to its rotational motion about two perpendicular directions at right angles to its axis. See Fig. 26.11b. The mean kinetic energy of a diatomic molecule is therefore $\frac{5}{2}kT$. The total kinetic energy of 1 mole of diatomic molecules is therefore $\frac{5}{2}nRT$.

Therefore, the total kinetic energy of n moles of diatomic molecules is $\frac{5}{2}nRT$.

Hence the internal energy, U , of n moles of an ideal gas of diatomic molecules is $\frac{5}{2}nRT$.

$$U = \frac{5}{2}nRT$$

If the temperature of an ideal gas changes by ΔT , the change of its internal energy, ΔU , is therefore equal to $\frac{5}{2}nR\Delta T$. Therefore, to raise the temperature of 1 mole by 1 K, the internal energy must increase by $\frac{5}{2}R$. Since this is equal to the heat supplied at constant volume, it follows that

$$C_v = \frac{5}{2}R$$

Also, since $C_p = C_v + R$, then

$$C_p = \frac{7}{2}R$$

Therefore, the ratio of molar heat capacities, γ , can be written as

$$\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$$

for a diatomic gas.

A **polyatomic molecule** (i.e. a molecule with more than two atoms) has six degrees of freedom, five of them being the same as for a diatomic molecule. The sixth is due to rotation in the third possible perpendicular direction. See Fig. 26.11c. The mean kinetic energy of a polyatomic molecule is therefore $\frac{6}{2}kT$.

Therefore, the total kinetic energy of n moles of polyatomic molecules is $\frac{6}{2}nRT$.

Hence the internal energy, U , of n moles of an ideal gas of polyatomic molecules is $\frac{6}{2}nRT$.

$$U = \frac{6}{2}nRT$$

If the temperature of an ideal gas changes by ΔT , the change of its internal energy, ΔU , is therefore equal to $\frac{6}{2}nR\Delta T$. Therefore, to raise the temperature of 1 mole by 1 K, the internal energy must increase by $\frac{6}{2}R$. Since this is equal to the heat supplied at constant volume, it follows that

$$C_v = \frac{6}{2}R$$

Also, since $C_p = C_v + R$, then

$$C_p = \frac{8}{2}R$$

Therefore, the ratio of molar heat capacities, γ , can be written as

$$\gamma = \frac{C_p}{C_v} = \frac{8}{6} = 1.33$$

for a polyatomic gas.

Adiabatic changes of an ideal gas

An adiabatic change is defined as one in which no heat transfer occurs. In practice, this could be due to a change being very rapid or due to perfect insulation.

- If an ideal gas expands adiabatically, it does work and therefore its internal energy decreases. It therefore cools.
- If an ideal gas is compressed adiabatically, work is done on the gas and therefore its internal energy rises. Its temperature therefore increases.

Fig. 26.12 shows how the pressure and volume of a fixed quantity of an ideal gas vary for both adiabatic changes and isothermal changes (i.e. constant temperature). The adiabatic curves are steeper than the isothermal curves because the temperature changes when an adiabatic expansion or compression occurs.

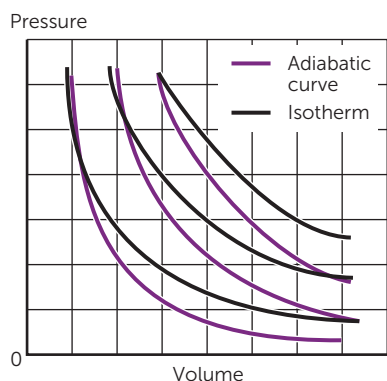


Fig. 26.12 Comparison of adiabatic and isothermal changes

- The area under a pressure against volume curve gives the work done, as shown in Fig. 26.13. This is because the work done in each small volume change, δV , is equal to $p\delta V$, which is represented by the strip of width, δV , under the curve. Therefore the total work done is represented by the total area under the curve.

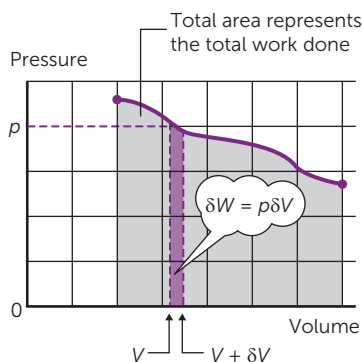


Fig. 26.13 Area under a pV curve

- The pressure and volume of an ideal gas before and after an adiabatic change are related by the equation:

$$pV^\gamma = \text{constant}$$

where γ is the ratio of molar heat capacities.

Proof of $pV^\gamma = \text{constant}$

To prove this equation, consider the First Law of Thermodynamics in the form $\Delta Q = nC_V\Delta T + p\Delta V$.

For an adiabatic change, $\Delta Q = 0$, thus $nC_V\Delta T + p\Delta V = 0$.

$$\therefore p\Delta V = -nC_V\Delta T \quad (1)$$

From the ideal gas equation, $pV = nRT$, then

$$(p + \Delta p)(V + \Delta V) = nR(T + \Delta T)$$

By subtraction, $V\Delta p + p\Delta V = nR\Delta T$

$$\begin{aligned} \therefore V\Delta p &= nR\Delta T - p\Delta V \\ &= nR\Delta T + nC_V\Delta T = nC_p\Delta T \end{aligned}$$

$$V\Delta p = nC_p\Delta T \quad (2)$$

Combining equations 1 and 2 gives

$$\begin{aligned} V\Delta p &= \frac{C_p}{C_V} nC_V\Delta T \\ &= \gamma nC_V\Delta T = -\gamma p\Delta V \end{aligned}$$

Therefore $V\Delta p = -\gamma p\Delta V$

$$\therefore \frac{\Delta p}{p} = \frac{-\gamma \Delta V}{V}$$

Integrating both sides gives

$$\log p = -\gamma \log V + \text{constant (since the integral of } \frac{\Delta x}{x} = \log x).$$

Hence $\log pV^\gamma = \text{constant}$

$$\therefore pV^\gamma = \text{constant}$$

26.2 WORKED EXAMPLE

$\gamma = 1.4$ for air

A cylinder contains 100 cm^3 of air at a pressure of 100 kPa and a temperature of 20°C . The air is compressed adiabatically to 50 cm^3 . Calculate

- the pressure of the air after compression,
- the temperature of the air after compression.

Solution

a) $pV^\gamma = \text{constant}$, hence $p \times 50^{1.4} = 100 \times 100^{1.4}$

$$\therefore p \times 239 = 100 \times 631$$

$$\therefore p = \frac{100 \times 631}{239} = 264 \text{ kPa.}$$

b) $pV = nRT$

$$\therefore pV = \text{constant, so } \frac{264 \times 50}{T} = \frac{100 \times 100}{(273 + 20)}$$

$$T = \frac{264 \times 50 \times 293}{100 \times 100} = 387 \text{ K} = 114^\circ\text{C}$$

26.3 QUESTIONS

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

- 1 A cylinder contains 240 cm^3 of a monoatomic gas ($\gamma = 1.67$ for a monoatomic gas) at a pressure of 80 kPa and a temperature of 17°C .
 - a) The gas is expanded by heating to a volume of 360 cm^3 at constant pressure. (i) Calculate the temperature of the gas at the end of this change. (ii) Calculate the work done, change of internal energy and heat transfer that takes place. (iii) Represent the change on a graph of pressure against volume.
 - b) (i) If the gas in (a) had expanded to a volume of 360 cm^3 at constant temperature, what would its final pressure have been? (ii) Represent this change on the same graph, labelling it 'constant temperature'. (iii) Use the graph to estimate the work done by the gas as a result of this change. Hence determine the heat transfer for this expansion.
 - c) (i) If the gas in (a) had expanded adiabatically to a volume of 360 cm^3 , what would its final pressure and temperature have been? (ii) Represent this change on your graph, labelling it 'adiabatic change'. (iii) Calculate the change of internal energy in c (i) and hence the work done by the gas.
- 2 a) A cylinder contains 1200 cm^3 of air at a pressure of 110 kPa and a temperature of 15°C . Calculate (i) the number of moles of air in the cylinder, (ii) the mass of this air, (iii) the internal energy of this air.
 - b) The air in (a) is compressed adiabatically to a volume of 150 cm^3 . Calculate (i) the pressure and the temperature of the air immediately after compression, (ii) the change of internal energy of the air, (iii) the work done on the air to compress it adiabatically.
The molar mass of air
= 0.029 kg, $\gamma = 1.4$ for air.

Enthalpy

In a chemical reaction or a physical change of state, changes usually take place at constant pressure. The heat transfer that takes place is not equal to the change of internal energy of the substances involved because work is done as a result of volume changes.

The enthalpy, H , of a substance is defined by the equation $H = U + pV$.

For a change at constant pressure, the change of enthalpy

$$\Delta H = \Delta U + p\Delta V$$

This change of enthalpy is therefore equal to the heat transfer, ΔQ , since $\Delta Q = \Delta U + p\Delta V$ from the First Law of Thermodynamics. The unit of enthalpy is the joule.

Notes

- 1 In chemical reactions, the enthalpy change is usually expressed in kilojoules per mole (kJ mol^{-1}). Where a reaction can take place by different routes, the enthalpy change is the same whichever route the reaction takes.
- 2 In a physical change of state, latent heat is released when a vapour condenses or a liquid freezes. Latent heat must be supplied to vaporise a liquid or to melt a solid. The enthalpy of the substance changes when it changes its physical state. The change of enthalpy is the latent heat involved in the change. Thus the fact that the specific latent heat of ice is 336 J g^{-1} means that the enthalpy of fusion of water is 336 J g^{-1} or 6.05 kJ mol^{-1} (since the molar mass of water = 0.018 kg).

26.4 The Second Law of Thermodynamics

Energy changes

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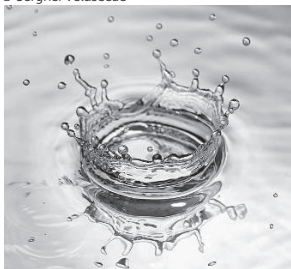


Fig. 26.14 Energy spreading out

Energy tends to become less useful when it changes from any one form into other forms. For example, fuels contain energy in concentrated form. When a fuel burns, chemical energy is transformed into heat and light. Some, but not all, of this energy can be stored and used later. For example, the fuel could be used to drive an engine which raises a weight. The potential energy of the weight can be used to do work later. However, the amount of work that the weight could do (i.e. useful energy) is less than the energy released from the fuel. Although energy is conserved in any change, the amount of useful energy is less after the change than before because some energy is wasted (e.g. due to friction).

A falling ball is another example where energy is changing from one form into other forms, namely from potential energy to kinetic energy as it falls. If the ball is perfectly elastic, it returns to the height it was released from. Such reversible changes require perfect elasticity and zero friction. In practice, such conditions as zero friction are usually unattainable and therefore energy is wasted in most transformations. For example, the energy of a bouncing ball would eventually be converted to internal energy of the surroundings.

Engines at work

An engine is designed to make objects move. The fuel supplied to an engine enables it to do work. In most engines, fuel is burned to release chemical energy in the form of heat and light. The engine uses heat from the fuel to raise the temperature of a fluid to make the fluid do work. A high temperature ‘reservoir’ is essential to heat the fluid. A low temperature reservoir is also essential to ‘draw’ the fluid through the engine. See Fig. 26.15. Thus some of the heat transfer from the high temperature reservoir must be supplied by the hot fluid to the low temperature reservoir if the engine is to work. No heat engine can therefore be 100% efficient. It is therefore impossible for a heat engine to convert all the heat supplied to it into work. This statement is known as the **Second Law of Thermodynamics**.

The efficiency of a heat engine, η , can be written as

$$\eta = \frac{\text{work done by the engine, } W}{\text{heat supplied to the engine, } Q_1}$$

Since the heat lost to the low temperature reservoir, $Q_2 = Q_1 - W$, then $W = Q_1 - Q_2$. Hence

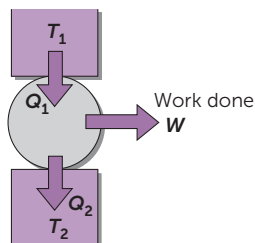
$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Examples of heat engines at work

A steam turbine

Steam is raised as a result of using a fuel to heat water. Steam jets at high pressure are used to make the turbine wheels rotate, as shown in Fig. 26.16. Steam at low pressure emerges from the turbine engine and this is condensed back to water which is then reused. A low temperature ‘reservoir’ is therefore essential to keep the steam

High temperature



Low temperature

Fig. 26.15 A heat engine

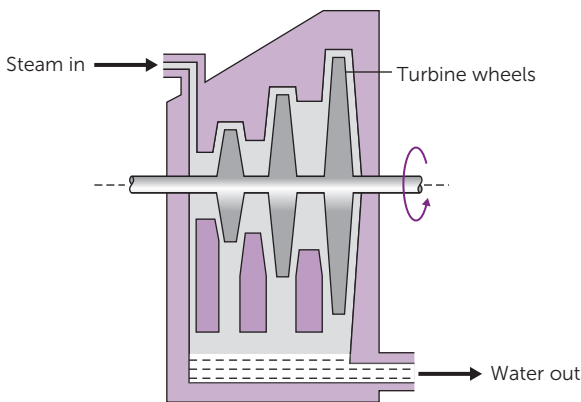


Fig. 26.16 A steam engine

moving through the engine. Without the low temperature reservoir provided by the cooling water, the steam would not condense and the inflowing steam would be unable to do work.

A thermopile engine

A thermopile generates an e.m.f. when there is a temperature difference between its two faces. Fig. 26.17 shows a thermopile between a reservoir of hot water and a reservoir of cold water. The thermopile may be used to operate an electric motor provided the cold water remains colder than the hot water. Heat flow to the cold water from the thermopile must therefore take place which means that not all the heat supplied to the thermopile from the hot water is converted to work.

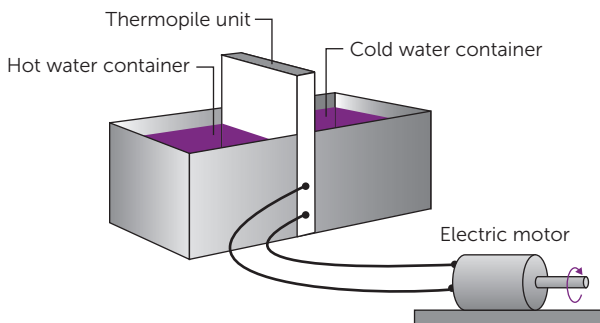


Fig. 26.17 A thermopile engine

The internal combustion engine and the jet engine are two further examples of heat engines. In both cases, the surroundings act as the low temperature reservoir. If an internal combustion engine or a jet engine was insulated from its surroundings, it would heat up and stop working sooner or later.

Reversible engines

The most efficient type of engine is a reversible engine, a theoretical engine which doesn't waste energy and can be operated in reverse. Suppose a reversible engine operating between a high temperature reservoir, T_1 , and a low temperature reservoir, T_2 , is used directly to drive a similar engine operating in reverse. Fig. 26.18 shows the idea. No net heat transfer occurs from either reservoir and so no energy is wasted.

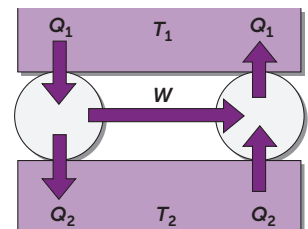


Fig. 26.18 Reversible engines

If a more efficient engine were possible, it could be used to cause a net heat transfer from the cold reservoir to the hot reservoir without any overall work being done. Since such heat transfer is impossible without work being done, an engine more efficient than a reversible engine is not possible.

The efficiency of a heat engine

When a reversible engine does work, W , and heat transfer, Q_1 , from the hot reservoir occurs, the heat transfer Q_2 to the cold reservoir is equal to $Q_1 - W$. Hence the work done $W = Q_1 - Q_2$. The ratio $Q_2 : Q_1$ depends on the ratio of the reservoir temperatures and can be shown to be equal to the ratio of absolute temperatures of the reservoirs, as below:

$$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

where T_1 is the temperature of the hot reservoir and T_2 is the temperature of the cold reservoir.

Therefore the efficiency of a reversible engine, η_R , can be written as

$$\begin{aligned}\eta_R &= \frac{W}{Q_1} \\ &= 1 - \frac{Q_2}{Q_1} \\ &= 1 - \frac{T_2}{T_1} \\ &= \frac{T_1 - T_2}{T_1}\end{aligned}$$

This is the maximum possible efficiency of any engine operating between temperatures T_1 and T_2 .

Therefore the efficiency of an engine operating between temperatures T_1 and T_2

$$\eta \leq \frac{T_1 - T_2}{T_1}$$

26.3 WORKED EXAMPLE

A heat engine operates between reservoirs at 750 K and 300 K, accepting heat from the hot reservoir at a rate of 2000 W and doing work at a rate of 600 W. Calculate

- the heat wasted each second,
- the efficiency of this engine,
- The maximum possible efficiency an engine could have, operating between 750 K and 300 K.

Solution

a) $Q_2 = Q_1 - W = 2000 - 600 = 1400 \text{ W}$

b) Efficiency = $\frac{600}{2000} = 0.30$

c) Maximum possible efficiency = $\frac{750 - 300}{750} = \frac{450}{750} = 0.60$

26.4 QUESTIONS

- 1 a) A vehicle uses fuel at a rate of 15 kms per litre when its speed is 30 m s^{-1} .
 (i) Calculate how long it takes to use a litre of fuel at this rate. (ii) The fuel releases 20 MJ of heat per litre when it is burned. How much energy is released by the fuel per second when it is used at the above rate?
 b) The output power of the vehicle at 30 m s^{-1} is 6 kW. Calculate the overall efficiency of the vehicle engine and transmission system.
- 2 A heat engine with a power output of 40 W operates between reservoirs at temperatures of 400 K and 300 K. The cooling system removes heat from the engine at a rate of 160 W.
 a) Calculate the heat supplied per second by the high temperature reservoir.
 b) Calculate (i) its efficiency, (ii) the maximum efficiency of an engine operating between these two temperatures.

SUMMARY

- Temperature scales

Absolute temperature, $T = \frac{pV}{(pV)_{\text{Tr}}} \times 273.16 \text{ K}$, Celsius temperature, $t = T - 273.15$

- The Laws of Thermodynamics

Zeroth law: If no heat transfer occurs between two bodies in thermal equilibrium, they are at the same temperature.

First Law: $\Delta Q = \Delta U + \Delta W$

Second Law: It is not possible for a heat engine to convert all the heat supplied to it into work.

- Thermodynamic changes

An isothermal change is a change at constant temperature.

An adiabatic change is a change in which no heat transfer occurs.

An enthalpy change is heat supplied or released at constant pressure.

- Thermodynamics of gases

1 Work done by a gas $= p\Delta V$

2 Molar heat capacities: $C_p - C_v = R$ where C_v is the heat supplied to raise the temperature of 1 mole by 1 K at constant volume, and C_p is the heat supplied to raise the temperature of 1 mole by 1 K at constant pressure.

3 For a monoatomic gas, $C_v = \frac{3}{2}RT$ and $C_p = \frac{5}{2}RT$, $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

For a diatomic gas, $C_v = \frac{5}{2}RT$ and $C_p = \frac{7}{2}RT$, $\gamma = \frac{C_p}{C_v} = \frac{7}{5}$

For a polyatomic gas, $C_v = \frac{6}{2}RT$ and $C_p = \frac{8}{2}RT$, $\gamma = \frac{C_p}{C_v} = \frac{8}{6}$

4 For an adiabatic change of an ideal gas, $pV^\gamma = \text{constant}$

- Heat engines

Efficiency $= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$, Maximum possible efficiency $= \frac{T_1 - T_2}{T_1}$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

26.1

- a) The pressure of the gas in a constant volume gas thermometer, relative to atmospheric pressure, was -6 kPa at ice point and $+29$ kPa at steam point. When the gas pressure was $+20$ kPa, what was the temperature of the gas thermometer in
- $^{\circ}\text{C}$,
 - K?
- b) The e.m.f. of a thermocouple was amplified using a d.c. amplifier. The amplifier was adjusted so its output voltage was zero when the thermocouple was at ice point and 100 mV when the thermocouple was at steam point. The output voltage, V , was found to vary with the Celsius temperature, t , of the thermocouple in accordance with the equation $V = 1.25t(1 - 20 \times 10^{-4}t)$ mV. When the Celsius temperature was 50°C , what was
- the output voltage,
 - the centigrade reading of this thermocouple thermometer?

26.2 Complete the following table for the energy changes of an ideal gas.

$\Delta U / \text{J}$	$\Delta Q / \text{J}$	p / kPa	$\Delta V / \text{m}^3$
1000	(a)	100	$+2.0 \times 10^{-3}$
(b)	-600	150	-2.0×10^{-3}
-800	200	100	(c)
1200	400	(d)	-5.0×10^{-3}

26.3 A glass bulb contains 60 cm^3 of inert gas consisting of monoatomic molecules at a pressure of 10 kPa and a temperature of 290 K.

- a) Calculate
- the number of moles of gas in the bulb,
 - the internal energy of this gas.
- b) The gas is heated to a temperature of 380 K. Calculate
- its pressure at this temperature,
 - its increase of internal energy.

26.4

- a) (i) Calculate the number of moles of an ideal gas in a volume of 100 cm^3 ($= 1.00 \times 10^{-4} \text{ m}^3$) at a pressure of 101 kPa and a temperature of 20°C .
- (ii) Air consists of diatomic molecules of nitrogen and oxygen. Calculate the internal energy of the air in a cylinder of volume 100 cm^3 at the above temperature and pressure.
- b) (i) The air in the cylinder in (a)(ii) is compressed adiabatically from its initial pressure of 101 kPa to a volume of 20 cm^3 . Calculate the pressure and temperature of the air in this cylinder after the compression.

(ii) Calculate the internal energy of the air in the cylinder at this new temperature.

(iii) Hence calculate the work done on the air when it was compressed adiabatically.

26.5

- a) An engine operating between 450 K and 300 K is supplied with heat at a rate of 2000 J s^{-1} and delivers power at a rate of 600 W. Calculate
- the heat supplied per second by the engine to the low temperature reservoir,
 - the efficiency of the engine,
 - the maximum possible efficiency of any heat engine working between these two temperatures.
- b) In a certain type of heat engine, air, initially at a pressure of 100 kPa, a volume of 0.0020 m^3 and a temperature of 300 K, is heated at constant volume to a temperature of 1000 K. The air is then allowed to expand adiabatically to a pressure of 100 kPa. It is then compressed at constant pressure back to its initial volume. Fig. 26.19 shows the process.

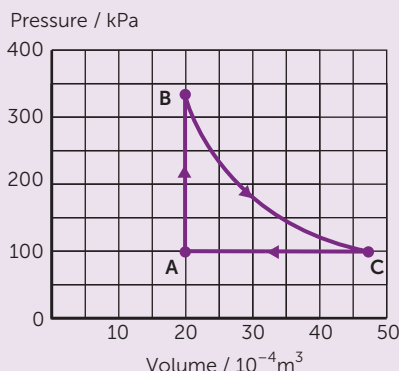


Fig. 26.19

(i) Show that the pressure is 333 kPa at the end of the first stage and the temperature at the end of the second stage is 706 K.

(ii) Calculate the maximum possible efficiency of any heat engine operating between 1000 K and 300 K.

(iii) Use the graph to estimate the work done by the engine in one cycle.

(iv) Calculate the heat gain by the gas.

(v) Estimate the efficiency of the engine.

(γ for air $= 1.4$)

Uniform Circular Motion

OBJECTIVES

After working through this unit, you should be able to:

- convert from degrees to radians
- calculate the frequency and the angular speed of an object in uniform circular motion from its time period
- calculate its speed from its time period and the radius of rotation
- explain why an object in uniform circular motion experiences a centripetal acceleration
- calculate the centripetal acceleration of an object in uniform circular motion from its speed and the radius of rotation
- analyse the motion of an object in uniform circular motion in terms of the forces acting on the object

CONTENTS

- 27.1 Circular measures
- 27.2 Centripetal acceleration
- 27.3 Reactions and rides
- Summary
- Revision questions

27.1 Circular measures

Angles and arcs

- The **radian** is a unit used to measure angles, defined from $360^\circ = 2\pi$ radians. In other words, the angle all the way around a circle is 2π radians (abbreviation 'rad').
- An **arc** of a circle is the curved edge of a segment of the circle, as shown in Fig. 27.1. The arc subtends an angle θ to the centre of the circle. If the angle θ is in radians, the length s of the arc is $r\theta$.

$$\text{Arc length, } s = r\theta$$

If the angle θ is small, the segment is almost the same as a triangle in which the length of the smallest side is equal to $r \sin \theta$. It therefore follows that for small angles

$$\sin \theta = \theta \text{ in radians}$$

This is known as the small angle approximation and it holds up to about 10° . This approximation will be used later in this unit. Table 27.1 shows some values of $\sin \theta$ and θ in radians upto 20° . The difference between $\sin \theta$ and θ in radians becomes too large to ignore over 10° .

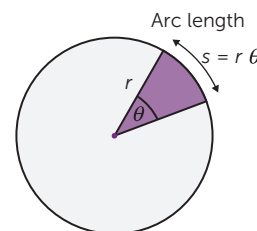
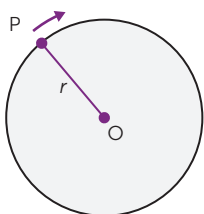


Fig. 27.1 Angles and arcs

Table 26.1 Values for $\sin \theta$ and θ in radians up to 20°

θ in degrees	0.0000	5.0000	10.0000	15.0000	20.0000
θ in radians	0.0000	0.0873	0.1745	0.2618	0.3491
$\sin \theta$	0.0000	0.0872	0.1736	0.2588	0.3420

Angular speed

**Fig. 27.2** Angular speed

Notes

- 1 The unit of ω is the radian per second (rad s^{-1}).
- 2 The speed, v , and the angular speed, ω , are related by the equation $v = \omega r$.

Consider a point object, P, moving on a circular path of radius r , as shown in Fig. 27.2.

- The **frequency** of rotation, f , is defined as the number of revolutions per second made by the object.

$$f = \frac{1}{T}$$

where T is the time for 1 rotation. The unit of frequency is the hertz (Hz), equal to 1 revolution per second.

- The **speed** of the object, v , can be written as

$$v = \frac{\text{circumference}}{\text{time period}} = \frac{2\pi r}{T}$$

- The **angular speed**, ω , is defined as the angle swept out per second by the radial line OP. Since this line turns through 2π radians in time T ,

$$\omega = \frac{2\pi}{T}$$

27.1 WORKED EXAMPLE

An object on the equator goes round once every 24 hours. Assuming the Earth is a sphere of radius 6350 km, calculate

- a) the speed,
- b) the angular speed of a point on the equator,
- c) the distance moved by such a point in exactly 1 hour.

Solution

$$\text{a) } v = \frac{2\pi r}{T} = \frac{2\pi \times 6.35 \times 10^6 \text{ m}}{24 \times 60 \times 60 \text{ s}} = 462 \text{ m s}^{-1}$$

$$\text{b) } \omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{24 \times 60 \times 60 \text{ s}} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

$$\text{c) } s = vt = 462 \times 3600 = 1.66 \times 10^6 \text{ m}$$

27.1 QUESTIONS

- 1 The drum of a spin drier has a diameter of 0.40 m and its maximum rate of rotation is 800 revolutions per minute. When it is turning at maximum speed, calculate
 - a) the time it takes to turn through one revolution,
 - b) its angular speed,
 - c) the speed of an object on the drum surface.

- 2 An electric motor rotates at 3000 revolutions per minute. Calculate (a) the time taken for 1 rotation, (b) the angular speed of the motor.
- 3 A 2.0 m length of cassette tape is wound onto a spool at a steady rate of rotation of 2 turns per second. The spool diameter is 40 mm.
- a) Calculate (i) the circumference of the spool, (ii) the number of turns the spool makes to wrap this length of tape on. Assume the number of turns makes no difference to the diameter of the spool with the tape wound on.
- b) Calculate (i) the speed of the tape, (ii) the time it takes to wind onto the spool.
- c) Calculate the angle between each end of the tape and the centre of the spool (i) in radians, (ii) in degrees.

27.2 Centripetal acceleration

Speed and velocity

- **Speed** is rate of change of distance with time. Speed is a scalar quantity because it has no direction.
- **Velocity** is speed in a certain direction. Velocity is a vector because it has a direction.

For an object moving at constant speed on a circular path, its velocity continually changes as it changes direction. Since acceleration is defined as rate of change of velocity, it follows that an object in uniform circular motion experiences acceleration because its velocity changes direction.

The velocity of an object in uniform circular motion is along a tangent to the circle. As Fig. 27.3 shows, if the object moves around the circle through a certain angle, the velocity direction moves through the same angle.

Consider the object, P, at point A, at time t , and at point B, at time $t + \delta t$, where δt is a small interval of time.

- The radial line, OP, turns through angle $\delta\theta$ in time δt . Hence the angular speed $\omega = \delta\theta \div \delta t$, which therefore gives $\delta\theta = \omega\delta t$.
- The arc length, $AB = \delta s = v\delta t$ (using distance = speed $\times$ time) $= \omega r\delta t$ since $v = \omega r$.
- Let the velocity at A be represented by vector v_A and the velocity at B be represented by vector v_B . If the two velocity vectors form a triangle as in Fig. 27.4, the shortest side of the triangle represents the change of velocity, $\delta v = v_B - v_A$ which is towards the centre of the circle. The lengths of the two other sides both represent the speed v .
- The acceleration of the object is referred to as 'centripetal' (which means 'towards the centre'). This is because the acceleration direction is towards the centre of the circle since acceleration is change of velocity per unit time.
- Because the velocity vector triangle is similar to triangle AOB (i.e. both have two equal sides and the same angle between the two equal sides), the ratio of the short side to either long side is the same for both triangles, i.e.

$$\frac{\delta}{v} = \frac{\delta s}{r}$$

$$\text{hence } \delta v = v \frac{\delta s}{r} = v \frac{v\delta t}{r} = \frac{v^2\delta t}{r}$$

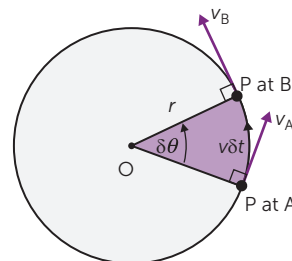


Fig. 27.3 Velocity directions

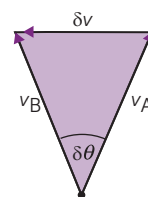


Fig. 27.4 Change of velocity

Therefore centripetal acceleration, a , can be written as

$$a = \frac{\text{change of velocity } \delta v}{\text{time taken } \delta t} = \frac{v^2}{r}$$

towards the centre of the circle. The equation may be written as

$$a = -\frac{v^2}{r}$$

where the minus sign signifies 'towards the centre'.

Since $v = \omega r$, then centripetal acceleration,

$$a = -\frac{v^2}{r} = -\frac{\omega^2 r^2}{r} = -\omega^2 r$$

$$\mathbf{a} = -\frac{v^2}{r} = -\omega^2 \mathbf{r}$$

Notes

- 1 The kinetic energy of an object in uniform circular motion is constant, even though the object is acted upon by the centripetal force. Because the force is towards the centre of the circle and the velocity is tangential, no work is done by the force because there is no movement towards or away from the centre of the circle.
- 2 If the centripetal force was suddenly removed from an object in uniform circular motion, the object would move off at a tangent to the circle. This is because the velocity at any point is tangential.

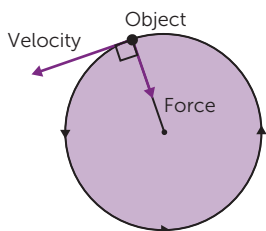


Fig. 27.5 Force and velocity

27.2 WORKED EXAMPLE

Calculate the centripetal acceleration of a car moving at a constant speed of 12 m s^{-1} around a roundabout of radius 50 m.

Solution

$$\begin{aligned} \text{The centripetal acceleration, } a &= \frac{v^2}{r} = \frac{12 \times 12}{50} \\ &= 2.88 \text{ m s}^{-2} \text{ towards the centre.} \end{aligned}$$

Centripetal force

An object in uniform circular motion must be acted on by a resultant force towards the centre of the circle because its acceleration is towards the centre of the circle. The resultant force is referred to as the **centripetal** force because its direction is towards the centre. Using Newton's second law in the form $F = ma$ therefore gives

$$\text{Centripetal force, } F = ma = -\frac{mv^2}{r}$$

27.2 QUESTIONS

- 1 A vehicle of mass 800 kg moves at a speed of 20 m s^{-1} on a circular path of radius 30 m around a bend. Calculate
 - a) the centripetal acceleration of the vehicle,
 - b) the centripetal force on the vehicle.
- 2 Calculate the maximum speed at which the vehicle in Q1 could travel safely on this path if the maximum frictional force for no slip between the tyres and the road is $0.5 \times$ the vehicle's weight.
- 3 The centripetal force on a satellite in circular orbit above the Earth is provided

by the force of gravity (i.e. its weight). This can be expressed by means of the equation

$$mg = \frac{mv^2}{r}$$

where m is the mass of the satellite, v is its speed and r is the radius of orbit.

- a) Prove from the above equation that $v = \sqrt{gr}$.
- b) Hence calculate the speed of a satellite in an orbit at a height of 100 km above the Earth's surface, assuming $g = 9.8 \text{ m s}^{-2}$ at this height. The Earth's mean radius is 6400 km.

27.3 Reactions and rides

Fairground rides are designed to take account of the forces acting on the moving parts and on passengers. The structures and materials used must be capable of withstanding the effects of circular motion.

The roller coaster

Over the top

When a carriage travels over a convex section of a track of radius of curvature r the normal reaction of the track on the carriage is less than the weight of the carriage. See Fig. 27.6. At the highest point, the normal reaction, N , is directed vertically upwards, in the opposite direction to the weight, mg . The resultant force on the carriage is therefore $mg - N$ towards the centre of curvature of the track.

Using $F = ma$ therefore gives

$$mg - N = \frac{mv^2}{r} \text{ since } a = \frac{v^2}{r}$$

towards the centre. Hence the normal reaction

$$N = mg - \frac{mv^2}{r}$$

Notes

- 1 The normal reaction is less than the weight.
- 2 The maximum speed, $v_{\max}$, at which the carriage is still in contact with the track is when $N = 0$.

$$\text{Hence } \frac{mv_{\max}^2}{r} = mg, \text{ giving } v_{\max} = \sqrt{gr}$$

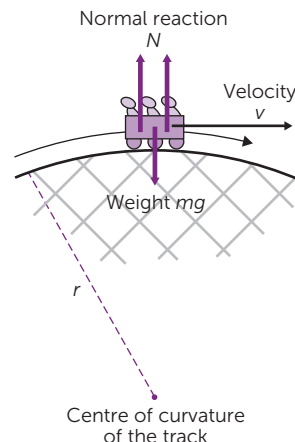


Fig. 27.6 Over the top

In a dip

When a carriage travels along a concave section of a track of radius of curvature, r , the normal reaction of the track on the carriage is more than the weight of the carriage. See Fig. 27.7. At the lowest point, the normal reaction, N , is directed vertically upwards, in the opposite direction to the weight, mg .

The resultant force on the carriage is therefore $N - mg$ towards the centre of curvature of the track.

Using $F = ma$ therefore gives

$$N - mg = \frac{mv^2}{r} \text{ since } a = \frac{v^2}{r}$$

towards the centre. Hence the normal reaction

$$N = mg + \frac{mv^2}{r}$$

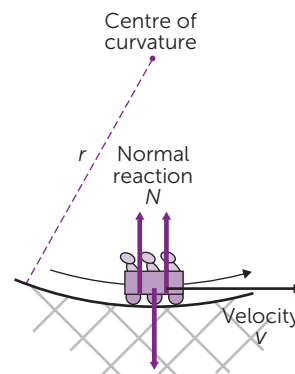


Fig. 27.7 In a dip

Notes

- 1 The normal reaction is more than the weight.
- 2 The extra 'g-force' on the carriage is $mv^2 \div r$. A person in the carriage will experience an extra support force equal to $mv^2 \div r$ from the carriage, equivalent to increasing g by $v^2 \div r$.

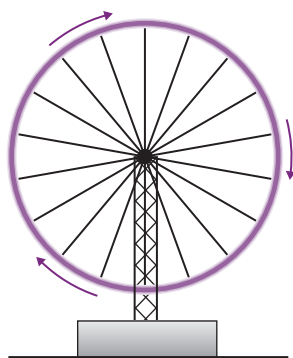


Fig. 27.8 The gravity wheel

The gravity wheel

The wheel starts rotating with its plane horizontal and the passengers standing around the inside of the rim. When its frequency of rotation is sufficiently high, the wheel axis is tilted gradually until it points in a fixed horizontal direction. The passengers now move on a vertical circle, as shown in Fig. 27.8, held against the inside of the wheel's rim by the combined effect of weight and the normal reaction of the wheel.

At the top

The normal reaction, N_1 , on each passenger is downwards, in the same direction as the passenger's weight, mg . See Fig. 27.9. The resultant force is therefore $N_1 + mg$ towards the centre of rotation. Using $F = ma$ and $a = v^2 \div r$ therefore gives

$$N_1 + mg = \frac{mv^2}{r}$$

where v is the speed of the rim and r is its radius.

$$N_1 = \frac{mv^2}{r} - mg$$

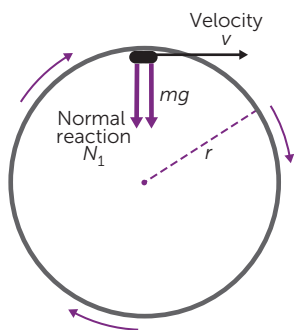


Fig. 27.9 At the top

Notes

- 1 For a passenger at the top to remain in contact with the rim, $N_1 > 0$. Therefore the minimum speed, $v_{\min}$, for this condition is given by

$$\frac{mv_{\min}^2}{r} - mg \text{ which gives } v_{\min} = \sqrt{gr}$$

- 2 If the speed becomes less than $\sqrt{gr}$ while the wheel is vertical, the passengers will fall off the inside of the rim before reaching the top unless strapped in.

At the bottom

The normal reaction, N_2 , on each passenger is upwards, in the opposite direction to the passenger's weight, mg . See Fig. 27.10. The resultant force is therefore $N_2 - mg$ towards the centre of rotation. Using $F = ma$ and $a = v^2 \div r$ therefore gives

$$N_2 - mg = \frac{mv^2}{r}$$

where v is the speed of the rim and r is its radius.

$$N_2 = \frac{mv^2}{r} + mg$$

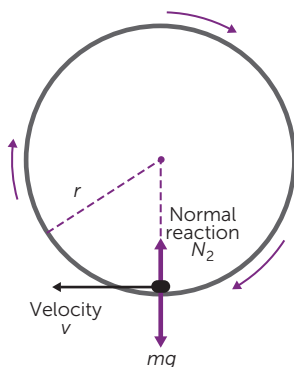


Fig. 27.10 At the bottom

Notes

- 1 The normal reaction on a passenger at the bottom exceeds the passenger's weight. This extra 'g-force' is equal to $mv^2 \div r$, effectively increasing g by $v^2 \div r$.
- 2 The maximum speed of rotation is governed by how much extra 'g-force' a passenger can withstand without any adverse health effects.

The very long swing

In this ride, one or two passengers are suspended from the end of a long cable and then winched until the cable is almost horizontal. The cable is then released and the

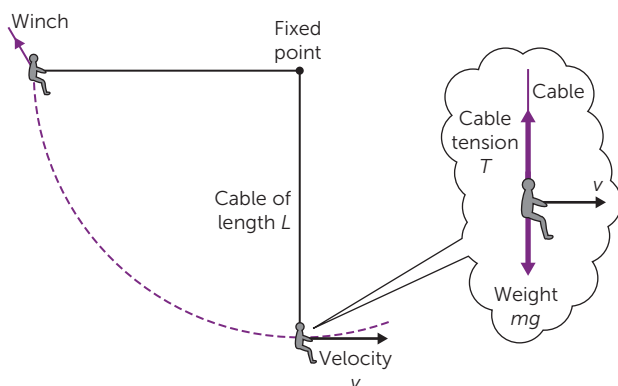


Fig. 27.11 A big swing

passengers swing from side to side with a gradually decreasing amplitude as shown in Fig 27.11.

- The speed v , at the lowest point of a passenger of mass, m , may be calculated by equating the gain of kinetic energy to the loss of potential energy, i.e. $\frac{1}{2}mv^2 = mgh$, where h is the height drop.
- At the lowest point, the tension, T , in the cable acts directly upwards on the passengers, in the opposite direction to their weight. The resultant force is therefore $T - mg$ towards the centre of rotation, where m is the total mass suspended from the cable. Using $F = ma$ and $a = v^2 \div L$ therefore gives

$$T - mg = \frac{mv^2}{L}$$

where L , the cable length, is equal to the radius of rotation.

Hence the tension in the cable in this position is given by the equation

$$T = mg + \frac{mv^2}{L}$$

This is the maximum tension in the cable.

Around the curves

The track of a roller coaster is banked where it curves horizontally. This is so that the carriages can travel around the curves at high speed without flying off the curve. The same principle is used for rail tracks and motorways.

Fig. 27.12 shows the front view of a vehicle on a banked track of radius of curvature r . The track is at angle θ to the horizontal. Assuming there is no sideways friction between the vehicle and the track, the horizontal component of the normal reaction, N , of the track on the vehicle, $N \sin \theta$,

$$N \sin \theta = \frac{mv^2}{r}$$

$$N \cos \theta = mg$$

$$\begin{aligned} \text{Hence } \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ &= \frac{mv^2}{mgr} \\ &= \frac{v^2}{gr} \end{aligned}$$

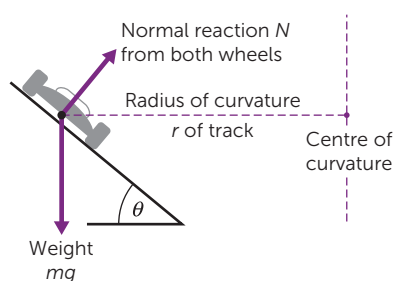


Fig. 27.12 Around a curve

Notes

1. The condition described on the previous page assumes no sideways friction. If the vehicle takes the curve faster, then it would tend to move up the track. If the vehicle takes the curve more slowly, it would tend to slide down the track. At the speed given by the above equation, there is no tendency to slide up or down the track.
2. The same equations apply to an aircraft turning in a horizontal circle of radius r . As shown in Fig. 27.13, the lift force, L , which is perpendicular to the wings, plays the same part as the normal reaction, N , on a banked track.

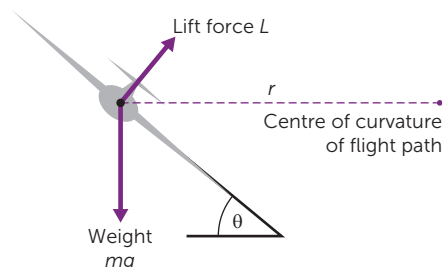


Fig. 27.13 An aircraft banking

The horizontal component of the lift force provides the centripetal force. There is no sideways friction force as on a banked track so the pilot must ensure the angle of banking matches its speed exactly, according to the above equation.

27.3 QUESTIONS

1. A truck of total mass 2000 kg travelling at a speed of 15 m s^{-1} goes over an arched bridge of radius of curvature 25 m.
 - a) Calculate the force of the truck on the bridge when the truck is at the top of the arch.
 - b) Calculate the maximum speed at which the truck could pass over the bridge without losing contact with the road.
2. A light aircraft pulls out of a descent, having reached a maximum speed of 95 m s^{-1} at the lowest point of the descent. The radius of curvature of its descent was 450 m. Calculate the centripetal acceleration of the aircraft at the lowest point of the descent and determine the maximum 'g' force suffered by the pilot.

SUMMARY

- Arc length, $s = r\theta$, where θ is in radians
 $360^\circ = 2\pi$ radians
- Angular speed, $\omega = \frac{2\pi}{T} = 2\pi f$
- Centripetal acceleration, $a = \frac{-v^2}{r}$
- Centripetal force, $F = ma = \frac{-mv^2}{r} = -m\omega^2 r$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

27.1 A satellite moves around the Earth once every 2 hours in a circular orbit of radius 8000 km. Calculate

- its angular speed,
- its speed,
- its centripetal acceleration.

27.2 The armature coil of an electric motor has a diameter of 0.08 m. When it rotates at a frequency of 50 Hz, calculate

- its angular speed,
- its speed,
- its centripetal acceleration.

27.3 A vehicle of mass 1200 kg travels around a roundabout of radius 60 m.

- Calculate the centripetal acceleration of the vehicle when it travels at a speed of 15 m s^{-1} around the roundabout.
- If the maximum frictional force between the vehicle tyres and the road is $0.5 \times$ its weight, calculate the maximum speed at which the vehicle could go around the roundabout without sliding outwards.

27.4 The track of a fairground roller coaster descends from the highest point to a dip which is 45 m below.

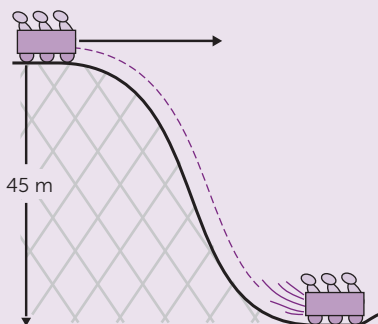


Fig. 27.14

- (i) Assuming its speed at the highest point is negligible, show that the maximum speed of a carriage at the lowest point of the dip is 30 m s^{-1} .
(ii) What other assumption is it necessary to make in (a)(i)?
- At the dip, the track is concave in shape and has a radius of curvature of 25 m.

(i) Calculate the centripetal acceleration of a carriage passing through the dip at a speed of 30 m s^{-1} at the lowest point.

(ii) Calculate the ratio of the centripetal force to the weight when the carriage is at the lowest point of the dip.

27.5 A fairground roundabout consists of eight, two-seat 'aircraft', each suspended from a steel cable of length 5.0 m which is attached at its upper end to the end of a horizontal steel beam. Each beam extends 7.5 m from a vertical steel pillar which is along the axis of rotation of the roundabout. When the roundabout rotates at its maximum frequency of rotation, the steel cables are at an angle of 30° to the vertical.

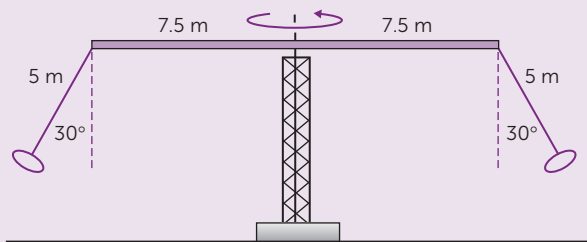


Fig. 27.15

- Calculate the radius of rotation of each 'aircraft' at this angle.
- Show that the speed of each aircraft, when the cable is at this angle, is 7.5 m s^{-1} .
- Calculate the time taken for one rotation at this speed.

27.6

- An aircraft travelling at a constant speed of 210 m s^{-1} travels horizontally on a circular path of radius 6400 m. Calculate the angle of banking of its wings when it turns in this way.
- A different aircraft performs a vertical loop of diameter 500 m.
 - At its highest point, why is the direction of the lift force on its wings downwards?
 - If the lift force at its highest point in the loop is to equal its weight, show that its speed at the top of the loop must be 70 m s^{-1} .

UNIT

28

Gravitation

CONTENTS

- 28.1 Newton's theory of gravity
- 28.2 Gravitational field strength
- 28.3 Gravitational potential energy
- 28.4 Satellite motion
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- explain the cause of an object's weight
- state and use Newton's law of gravitation to calculate the force between two masses
- define gravitational field strength and state its unit
- relate acceleration due to gravity, g , to gravitational field strength
- describe the variation of g with distance from the centre of a spherical planet
- calculate the potential energy of an object in a gravitational field
- explain what is meant by escape speed and calculate the escape speed from a planet
- explain the motion of a satellite in a circular orbit and relate its time period to the radius of its orbit
- carry out calculations relating the time period of a satellite to the radius of its orbit

28.1 Newton's theory of gravity

Release an object from rest above the ground and it falls down, pulled by the Earth's gravity. The Earth attracts the object, so unless the object is supported above the ground, it moves towards the Earth. The object attracts the Earth with an equal and opposite force but because the Earth is so much more massive, the Earth is not moved by the object.

Gravity acts between any two objects, causing a force of attraction which tries to pull the two objects together. The universal nature of the force of gravity, acting between any two objects, was first realised by Isaac Newton in the 17th century. He devised a mathematical theory of gravity which he used to explain a wide range of phenomena, including the motion of the planets and why comets reappear. He was able to use the theory to make many predictions, including the times of forthcoming tides, eclipses and reappearance of comets.

Newton's theory of gravitation states that a force of attraction due to gravity exists between any two objects and that the force of gravity between two point objects is

- 1 proportional to the product of the masses of the objects,
- 2 inversely proportional to the square of their distance apart.

Thus for two point masses, m_1 and m_2 , at distance, r , apart, the force of attraction, F , can be written as

$$F \propto \frac{m_1 m_2}{r^2}$$

This proportionality relationship, shown in Fig. 28.1, is made into an equation by introducing a constant of proportionality, G , the Universal Constant of Gravitation. This equation, as stated below, is referred to as Newton's law of gravitation.

$$F = G \frac{m_1 m_2}{r^2}$$

where F = the force of gravitational attraction and r = the distance between point masses m_1 and m_2 .

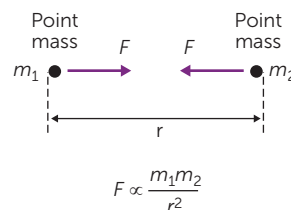


Fig. 28.1 Newton's law of gravitation

Notes

- 1 Newton extended his theory to objects which are not points. He showed that the equation gives the force of gravity between any two objects, provided r is the distance between their centres of gravity.
- 2 The equation is an example of an inverse square law of force in which the force is proportional to the inverse of the square of the distance. For example, if the distance is doubled, the force decreases to one quarter of what it was.
- 3 The unit of G is the newton metre² kilogram⁻², corresponding to $G = Fr^2 \div m_1 m_2$ from the equation. Note that mass must be in kilograms and distances in metres when using Newton's law of gravitation.
- 4 The value of G is $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. This was first determined by Henry Cavendish in the 18th century. He made a torsion balance consisting of a horizontal rod which carried a small metal ball at either end, as shown in Fig. 28.2. The rod was suspended by a thin wire. Two large lead spheres were brought near either end of the rod so as to attract the metal balls and make the rod twist.

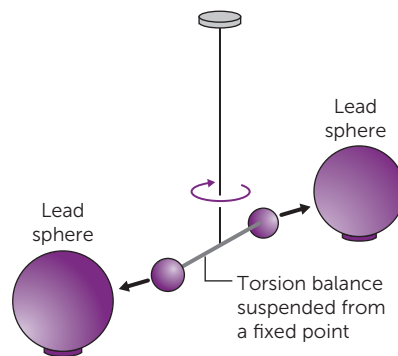


Fig. 28.2 Measurement of G

28.1 WORKED EXAMPLE

Calculate the force of attraction between a lead sphere of mass 2.00 kg and
 a) an object of mass 6.00 kg when the centres of gravity are 0.100 m apart,
 b) the Earth when the object is on the surface of the Earth. The Earth has a mass of $5.98 \times 10^{24} \text{ kg}$ and a radius of 6380 km.

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

Solution

$$\text{a) } F = G \frac{m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 2.00 \times 6.00}{(0.100)^2} = 8.00 \times 10^{-8} \text{ N}$$

$$\text{b) } F = G \frac{m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 2.00 \times 5.98 \times 10^{24}}{(6380 \times 10^3)^2} = 19.6 \text{ N}$$

The nature of gravity

The force of gravitational attraction exists between any two objects unless they are without mass. Each object is surrounded by a gravitational field due to its own mass. See Fig. 28.3. Any other object in this field experiences a force due to the field. For example, the gravitational field of the Earth causes an object at rest on the Earth's surface to be held on the Earth.

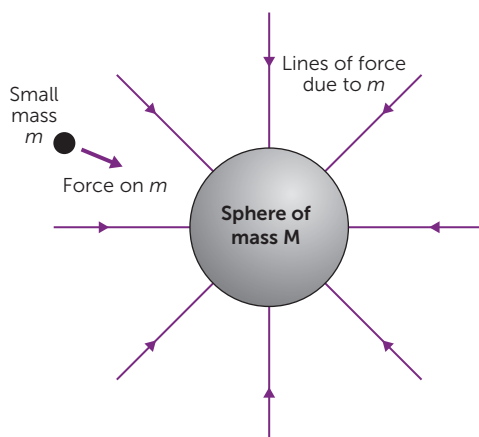


Fig. 28.3 In a gravitational field

The Earth's gravitational field stretches to infinity. According to Newton's Law of Gravitation, no matter how far apart two objects are, the force of gravity between them is never zero. The force of gravitational attraction between any two objects becomes weaker and weaker as the two objects move further and further apart. The force tends to zero as the distance apart becomes larger and larger, but is nevertheless non-zero except at infinite separation. Thus it is not possible, in theory, to escape from the Earth's gravitational field or the gravitational field of any other object, although in practice the force would eventually become negligibly small if the distance from the Earth became ever larger.

Zero gravity

The Earth's gravity stretches to infinity, but it isn't necessary to go to infinity to experience zero gravity. An object at any point along the line between the Earth and the Moon would experience a force due to the Earth and a force in the opposite direction due to the Moon. At one particular point along this line, as shown in Fig. 28.4, the two forces are equal and opposite.

Let D represent the distance between the centre of the Earth and the centre of the Moon. Let d represent the distance from the centre of the Earth to point P, the position where the force due to the Earth is equal and opposite to the force due to the Moon.

The force, F_1 , due to the Earth on a small object of mass, m , at P, can be written as

$$F_1 = \frac{GM_1m}{d^2}$$

where M_1 = the mass of the Earth.

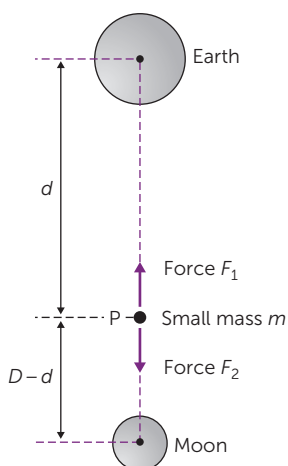


Fig. 28.4 Zero gravity

The force, F_2 , due to the Earth on a small object of mass, m , at P, can be written as

$$F_2 = \frac{GM_2m}{(D-d)^2}$$

where M_2 = the mass of the Moon. Since $F_1 = F_2$ at point P, then

$$\frac{GM_1m}{d^2} = \frac{GM_2m}{(D-d)^2}$$

Hence

$$\frac{(D-d)^2}{d^2} = \frac{M_2}{M_1}$$

Since $M_1 = 5.98 \times 10^{24}$ kg and $M_2 = 7.35 \times 10^{22}$ kg, then

$$\frac{M_2}{M_1} = \frac{7.35 \times 10^{22}}{5.98 \times 10^{24}} = 1.23 \times 10^{-2}$$

$$\therefore \frac{D-d}{d} = \sqrt{(1.23 \times 10^{-2})} = 0.11$$

$$\therefore D-d = 0.11d,$$

which gives

$$1.11d = D,$$

thus

$$d = 0.90D.$$

Since $D = 384\,000$ km, then

$$\begin{aligned} d &= 0.90 \times 384\,000 \text{ km} \\ &= 346\,000 \text{ km.} \end{aligned}$$

Point P is therefore a distance of 346 000 km from the centre of the Earth.

Note

A spring balance may be used to measure the force of gravity on an object. If a spring balance supporting an object is released so the spring balance and the object fall freely, the reading on the spring balance becomes zero as soon as it is released. The object is unable to exert a force on the spring balance when they are both in free fall. In effect, this is the same as if the force of gravity due to the Earth vanished. An object in free fall above the Earth is therefore effectively in zero gravity.

28.1 QUESTIONS

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

Use the following data in the questions below:

Earth's mass = 5.98×10^{24} kg,
 Earth's radius = 6380 km.
 Moon's mass = 7.35×10^{22} kg.
 Mean distance from the Earth to the Moon = 384 000 km.
 Sun's mass = 1.99×10^{30} kg,
 Mean distance from the Earth to the Sun = 150 000 000 km.
 Moon's radius = 1740 km,
 Sun's radius = 696 000 km.

- 1 Calculate the force of gravitational attraction between
 - a) the Earth and a person of mass 70 kg on the Earth,
 - b) the Earth and the Moon,
 - c) the Moon and a person of mass 70 kg on the Earth.
- 2 a) Calculate the force due to the Sun on an object of mass 1.0 kg on the Earth.
 b) How far above the Earth would an object need to be to experience an equal and opposite force from the Sun?



Fig. 28.5

28.2 Gravitational field strength

Notes

- 1 The object must be small enough not to affect the distribution of mass that creates the field.
- 2 Gravitational field strength is measured in newtons per kilogram. For example, the gravitational field strength of the Earth on its surface is 9.8 N kg^{-1} .
- 3 Gravitational field strength is a vector quantity because it has magnitude and direction.

A gravitational field is a region of space where an object experiences a force due to its mass. For example, the gravitational field of the Earth causes a force on any object near the Earth. The gravitational field of the Earth is due to its own mass and the field acts on any other mass.

The strength of a gravitational field, g , is defined as the force per unit mass on a small object placed in the field.

$$\text{Gravitational field strength } g = \frac{F}{m}$$

where F = force of gravity on a small object of mass m .

The weight of an object is the force of gravity on it.

For an object of mass m , its weight, W , is therefore given by

$$W = mg$$

where g is the gravitational field strength at the position of the object.

For example, a person of mass 70 kg would weigh approximately 700 N on the surface of the Earth since $g = 9.8 \text{ N kg}^{-1}$ at the Earth's surface. However, the same person on the Moon, where the gravitational field strength is 1.6 N kg^{-1} , would weigh just 102 N ($= 1.6 \text{ N kg}^{-1} \times 70 \text{ kg}$).

- An **object in free fall** above the Earth accelerates due to the force of gravity on it. The acceleration of free fall is equal to g , the gravitational field strength. This is because $\text{acceleration} = \text{force} \div \text{mass} = mg \div m = g$.
- A **line of force** of a gravitational field is the line along which a small mass would move if it were free to move. Fig. 28.6 shows the lines of force surrounding a sphere. The lines point towards the centre of the sphere because any small mass in the field would be attracted towards the centre of the sphere. This is an example of a radial field because the lines of force point to the centre.

The force, F , on a small mass, m , at distance, r , from the centre of a sphere of mass, M , is given by Newton's Law of Gravitation,

$$F = \frac{GMm}{r^2}$$

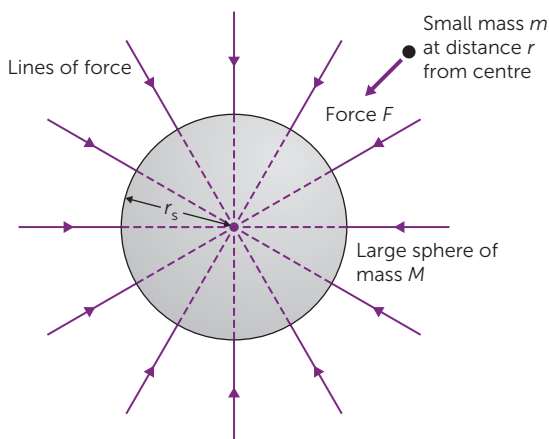


Fig. 28.6 Lines of force

Since gravitational field strength, g , is defined as force per unit mass, then

$$g = \frac{F}{m} = \frac{GM}{r^2} \text{ at distance, } r, \text{ from the centre of the sphere.}$$

$$g = \frac{GM}{r^2} \text{ at distance, } r, \text{ from the centre of a sphere of mass, } M.$$

Notes

1 The formula for g is the same as if the sphere's mass, M , was concentrated at the centre of the sphere.

2 The gravitational field strength at the surface of the sphere, g_s , can be written as

$$g_s = \frac{GM}{r_s^2}$$

where r_s is the radius of the sphere.

3 The variation of g with distance, r , is according to the inverse square law.

Fig. 28.7 shows how g decreases with increasing distance, r .

$$\text{Since } GM = g_s r_s^2, \text{ then } g = \frac{g_s r_s^2}{r^2}$$

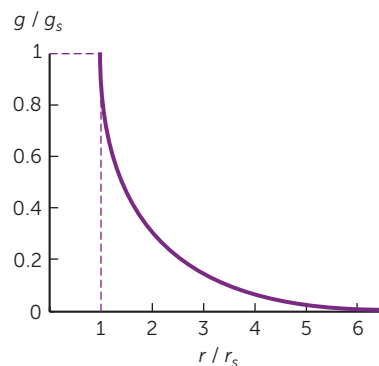


Fig. 28.7 The variation of g with distance r from the centre of a spherical object

28.2 WORKED EXAMPLE

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

a) Show that the gravitational field strength of the Earth on its surface is 9.8 N kg^{-1} .

b) Calculate the gravitational field strength of the Earth at a height of 1000 km above the Earth.

Earth's mass = $5.98 \times 10^{24} \text{ kg}$, Earth's radius = 6380 km.

Solution

$$\text{a) } g_{\text{surface}} = \frac{GM}{r_E^2} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6380 \times 10^3)^2} = 9.80 \text{ N kg}^{-1}$$

$$\text{b) } r = 6380 + 1000 \text{ km} = 7380 \text{ km}$$

$$\therefore g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(7380 \times 10^3)^2} = 7.32 \text{ N kg}^{-1}$$

28.2 QUESTIONS

Use the data in the question box on p. 443 for the following questions.

- Calculate the gravitational field strength due to the Sun
 - at the surface of the Sun,
 - at the surface of the planet Mercury, a distance of 57.9 million kilometres from the Sun.
- Jupiter has a mass of $1.90 \times 10^{27} \text{ kg}$ and moves around the Sun at a distance of 778 million kilometres. Calculate the distance from the Sun at which the gravitational field strength due to the Sun is equal and opposite to the gravitational field strength due to Jupiter.

28.3 Gravitational potential energy

The potential energy of an object is raised when it is lifted up above the Earth's surface.

- Provided its height gain is small enough so the change of gravitational field strength over height, h , is negligible, the change of potential energy = force of gravity $\times$ height gain = mgh .
- If the change of gravitational field strength is not negligible, the force of gravity on an object decreases with height. Fig. 28.8 shows how the force, F , varies with distance, r , from the centre of the Earth, in accordance with Newton's Law of Gravitation.

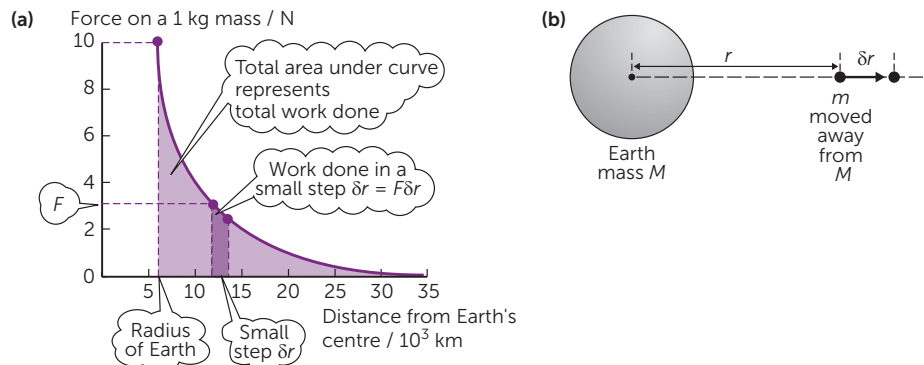


Fig. 28.8 Doing work (a) graph of force versus distance near the Earth, (b) gaining potential energy

Consider an object of mass m that moves away from the centre of the Earth from distance r to distance $r + \delta r$. Provided δr is much less than r , the force of gravity on the object, F , can be written as

$$F = mg = \frac{GMm}{r^2}$$

where M is the mass of the Earth. The change of potential energy of the object, $\delta(\text{p.e.})$ can be written as

$$\delta(\text{p.e.}) = F\delta r = \frac{GMm}{r^2} \delta r$$

However

$$\frac{1}{r} - \frac{1}{r + \delta r} = \frac{r + \delta r - r}{r(r + \delta r)} = \frac{\delta r}{r^2}$$

when δr is much less than r . Therefore the change of potential energy

$$= \frac{GMm}{r} - \frac{GMm}{r + \delta r}$$

Since the change of potential energy = the potential energy at $r + \delta r$ - the potential energy at r , the potential energy at $r = -GMm \div r$ and the potential energy at $r + \delta r$ can be written as $-GMm \div (r + \delta r)$

$$\text{Potential energy} = \frac{-GMm}{r}$$

where r = the distance between the centres of mass.

Notes

- 1 The potential energy varies as the inverse of the distance, r . As this distance increases, the potential energy becomes less negative and tends to zero at infinity.
- 2 The work done to move a mass, m , from distance, r , to infinity is equal to its change of potential energy from $-GMm/r$ at distance r to zero at infinity.
- 3 The area under the curve in Fig. 28.8a represents the work done to move mass, m , from distance, r , to infinity. This is because the work done in a small step, δr , is equal to $F\delta r$, the area of a strip under the curve. So the total work done is represented by the area of all the strips.
- 4 Since the surface gravitational field strength, $g_s = GM \div r_s^2$, where r is the radius of the Earth, then $GM = g_s r_s^2$, hence the potential energy at r can be written as
$$\frac{-mg_s r_s^2}{r}$$

28.3 WORKED EXAMPLE

Calculate the work done to move a 1.0 kg mass from the surface of the Earth to infinity. The gravitational field strength at the surface of the Earth = 9.8 N kg^{-1} and the radius of the Earth = 6380 km.

Solution

For $m = 1.0 \text{ kg}$ on the Earth's surface,

$$\begin{aligned}\text{its p.e.} &= \frac{-GMm}{r_s} = -\frac{mg_s r_s^2}{r_s} = -mg_s r_s \\ &= -1.0 \times 9.8 \times 6380 \times 10^3 \\ &= -62.5 \text{ MJ}\end{aligned}$$

Since its p.e. at infinity is zero, the work done to move a 1.0 kg mass to infinity from the Earth is therefore 62.5 MJ.

Escape speed

When a rocket is launched into space from the Earth, it is given sufficient kinetic energy to enable it to overcome the Earth's gravity. If the rocket does not gain sufficient kinetic energy, it falls back to the Earth's surface.

The escape speed, v_{esc} , of a projectile is the minimum speed it must be given to escape from the Earth to infinity.

For a projectile of mass, m , to escape, its initial speed, v , must be large enough so that its initial kinetic energy, $\frac{1}{2}mv^2$, exceeds GMm/r , the work done to escape from the surface to infinity. Hence

$$\begin{aligned}\frac{1}{2}mv_{\text{esc}}^2 &= \frac{GMm}{r_s} \\ \therefore \text{the escape speed, } v_{\text{esc}} &= \sqrt{\left(\frac{2GM}{r_s}\right)} \\ &= \sqrt{(2g_s r_s)}\end{aligned}$$

Black holes

A black hole is a massive object which has a gravitational field so strong that not even light can escape. Since no object can travel faster than light, it follows that nothing can escape from inside a black hole.

The **event horizon** of a black hole is the closest an object can approach the black hole without becoming trapped.

The escape speed from the surface of a sphere of mass, M , and radius, R , is given by the equation

$$v_{\text{esc}}^2 = \frac{2GM}{R}$$

Since the escape speed cannot exceed c , the speed of light in free space, the sphere would be a black hole if $2GM \div R$ exceeds c^2 . In other words, for a sphere to be a black hole, its radius must be less than or equal to $2GM \div c^2$.

28.4 WORKED EXAMPLE

Show that a sphere of mass equal to the mass of the Earth ($= 5.98 \times 10^{24} \text{ kg}$) would need have a radius of 9 mm to be a black hole.

$$c = 3.0 \times 10^8 \text{ m s}^{-1}, G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

Solution

$$R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(3.0 \times 10^8)^2} = 8.9 \times 10^{-3} \text{ m}$$

28.3 QUESTIONS

Use the data in the question box on p. 443 for the following questions.

- 1 Calculate the work needed to move a 1.0 kg mass from
 - a) the Moon's surface to infinity,
 - b) the Sun's gravitational field at a point on the Earth's orbit to infinity.
- 2 Calculate the radius to which the Sun would need to be compressed to become a black hole.
 $c = 3.0 \times 10^8 \text{ m s}^{-1}$.

28.4 Satellite motion

Consider a satellite in a circular orbit around the Earth as in Fig. 28.9. The centripetal force needed to keep the satellite on its circular path is provided by the force of gravity on the satellite due to the Earth.

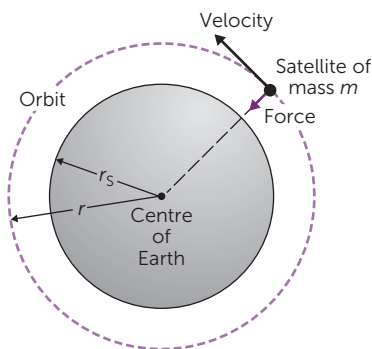


Fig. 28.9 The orbit of a satellite

If the force of gravity suddenly vanished, the satellite would fly off at a tangent. A satellite in a circular orbit moves around the Earth at constant speed because the force on it is perpendicular to its direction of motion and therefore no work is done on the satellite.

The direction of motion of the satellite changes continuously as it moves around the Earth. At any point on its orbit, its velocity is tangential and its acceleration is towards the centre, in the same direction as the force of gravity on the satellite.

For a satellite of mass, m , moving at steady speed, v , on a circular orbit of radius, r , about the Earth, its acceleration is given by the centripetal acceleration formula, $a = v^2 \div r$, from p. 434. The force on the satellite is given by Newton's law of

gravitation, $F = GMm/r^2$, where M is the mass of the Earth. Using Newton's second law of motion,

$$F = ma$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Rearranging this equation gives

$$v^2 = \frac{GM}{r}$$

Since $v = \frac{\text{circumference}}{\text{time for 1 complete orbit, } T}$

$$= \frac{2\pi r}{T}$$

$$\text{then } \frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$$

$$\text{Hence } r^3 = \frac{GM}{4\pi^2} T^2$$

28.5 WORKED EXAMPLE

$G = 6.67 \times 10^{-11} \text{ N m}^2\text{kg}^{-2}$, radius of the Earth = 6380 km

The first artificial satellite was launched into an orbit 200 km above the Earth, moving around the Earth once every 88 minutes. Use this data to calculate the mass of the Earth.

Solution

Rearrange

$$r^3 = \frac{GM}{4\pi^2} T^2$$

to give

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 (6580 \times 10^3)^3}{6.67 \times 10^{-11} \times (88 \times 60)^2} = 6.0 \times 10^{24} \text{ kg}$$

Notes

- 1 The equation opposite applies to any situation where a satellite is in orbit about a much more massive spherical body of mass, M .
- 2 Given values of r and T , the mass, M , of the central body can be calculated.
- 3 Since the surface gravitational field strength, $g_s = GM \div r_s^2$, where r_s is the radius of the central body, then the equation can be written as

$$r^3 = \frac{g_s r_s^2}{4\pi^2} T^2$$

Kepler's Laws of Planetary Motion

Johannes Kepler was a 16th century astronomer who observed and measured the motion of the planets and established three laws describing their motion. The same laws also describe the motion of comets in the Solar System.

- **Kepler's first law** states that each planet moves around the Sun on an elliptical orbit.

Fig. 28.10 shows an elliptical orbit. The Sun is at one of the two 'focal points' of the ellipse. The degree of ellipticity of the orbit of each planet differs from an almost circular orbit for the Earth to the orbit for Pluto, the outermost planet. Pluto's orbit is so elliptical that it moves closer to the Sun than its nearest neighbour, Neptune, for part of its orbit.

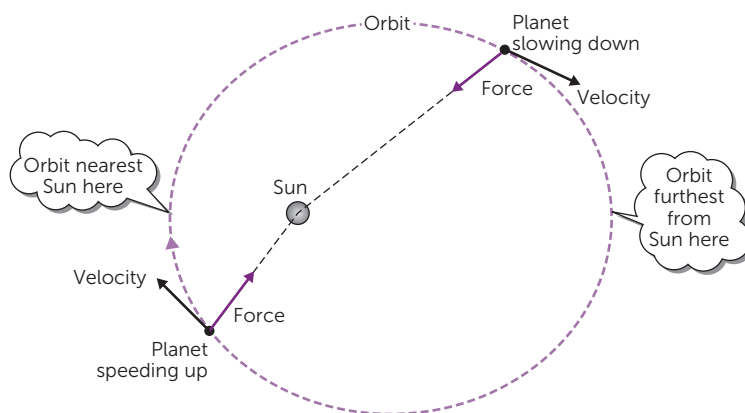


Fig. 28.10 An elliptical orbit

- **Kepler's second law** states that the line from the Sun to a planet sweeps out equal areas in equal times as the planet moves around the Sun. [Fig. 28.11](#) shows the idea. A comet moves faster as it approaches the Sun because the force of gravity on it increases. It slows down as it moves away from the Sun because the force of gravity decreases.

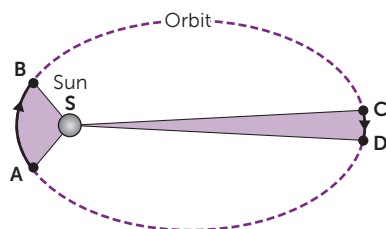


Fig. 28.11 Kepler's second law

If the time from **A** to **B**
equals the time from **C** to **D**
area **SAB** = area **SCD**

Note

One astronomical unit (A.U.) is defined as the mean distance from the Earth to the Sun and is equal to 1.496×10^8 km.

- **Kepler's third law** states that the cube of the mean distance from a planet to the Sun is proportional to the square of the time it takes to move around the Sun. See [Table 28.1](#). Newton devised the theory of gravitation to explain Kepler's third law.

Table 28.1 Planetary data

[illegible]

Geostationary satellites

A geostationary satellite is a satellite that takes exactly 24 hours to go around a circular orbit directly above the equator. Such a satellite is always directly above the same point of the equator as it moves around at the same rate as the Earth spins.

Geostationary satellites are used for communications and satellite TV broadcasting because transmitters and receivers on the Earth do not need to be realigned once they have been pointed towards the satellite.

The height, H , of a geostationary satellite can be calculated using the equation

$$r^3 = \frac{g_s r_s^2}{4\pi^2} T^2$$

from p. 449, where g_s is the surface gravitational field strength of the Earth and r_s is the radius of the Earth. Substituting $T = 24$ hours $= 24 \times 3600$ s, $g_s = 9.8 \text{ N kg}^{-1}$ and $r_s = 6380 \times 10^3$ m, gives

$$r^3 = \frac{g_s r_s^2}{4\pi^2} T^2 = \frac{9.8 \times (6380 \times 10^3)^2 \times (24 \times 3600)^2}{4\pi^2} = 7.54 \times 10^{22} \text{ m}^3$$

Hence $r = 4.225 \times 10^7 \text{ m} = 42\,250 \text{ km}$, $\therefore H = 42\,250 - 6\,380 = 35\,870 \text{ km}$

28.4 QUESTIONS

Use the data in the question box on p. 443 for the following questions.

- 1 A satellite orbits the Earth in a circular orbit.
 - a) Calculate its speed and its time period if its height is 1000 km.
 - b) Calculate its speed and height if its time period is exactly 4 hours.
- 2 a) Calculate the time period of a satellite in circular orbit around the Moon at a height of 100 km.
 b) Calculate the height and time period of a satellite moving at a speed of 1.0 km s^{-1} in a circular orbit around the Moon.

SUMMARY

- **Newton's law of gravitation** $F = \frac{Gm_1m_2}{r^2}$
- **Gravitational field strength** $g = \frac{F}{m}$
- **At distance, r , from the centre of a spherical mass, M**
 - 1 $g = \frac{GM}{r^2}$
 - 2 PE of a small mass, $m = \frac{-GMm}{r}$
 - 3 Escape speed $= \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$
- **For a satellite in a circular orbit round a spherical body of mass, M**
 - 1 Its speed, $v = \sqrt{\left(\frac{GM}{r}\right)} = \frac{2\pi r}{T}$
 - 2 $r^3 = \frac{GMT^2}{4\pi^2}$ where r = radius of orbit and T = time per orbit.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

$$G = 6.67 \times 10^{-11} \text{ N m}^2\text{kg}^{-2}, 1 \text{ A.U.} = 1.50 \times 10^8 \text{ km}$$

Use the following data, where appropriate, in the questions below:

$$\text{Earth's mass} = 5.98 \times 10^{24} \text{ kg}$$

$$\text{Earth's radius} = 6380 \text{ km}$$

$$g_s = 9.80 \text{ N kg}^{-1}$$

$$\text{Moon's mass} = 7.35 \times 10^{22} \text{ kg}$$

$$\text{Mean distance from the Earth to the Moon} = 384\,000 \text{ km}$$

$$\text{Sun's mass} = 1.99 \times 10^{30} \text{ kg}$$

$$\text{Mean distance from the Earth to the Sun} = 150\,000\,000 \text{ km}$$

$$\text{Moon's radius} = 1740 \text{ km}$$

$$\text{Sun's radius} = 696\,000 \text{ km}$$

28.1 When a solar eclipse occurs, the Moon is directly between the Sun and the Earth, as in Fig. 28.12.



Fig. 28.12

- a) Calculate the force on the Moon due to
 - (i) the Earth,
 - (ii) the Sun.
- b) Hence calculate the overall force on the Moon when it is directly between the Sun and the Earth.

28.2 The mass of Jupiter is $318 \times$ the mass of the Earth and its mean distance from the Sun is 5.20 A.U.

- a) Calculate the gravitational field strength of Jupiter at a distance of 1 A.U. from the Sun.
- b) When Jupiter is at opposition, it lies in the opposite direction to the Sun.
 - (i) Show that the gravitational field strength of Jupiter is equal and opposite to that of the Earth at a distance of 0.22 A.U. from the Earth.
 - (ii) Calculate the gravitational field strength of the Sun at this position.

28.3

- a) (i) The potential energy per unit mass of an object on the Earth is -62.5 MJ kg^{-1} due to the Earth's gravity. Calculate the escape speed from the Earth.
 (ii) The potential energy per unit mass of an object on the surface of the Moon is -2.8 MJ kg^{-1} . Calculate the escape speed from the Moon.
- b) A spacecraft of mass 1000 kg travelled from the Moon's surface to the Earth. Assuming it was launched from the Moon with just sufficient kinetic energy to enable it to escape, calculate
 - (i) its kinetic energy just before reaching the Earth,
 - (ii) its speed just before reaching the Earth.

28.4 Mars has a mass of $0.108 \times$ the mass of the Earth, and a radius of 3400 km.

- a) Calculate
 - (i) the gravitational field strength at the surface of Mars,
 - (ii) the escape speed from Mars.
- b) Mars has two moons, Phobos and Deimos.
 - (i) Phobos orbits Mars in a circular orbit once every 7.65 hours. Calculate the height of Phobos above the surface of Mars.
 - (ii) Deimos orbits Mars in a circular orbit at a height of 20 100 km above the surface of Mars. Calculate the time it takes to complete one orbit.

28.5

- a) Neptune has two moons, Triton and Nereid. Triton orbits Neptune at a distance of 354 000 km once every 5.88 days.
 - (i) Calculate the mass of Neptune.
 - (ii) Nereid orbits Neptune once every 359 days. Calculate the distance from Neptune to Nereid.
- b) A satellite orbits the Earth at a height of 1680 km in a circular orbit that takes it over the Earth's poles.
 - (i) Calculate its time period.
 - (ii) Calculate the distance along the equator between successive transits in the same direction across the equator.

Simple Harmonic Motion

29

OBJECTIVES

After working through this unit, you should be able to:

- describe examples of oscillating systems
- explain what is meant by the amplitude and the time period of an oscillating system
- describe without calculations how the acceleration, velocity and displacement of an oscillating system change with time
- explain what is meant by simple harmonic motion and relate it to circular motion
- define angular frequency and use it to relate the acceleration to the displacement for an object in simple harmonic motion
- use formulae for the time period of a mass oscillating on a spring and a simple pendulum
- distinguish between free and damped oscillations
- describe what is meant by forced oscillations and resonance

CONTENTS

- 29.1 Oscillating motion
- 29.2 Sine waves
- 29.3 Forces in oscillating systems
- 29.4 Resonance
- Summary
- Revision questions

29.1 Oscillating motion

Oscillating systems

Any oscillating system moves to and fro repeatedly. Two simple systems are described below. By studying such simple systems, we can understand the motion of more complicated oscillating systems.

- If a pendulum is displaced from its equilibrium position and released, it oscillates to and fro, repeatedly passing back and forth through its equilibrium position. See Fig. 29.1. Eventually, the oscillations die away and it stops moving altogether. Each complete cycle of its oscillating motion takes it from one side of equilibrium to the other side and back again.
- A weight suspended on a spring will oscillate if it is pulled down from equilibrium then released. See Fig. 29.2 overleaf. Each complete cycle of oscillation takes it from its lowest position to its highest position then back to its lowest position. In practice, the oscillations die away gradually and it eventually stops moving.

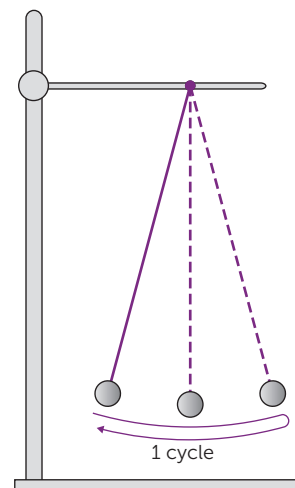


Fig. 29.1 A simple pendulum

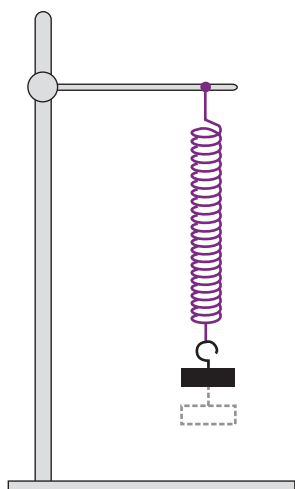


Fig. 29.2 An oscillating spring

Measuring oscillations

Any oscillating object oscillates about a certain position which is therefore the centre of the oscillating motion. This position is usually the equilibrium position of the object, the point where the object would stay if released at rest at that point.

- The **displacement** of an oscillating object at any instant is the distance and direction of the object from its equilibrium position. Fig. 29.3 shows how the displacement of an oscillating object varies with time. The displacement is positive in one direction and negative in the opposite direction.

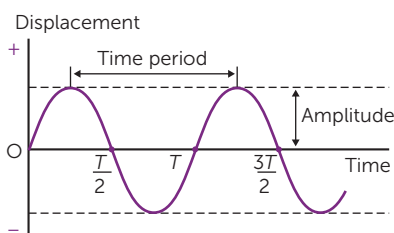


Fig. 29.3 Displacement versus time for an oscillating object

- The **amplitude of oscillations** is the maximum displacement from equilibrium. In the example of the simple pendulum and the oscillating spring, the amplitude gradually decreases due to air resistance. If there was no air resistance, the oscillations would continue indefinitely. Air resistance causes friction on the moving parts, gradually reducing the amplitude of the oscillations to zero.
- The **time period of oscillations** is the time for one complete cycle of oscillations. This is the least time taken by an oscillating object to move from a certain position and velocity back to the same position and velocity. For example, the time period of a simple pendulum is equal to the time taken by the pendulum to pass in a certain direction through equilibrium to the next pass through equilibrium in the same direction.

Velocity and acceleration

When an oscillating pendulum is moving through its equilibrium position, its speed is at its greatest and its acceleration is zero. At maximum displacement from equilibrium, its speed is zero and the magnitude of its acceleration is at its greatest.

Velocity versus time

Since velocity is defined as rate of change of displacement, the velocity of an oscillating object can be determined from the gradient of the displacement versus time graph for the object. Fig. 29.4 shows the velocity versus time graph corresponding to Fig. 29.3.

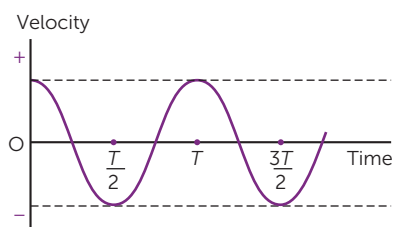


Fig. 29.4 Velocity versus time for an oscillating object

- At zero displacement, the gradient of Fig. 29.3 is a maximum, either in the positive direction or the negative direction, corresponding to the two possible directions of the object as it moves through equilibrium.
- At maximum displacement, the gradient of Fig. 29.3 is zero, corresponding to zero speed at maximum displacement in either direction. The object changes its direction at maximum displacement so the velocity changes from + to –, or – to +.

Acceleration versus time

Since acceleration is rate of change of velocity, the acceleration of an oscillating object can be determined from the gradient of the velocity versus time graph for the object. Fig. 29.5 shows the acceleration versus time graph corresponding to Fig. 29.4.

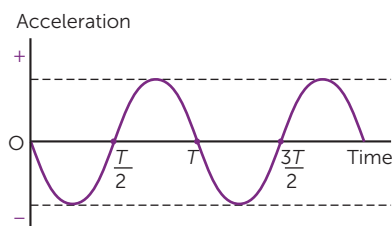


Fig. 29.5 Acceleration versus time for an oscillating object

- At zero velocity, the gradient of Fig. 29.4 is a maximum, either in the positive direction or in the negative direction, corresponding to maximum displacement in either direction where the object turns around.
- At maximum velocity, the gradient of Fig. 29.4 is zero, corresponding to zero acceleration which occurs when the object passes through equilibrium.
- Compare Fig. 29.5 and Fig. 29.3. When the displacement is positive, the acceleration is negative. When the displacement is negative, the acceleration is positive. In other words, when the displacement is in one direction, the acceleration is in the opposite direction.

29.1 QUESTIONS

- 1 A pendulum shown in Fig. 29.6 oscillates in a line which lies in a vertical plane from north to south.
 - a) When the pendulum is at its northernmost position, what is the direction of (i) its displacement from the centre of oscillations, (ii) its acceleration?
 - b) What will be the direction of the displacement and the velocity of the pendulum a quarter of a cycle after it is at its northernmost position?
- 2 A child is bouncing up and down on a trampoline.
 - a) When the child is at the lowest possible position, what can you say about (i) the direction of the acceleration, (ii) the magnitude of the velocity?
 - b) When the child is at the highest possible position, what can you say about (i) the direction of the acceleration, (ii) the magnitude of the velocity?

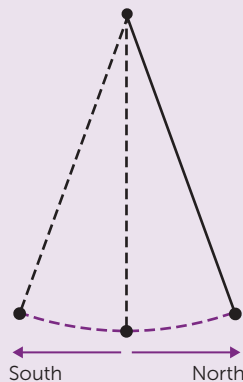


Fig. 29.6

29.2 Sine waves

Making a sine wave

Consider point P, moving on a circular path, as shown in Fig. 29.7. Thus the coordinates of point P in Fig. 29.7 are $x = r \cos \theta$ and $y = r \sin \theta$, where r is the radius of the circle and θ is the angle between the line OP and the x -axis.

Fig. 29.7 also shows how its y -coordinate ($= r \sin \theta$) changes as θ increases from 0 to 360° as P moves around. The curve of Fig. 29.7 is called a sine curve because it is generated by the sine function. Its shape is described as 'sinusoidal'.

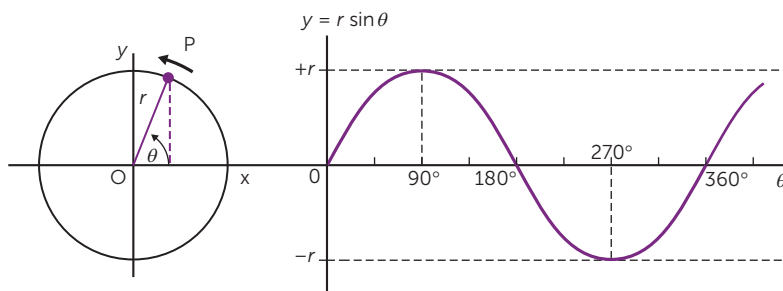


Fig. 29.7 Making a sine curve

This type of curve describes the oscillating motion of a loaded spring and small oscillations of a simple pendulum. Oscillating motion described by a sinusoidal curve is called **simple harmonic motion**.

The link between circular motion and simple harmonic motion can be seen if the shadow of an object, P, moving on a circular path is observed adjacent to the shadow of an oscillating object, Q, as in Fig. 29.8. The shadows are projected on to a vertical screen perpendicular to the plane of the circle. The frequency of rotation of object P is adjusted to match the frequency of oscillation of object Q. The two shadows keep up with each other, demonstrating that the motion of the oscillating object is the same as the y -coordinate of the object in circular motion (i.e. sinusoidal).

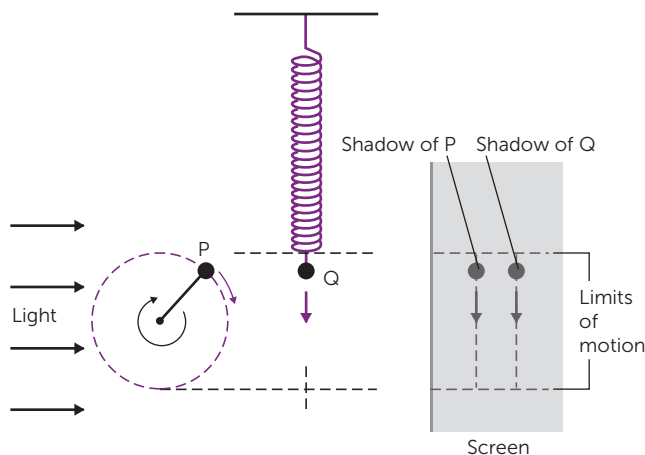


Fig. 29.8 Linking circular motion and simple harmonic motion

To explore how the acceleration of an oscillating object is related to its displacement, consider the acceleration of object P in circular motion in Fig. 29.8.

The acceleration of P depends on the speed, v , of the object and the circle radius, r , in accordance with the equation for centripetal acceleration

$$a_p = \frac{-v^2}{r}$$

where the minus sign indicates that the acceleration is towards the centre.

This may be written as $a_p = -\omega^2 r$, where the angular speed, ω , of object P is defined as

$$\omega = \frac{2\pi}{T} = \frac{v}{r}$$

and T is the time for 1 complete oscillation.

Since the vertical component of the acceleration $= a_p \sin \theta$ and $y = r \sin \theta$, then the vertical component of the acceleration $= -\omega^2 r \sin \theta = -\omega^2 y$. This last equation therefore describes how the acceleration of an object in simple harmonic motion varies with its displacement from equilibrium.

Definition of simple harmonic motion

Simple harmonic motion is oscillating motion of an object in which the acceleration of the object is

- 1 always in the opposite direction to the displacement from equilibrium,
- 2 proportional to the displacement from equilibrium.

The acceleration and displacement are linked by the equation:

$$\text{acceleration} = -\omega^2 \times \text{displacement}$$

where $\omega = 2\pi/T$.

Notes

- 1 The acceleration is therefore always negative when the displacement is positive and is always positive when the displacement is negative.
- 2 The maximum speed, $v_{\text{MAX}} = \omega r$ where r is the amplitude of the oscillations. This follows from Fig. 29.8 because the oscillating object moves as fast as the object moving around the circle.
- 3 The displacement varies sinusoidally with time as shown in Fig. 29.7.
- 4 The time period, T , is independent of the amplitude for any object oscillating in simple harmonic motion.
- 5 The unit of ω is radian second⁻¹ (rad s⁻¹) since 2π is the angle around a circle in radians and T is in seconds. In the context of simple harmonic motion, ω is called the **angular frequency**.

29.1 WORKED EXAMPLE

An object oscillates in simple harmonic motion with a time period of 5.0 seconds and an amplitude of 0.040 m. Calculate

- a) its angular frequency,
- b) its maximum speed,
- c) its maximum acceleration.

Solution

a) $\omega = \frac{2\pi}{T} = 1.26 \text{ rad s}^{-1}$

b) $v_{\text{MAX}} = \omega r = 1.26 \times 0.040 = 0.050 \text{ m s}^{-1}$

c) Maximum acceleration, a_{MAX} , is at maximum displacement which is 0.040 m.

Hence $a_{\text{MAX}} = -\omega^2 \times \text{maximum displacement}$

$$= -1.26^2 \times 0.040$$

$$= 6.3 \times 10^{-2} \text{ m s}^{-2}$$

29.2 QUESTIONS

- 1 A mass on a spring oscillates along a vertical line, taking 12 s to complete 10 oscillations.
 - a) Calculate (i) its time period, (ii) its angular frequency.
 - b) Its height above the floor varies from a minimum of 1.00 m to a maximum of 1.40 m. Calculate (i) its amplitude, (ii) its maximum velocity, (iii) its acceleration when it is at its lowest position.
- 2 The motion of a vibrating blade is 'frozen' by illuminating it with a stroboscope (a flashing light). The least stroboscope frequency at which this occurs is 40 Hz.
 - a) Explain why the blade appears stationary at this frequency.
 - b) Calculate (i) the time period, (ii) the angular frequency of the vibrations.
 - c) The free end of the blade vibrates with an amplitude of 8.0 mm. Calculate the maximum speed of the free end of the blade.

29.3 Forces in oscillating systems

When an object is oscillating, the forces acting on it try to restore it to its equilibrium position. When the object is moving away from its equilibrium position, the forces on it slow it down and reverse its direction of motion. When the object is moving towards equilibrium, the forces on it speed it up and make it overshoot the equilibrium position.

If the object is oscillating with simple harmonic motion, its acceleration, a , depends on its displacement, s , in accordance with the equation

$$a = -\omega^2 s$$

where $\omega = 2\pi/T$.

Applying Newton's second law of motion in the form 'force = mass $\times$ acceleration' gives the following equation for the restoring force, F , on an oscillating object:

$$F = -m\omega^2 s = -ks$$

where $k = m\omega^2$.

It therefore follows that any oscillating system moves with simple harmonic motion if the restoring force, F , is

- 1 proportional to the displacement from the equilibrium position;
- 2 always directed towards the equilibrium position.

The oscillations of a loaded spring

Hooke's Law states that the tension, T , in a spring is proportional to its extension from its natural length. This may be expressed as $T = ke$, where e is the extension and k is a constant, referred to as the spring constant. See Fig. 29.9.

Consider a spring suspended vertically from a fixed point, supporting an object of mass, m , at its lower end, as in Fig. 29.10.

- When the object is at rest at its equilibrium position, the tension in the spring at equilibrium, T_0 , is equal and opposite to the weight, mg , of the object.

$$T_0 = mg$$

- When the object is oscillating, the tension in the spring changes as the length of the spring changes. When the object is at displacement, s , from equilibrium, the restoring force is provided by the change of tension from equilibrium, ΔT .

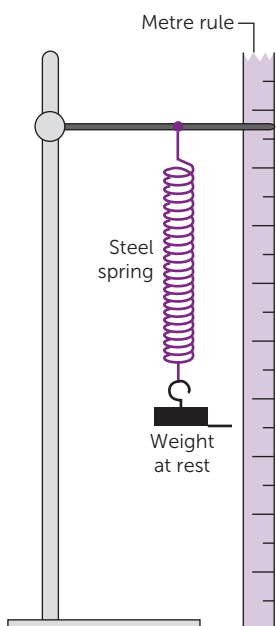
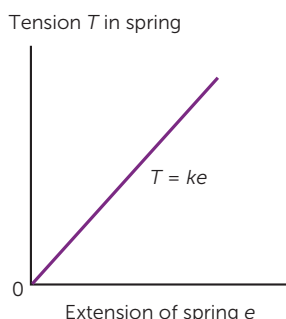


Fig. 29.9 Testing Hooke's Law

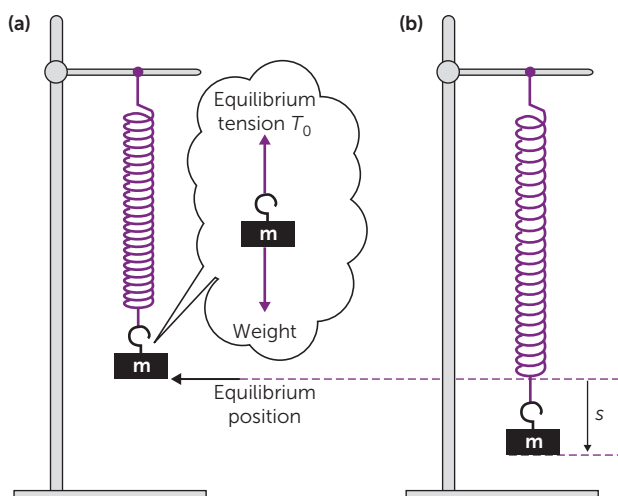


Fig. 29.10 The oscillations of a loaded spring
(a) at rest, (b) oscillating

Applying Hooke's Law to this change of tension gives $\Delta T = -ks$, where the minus sign signifies that the change of tension is in the opposite direction to the displacement. The acceleration, a , can be written as

$$a = \frac{\text{restoring force}}{\text{mass}} = \frac{\Delta T}{m} = \frac{-k}{m}s$$

This is the same as the equation for simple harmonic motion, $a = -\omega^2 s$, where $\omega^2 = k \div m$. A loaded spring therefore oscillates with simple harmonic motion. Furthermore, its time period, T , is related to the mass and the spring constant by the following equation:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

29.2 WORKED EXAMPLE

$$g = 9.8 \text{ m s}^{-2}$$

A steel spring suspended from a fixed point supports a scale pan of mass 0.050 kg at equilibrium.

- The scale pan descends 40 mm to a new equilibrium position when a 1.0 N weight is placed on it. Calculate (i) the spring constant, (ii) the total mass of the scale pan and the 1.0 N weight.
- The scale pan with the 1.0 N weight on it is pulled a distance of 15 mm downwards from equilibrium then released. Calculate (i) the angular frequency of the oscillations, (ii) the time period of the oscillations, (iii) the maximum speed of the scale pan.

Solution

a) (i) Rearranging $F = ke$ gives $k = \frac{F}{e} = \frac{1.0}{0.040} = 25 \text{ N m}^{-1}$

(ii) Mass of 1.0 N weight = $\frac{\text{weight}}{g} = \frac{1.0}{9.8} = 0.10 \text{ kg}$ $\therefore$ Total mass = 0.15 kg

29.2 WORKED EXAMPLE *continued***Solution**

$$\text{b) (i) } \omega = \sqrt{\left(\frac{k}{m}\right)} = \left(\frac{25}{0.15}\right)^{1/2} = 13 \text{ rad s}^{-1}$$

$$\text{(ii) } T = \frac{2\pi}{\omega} = 0.49 \text{ s}$$

$$\text{(iii) The amplitude of the oscillations} = 15 \text{ mm} = 0.015 \text{ m}$$

$$\therefore v_{\text{MAX}} = \omega r = 13 \times 0.015 = 0.195 \text{ m s}^{-1}$$

The oscillations of a simple pendulum

A simple pendulum consists of a small bob on a thread. When the bob is displaced from its equilibrium position with the thread taut and then released, it oscillates about its equilibrium position in a fixed vertical plane.

Consider the forces on a bob of mass, m , when the thread is at angle θ to the vertical, as in Fig. 29.11. The weight can be resolved into a component, $mg \cos \theta$, parallel to the thread and $mg \sin \theta$, perpendicular to the thread.

The restoring force on the bob, $F = -mg \sin \theta$, since the component of weight at right angles to the thread acts towards the equilibrium position of the bob.

Hence the acceleration of the bob, a , can be written as

$$\begin{aligned} a &= \frac{\text{restoring force}}{\text{mass}} \\ &= \frac{-mg \sin \theta}{m} = -g \sin \theta \end{aligned}$$

From Fig. 29.11,

$$\sin \theta = \frac{s}{L}$$

where L is the length of the thread and s is the horizontal displacement of the bob from equilibrium.

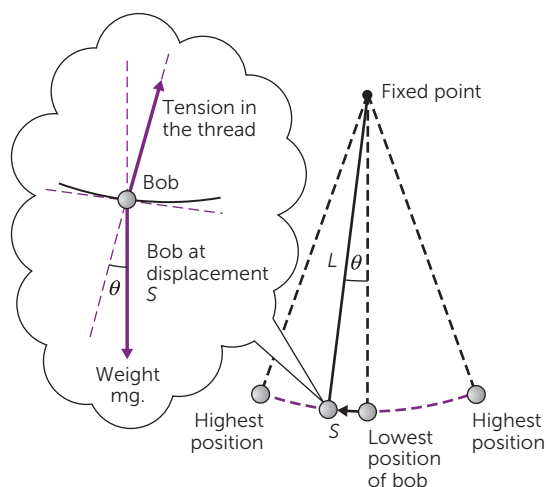


Fig. 29.11 The simple pendulum

However, $x = s$, provided θ does not exceed about 10° .

$$\therefore \sin \theta = \frac{s}{L}$$

for small oscillations, hence

$$a = -g \sin \theta = -\frac{g}{L}s = -\omega^2 s, \text{ where } \omega^2 = \frac{g}{L}$$

A simple pendulum therefore oscillates with simple harmonic motion, provided θ does not exceed about 10° . Furthermore, its time period, T , is related to the length of the thread by the following equation:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

29.3 WORKED EXAMPLE

$$g = 9.8 \text{ m s}^{-2}$$

Calculate the length of a pendulum that has a time period of exactly 1 second.

Solution

$$\text{Rearrange } T = 2\pi\sqrt{\frac{L}{g}} \text{ to give } L = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 1^2}{4\pi^2} = 0.25 \text{ m}$$

Experiment to measure g using a simple pendulum

- 1 Measure the length, L , of a simple pendulum from the fixed support to the centre of the bob. Determine the time period, T , of small oscillations of the simple pendulum by timing 20 complete oscillations three times.
- 2 Calculate an average value for the time period. Repeat the procedure for five further lengths.
- 3 Plot a graph of T^2 on the vertical axis against L on the horizontal axis as shown in Fig. 29.12. Since

$$T^2 = \frac{4\pi^2}{g}L$$

the graph should give a straight line through the origin with a gradient equal to $4\pi^2 \div g$. Hence $g = 4\pi^2 \div$ the gradient of the graph.

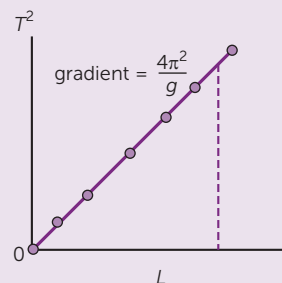


Fig. 29.12 Measurement of g

29.3 QUESTIONS

$$g = 9.8 \text{ m s}^{-2}$$

- 1 A steel spring suspended from a fixed point supports a 0.20 kg mass hung from its lower end. The stone is displaced downwards from its equilibrium position by a distance of 25 mm then released. The time for 20 oscillations is measured at 22.0 s. Calculate
 - a) its time period,
 - b) its angular frequency,
 - c) its maximum speed,
 - d) the maximum tension in the spring.
- 2 a) Calculate the time period of a simple pendulum of length 2.0 m.
 b) When a simple pendulum of length 2.0 m is in oscillating motion, its thread makes a maximum angle with its equilibrium position of 5° . Calculate (i) the maximum height of the bob above its equilibrium position, (ii) the maximum speed of the bob.

29.4 Resonance

The energy of an oscillating system

The energy of an oscillating system changes every half cycle from kinetic energy at maximum speed to potential energy at maximum displacement and back to kinetic energy. At any instant, the total energy of the system is the sum of its kinetic energy and its potential energy.

Free oscillations

If there are no frictional forces present, the total energy remains constant and the amplitude is unchanged. See Fig. 29.13. The oscillations are described as **free** because there are no forces present that would cause the energy of the system to be dissipated.

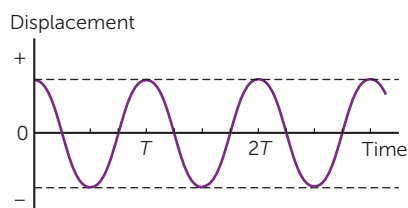


Fig. 29.13 Free oscillations

Damped oscillations

The oscillations of a simple pendulum gradually die away as air resistance causes the energy of the simple pendulum to be gradually reduced to zero. The amplitude becomes smaller and smaller, as shown in Fig. 29.14. The presence of air resistance ‘damps’ the oscillations. If it were possible to remove air resistance, the amplitude would remain constant as the pendulum would not lose its energy. The degree of damping depends on the maximum damping force in relation to the maximum restoring force.

- **Light damping** is where the damping force is much smaller than the maximum restoring force. The amplitude gradually becomes smaller and smaller as the damping force gradually dissipates the energy of the system, as shown in Fig. 29.14.

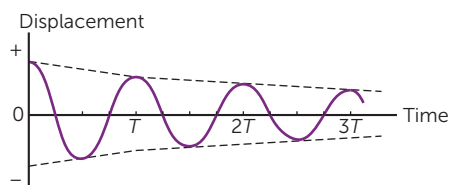


Fig. 29.14 Light damping

- **Heavy damping** is where the system returns gradually to equilibrium without oscillating, after being displaced then released. For example, a mass on a spring in a tube of viscous oil would return slowly to equilibrium without oscillating if released from a non-equilibrium position. Fig. 29.15 shows how the displacement of a heavily damped object changes after it has been released.

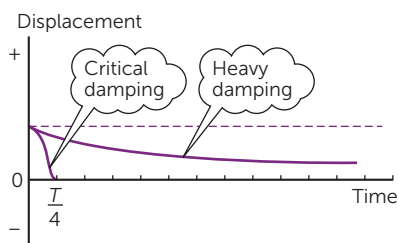


Fig. 29.15 More damping

- **Critical damping** is where the system returns to equilibrium in as little time as possible without oscillating, as shown in Fig. 29.15. This type of damping is an important design feature of an analogue meter as it ensures the pointer of such a meter does not oscillate and does not take too long to reach its correct position when the current changes.

Forced oscillations

If a child on a swing is given a push every time the child reaches maximum height, the amplitude of the swing increases and increases. The force of the push supplies energy to the child so the total energy of the child increases. The push force needs to be applied at the same frequency as the natural frequency of oscillation of the spring.

The amplitude of any oscillating system subjected to a periodic force becomes very large if the periodic force supplies energy at a certain point in every cycle of oscillation. This effect is known as **resonance** and the frequency of the periodic force at which it occurs is called the **resonant frequency**. At any other frequency, the system is forced to oscillate with a small variable amplitude without resonating.

Fig. 29.16 shows how an oscillator may be used to apply forced oscillations to a mass held between two vertical springs. The frequency of the oscillator is altered

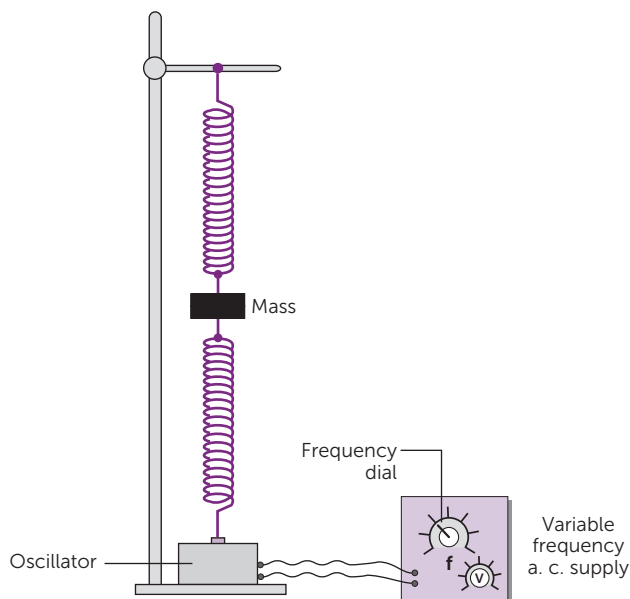


Fig. 29.16 Forced oscillations

gradually until it is equal to the resonant frequency of the system. When this occurs, the amplitude of oscillations increases until the energy loss due to damping equals the energy supplied by the oscillator. The lighter the damping, the greater the amplitude.

The resonant frequency is equal to the natural frequency of oscillation of the system, only if the damping is very light.

Further examples of mechanical resonance

- 1 The Tacoma bridge collapse on 7 November 1940 was due to forced oscillations caused by high winds that created periodic eddies. Each eddy caused the bridge to sway. On the day of its destruction, the wind speed was such that the frequency of the eddies was the same as the natural frequency of the bridge span twisting and swinging from side to side. The twisting motion caused more eddies, which caused more twisting until the bridge destroyed itself. Since then, bridge engineers have incorporated damping mechanisms to prevent resonance.
- 2 A wine glass shatters when subjected to sound at a certain frequency. The sound waves make the wine glass oscillate and if the sound frequency is matched to the natural frequency of the wine glass, the glass shatters.
- 3 A vehicle panel that vibrates at a certain engine frequency has its own natural frequency of oscillation. The panel is forced to vibrate by the engine's motion. When the frequency of the engine is equal to the natural frequency of vibration of a panel, the panel vibrates at resonance.

See www.palgrave.com/foundations/breithaupt to model oscillating motion using a spreadsheet.

29.4 QUESTIONS

- 1 A washing machine vibrates excessively at a particular speed.
 - a) Explain why this occurs at a certain speed.
 - b) Describe how such excessive vibrations could be stopped.
- 2 The chassis of a car of mass 1200 kg is 20 cm clear of the ground when the car is unloaded. When the car carries four people of combined mass 300 kg, the chassis is only 15 cm clear of the ground.
 - a) Calculate (i) the additional weight on each of the four wheels of the car when it carries four people, (ii) the spring constant of the suspension spring of each wheel.
 - b) (i) Show that the time period of the oscillations of the suspension system is 1.0 s. (ii) Explain why the car oscillates uncomfortably when it travels at a speed of 10 m s^{-1} over speed bumps spaced 10 m apart.

SUMMARY

- **Amplitude** is the maximum displacement from equilibrium.
- **Time period**, T , is the time for one complete cycle of oscillation.
- **Frequency**, $f = \frac{1}{T}$
- **Acceleration** = $-\omega^2 \times \text{displacement}$, where ω = angular frequency = $\frac{2\pi}{T}$
- **Maximum speed**, $v_{\text{MAX}} = \omega \times \text{amplitude}$.

• **Time period formulae**

1 Mass, m , on a spring, $T = 2\pi\left(\frac{m}{k}\right)^{1/2}$, where k is the spring constant.

2 Simple pendulum of length L , $T = 2\pi\left(\frac{L}{g}\right)^{1/2}$

- **Resonance** for light damping occurs when the frequency of the applied force is equal to the natural frequency of oscillation of the system.

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

$$g = 9.8 \text{ m s}^{-2}$$

29.1 An object oscillates vertically with an amplitude of 100 mm in simple harmonic motion.

- a) The object takes 15.0 s to undertake 20 oscillations. Calculate
- (i) its time period of oscillation,
 - (ii) its maximum velocity,
 - (iii) its maximum acceleration.
- b) The displacement, s , of the object varies with time in accordance with the equation, $s = r \sin \omega t$, where r is the amplitude of oscillations and T is the time period.
- (i) Sketch a graph to show how the displacement varies with time in accordance with the equation, $s = r \sin \omega t$.
 - (ii) Indicate on your graph a point where the speed is a maximum. Label this point X.
 - (iii) Indicate on your graph a point where the acceleration is a maximum. Label this point Y.

29.2 A vertical spring is suspended from a fixed point. When an object of mass 0.30 kg is suspended from the lower end of the spring, the spring stretches by 90 mm.

- a) Calculate the spring constant of the spring.
- b) The object on the spring is pulled vertically down by a distance of 0.060 m from its equilibrium position, then released. Calculate
- (i) the time period of oscillations of the spring,
 - (ii) the maximum speed of the object.

29.3 In a vehicle test, a loaded vehicle of total mass 900 kg travelled over a bump on the road surface at constant speed. A passenger in the car counted two complete cycles of oscillation in 3.0 s after passing over the bump.

- a) Calculate (i) the time period of the oscillations, (ii) the spring constant of the suspension spring at each wheel.
- b) The mass of the unloaded vehicle and driver is 750 kg. Calculate the time period of oscillations of the unloaded vehicle and driver.

29.4 A simple pendulum consists of a small sphere of diameter 2.0 cm attached to a thread of length 1.18 m.

- a) Calculate the time period of small oscillations of this pendulum.
- b) (i) The pendulum bob is displaced from equilibrium with the string taut until the thread makes an angle of 10° with a vertical line. Calculate the displacement of the bob at this position.
- (ii) The bob is then released from this position. Calculate its maximum speed as it passes through its lowest position.

29.5 A trailer body has a mass of 400 kg and is mounted on four springs, one at each corner. The floor of the trailer body is lowered by a distance of 20 mm when four 30 kg bags of sand are evenly placed in the trailer.

- a) Calculate
- (i) the total weight of the bags of sand,
 - (ii) the spring constant of each spring,
 - (iii) the time period of oscillations of the loaded trailer.
- b) Explain why the trailer oscillates considerably when it is driven at a certain speed over a series of speed bumps.

UNIT

30

Rotational Dynamics

CONTENTS

- 30.1 Angular motion
- 30.2 Moment of inertia
- 30.3 Rotational kinetic energy
- 30.4 Angular momentum
- 30.5 Comparison of linear and rotational motion
- Summary
- Revision questions

OBJECTIVES

After working through this unit, you should be able to:

- explain what is meant by angular speed and angular acceleration
- explain what is meant by moment of inertia and relate it to angular acceleration and torque
- calculate the moment of inertia about a certain axis of an object which has a simple shape given the relevant formula for its moment of inertia.
- calculate the kinetic energy and the angular momentum of a rotating object, given its angular speed and the relevant formula for its moment of inertia
- describe how to measure the moment of inertia of a flywheel and a freely rotating turntable
- state and use the principle of conservation of angular momentum
- identify and compare similarities between angular motion and linear motion

30.1 Angular motion

Notes

- 1 Angular speed is sometimes expressed in **revolutions per minute** or **rpm**. To convert to rad s^{-1} , multiply by $2\pi/60$ as there are 2π radians per revolution and 60 seconds per minute.
- 2 **Angular velocity** is angular speed in a certain direction of rotation.

There are many examples of rotating objects around us. Examples include the wheels of a moving car, a flywheel in motion and the rotation of the armature coil of an electric motor.

For any rotating object,

- its **frequency of rotation**, f , is the number of revolutions it turns through per second. For example, if a flywheel turns through 15 revolutions in 5 seconds, its frequency of rotation is 3 revolutions per second, expressed in revolutions per second or revolutions per minute.
- its **period of rotation**, T , is the time it takes to rotate through one complete revolution. Note that $T = 1 \div f$.

The **angular speed**, ω , of a rotating object is defined as the angle it turns through per second.

For an object rotating at constant angular speed, in 1 complete revolution, it turns through an angle of 360 degrees or 2π radians. If its time period T is known, its angular speed can be calculated using the equation

$$\text{angular speed } \omega = \frac{2\pi}{T}$$

where T is its period of rotation.

The unit of angular speed is the radian per second (rad s^{-1}).

Angular acceleration

When the angular speed of a rotating object changes, for example if it is made to rotate faster, the object undergoes angular acceleration.

The **angular acceleration**, α , is defined as the change of its angular velocity per second.

Consider a disc rotating about a fixed axis as shown in Fig. 30.1.

- When the disc rotates at constant angular speed, its angular acceleration is zero because its angular speed does not change. Every part of the flywheel has same angular speed.
- When the angular speed of the disc changes, every part of a rotating object experiences the same change of angular speed. If the disc speeds up from an initial angular speed ω_1 to an angular speed ω_2 in time t , then

$$\text{its angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t}$$

The unit of angular acceleration is the radian per second per second (rad s^{-2})

Consider a point P on the rotating disc, as shown in Fig. 30.2. P continually moves in a circle of radius r , where r is the distance from P to the axis of rotation. In each complete revolution, P moves a distance equal to $2\pi r$, the circumference of the circle.

- 1 The speed v of P on the disc depends on its distance from the axis as well as how fast the disc is turning (i.e. its angular speed). Therefore its speed,

$$v = \frac{\text{circumference, } 2\pi r}{\text{period of rotation, } T} = \omega r$$

where its angular speed

$$\omega = \frac{2\pi}{T}$$

- 2 The direction of motion of P continually changes as the disc turns. At any given instant, the direction of motion of P is along the tangent to the circle.
- 3 If the angular speed of the disc changes, the speed of P changes. If the angular speed changes from ω_1 to ω_2 in time t , the speed of P changes from $u = \omega_1 r$ to an angular speed $v = \omega_2 r$ in that time t .

Therefore, the acceleration, a , of P

$$\begin{aligned} &= \frac{(v - u)}{t} = \frac{\omega_2 R - \omega_1 R}{t} \\ &= \frac{(\omega_2 - \omega_1)R}{t} \\ &= \alpha R \end{aligned}$$

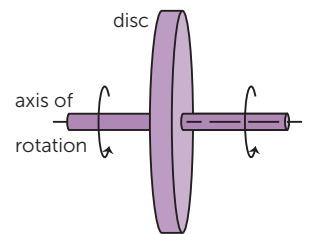


Fig. 30.1 A rotating disc

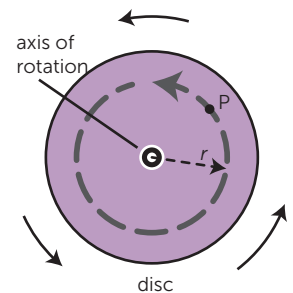


Fig. 30.2 Motion of a point on the disc

Note

Every part of a rotating body experiences a centripetal acceleration equal to $\omega^2 r$ which acts directly towards the centre of rotation of that point.

In general, in any rotating body, for any point P at perpendicular distance r from the axis of rotation,

its speed, $v = \omega r$ where ω is the angular speed of the body

its acceleration $a = \alpha r$ where α is the angular acceleration of the body

30.1 WORKED EXAMPLE

The angular speed of a flywheel is increased from 3.0 to 12.0 revolutions per minute in 50s. The radius of the flywheel is 0.040 m. Calculate

- the initial angular speed in radians per second,
- the angular speed in radians per second after 50 s,
- the angular acceleration of the flywheel,
- the acceleration of a point on the rim along its circular path.

Solution

a) Initial angular speed, $\omega_1 = 3.0 \times 2\pi/60$

$$= 0.31 \text{ rad s}^{-1}$$

b) Angular speed after 50 s, $\omega_2 = 12.0 \times 2\pi/60$

$$= 1.26 \text{ s}^{-1}$$

c) Angular acceleration, $\alpha = \frac{(\omega_2 - \omega_1)}{t}$

$$= \frac{1.26 - 0.31}{50} = 0.019 \text{ rad s}^{-2}$$

d) Acceleration tangential to the rim, $a = \alpha R = 0.019 \times 0.040$

$$= 7.6 \times 10^{-4} \text{ m s}^{-1}$$

Equations for constant angular acceleration

These are similar to the equations for straight line motion with constant acceleration.

- From the definition of angular acceleration α above, we obtain by rearrangement,

$$\omega = \omega_0 + \alpha t$$

where ω_0 = initial angular speed, and ω = angular speed at time t .

- The average value of the angular speed is obtained by averaging the initial and final values, giving an average value of $\frac{1}{2}(\omega + \omega_0)$.

The angle through which the object turns, the angular displacement θ , is equal to the average angular speed $\times$ the time taken which gives

$$\theta = \frac{1}{2}(\omega + \omega_0)t$$

- The equations in 1 and 2 can be combined to eliminate ω to give

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

- Alternatively, the equations in 1 and 2 may be combined to eliminate t , giving

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Note

The number of turns for an angular displacement θ in radians $= \theta/2\pi$.

Note

These four equations are directly comparable with the four linear dynamics equations for constant acceleration a :

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Using the 'angular equations' is made much easier by 'translating' between linear and angular terms, as follows:

Linear equations		Angular equations
displacement		angular displacement
s	$\longleftrightarrow$	θ
speed or velocity		angular speed
u	$\longleftrightarrow$	ω_0
v	$\longleftrightarrow$	ω
acceleration		angular acceleration
a	$\longleftrightarrow$	α
For example		
$v = u + at$	$\longleftrightarrow$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\longleftrightarrow$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\longleftrightarrow$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

30.1 QUESTIONS

- A flywheel accelerates uniformly from rest to 12 rad s^{-1} in 60 seconds. Calculate
 - its angular acceleration,
 - (i) the angle it turned through in radians in this time, (ii) the number of turns it made.
- A spin drier tub accelerated uniformly from rest to an angular speed of 1100 revolutions per second in 40 seconds. Calculate
 - the angle which the tub turned through in radians in this time,
 - the number of turns it made as it accelerated.

30-2 Moment of inertia

Torque

To make a flywheel rotate, a turning force must be applied to it. The turning effect of the force depends not just on the force but also on where it is applied. The torque of a turning force is the moment of the force about the axis. Therefore,

$$\text{Torque} = \text{force} \times \text{perpendicular distance from the axis to the line of action of the force}$$

The unit of torque is the newton metre (N m).

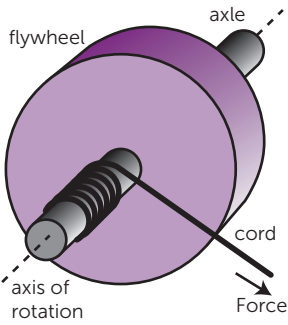


Fig. 30.3 Applying a torque

In Fig. 30.3, a cord wrapped round the axle of the flywheel is used to apply a turning force to make the flywheel rotate. The axis of rotation is along the centre of the axle. Because the force acts on the axle at its perimeter, the perpendicular distance from the centre of the axle to the line of action of the force is equal to the radius of the axle. The torque is therefore equal to the force $\times$ the radius of the axle.

Inertia

The **inertia** of an object is its resistance to change of its motion. If a large torque is required to start a flywheel turning, the flywheel must have considerable inertia. In other words, its resistance to change of its motion is large. Every object has inertia because every object has mass. However, the inertia of a rotating body depends on the distribution of mass as well as the amount of mass.

To appreciate why the distribution of mass affects the inertia of a rotating object, consider again the flywheel shown in Fig. 30.3. Suppose it is initially at rest and a torque is applied to it to make it rotate. Assuming there is no friction on the axle bearings, the applied torque will increase its angular speed. When the torque is removed, the angular speed stops increasing so it turns at constant frequency once the torque is removed.

The flywheel in Fig. 30.3 may be thought of as a network of many point masses, m_1, m_2, m_3 , etc., at distances r_1, r_2, r_3 , etc from the axis, as shown in Fig. 30.4. Each point turns on a circular path about the axis.

At angular speed ω , the speed of each point is given by $v = \omega r$ so

the speed of m_1 is ωr_1 ,

the speed of m_2 is ωr_2

the speed of m_3 is ωr_3 etc.

When the body speeds up, every point in it accelerates. If the angular acceleration of the body is α , then the acceleration of each point mass is given by $a = \alpha r$ so

the acceleration of m_1 is αr_1 ,

the acceleration of m_2 is αr_2

the acceleration of m_3 is αr_3 etc.

Using $F = ma$, the force needed to accelerate each point mass is therefore given by

$F_1 = m_1 \alpha r_1$ for m_1 ,

$F_2 = m_2 \alpha r_2$ for m_2

$F_3 = m_3 \alpha r_3$ for m_3 etc.

The moment needed for each point mass to be given angular acceleration α is given by force $F \times$ distance r since the moment = force $\times$ perpendicular distance from the point mass to the axis. Thus

the moment for $m_1 = (m_1 \alpha r_1) r_1$,

the moment for $m_2 = (m_2 \alpha r_2) r_2$

the moment for $m_3 = (m_3 \alpha r_3) r_3$ etc.

The total moment (torque T) needed to give the body angular acceleration α = the sum of the individual moments needed for all the point masses.

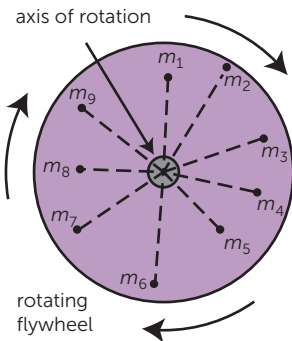


Fig. 30.4 A network of masses

Therefore, if there are N point masses in total,

$$\begin{aligned} \text{the torque } T &= (m_1 r_1^2) \alpha + (m_2 r_2^2) \alpha + (m_3 r_3^2) \alpha + \dots + (m_N r_N^2) \alpha \\ &= [(m_1 r_1^2) + (m_2 r_2^2) + (m_3 r_3^2) + \dots + (m_N r_N^2)] \alpha \\ &= I \alpha \end{aligned}$$

where $I = [(m_1 r_1^2) + (m_2 r_2^2) + (m_3 r_3^2) + \dots + (m_N r_N^2)]$ is the **moment of inertia** of the body about the axis of rotation.

The summation $(m_1 r_1^2) + (m_2 r_2^2) + (m_3 r_3^2) + \dots + (m_N r_N^2)$ is written in 'short form' as $\Sigma m r^2$ (pronounced 'sigma m r squared')

The moment of inertia I of a body about a given axis is defined as $\Sigma m r^2$ for all the points in the body, where m is the mass of each point and r is its perpendicular distance from the axis.

The unit of I is kg m^2 .

In general, T is the resultant torque. For example, if a torque T_1 is applied to a flywheel which is also acted on by a frictional torque T_2 , the resultant torque is $T_1 - T_2$.

Therefore when a body undergoes angular acceleration α , the resultant torque T acting on it is given by

$$T = I \alpha$$

30.2 WORKED EXAMPLE

A flywheel of moment of inertia 0.55 kg m^2 is accelerated uniformly from rest to an angular speed of 5.7 rad s^{-1} in 4.9 s . Calculate the resultant torque acting on the flywheel during this time.

Solution

$$\text{Angular acceleration } \alpha = \frac{(\omega_2 - \omega_1)}{t} = \frac{5.7 - 0}{4.9} = 1.16 \text{ rad s}^{-2}$$

$$\text{Resultant torque } T = I \alpha = 0.55 \times 1.16 = 0.64 \text{ N m}$$

Moment of inertia and angular acceleration

When a resultant torque is applied to a body, the angular acceleration, α , of the body is given by $\alpha = T/I$. Thus the angular acceleration depends not just on the torque T but also on the moment of inertia I of the body about the given axis which is determined by the distribution of mass about the axis.

Two bodies of equal mass distributed in different ways will have different values of I . For example, compare a bicycle wheel and a disc of the same mass, as in Fig. 30.5.

About the axis shown,

- the moment of inertia of the wheel is approximately MR^2 where M is its mass and R is its radius. This is because most of the mass of the wheel is at the same distance ($= R$) from the axis. Assuming the mass of the hub and the spokes is a very small proportion of the total mass, the moment of inertia I about the axis shown, $\Sigma m r^2$, is therefore MR^2 .

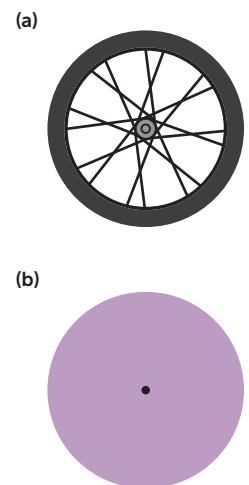


Fig 30.5 Distribution of mass comparison **a)** a bicycle wheel **b)** a uniform disc

- the moment of inertia of the disc about the same axis is less than MR^2 because the mass of the disc is distributed between the centre and the rim. Detailed theory shows that the value of I for the disc about the axis shown is $\frac{1}{2}MR^2$.

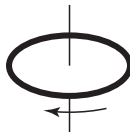
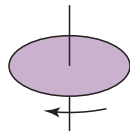
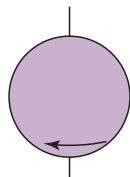
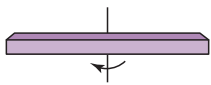
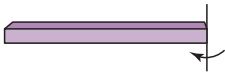
In general,

the further the mass is distributed from the axis, the greater is the moment of inertia about that axis.

The **moment of inertia** about a given axis of an object with a simple geometrical shape can be calculated using an appropriate mathematical formula for that shape and axis. In general terms, such formulas include geometrical factors as well as mass.

Table 30.1 gives the formula for the moment of inertia about a given axis for some different objects. The formulas are derived using the mathematical technique of integration to obtain $\sum mr^2$ for all points in the body. The emphasis in physics and engineering is on the use rather than the derivation of the formulas. By using the appropriate formula, the moment of inertia of a simple shape can be calculated from its dimensions and mass. Then, when subjected to torque, the effect on the motion can be determined.

Table 30.1

Object	Axis	Moment of inertia	Diagram
A thin hoop of mass M and radius R	through the centre of the hoop perpendicular to its plane	MR^2	
A uniform disc of mass M and radius R	through its centre perpendicular to its plane	$\frac{1}{2}MR^2$	
A sphere of mass M and radius R	through its centre	$\frac{2}{5}MR^2$	
a uniform beam of length L and mass M	through its centre perpendicular to its length	$\frac{1}{12}ML^2$	
a uniform beam of length L and mass M	through one end perpendicular to its length	$\frac{1}{3}ML^2$	

30.3 WORKED EXAMPLE

A solid circular disc of mass 28 kg and radius 0.12 m is rotating at an angular speed of 14 rad s^{-1} about an axis as shown in Fig. 30.6 when a 'braking' torque is applied to it which brings it to rest in 6.7 s. Calculate

- its angular deceleration when the braking torque is applied,
- the moment of inertia of the disc about the axis shown,
- the resultant torque that causes it to decelerate.

Moment of inertia of disc about the axis shown = $\frac{1}{2}MR^2$

Solution

- Angular acceleration $\alpha = \frac{(\omega_2 - \omega_1)}{t} = \frac{14 - 0}{6.7} = 2.1 \text{ rad s}^{-2}$
- $I = \frac{1}{2}MR^2 = 0.5 \times 28.0 \times 0.122 = 0.20 \text{ kg m}^2$
- Resultant torque $T = I\alpha = 0.20 \times 2.1 = 0.42 \text{ N m}$

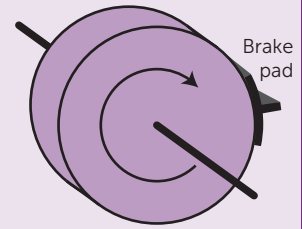


Fig. 30.6

30.2 QUESTIONS

- The rotating part of an electric fan has a moment of inertia of 0.78 kg m^2 . The rotating part is accelerated uniformly from rest to an angular speed of 4.7 rad s^{-1} in 9.6 s. Calculate the resultant torque acting on the fan during this time.
- A solid circular disc of mass 6.4 kg and radius 0.075 m is mounted on an axis as in Fig. 30.3. A force of 6.5 N is applied tangentially to the disc at its rim as shown in Fig. 30.3 to accelerate the disc from rest.
 - Show that (i) the moment of inertia of the disc about this axis is 0.018 kg m^2 , (ii) the torque applied to the disc is 0.49 Nm
 - The force is applied for 11.0 s. Calculate (i) the angular acceleration of the disc during this time, (ii) the angular speed of the disc at the end of this time, (iii) the number of turns made by the disc in this time.

Moment of inertia of disc about the axis shown = $\frac{1}{2}MR^2$

30-3 Rotational kinetic energy

To make an initially stationary body rotate about a fixed axis, it is necessary to apply a torque to the body. The torque does work on the body and gives the body kinetic energy. The greater the kinetic energy of the body, the faster it rotates.

The kinetic energy, E_K , of a body rotating at angular speed ω is given by

$$E_K = \frac{1}{2}I\omega^2$$

where I is its moment of inertia about the axis of rotation.

To prove this equation, consider the body as a network of point masses m_1, m_2, m_3, m_4 , etc. When the body rotates at angular speed ω , the speed of each point mass is given by $v = \omega r$ (see Fig. 30.4).

speed of $m_1 = \omega r_1$, where r_1 is the distance of m_1 from the axis,

speed of $m_2 = \omega r_2$, where r_2 is the distance of m_1 from the axis,

speed of $m_3 = \omega r_3$, where r_3 is the distance of m_3 from the axis, etc,

.....

speed of $m_N = \omega r_N$, where r_N is the distance of m_N from the axis.

Since the kinetic energy of each point mass is given by $\frac{1}{2}mv^2$, then

$$\text{kinetic energy of } m_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\omega^2r_1^2,$$

$$\text{kinetic energy of } m_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\omega^2r_2^2, \text{ etc}$$

$$\text{kinetic energy of } m_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}m_3\omega^2r_3^2, \text{ etc}$$

.....

$$\text{kinetic energy of } m_N = \frac{1}{2}m_Nv_N^2 = \frac{1}{2}m_N\omega^2r_N^2.$$

Hence the total kinetic energy

$$= \frac{1}{2}m_1\omega^2r_1^2 + \frac{1}{2}m_2\omega^2r_2^2 + \frac{1}{2}m_3\omega^2r_3^2 \dots + \frac{1}{2}m_N\omega^2r_N^2$$

$$= \frac{1}{2}[(m_1r_1^2) + (m_2r_2^2) + (m_3r_3^2) \dots + (m_Nr_N^2)]\omega^2$$

$$= \frac{1}{2}I\omega^2$$

$$\text{since } I = [(m_1r_1^2) + (m_2r_2^2) + (m_3r_3^2) \dots + (m_Nr_N^2)].$$

The equation $E_K = \frac{1}{2}I\omega^2$ enables us to calculate how much energy a rotating object 'stores' due to its rotational motion. In addition, it shows that the kinetic energy of a rotating object is proportional to

- its moment of inertia about the axis of rotation,
- the square of its angular speed.

About flywheels

A flywheel is used in a machine (or an engine) to keep the moving parts moving when the load on the machine increases and it has to do more work. For example, when a metal press is used to make a shaped object from a sheet of thin metal, the press is able to do the necessary work because the flywheel keeps it moving.

Flywheels are also fitted in some vehicles to store kinetic energy when the vehicle brakes are applied and it slows down. Instead of energy being transferred as heat to the surroundings, some of the vehicle's kinetic energy is transferred to an 'on-board' flywheel – to be returned to vehicle when the accelerator pedal is pressed.

The greater the moment of inertia of a flywheel, the more kinetic energy it can store for a given angular speed. The moment of inertia depends on the distribution of mass about the axis. Fig. 30.7 shows the cross-section of two flywheels X and Y which have the same mass and radius. The outer part of flywheel X is thicker than the inner part whereas flywheel Y has a uniform thickness. X has a greater moment of inertia than Y because its mass is distributed on average further from the axis of rotation than that of Y.

Work done

Work must be done to make an object that is initially at rest rotate. In Fig. 30.8, a disc is turned by pulling on a cord wrapped round the axle. Work is done by the force pulling the cord.

The work done by the force on the cord is given by the force $\times$ the distance moved by the force. To turn the disc through an angle θ in radians, the cord must be pulled through a distance $s = r\theta$ where r is the radius of the angle. Since the work done, W , by a force, F , pulling the cord is equal to Fs ,

$$W = Fs = Fr\theta = T\theta$$

where $T = Fr$ = the torque acting on the disc

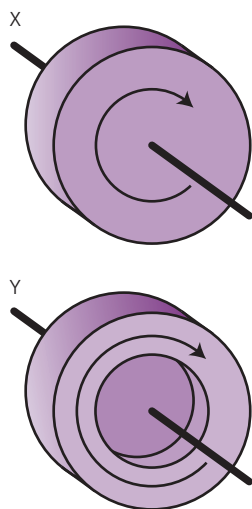


Fig. 30.7 Comparing two flywheels

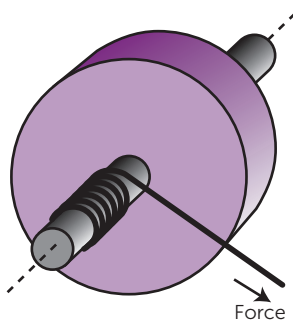


Fig. 30.8 Work done by a torque

When a body is turned through angle θ by a constant torque T ,

the work done by the torque, $W = T\theta$

Assuming there is no frictional torque, the applied torque $T = I\alpha$, where α is the angular acceleration of the disc.

Therefore, the work done W by the applied torque is given by

$$W = T\theta = (I\alpha)\theta$$

From the dynamics equations for angular motion, $\omega^2 = \omega_0^2 + 2\alpha\theta$ where ω_0 is the initial angular speed of the disc and ω is the angular speed after the disc has turned through angle θ . Rearranging this equation gives

$$\alpha\theta = \frac{1}{2}\omega^2 - \frac{1}{2}\omega_0^2$$

Therefore

$$\begin{aligned} W &= (I\alpha)\theta = I\left(\frac{1}{2}\omega^2 - \frac{1}{2}\omega_0^2\right) \\ &= \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 \\ &= \text{the gain of kinetic energy.} \end{aligned}$$

Therefore, in the absence of friction, the work done by the torque is equal to the gain of rotational kinetic energy of the body.

Power

For a body rotating at constant angular speed ω , the power P delivered by a torque T acting on the body is given by

$$P = T\omega$$

To see how this equation arises, consider a constant torque T acting on a body for a time t .

If the body turns through an angle θ in this time, the work done W by the torque is given by $W = T\theta$.

Since the power P delivered by the torque is the rate of work done by the torque, then $P = W/t$.

$$\text{Hence } P = \frac{W}{t} = \frac{T\theta}{t} = T\omega$$

where the angular speed $\omega = \theta/t$.

If a rotating body is acted on by an applied torque T_1 which is equally opposed by a frictional torque T_F , the resultant torque is zero so its angular speed ω is constant.

The power P delivered by the applied torque T_1

$$\begin{aligned} &= \frac{\text{work done, } W}{\text{time taken, } t} \\ &= \frac{T_1\theta}{t} = T_1\omega \end{aligned}$$

The work done per second by the frictional torque

$$T_F = \frac{T_F\theta}{t} = T_F\omega$$

In this situation, the rate of transfer of energy by the applied force is equal to the rate of transfer of energy to the surroundings by the frictional force. So the rotating body does not gain any kinetic energy.

30.4 WORKED EXAMPLE

A flywheel is rotating at an angular speed of 310 rad s^{-1} on a fixed axle which is mounted on frictionless bearings. The moment of inertia of the flywheel and the axle about the axis of rotation is 0.073 kg m^2 .

- Calculate the rotational kinetic energy of the flywheel when it rotates at 310 rad s^{-1} .
- When a braking torque of 1.4 Nm is applied to its rim, the flywheel is brought to rest. Calculate the number of turns the flywheel makes as it decelerates to a standstill.

Solution

a) $E_K = \frac{1}{2}I\omega^2 = 0.5 \times 0.073 \times 310^2 = 3500 \text{ J}$

- b) The work done by the braking torque = loss of kinetic energy ΔE_K

Therefore $T\theta = \Delta E_K$ hence $1.4\theta = 3500$

Hence $\theta = \frac{3500}{1.4} = 2500 \text{ rad} = \frac{2500}{2\pi} = 400 \text{ turns}$

Experiment to measure the moment of inertia I of a flywheel

A weight attached to a cord is to be used to make the flywheel rotate from rest as shown in Fig. 30.9. The length of the cord should be such that the cord is no longer wrapped round the axle when the weight reaches the floor.

Use the smallest possible weight, W_0 , that will make the flywheel rotate at constant angular speed. Record this amount of weight as it provides just enough torque to overcome friction on the flywheel at its bearings.

Add a further weight W_1 to the weight hanger so that the flywheel accelerates when released. The following measurements then need to be made.

- the distance, h , fallen by the weights from release to when it reaches the floor,
- the diameter, d , of the axle,
- the time taken, t , for the weights to drop to the floor,
- the number of turns, n , made by the flywheel after the weight reaches the floor

These measurements are used in the following calculations:

- The number of turns made by the flywheel as the string unwraps, $N = h/\pi d$.
- The maximum angular speed of the flywheel, $\omega = 2 \times$ the average angular speed $= 2 \times (2\pi N)/t$.
- The speed of the object at the instant the cord detaches, $v = \omega d/2$.
- The kinetic energy gained by the flywheel, $\Delta E_{KF} = \frac{1}{2}I\omega^2$, to be calculated in terms of I .
- The kinetic energy gained by the weights, $\Delta E_{K0} = \frac{1}{2}Mv^2$ where $v = \omega d/2$ and M is the mass of the total weight supported by the cord.
- The potential energy lost by the weight W_1 , $\Delta E_P = W_1 h$.

Assuming the energy loss due to friction at the flywheel bearings is equal to the potential energy lost by the weight W_0 , the potential energy lost by weight $W_1 =$ the kinetic energy gained by the flywheel + the kinetic energy gained by the falling weights,

$$W_1 h = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

The moment of inertia of the flywheel can therefore be calculated from the equation above.

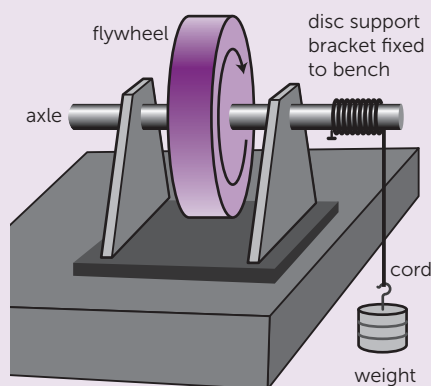


Figure 30.9 Measuring the moment of inertia of a rotating disc

30.3 QUESTIONS

- 1 A flywheel is rotating at an angular speed of 8.0 rad s^{-1} on a fixed axle which is mounted on frictionless bearings. The moment of inertia of the flywheel and the axle about the axis of rotation is 0.053 kg m^2 . Calculate
 - a) the rotational kinetic energy of the flywheel when it rotates at 8.0 rad s^{-1} ,
 - b) (i) the torque needed to accelerate the flywheel from rest to an angular speed of 8.0 rad s^{-1} in 15 s, (ii) the angle in radians which the flywheel turns through in this time while it is being accelerated.
- 2 In an experiment to measure the moment of inertia of a flywheel, a 0.8 N weight on a cord was used as shown in Fig 30.9 to make the flywheel rotate at constant angular speed. When the test was repeated with a 6.4 N weight added to the weight hanger, the weights accelerated the flywheel from rest and fell to the floor through a vertical distance of 0.92 m in 4.5 s .
 - a) Calculate (i) the potential energy lost by the 6.4 N weight in descending 0.91 m , (ii) the speed and the kinetic energy of the weights just before they reached the floor.
 - b) The axle diameter was 8.5 mm . (i) Show that the flywheel reached an angular speed of 96 rad s^{-1} just before the weights reached the floor. (ii) Show that the kinetic energy of the flywheel was 5.8 J just before the weights reached the floor. (iii) Hence calculate the moment of inertia of the flywheel.

30.4 Angular momentum

An ice skater spinning rapidly is a dramatic sight. The skater turns slowly at first, then quite suddenly goes into a rapid spin. This sudden change is brought about by the skater pulling both arms (and possibly a leg!) towards the axis of rotation. In this way the moment of inertia of the skater about the axis is reduced. As a result, the skater spins faster. To slow down, the skater only needs to stretch his or her arms and maybe a leg. In this way, the moment of inertia is increased. So the skater slows down.

To understand such effects, consider a rotating body with no resultant torque on it. Provided its moment of inertia stays the same, then its angular speed ω does not change. This can be seen by considering the equation $T = I\alpha$ where α is the angular acceleration. If the resultant torque T is zero, then the angular acceleration α is zero so the angular speed is constant.

In more general terms, the equation $T = I\alpha$ may be written as

$$T = \frac{d(I\omega)}{dt}$$

where d/dt is the formal mathematical way of writing 'change per unit time'.

The quantity $I\omega$ is the **angular momentum** of the rotating body. So the above equation tells us that resultant torque T is the rate of change of angular momentum of the rotating object.

Angular momentum of a rotating object $= I\omega$

where I is the moment of inertia of the body about the axis of rotation and ω is its angular speed.

The unit of angular momentum is $\text{kg m}^2 \text{ rad s}^{-1}$ or N m s (i.e. the unit of torque $\times$ the unit of time).

- If the resultant torque on a rotating body is zero, then

$$\frac{d(I\omega)}{dt} = 0$$

which means its angular momentum $I\omega$ is constant. In other words, the angular momentum is conserved.



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Figure 30.10 A spinning ice skater

In the ice skater example, the moment of inertia of the ice skater suddenly decreases when he or she pulls the arms in. Since the angular momentum is constant, the sudden decrease in the moment of inertia causes the angular speed to increase. In specific terms, if the moment of inertia changes from I_1 to I_2 , the angular speed will change from ω_1 to ω_2 such that $I_1\omega_1 = I_2\omega_2$.

- If the resultant torque is constant and the object's moment of inertia I does not change, the object undergoes uniform angular acceleration. If it accelerates from rest to angular speed ω in time t , the resultant torque acting on it $T = I\omega/t$. Hence at time t , its angular momentum $I\omega = Tt$.

Notes

The angular momentum of a particle moving about a fixed point is defined as its momentum $\times$ its distance from the fixed point.

- For a small mass m rotating at angular speed ω at distance r from the axis, its momentum is $m\omega r$ (as its speed $v = \omega r$) so its angular momentum is $m\omega r^2$.
- For a network of small masses $m_1, m_2, \dots, m_N$ which make up a rigid body, the total angular momentum = $(m_1r_1^2\omega) + (m_2r_2^2\omega) + \dots + (m_Nr_N^2\omega)$

$$= [(m_1r_1^2) + (m_2r_2^2) + \dots + (m_Nr_N^2)]\omega = I\omega$$

Conservation of angular momentum

In the spinning ice skater example, the angular momentum of the ice skater is constant because the resultant torque is zero. In other words, the angular momentum is conserved.

Where a system is made up of more than one spinning body and two of the bodies interact (e.g. collide), one will lose angular momentum to the other. **If the resultant torque on the system is zero, the total amount of angular momentum must stay the same so is conserved.** This is known as the principle of conservation of angular momentum.

Further examples of angular momentum

- 1 **When a spinning satellite is taken on board a space station for repair, rocket motors are used to prevent the space station from spinning.** Otherwise, some of the angular momentum of the satellite would be transferred to the space station when the satellite is stopped.

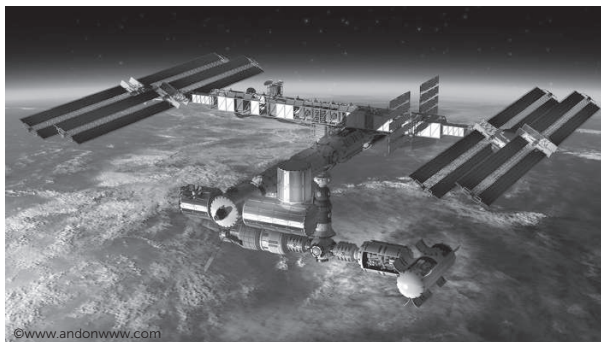


Fig. 30.11 A space station

In this example

- the total angular momentum before the satellite is taken on board = $I_1\omega_1$, where I_1 is the moment of inertia of the satellite and ω_1 is its initial angular speed,
- the total angular momentum after the satellite has been taken on board and stopped = $(I_1 + I_2)\omega_2$, where ω_2 is the final angular speed and $(I_1 + I_2)$ is the total moment of inertia about the axis of rotation of the satellite where I_2 is the moment of inertia of the space lab.

According to the conservation of momentum,

$$(I_1 + I_2)\omega_2 = I_1\omega_1$$

Rearranging this equation gives

$$I_2\omega_2 = I_1\omega_1 - I_1\omega_2$$

This rearranged equation shows that the angular momentum gained by the space lab is equal to the angular momentum lost by the satellite.

- 2 **Pulsars are rapidly spinning stars.** They were first discovered by astronomers in 1967. Regular pulses of radio energy were detected from these stars. Such stars are rapidly rotating neutron stars which emit radio energy in a beam at an angle to the axis. Each time the beam sweeps round to point towards Earth, radio energy is directed towards us, rather like a light beam from a lighthouse.

Neutron stars are the remnants of large stars. When a massive star runs out of fuel, a huge explosion takes place. The remnants of the explosion are pulled in together by their gravitational attraction, perhaps equivalent to a mass equal to the Sun shrinking to only 15 km or so in diameter. The moment of inertia is therefore made much smaller so the angular speed increases. The pulse frequency from a pulsar is of the order of 1 to 10 Hz, so the rate of rotation is of that order, much much greater than the Sun's rate of rotation which is about once every 25 days!

Experiment to measure the moment of inertia of a freely-rotating turntable

If a small object dropped onto a freely-rotating turntable, as shown in Fig. 30.12, so that it sticks to the turntable, the object gains angular momentum and the turntable loses angular momentum.

The total angular momentum of the turntable before the object is dropped onto it = $I_1\omega_1$, where I_1 is the moment of inertia of the turntable and ω_1 is its initial angular speed.

The total angular momentum after the object has been dropped onto the turntable = $(I_1 + I_2)\omega_2$, where ω_2 is the final angular speed and $(I_1 + I_2)$ is the total moment of inertia about the axis of rotation of the turntable.

If the mass of the object is m and its perpendicular distance from the axis of rotation of the turntable is r , then the moment of inertia I_2 of the object about the axis of rotation = mr^2 .

According to the conservation of momentum,

$$(I_1 + mr^2)\omega_2 = I_1\omega_1$$

Rearranging this equation gives

$$I_1 = \frac{mr^2\omega_2}{\omega_1 - \omega_2}$$

Hence I_1 can be found by measuring m , r , ω_1 and ω_2 .

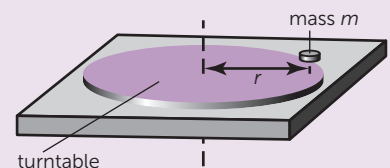


Fig. 30.12 Measuring the moment of inertia of a freely-rotating turntable

Comparison of linear and rotational motion

When analysing a rotational dynamics situation, it is sometimes useful to compare the situation with an equivalent linear situation. For example, the linear equivalent of a torque T used to change the angular speed of flywheel is a force F used to change the speed of an object moving along a straight line. If the change takes place in time t

- the change of momentum of the object $= Ft$
- the change of angular momentum of the flywheel $= Tt$

Table 30.2 summarises the comparison between linear and rotational motion.

Table 30.2 Comparison between linear and rotational motion	
Linear motion	Rotational motion
displacement s	angular displacement θ
speed and velocity v	angular speed ω
acceleration a	angular acceleration α
mass m	moment of inertia I
momentum mv	angular momentum $I\omega$
force F $F = ma$ $F = \frac{d}{dt}(mv)$	torque $T = Fd$ $T = I\alpha$ $T = \frac{d}{dt}(I\omega)$
kinetic energy $\frac{1}{2}mv^2$	kinetic energy $\frac{1}{2}I\omega^2$
work done Fs	work done $T\theta$
power Fv	power $T\omega$

30.4 QUESTIONS

- A vehicle wheel has a moment of inertia of $6.5 \times 10^{-2} \text{ kg m}^2$ and a radius of 0.32 m.
 - Calculate the angular momentum of the wheel when the vehicle is travelling at a speed of 30 m s^{-1} .
 - When the brakes are applied, the vehicle speed decreases from 30 m s^{-1} to zero in 13 s. Calculate the resultant torque on the wheel during this time.
- A metal disc X on the end of an axle rotates freely at 720 revolutions per minute. The moment of inertia of the disc and the axle is 0.064 kg m^2 .
 - Calculate the angular momentum of the disc and the axle.
 - After a second disc Y that is initially stationary is engaged by X, both discs rotate at 520 revolutions per second. Calculate the moment of inertia of Y.
 - Show that the total loss of kinetic energy is 1.3 J.

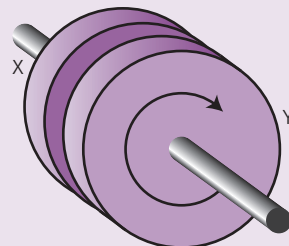


Fig. 30.13

SUMMARY

Definitions

- **Torque** = force $\times$ perpendicular distance from the axis to the line of action of the force
- **The moment of inertia I of a body about a given axis = Σmr^2 for all the points in the body**, where m_i is the mass of each point and r_i is its perpendicular distance from the axis.
- **The Principle of Conservation of Angular Momentum** states that if the resultant torque on the system is zero, the total amount of angular momentum is conserved. (i.e. does not change).

Equations

- **Angular speed** $\omega = \frac{2\pi}{T}$
- **Angular acceleration** $\alpha = \frac{(\omega_2 - \omega_1)}{t}$
- **Angular dynamics equations**

$$\omega = \omega_0 + \alpha t$$

$$\theta = \frac{1}{2}(\omega + \omega_0)t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$
- **Resultant torque** $T = I\alpha$
- **Work done by the torque** $W = T\theta$
- **Power delivered by a torque** $= T\omega$
- **Angular momentum** $= I\omega$

REVISION QUESTIONS

See Further questions (p. 482) for more questions on this unit.

30.1 A child's spinning top spinning at a frequency of 24 Hz decelerated uniformly to rest in 29 s. Calculate

- its initial angular speed,
- its angular deceleration,
- the number of turns it made when it decelerated.

30.2 Fig. 30.14 shows cross-sections of two discs X and Y which have the same mass and radius. State and explain which disc has the greater moment of inertia about the axis.

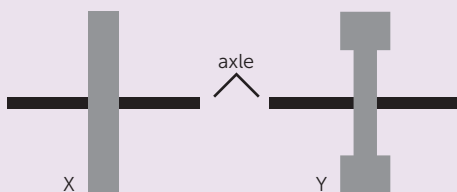


Fig. 30.14

30.3 A flywheel is accelerated by a constant torque for 15 s from rest. During this time it makes 36 turns. It then slows down to a standstill 71 s after the torque is removed, making 56 turns during this time.

- (i) Show that in the time it accelerated, its angular acceleration was 2.0 rad s^{-2} .
(ii) Show that in the time it slowed down, its angular deceleration was 0.14 rad s^{-2} .
- The torque applied to it when it accelerated was 28 Nm.
(i) Calculate the moment of inertia of the flywheel.
(ii) Show that the frictional torque that slowed it down was 1.8 Nm.

30.4 A ball released at the top of a slope rolls down the slope and continues on a flat horizontal surface until it stops. Discuss the energy changes of the ball from the moment it is released to when it stops.

30.5 A frictionless turntable is set rotating at a steady angular speed of 15 rad s^{-1} . A small 0.18 kg mass is dropped onto the disc from rest just above it, at a distance of 0.22 m from the centre of the disc. As a result, the angular speed of the turntable decreases to 11 rad s^{-1} . Calculate the moment of inertia of the turntable about its axis of rotation.

Further Questions

(Numerical answers only are supplied on p. 534)

Unit 1

Q1.1

- a) Calculate the components parallel and perpendicular to a horizontal line from north to south of a horizontal force of magnitude 8 N acting
 - (i) 30° E of due north,
 - (ii) 70° W of due north.
- b) Calculate the magnitude and direction of the resultant force on a point object due to a force of 10 N acting due North and a force of 8 N acting
 - (i) due South,
 - (ii) due East,
 - (iii) 60° E of due North.

Q1.2 Calculate the force necessary to maintain a point object of weight 6.0 N in equilibrium when it is also acted on by

- a) an upward vertical force of magnitude 5.0 N,
- b) a horizontal force of magnitude 5.0 N.

Q1.3 A uniform wooden plank of length 1.60 m and weight 80 N rests horizontally on two bricks X and Y, where X is at a distance of 0.40 m from one end and Y is at a distance of 0.20 m from the other end.

- a) Sketch a free body diagram showing the forces acting on the plank.
- b) Calculate the support force acting on the plank due to each brick.
- c) A child of weight 200 N stands on the plank at its midpoint and walks along the plank towards the end beyond X. Calculate the distance of the child from the mid-point at which the plank lifts off the brick at Y.

Q1.4

- a) A garden rake consists of a 8.0 N handle of length 1.50 m and a metal 'comb' of weight 12.0 N and of width 0.30 m attached to the handle at its midpoint. Calculate the distance from the free end of the rake to its centre of gravity.
- b) A wheelbarrow is designed so that its handles are 1.2 m from its wheel base. The wheelbarrow is loaded with a 280 N bag of sand which is placed 0.30 m from its wheel base. Sketch the arrangement and calculate the force that must be applied to its handles to raise it.

Unit 2

$$g = 9.8 \text{ m s}^{-2}$$

Q2.1 A vehicle was travelling at 25 m s^{-1} when the driver saw traffic lights, 65 m away, turn red. The driver applied the brakes and the car decelerated uniformly to stop at the lights.

- a) The driver's reaction time before applying the brakes was 0.6 s. Calculate the distance travelled by the car in this time moving at 25 m s^{-1} .
- b) Calculate the deceleration of the vehicle during the period when the brakes were applied.

Q2.2 A train travelling between two stations accelerated from rest to a speed of 15 m s^{-1} for 75 s then moved at constant speed for 240 s before decelerating to a standstill in 60 s.

- a) Sketch a speed versus time graph for this journey.
- b) Calculate
 - (i) the acceleration,
 - (ii) the distance moved, in each of the three parts of the journey.
- c) Calculate the distance apart of the two stations and the average speed of the train for the journey between the two stations.

Q2.3 A stone released from rest at the top of a well hits the water in the well 1.4 s later.

- a) Calculate
 - (i) the speed,
 - (ii) the distance fallen by the stone just before impact with the water surface.
- b) Sketch a speed–time graph to represent the motion of the stone from when it was released to when it hit the water surface.

Q2.4 A tennis ball is thrown vertically upwards and returns to the thrower 3.2 s later.

- a) Calculate
 - (i) the time it takes to reach maximum height,
 - (ii) the maximum height reached,
 - (iii) its speed of projection,
 - (iv) its speed just before returning to the thrower.
- b) Sketch a velocity–time graph to represent the motion of the tennis ball.

Unit 3

$$g = 9.8 \text{ m s}^{-2}$$

Q3.1

- a) An object of mass 5.0 kg, initially at rest, is acted on by a resultant force of 15 N for 10 s. Calculate
- its acceleration,
 - its speed after 10 s,
 - its momentum after 10 s.
- b) A ball of mass 0.15 kg, moving at right angles to a vertical wall at a speed of 14 m s^{-1} , rebounds from the wall at a speed of 10 m s^{-1} .
- Calculate the change of momentum of the ball.
 - The ball was in contact with the wall for 25 ms. Calculate the force of impact of the ball on the wall.

Q3.2

- a) A brick of mass 4.7 kg, dropped from rest at a height of 8.4 m, falls onto a bed of sand, causing an indentation in the sand of 42 mm. Calculate
- the speed of the brick just before it hits the sand,
 - its deceleration in the sand,
 - the impact force.
- b) A lorry of total mass 38 000 kg, travelling at a speed of 31 m s^{-1} is brought to rest in a distance of 185 m when its brakes are applied. Calculate
- the deceleration of the lorry,
 - the braking force.

Q3.3

- a) Water emerges from a hosepipe at a speed of 11 m s^{-1} at a flow rate of 0.17 kg s^{-1} . Calculate
- the momentum per second carried away by the water from the hosepipe,
 - the force on the hosepipe due to this rate of loss of water.
- b) A cannon of mass 340 kg is designed to fire a cannon ball of mass 12.5 kg at a speed of 74 m s^{-1} from a barrel of length 1.35 m. Calculate
- the acceleration of the ball in the barrel,
 - the force on the ball,
 - the recoil velocity of the cannon.

- Q3.4** A rail wagon of mass 1200 kg, moving at a steady speed of 1.8 m s^{-1} , collides with another wagon of mass 900 kg and couples to it. Calculate the velocity of the two wagons immediately after the impact if the lighter wagon was initially
- stationary,
 - moving at a speed of 2.5 m s^{-1} in the opposite direction to the other wagon.

Unit 4

$$g = 9.8 \text{ m s}^{-2}$$

Q12.1

- a) Calculate the change of potential energy and kinetic energy of a ball of mass 0.30 kg, released from rest at a height of 2.4 m above the floor, when it is
- 1.2 m above the floor,
 - just above the floor.

- b) A ball of mass 0.080 kg, released from a height of 1.50 m above a tiled floor, rebounds to a height of 1.20 m above the floor. Calculate
- its kinetic energy and speed immediately before the impact,
 - its kinetic energy and speed immediately after the impact,
 - its loss of kinetic energy due to the impact.

Q4.2 A roller coaster car of total mass (including passengers) 2500 kg descends from a height of 55 m along a steep incline onto a level section of track.

- Calculate its loss of potential energy due to this descent.
- Its speed before descent was 3 m s^{-1} and its speed immediately after the descent was 25 m s^{-1} . Calculate its gain of kinetic energy as a result of the descent.
- Calculate the loss of total energy of the roller coaster in this descent.

Q4.3

- a) Estimate the average power from a tidal power station operating at an efficiency of 0.40, which traps sea water over a level area of 15 m^2 when high tide is 3.0 m above low tide. The density of sea water = 1050 kg m^{-3} .
- b) A certain type of fuel is capable of releasing 30 MJ of energy per kilogram. A vehicle travelling at a steady speed of 25 m s^{-1} uses this type of fuel at a rate of 12 km per kilogram. Estimate the input power from the fuel to the engine.

Q4.4 A vehicle of mass 1100 kg, moving at a speed of 20 m s^{-1} , collided with a stationary vehicle of mass 900 kg. The two vehicles locked together as a result of the impact.

- Calculate

 - the speed of the two vehicles immediately after the impact,
 - the loss of kinetic energy due to the impact.

- The impact caused the two vehicles to shorten by a combined length of 1.24 m. Calculate the loss of kinetic energy per metre during the impact and hence determine the impact force.

Unit 5

$$\text{The Avogadro constant} = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Q5.1 The relative atomic masses of hydrogen, carbon and oxygen are 1, 12 and 16 respectively.

- Calculate the molar mass of

 - water, H_2O ,
 - carbon dioxide, CO_2 .

- (i) Calculate the mass of a water molecule.
(ii) Estimate the linear size of a water molecule. The density of water = 1000 kg m^{-3} .

Q5.2

- What type of bond holds the atoms together in

 - a molecule of carbon dioxide,
 - a sodium chloride crystal?

- b) (i) Describe how the arrangement of molecules in a solid differs from the arrangement in a liquid.
 (ii) When a pure substance melts, its temperature does not change. Explain why thermal energy must be supplied to melt it when there is no change of temperature.

Q5.3

- a) The isotopes of an element have different physical properties such as atomic mass.
 (i) What is meant by the phrase 'isotopes of an element'?
 (ii) Why do the isotopes of an element form identical bonds when they react with other elements?
 b) Every neon atom in neon gas contains 10 protons and either 10 or 12 neutrons.
 (i) Write down the symbol for each of these two isotopes of neon.
 (ii) The relative atomic mass of neon gas is 20.2. Calculate the proportion of each isotope in neon gas.

Q5.4 Solid aluminium has a density of 2700 kg m^{-3} and its relative atomic mass is 27.

- a) Calculate
 (i) the molar volume of solid aluminium,
 (ii) the number of atoms in a volume of 1.0 cm^3 of aluminium.
 b) Estimate the distance between the centres of adjacent aluminium atoms in solid aluminium.

Unit 6**Q6.1**

- a) Calculate the expansion of a brass bar of length 2.50 m when its temperature is raised from 20°C to 50°C .
 b) An elevator in a vertical shaft is supported by several steel cables, each of length 200 m. Calculate the extension of each cable due to a 20 K change of temperature. Values of linear expansivity: brass 1.9×10^{-5} , steel 1.1×10^{-5} .

Q6.2

- a) 52 kg of water at 15°C in an insulated copper tank of mass 15 kg is heated by a 4.0 kW immersion heater for 30 minutes. Calculate
 (i) the electrical energy supplied in this time,
 (ii) the final temperature of the water, assuming no heat loss occurs.
 b) Estimate the length of time taken to boil away 0.5 kg of water in a 3.0 kW electric kettle.
 Specific heat capacities: water $4200 \text{ J kg}^{-1} \text{ K}^{-1}$, copper $390 \text{ J kg}^{-1} \text{ K}^{-1}$. Specific latent heat of vaporisation of water = 2.3 MJ kg^{-1} .

Q6.3

- a) Describe two methods of reducing heat loss from a house in winter. For each method, state which physical process is responsible for this reduction.
 b) Calculate the heat loss per second through a glass window pane of area 0.80 m^2 and thickness 6.0 mm when the temperature difference across the window pane is 8.0°C . The thermal conductivity of glass = $0.12 \text{ W m}^{-2} \text{ K}^{-1}$.

Q6.4

- a) Use Wien's Law to estimate the temperature of the light-emitting surface of the Sun, assuming the wavelength of light from the Sun at peak intensity is 500 nm.
 b) (i) The intensity of solar radiation at the Earth, which is $1.5 \times 10^{11} \text{ m}$ from the Sun, is 1.4 kW m^{-2} . Use this data to estimate the energy per second radiated from the Sun.
 (ii) The Sun's diameter is $1.4 \times 10^9 \text{ m}$. Use this data and your estimate in (a) of the Sun's temperature to calculate the emissivity of the Sun. The Stefan constant = $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

Unit 7**Q7.1**

- a) Sketch curves to show how the extension varies with tension for
 (i) a steel spring when it is stretched,
 (ii) a rubber band when it is stretched.
 b) A vertical steel spring is fixed at its upper end and supports a weight of 2.0 N at its lower end. When an extra 1.0 N weight is added to its lower end, its length increases from 405 mm to 445 mm.
 (i) Calculate its length when it is unloaded.
 (ii) The spring is unloaded and an object of unknown weight is hung from its lower end, causing its length to increase to 500 mm. Calculate the weight of this object.

Q7.2

- a) What is meant by the breaking stress of a material?
 b) A wire of diameter 0.25 mm snapped when the tension in it reached 49 N.
 (i) Calculate the breaking stress of the material of this wire.
 (ii) Calculate the tension at which a wire of identical material of diameter 0.36 mm would snap.

Q7.3 A vertical steel wire of diameter 0.28 mm and of length 1.8 m is fixed at its upper end.

- a) Calculate the stress in the wire when it supports a 20 N weight.
 b) Calculate the increase in the length of this wire if a weight of 50 N is attached to its lower end. The Young modulus of steel = $2.0 \times 10^{11} \text{ Pa}$.

Q7.4 A wire of diameter 0.26 mm and of length 5.00 m extends by 30 mm when the tension in it is increased from 0 to 40 N.

- a) Calculate the Young modulus of the metal.
 b) Calculate the energy stored at a tension of 40 N.

Unit 8**Q8.1**

- a) Calculate the pressure at the base of an upright concrete post of weight 400 N and of dimensions $1.6 \text{ m} \times 0.10 \text{ m} \times 0.10 \text{ m}$.
 b) Calculate the pressure the post in (a) would exert on level ground if it was laid flat.

Q8.2

- a) Calculate the pressure due to sea water of density 1050 kg m^{-3} at a depth of 5.0 m below the surface.
 $g = 9.8 \text{ m s}^{-2}$
- b) Explain why a deep sea diver could not inhale air from the surface via a long tube.

Q8.3

- a) Explain why a small amount of air in a vehicle brake system reduces the effectiveness of the brakes.
- b) In a hydraulic brake system, a force of 200 N is applied to the master piston which has a cross-sectional area of 4.0 cm^2 .
- (i) Calculate the pressure, in pascals, on the brake fluid in the piston due to this force.
- (ii) Each slave piston has a cross-sectional area of 60 cm^2 . Calculate the force exerted by each slave piston.

Q8.4 A flat barge floats with its deck 1.20 m above the water when unloaded. When it is loaded with material of total weight 18 kN, it floats 50 mm lower in the water.

- a) Calculate the pressure due to 50 mm of water. The density of water = 1000 kg m^{-3} . $g = 9.8 \text{ m s}^{-2}$.
- b) Estimate the area of the barge hull.
- c) If the displacement of the boat is not allowed to exceed 200 mm, calculate the maximum load it can carry.

Unit 9
Q9.1

- a) List the following types of electromagnetic waves in order of increasing frequency.
- | | |
|---------------------|------------|
| infra-red radiation | microwaves |
| visible light | X-rays |
- b) The speed of electromagnetic waves in free space is $3.00 \times 10^8 \text{ m s}^{-1}$. Calculate
- (i) the frequency of light of wavelength 580 nm,
- (ii) the wavelength of radio waves of frequency 95 MHz.

Q9.2

- a) (i) State which of the following types of waves are transverse waves.
- | | |
|-----------------------|-------------|
| electromagnetic waves | sound waves |
| waves on a string | water waves |
- (ii) Which one of the following properties is shown by transverse waves and not by longitudinal waves?
- | | | | |
|-------------|--------------|------------|------------|
| diffraction | polarisation | refraction | reflection |
|-------------|--------------|------------|------------|
- b) Sound waves of frequency 500 Hz travel through a substance at a speed of 1500 m s^{-1} .
- (i) Calculate the wavelength of these waves.
- (ii) Calculate the phase difference between two points that lie along the direction of propagation of the wave at a distance of 1.0 m apart.

Q9.3

- a) Explain why the wider a satellite dish is
- (i) the stronger the signal it is capable of collecting,
- (ii) the more difficult it is to align it to obtain maximum signal strength.

- b) Straight waves on the surface of a pool cross a straight boundary from shallow to deep water at an angle of 45° to the boundary.
- (i) Water waves travel faster in deep water than in shallow water. What change occurs to the wavelength of the water waves on crossing the boundary?
- (ii) Sketch a top view of a wavefront as it is crossing the boundary, indicating its direction of propagation on each side of the boundary.

Q9.4

- a) Describe with the aid of a diagram what is meant by a polarised wave on a vibrating string.
- b) Describe an experimental test that could be carried out using two polaroid squares to find out if light from a point light source is polarised.

Unit 10
Q10.1

- a) (i) State the approximate frequency range of the normal human ear.
- (ii) State an approximate value of frequency of sound which the ear is most sensitive to.
- b) A motor cycle produces a maximum loudness level of 80 dB as it passes by a roadside monitor. Calculate the loudness level produced by four such motor cycles passing by the same roadside monitor.

Q10.2

- a) With the aid of a diagram, describe and explain the principle of operation of an ultrasonic transducer.
- b) An ultrasonic transducer on the surface of a patient emits short pulses at a rate of 1000 Hz into the patient.
- (i) Calculate the time taken for an ultrasonic pulse travelling at a speed of 1500 m s^{-1} to travel from the transducer to the other side of the patient and back, a total distance of 0.60 m.
- (ii) Calculate the time between such a reflected pulse returning to the patient and the next pulse being transmitted.

Q10.3

- a) What change occurs in the note produced by a guitar string if the string
- (i) is slackened without change of length,
- (ii) is made shorter without change of tension?
- b) A string of length 1.80 m is made to vibrate in its fundamental mode at a frequency of 60 Hz.
- (i) Calculate the wavelength and speed of the waves on the string in this condition.
- (ii) The frequency is raised without changing the length or the tension of the string until two equally spaced nodes are observed between its ends. Sketch this pattern of vibration and calculate the frequency at which it occurs.

Q10.4 A pipe of length 0.70 m is closed at one end. A small speaker, connected to a variable frequency a.c. supply, is held near the open end of the pipe and the frequency is adjusted until the pipe resonates. The lowest frequency at which this occurs is 120 Hz.

- a) Explain why the pipe resonates at this frequency.
- b) (i) Estimate the speed of sound in the pipe.
(ii) What is the next highest frequency at which this pipe resonates?

Unit 11

Q11.1

- a) A light ray is directed at the flat surface of a glass block of refractive index 1.5 at an angle of incidence of
 - (i) 20° .
 - (ii) 50° .
 Calculate the angle of refraction in each case.
- b) (i) What is meant by total internal reflection of light?
(ii) Calculate the critical angle of glass of refractive index 1.5.
(iii) With the aid of a diagram, explain why light entering an optical fibre emerges from the other end even if the fibre is bent.

Q11.2

- a) In an experiment to demonstrate interference of light, a laser beam of wavelength 650 nm was directed at a pair of closely spaced slits. An interference pattern consisting of alternate bright and dark fringes was observed on a screen 1.5 m away from the slits, placed perpendicular to the direction of the beam.
 - (i) The distance across four fringe spaces was 6.0 mm. Calculate the fringe spacing.
 - (ii) Use your answer to (i) and the other data to calculate the slit spacing.
- b) The pair of slits was moved 0.5 m away from the screen. Calculate the distance across four fringe spaces for this arrangement.

Q11.3

- a) Determine the position and nature of the image of an object formed by a 150 mm focal length convex lens for an object distance of
 - (i) 100 mm,
 - (ii) 300 mm.
- b) A concave lens of focal length 0.20 m is used to view an object which is held 0.20 m from the lens. Calculate the position of the image formed and draw a ray diagram to show how the image is formed.

Q11.4

- a) Draw a ray diagram to show how a refracting telescope forms at infinity an image of a distant point object which is off the axis of the telescope.
- b) A telescope which has a magnifying power of X8 is used to observe two stars which are 0.5° apart. Calculate how far apart the two stars appear when viewed through the telescope.

- c) The objective lens of this telescope has a focal length of 0.60 m. What is the focal length of the eyepiece for a magnifying power of $\times 8$?

Unit 12

Q12.1

- a) (i) State the three primary colours in order of increasing wavelength.
(ii) State an approximate value of the wavelength of blue light.
- b) A parallel beam of white light directed into a glass prism emerges to form a spectrum on a white screen.
 - (i) What type of spectrum is formed?
 - (ii) Which colour of light on the screen is refracted least?

Q12.2 A parallel beam of monochromatic light is directed normally at a diffraction grating which has 600 lines per millimetre.

- a) Calculate the spacing between adjacent lines of this grating.
- b) Use the diffraction grating equation to calculate the angle of diffraction of the first order diffracted beam.
- c) Calculate the maximum order number.

Q12.3 A light source emits light of wavelength 430 nm and 550 nm. Parallel light from this source was directed at a diffraction grating which had 500 lines per millimetre.

- a) Calculate the angle between these two wavelengths in the second order spectrum.
- b) Calculate the maximum angle of diffraction and state which wavelength corresponds to this angle.

Q12.4

- a) State which type of electromagnetic radiation is used for each purpose below and which of its properties makes it suitable for that purpose.
 - (i) Communication from a satellite to the Earth.
 - (ii) Detection of movement of people and animals in darkness.
 - (iii) Image formation of a damaged bone.
- b) (i) Describe how amplitude modulation and frequency modulation differ.
(ii) Why is an infra-red signal capable of carrying more pulses per second than a microwave signal?

Unit 13

Q13.1 A plastic rod is charged by friction. When it is held over a negatively charged electroscope, the electroscope leaf rises.

- a) Use this observation to deduce the type of charge on the rod.
- b) Explain, in terms of electron transfer, how the rod became charged.

Q13.2

- a) Calculate the combined resistance of a $3.0\ \Omega$ resistor and a $6.0\ \Omega$ resistor
 - (i) in series with each other,
 - (ii) in parallel with each other.

- b) A $4.0\ \Omega$ resistor and a $6.0\ \text{V}$ battery of negligible internal resistance are connected in series with the combination in (a)
 (ii) Sketch the circuit diagram and calculate the current, p.d. and power dissipated for each resistor.

Q13.3 An electric kettle is designed to use $2.5\ \text{kW}$ of electrical power at $240\ \text{V}$.

- a) Calculate
 (i) the current through the kettle element when it operates normally,
 (ii) the charge passing through the kettle in 3 minutes,
 (iii) the electrical energy delivered to the kettle in 3 minutes.
 b) Given a $3\ \text{A}$ fuse, a $5\ \text{A}$ fuse and a $13\ \text{A}$ fuse, which one of these fuses would you select for this kettle?

Q13.4

- a) Sketch a graph to show how the current varies with p.d. for a silicon diode.
 b) A silicon diode is connected in its 'forward direction' in series with a $1.5\ \text{k}\Omega$ resistor and a $1.5\ \text{V}$ cell.
 (i) Sketch the circuit diagram for this arrangement.
 (ii) Assuming the p.d. across the silicon diode remains at $0.6\ \text{V}$ when it conducts, show that the current through the diode in this circuit is $0.60\ \text{mA}$.
 (iii) Calculate the power dissipated in the diode and in the resistor and the power supplied by the cell when the current through the diode is $0.60\ \text{mA}$.

Unit 14

Q14.1 A battery of e.m.f. $12.0\ \text{V}$ and internal resistance $1.5\ \Omega$ is connected to two $5.0\ \Omega$ resistors in parallel with each other.

- a) Sketch the circuit diagram and calculate the current through the battery.
 b) Calculate the p.d. across the parallel combination and the current through each of the resistors in the combination.
 c) (i) Calculate the power supplied by the battery and the power delivered to each $5.0\ \Omega$ resistor.
 (ii) Account for the difference between the power supplied by the battery and the total power delivered to the two $5.0\ \Omega$ resistors.

Q14.2

- a) A Wheatstone bridge circuit consists of a metre length of uniform wire, a $5.0\ \Omega$ standard resistance, S , and an unknown resistor, X , as in Fig Q14.2. When the bridge is balanced, the balance point is $450\ \text{mm}$ from the end of the wire connected to S . Calculate the resistance of X .

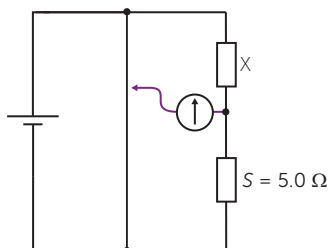


Fig. Q14.2

- b) A second $5.0\ \Omega$ resistance is connected in parallel with S . Calculate the new balance position.

Q14.3 A potential divider consisting of a $10\ \text{k}\Omega$ resistor and an LDR is connected to a $5.0\ \text{V}$ battery. A high-resistance voltmeter is connected across the $10\ \text{k}\Omega$ resistor.

- a) (i) Sketch the circuit diagram for this arrangement.
 (ii) When the LDR is in darkness, the voltmeter reading is $0.20\ \text{V}$. Calculate the resistance of the LDR in darkness.
 b) Describe and explain how the voltmeter reading would change when the LDR is exposed to light.

Q14.4

- a) Calculate the resistivity of a $2.5\ \text{m}$ length of metal wire of diameter $0.35\ \text{mm}$ which has a resistance of $12.5\ \Omega$.
 b) Calculate the length of wire necessary to make a $5.0\ \Omega$ resistor using wire of diameter $0.28\ \text{mm}$ and resistivity $5.2 \times 10^{-7}\ \Omega\ \text{m}$.

Unit 15

$$\epsilon_0 = 8.85 \times 10^{-12}\ \text{F m}^{-1}$$

Q15.1 A capacitor is connected in series with a battery, a switch, a milliammeter and a variable resistor. The switch is closed and the variable resistor is used to keep the current at a constant value of $1.2\ \text{mA}$. The switch is opened $60\ \text{s}$ after it has been closed.

- a) Calculate the charge stored by the capacitor in this time.
 b) The p.d. across the capacitor plates after $60\ \text{s}$ is measured at $3.3\ \text{V}$ using a digital voltmeter. Calculate the capacitance of the capacitor.

Q15.2

- a) Calculate the combined capacitance of a $6\ \mu\text{F}$ capacitor and a $12\ \mu\text{F}$ capacitor
 (i) in parallel,
 (ii) in series.
 b) The series combination in (a) is connected to a $6.0\ \text{V}$ battery. Calculate
 (i) the charge and energy stored in each capacitor,
 (ii) the total charge and energy stored.

Q15.3

- a) (i) Calculate the capacitance of a parallel plate capacitor consisting of two square, metal plates of length $0.26\ \text{m}$, separated by an air gap of width $2.0\ \text{mm}$.
 (ii) The capacitor in (i) was connected to a $12\ \text{V}$ battery. Calculate the charge and energy stored by the capacitor in this arrangement.
 b) In (a)(ii), the capacitor was disconnected from the battery and the plates moved together to a separation of $1.0\ \text{mm}$. Calculate
 (i) the new capacitance,
 (ii) the p.d. between the plates at $1.0\ \text{mm}$ apart.

Q15.4 A $4.7\ \mu\text{F}$ capacitor was connected to a $9.0\ \text{V}$ battery and then discharged through a $1.5\ \text{M}\Omega$ resistor. Calculate

- a) the initial charge and energy stored by the capacitor,
 b) the initial current,
 c) the charge and energy stored after $5.0\ \text{s}$.

Unit 16

Q16.1

- Sketch the symbol and write down the truth table for
 - an OR gate,
 - a NAND gate.
- Design a logic gate system that will switch an LED on when the light intensity in a room falls below a certain level and the temperature of the room falls below a certain value. Include a test switch in your system which will make the LED switch on when the switch is closed.

Q16.2

- An a.c. signal generator is connected between 0 V and the non-inverting input of an op-amp on open-loop. The inverting input is connected to 0 V. Sketch the circuit diagram and the output waveform, assuming the output voltage is limited to the range +15 V when the signal generator is adjusted to produce a sine wave voltage of peak value 1.0 V at a frequency of 1.0 kHz.
- In the circuit in (a), a $1.0\text{ M}\Omega$ resistor is connected between the output terminal of the op-amp and the inverting input. Also, the inverting input is disconnected from 0 V and reconnected via a $0.5\text{ M}\Omega$ resistor. Sketch the circuit diagram now and calculate the peak value of the output voltage for the same a.c. voltage as before applied between the non-inverting input and 0 V.

Q16.3

- Sketch a circuit diagram to show how an op-amp can be used as a voltage follower, to convert a low-resistance voltmeter to be a high-resistance voltmeter.
- Sketch the output voltage waveform of an astable multivibrator which operates at a frequency of 5 kHz.
 - Estimate a suitable resistance value for a 5 kHz multivibrator which is to be constructed using $0.1\text{ }\mu\text{F}$ capacitors.

Q16.4 Design an electronic thermometer capable of giving an output voltage that increases smoothly as the temperature of a thermistor increases over a limited range.

Unit 17

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}, e = 1.6 \times 10^{-19} \text{ C}$$

Q17.1

- Calculate the electric field strength and the electric potential at a distance of $3.0 \times 10^{-10} \text{ m}$ from a nucleus carrying a charge of $9.6 \times 10^{-19} \text{ C}$.
- Calculate the work done to remove an electron at the above position away from the nucleus to infinity.

Q17.2

- Calculate the force between a $+1.5 \text{ nC}$ point charge and a $+3.6 \text{ nC}$ point charge which are at a distance of 50 mm apart.
- Calculate the electric field strength and the potential at the midpoint of these two point charges.

Q17.3

- Calculate the electric field strength between two horizontal, parallel plates spaced 40 mm apart when the lower plate is earthed and the top plate is at a positive potential of 5.0 kV.
 - Sketch a graph to show how the potential varies from the earthed plate to the positive plate.
- A droplet carrying a charge of $+2.7 \times 10^{-15} \text{ C}$ falls from the top plate to the bottom plate. Calculate the work done by the electric field on this droplet.

Q17.4

- Sketch the pattern of lines of force near a negatively charged conducting sphere.
- A conducting sphere of radius 0.15 m is negatively charged to a potential of 4.6 kV. Calculate
 - the charge on the sphere,
 - the potential at a distance of 1.0 m from the sphere's surface.

Unit 18

$$\epsilon_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$$

Q18.1

- Sketch the pattern of magnetic field lines near
 - a bar magnet,
 - a long straight wire carrying a direct current.
- The magnetic flux density at a distance of 80 mm from a long straight wire of diameter 0.80 mm was $38 \text{ }\mu\text{T}$. Calculate
 - the current in the wire,
 - the magnetic flux density at the surface of the wire.

Q18.2

- A wire of length 0.065 m was placed perpendicular to the lines of force of a uniform magnetic field. When a current of 0.42 A was passed through the wire, a force of 1.50 mN was exerted on the wire. Calculate the magnetic flux density.
- The wire was turned so it made an angle of 40° with the lines of force of the magnetic field. Calculate the force on the wire in this position for the same current.

Q18.3

- In an experiment to measure the horizontal component of the Earth's magnetic flux density, B_H , a 500 turn solenoid of length 0.20 m was aligned with its axis horizontal along a line from north to south. A plotting compass was placed at one end of the solenoid. An increasing current was passed through the solenoid until the direction of the plotting compass reversed.
- The current at the point of reversal was 5.7 mA. Calculate the horizontal component of the Earth's magnetic flux density, B_H .
 - Sketch an end view of the end of the solenoid where the plotting compass was positioned, indicating the direction of current around the solenoid.

Q18.4

- a) Sketch a graph to show how the magnetisation of a ferromagnet in a solenoid varies with the solenoid current.
- b) Explain each of the following:
- (i) An iron bar is magnetised when it is placed in a direct current solenoid.
 - (ii) An iron bar can be demagnetised by removing it gradually from an a.c. solenoid.

Unit 19
Q19.1

- a) A coil of wire was connected to a centre-reading microammeter.
- (i) When a bar magnet with its north pole nearest the coil was moved towards the coil, the needle of the microammeter deflected briefly to the left of centre. Explain why this happened and describe what would have happened if the south pole of the bar magnet had been nearest the coil.
 - (ii) Explain why the needle did not deflect when the magnet was held at rest close to the coil.
- b) A rod of length 0.20 m, moving at a speed of 25 m s^{-1} , cuts the lines of force of a uniform magnetic field of flux density 38 mT at right angles to the lines of force. Calculate the e.m.f. induced in the rod.

Q19.2

- a) (i) With the aid of a diagram, explain the operation of an a.c. generator.
- (ii) Explain why the peak voltage from an a.c. generator occurs when the coil is parallel to the lines of force of the magnetic field.
- b) A 120 turn coil of length 0.15 m and width 0.04 m spins at a constant frequency of 20 Hz in a uniform magnetic field of flux density 85 mT. Calculate
- (i) the peak magnetic flux through the coil,
 - (ii) the peak voltage produced by the coil.

Q19.3

- a) Explain why the current passing through the armature of a d.c. motor connected to a battery increases when the motor speed is reduced.
- b) A d.c. electric motor with an armature of resistance 2.5Ω is connected to a 12 V battery of internal resistance 0.5Ω . When the motor rotates at a frequency of 25 Hz, the current in the circuit is 0.20 A. Calculate
- (i) the back e.m.f. at this frequency,
 - (ii) the current when the frequency is reduced to 15 Hz.

Q19.4 A transformer consists of a 60 turn primary coil and a 1200 turn secondary coil. A 240 V, 60 W light bulb is connected to the secondary coil.

- a) Calculate the primary voltage which would light the bulb normally.
- b) Calculate the maximum primary current to light this bulb normally.

Unit 20

Q20.1 The current in an a.c. circuit changes with time according to the equation I (in amperes) $= 1.60 \sin(100\pi t)$.

- a) Calculate the peak current and the frequency of this alternating current.
- b) Calculate the current in this circuit at $t =$
- (i) 2.50 ms,
 - (ii) 5.00 ms,
 - (iii) 12.0 ms.

Q20.2

- a) Calculate
- (i) the r.m.s. value of a sinusoidal current of peak value 2.4 A,
 - (ii) the peak voltage of a sinusoidal voltage of r.m.s. value 12.0 V.
- b) A 5.0Ω resistor is connected to a 50 Hz, 6.0 V r.m.s. alternating voltage supply unit. Calculate
- (i) the r.m.s. current through the resistor,
 - (ii) the mean power dissipated in the resistor.

Q20.3

- a) A $2.2 \mu\text{F}$ capacitor is connected in series with a $2.0 \text{ k}\Omega$ resistor and a 50 Hz, 6.0 V r.m.s. alternating voltage supply unit. Sketch the circuit and calculate
- (i) the r.m.s. current in the circuit,
 - (ii) the r.m.s. voltage across each component,
 - (iii) the power dissipated in the circuit.
- b) The capacitor in (a) is replaced by a 1.2 H inductor which has negligible resistance. Sketch this circuit and calculate
- (i) the r.m.s. current in the circuit,
 - (ii) the r.m.s. voltage across each component,
 - (iii) the power dissipated in the circuit.

Q20.4 A $4.7 \mu\text{F}$ capacitor is connected in series with a coil of inductance 7.5 mH and resistance 25Ω and a variable frequency a.c. supply unit.

- a) Calculate the resonant frequency of this circuit.
- b) The supply unit is adjusted to give an output at 1.5 kHz and 6.0 V r.m.s. Calculate
- (i) the circuit impedance,
 - (ii) the r.m.s. current,
 - (iii) the r.m.s. voltage across the coil.

Unit 21

$$e = 1.6 \times 10^{-19} \text{ C}, h = 6.6 \times 10^{-34} \text{ J s}, m_e = 9.1 \times 10^{-31} \text{ kg}$$

Q21.1

- a) With the aid of a diagram, explain how a beam of electrons is produced from a hot wire filament.
- b) Calculate the kinetic energy and speed of electrons produced in an electron tube operating at an anode voltage of 4.5 kV.

Q21.2 A narrow beam of electrons is directed into a uniform magnetic field of magnetic flux density 3.2 mT at 90° to the lines of force of the magnetic field. The beam forms a circle of diameter 78 mm.

- a) Calculate
 - (i) the speed and
 - (ii) the kinetic energy of the electrons in this beam.
- b) (i) Sketch the path of the beam in the field and indicate the direction of the magnetic field.
 (ii) Mark a point on the path of the beam and indicate the direction of the force due to the magnetic field on an electron in the beam at that point.
 (iii) Explain why the kinetic energy of each electron in the beam is constant, even though a force acts on each electron.

Q21.3 In an experiment to measure the charge on an oil droplet, an oil droplet was held stationary in a uniform electric field of strength 252 kV m^{-1} . When the field was switched off, the droplet descended at steady speed, taking 16.2 s to fall through a distance of 2.9 mm. Calculate

- a) the terminal speed of the droplet,
- b) the droplet's radius and mass,
- c) the charge carried by the droplet.

The viscosity of air $= 1.8 \times 10^{-5} \text{ N s m}^{-2}$, the density of the oil used $= 850 \text{ kg m}^{-3}$.

Q21.4

- a) Calculate the maximum frequency of X-rays produced by an X-ray tube operating at 55 kV.
- b) Light of wavelength 550 nm is directed at an earthed metal plate, causing photoelectric emission. This emission ceases if the plate potential is raised to +0.52 V. Calculate the work function and the threshold frequency of the metal surface.

Unit 22

Q22.1

- a) Explain, in terms of protons and neutrons, the change that takes place in an unstable nucleus when it emits
 - (i) an α -particle,
 - (ii) a β -particle.
- b) A sample of a radioactive isotope of half-life 15 hours has an initial activity of 0.64 MBq. Calculate its activity after
 - (i) 30 hours,
 - (ii) 7 days.

Q22.2 A Geiger counter was placed at a distance of 0.20 m from a radioactive source. The average count rate was measured at 25.20 s^{-1} . When a metal plate of thickness 2.00 mm was placed between the source and the counter, the count rate decreased to 9.40 s^{-1} . When the source was removed, the count rate was 0.40 s^{-1} .

- a) What type of radiation was emitted by the source? Explain your answer.
- b) Calculate the half value thickness of the metal.

Q22.3 Carbon 14 is a radioactive isotope which has a half-life of 5570 years. It decays to form nitrogen 14.

- a) Write down the equation for this radioactive change.
- b) Living wood has an activity of 270 Bq kg^{-1} due to the decay of carbon 14. A 1.2 g sample of wood from the hull of an ancient ship has an activity of 0.26 Bq. Estimate the age of the sample.

Q22.4 A point source of gamma radiation is at a distance of 0.50 m from the end of a GM tube of cross-sectional area $3.1 \times 10^{-4} \text{ m}^2$. The corrected count rate is 35.2 counts per second.

- a) Estimate the activity of the source.
- b) Calculate the corrected count rate if the source-tube distance is reduced to 0.20 m.

Unit 23

$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$, $e = 1.6 \times 10^{-19} \text{ C}$, $1u = 931 \text{ MeV}$. The Avogadro constant $= 6.02 \times 10^{23} \text{ mol}^{-1}$

Q23.1

- a) Estimate the least distance of approach of a 5.0 MeV α -particle to a nitrogen nucleus ($Z = 7$).
- b) Use your estimate in (a) to show that the density of the nucleus is much greater than the density of ordinary matter.

Q23.2

- a) (i) Explain what is meant by the binding energy of a nucleus.
 (ii) Sketch a graph to show how the binding energy per nucleon of a nucleus varies with the mass number of the nucleus.
- b) Calculate the binding energy per nucleon of
 - (i) a helium 4 nucleus ($Z = 2$),
 - (ii) a uranium 238 nucleus ($Z = 92$).

(Atomic masses: helium 4 $= 4.00260 \text{ u}$, uranium 238 $= 238.05076 \text{ u}$, proton mass $= 1.00728 \text{ u}$, neutron mass $= 1.00867 \text{ u}$, electron mass $= 0.00055 \text{ u}$)

Q23.3 Cobalt 60 ($Z = 27$) is a radioactive isotope that emits β -particles to form an isotope of nickel ($Z = 28$).

- a) Write down the equation representing the decay of this isotope.
- b) Calculate the Q-value for this decay. (Atomic masses: cobalt 60 $= 59.93381 \text{ u}$, nickel 60 $= 59.93078 \text{ u}$)
- c) Cobalt 60 has a half-life of 5.3 years. Calculate the activity of 1 gram of cobalt 60.

Q23.4

- a) In a nuclear reactor, uranium 235 nuclei undergo a controlled chain reaction of neutron-induced fission reactions. Explain what is meant by
 - (i) neutron-induced fission,
 - (ii) a controlled chain reaction of fission reactions.
- b) Explain the function of each of the following in a nuclear reactor:
 - (i) the moderator,
 - (ii) the control rods,
 - (iii) the coolant.

Unit 24

Q24.1 Describe and explain why a car has less drag on it if its windscreen is sloped compared with a vertical windscreen.

Q24.2 A river flows under a bridge at a speed of 0.86 m s^{-1} in a rectangular channel which is 4.2 m wide. The depth of water in the channel is 1.8 m .

- Calculate the volume of water that flows through the channel each second.
- Explain why the speed of the water flowing under the bridge is increased if part of the channel becomes blocked.

Q24.3

- In terms of the motion of its molecules, explain the cause of the viscosity of a viscous fluid.
- A gas of viscosity $1.4 \times 10^{-5} \text{ N s m}^{-2}$ flows through a pipe of internal diameter 13 mm and length 4.6 m . If the pressure difference between the ends of the pipe is 2.6 kPa , calculate the volume of gas that flows through the pipe each second.

Q24.4

- Explain the origin of the lift force that acts on the wings of an aircraft when it is in flight.
- An incompressible fluid flows through a pipe that has a narrow section of diameter which is half the diameter of the rest of the pipe.
 - Explain why the speed of the fluid is four times greater in the narrow section than in the rest of the pipe.
 - If the pressure in the narrow section is 1800 Pa lower than in the rest of the pipe, calculate the speed of the fluid in the rest of the pipe.

Density of the fluid = 920 kg m^{-3}

Unit 25

The Avogadro constant = $6.02 \times 10^{23} \text{ mol}^{-1}$,
 $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

Q25.1

- A piston contains 0.12 m^3 of dry air at 300 K and a pressure of 102 kPa . Calculate
 - the number of moles of air in the piston,
 - the pressure in the piston if the volume is reduced to 0.02 m^3 and the temperature is changed to 400 K .
- In a chemistry experiment, a volume of 18.7 cm^3 of gas was collected at 14°C and at a pressure of 105 kPa . Calculate the volume of this amount of gas at 0°C and 102 kPa pressure.

Q25.2

- A hand-operated pump of volume 100 cm^3 is designed to reduce the air pressure in a flask of volume 500 cm^3 . The air in the pump is initially at a pressure of 102 kPa . Calculate the pressure of the air in the flask after
 - a single stroke of the pump which increases the volume of air from 500 to 600 cm^3 ,
 - 10 strokes of the pump.
- Calculate the number of molecules per unit volume in a gas at a pressure of 10 kPa and a temperature of 300 K .

Q25.3

- State the kinetic theory equation for an ideal gas, defining the symbols you use.
 - State the assumptions of the kinetic theory of gases.
- A sealed cylinder of volume 0.25 m^3 contains nitrogen gas at a pressure of 120 kPa and a temperature of 290 K . Calculate
 - the number of moles of gas present,
 - the mass of gas present,
 - the r.m.s. speed of the gas molecules,
 - the mean kinetic energy of the gas molecules.
 The molar mass of nitrogen = 0.028 kg

Q25.4

- Define the root mean square speed of the molecules of a gas.
- Calculate the root mean square speed of the molecules of
 - oxygen (molar mass = 0.032 kg) at 15°C ,
 - oxygen at 0°C ,
 - hydrogen (molar mass = 0.002 kg) at 0°C .

Unit 26

Q26.1

- State what physical property is measured in
 - a liquid-in-glass thermometer,
 - a thermocouple thermometer.
- The pressure of a constant volume gas thermometer relative to atmospheric pressure was -9.2 kPa at 0°C and 27.4 kPa at 100°C . Calculate the temperature when the pressure was 5.1 kPa in
 - $^\circ\text{C}$,
 - K .

Q26.2

- A gas cylinder of volume 0.058 m^3 contained an ideal gas at 20°C at a pressure of 101 kPa . Calculate
 - the number of moles of gas in the cylinder,
 - the internal energy of the gas.
- The temperature of the cylinder was raised to 100°C . Calculate
 - the pressure of the gas,
 - the increase of internal energy of the gas.

Q26.3 Four moles of an ideal gas at 300 K were heated at a constant pressure of 100 kPa until its temperature was 350 K .

- Calculate
 - the initial volume of the gas,
 - the final volume of the gas,
 - the work done on the gas.
- Show that the increase of internal energy of the gas was 1.66 kJ .
 - Calculate the heat supplied to the gas.

Q26.4

- A piston containing 0.0301 m^3 of air at 102 kPa and 20°C was compressed adiabatically to a volume of 0.0040 m^3 . Calculate the pressure of the gas after the compression and show that its temperature was 384°C . (Assume γ for air = 1.4)
- Calculate the maximum efficiency of a heat engine operating between 20°C and 384°C .

Unit 27

Q27.1 A spin drier has a drum of diameter 0.35 m which is designed to spin at a speed of 600 turns per minute. Calculate

- the time taken for one turn at this frequency,
- the speed and centripetal acceleration of a point on the drum.

Q27.2 A vehicle fitted with tyres of diameter 0.45 m is travelling at a steady speed of 31 m s^{-1} . Calculate

- the time taken for each wheel to rotate once,
- the frequency of rotation of the wheels,
- the centripetal acceleration of a point on one of the tyres.

Q27.3 A vehicle of mass 950 kg travels over a bridge of radius of curvature 128 m at a steady speed of 24 m s^{-1} . Calculate

- the weight of the vehicle,
- the centripetal acceleration of the vehicle on the bridge,
- the force of the vehicle on the bridge as it passes over the highest point of the bridge.

Q27.4 A pendulum consists of a bob of mass 0.12 kg on the end of a string of length 0.95 m. The bob is released from a stationary position with the string taut and horizontal. Calculate

- the loss of potential energy of the bob between its initial position and its lowest position,
- the speed of the bob as it passes through its lowest position,
- the centripetal acceleration of the bob at its lowest position,
- the tension in the string as the bob passes through its lowest position.

Unit 28

See the question box on p. 443 for data for the questions below.

Q28.1

- Calculate the force of gravity on a 100 kg satellite in a circular orbit 3000 km above the Earth.
- Calculate the speed and time period of the satellite in (a).

Q28.2

- The gravitational field strength at the surface of the Earth is 9.8 N kg^{-1} . Calculate the height above the Earth at which the gravitational field strength of the Earth is 9.0 N kg^{-1} .
- Show that the escape speed from the Earth's surface is 11.2 km s^{-1} .

Q28.3

- Define gravitational potential.
- (i) Show that the gravitational potential at the surface of the Moon is -2.8 MJ kg^{-1} .
(ii) Calculate the escape speed off the lunar surface.

Q28.4

- Calculate the gravitational field strength of the Sun at the Earth's orbit.

- (i) Calculate the distance from the centre of the Sun to the point at which its gravitational field strength is equal to 9.8 N kg^{-1} .

(ii) Calculate the ratio of the distance in (i) to the mean distance from the Earth to the Sun.

Unit 29

$$g = 9.8 \text{ m s}^{-2}$$

Q29.1

- Define simple harmonic motion.
- A mass on a spring oscillates vertically, taking 25.2 s to complete 10 complete cycles of oscillation. Its height above the floor changes from 50 mm to 180 mm during each half cycle. Calculate
 - its time period,
 - its amplitude of oscillation,
 - its maximum speed.

Q29.2 A simple pendulum consists of a spherical bob of mass 0.052 kg and diameter 1.1 cm attached to a thread of length 920 mm.

- Calculate the time period of this pendulum.
- The bob is released at rest with the thread taut at an angle of 5° to the vertical. Calculate
 - the amplitude of oscillation,
 - the maximum speed of the bob,
 - the maximum acceleration of the bob.

Q29.3

- A 300 mm steel spring fixed at its upper end is used to support a 0.25 kg mass. Its equilibrium length when supporting this mass is 348 mm. Calculate
 - the stiffness constant of this spring,
 - the time period of vertical oscillations of the 0.25 kg mass on the spring.
- The 0.25 kg mass is displaced vertically by 20 mm from its equilibrium position and then released. Calculate
 - the maximum speed of the mass,
 - the minimum tension in the spring.

Q29.4

- What is meant by
 - free oscillations,
 - damped oscillations?
- Explain what is meant by resonance.
(ii) Describe an example of a mechanical system in resonance.

Unit 30

Q30.1 A vehicle with wheels of diameter 0.45 m decelerated uniformly from a speed of 24 m s^{-1} to a standstill in a distance of 60 m. Calculate

- the angular speed of each wheel when the vehicle was moving at 24 m s^{-1} ,
- (i) the time taken by the vehicle to decelerate to a standstill from a speed of 24 m s^{-1} .

- (ii) the number of turns each wheel made during the time the vehicle slowed down.
- c) the angular speed of the vehicle 1 second before it stopped.

Q30.2 Figure Q30.2 shows a rugby ball with an axis X through both ends and an axis Y through its centre perpendicular to X. State and explain whether or not its moment of inertia about X is less than or greater than its moment of inertia about Y.

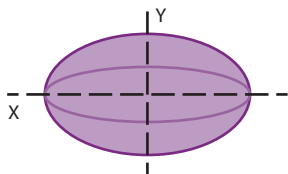


Fig. Q30.2

Q30.3 A flywheel fitted to a vehicle gains kinetic energy when the vehicle slows down and stops. The kinetic energy of the flywheel is used to make the vehicle start moving again.

- a) The flywheel is a uniform steel disc of diameter 0.31 m and thickness 0.08 m. Calculate
- the mass of the disc,
 - the moment of inertia of the flywheel.

Density of steel = 7800 kg m^{-3} , moment of inertia of a uniform flywheel = $\frac{1}{2}MR^2$

- b) (i) Calculate the kinetic energy of the flywheel when it is rotating at 3000 revolutions per minute.
- (ii) The kinetic energy of the flywheel can be converted to kinetic energy of motion of the vehicle in 30 s. Estimate the average power transferred from the flywheel.

Q30.4 A revolving door system consisting of four doors has a moment of inertia of 53 kg m^2 . The system is made to rotate at 8 revolutions per minute by applying a force F to one of the doors. When the force F is removed, frictional forces bring the system to rest in 4.0 s.

- a) (i) Calculate the angular deceleration of the system when the force F is removed.
- (ii) Calculate the frictional torque on the system.
- b) Calculate the power necessary to keep the system turning at 8.0 revolutions per minute. Assume the frictional torque is the same as in a)(ii).

Q30.5 A rotating flywheel is used via a clutch and gear system to supply extra power to a borehole drill. The moment of inertia of the flywheel is 550 kg m^2 .

- a) When the flywheel is rotating at 6000 revolutions per minute and the drill is at rest and free to rotate, the clutch is engaged and the flywheel's rate of rotation drops to 5500 revolutions per minute. Calculate the effective moment of inertia of the clutch and gear system and the borehole drill.
- b) In operation, the flywheel is used to supply extra power when the drill becomes stuck. On one such occasion, the clutch was engaged and the rate of rotation of the flywheel dropped from 6000 revolutions per minute to 4800 revolutions per minute.
- Calculate the loss of kinetic energy of rotation of the flywheel when the clutch was engaged.
 - Calculate the kinetic energy of rotation of the clutch and gear system and the drill immediately after the clutch was engaged.
 - Account for the difference between the answers to b)(i) and (ii)

Using Spreadsheets

A spreadsheet may be used to carry out repeatedly a sequence of calculations. Example 1 opposite is designed to illustrate how to set up a spreadsheet.

A general dynamics spreadsheet

The spreadsheet instructions that follow are designed to enable you to calculate the displacement, velocity and acceleration of any object at successive intervals, given its acceleration in terms of displacement and velocity.

The acceleration, a , is expressed as a general formula $a = \alpha - bv + cs$, where α , b and c are constants. Note that α represents the effect of constant forces, b represents the effect of a 'drag' force and c represents the effect of a restoring force. The values of the constants are inserted into the spreadsheet as well

as the time interval, dt . The shorter the time interval, the more accurate the calculations. However, more rows are then needed to reach a particular point in time.

The initial values of s , v and time t are inserted. The spreadsheet then automatically calculates s , v and a at subsequent times dt , $2dt$, $3dt$, $4dt$, etc. See Example 2 opposite and Example 3 on the following page. The results may be plotted as a graph using the related chart display.

The instructions to set up this spreadsheet using Microsoft[®] Excel are given below. Some changes may be necessary for other spreadsheet packages.

See www.palgrave.com/foundations/breithaupt to investigate motion using a spreadsheet.

Microsoft[®] Excel set up

Acceleration = $\alpha - bv + cs$; insert values of constants α , b and c into cells A1, B1 and C1 respectively.

Insert the column titles t , $t + dt$, v , s , a , dv , $v + dv$, ds , $s + ds$ into cells A2 to I2; use " to insert these labels.

Initial time, t , and time interval, dt : insert values into cells A3 and B3 respectively.

Initial displacement and velocity: insert values into cells D3 and C3 respectively.

Calculation of acceleration: insert the formula $+A\$1+B\$1*C3+C\$1*D3$ into cell E3 to calculate $\alpha + bv + cs$.

Calculation of change of velocity, dv , and final velocity, $v + dv$: insert $+E3*B\$3$ into cell F3 to calculate dv ; insert $+F3+C3$ into cell G3 to calculate $v + dv$.

Calculation of change of displacement, ds , and s :

insert $+0.5*(G3+C3)*B\$3$ into cell H3 to calculate ds ; insert $+H3+D3$ into cell I3 to calculate $s + ds$.

Calculation of time at the start of the next interval:

insert $+A3+B\$3$ into cell A4.

Calculation of time at the end of the interval ($t + dt$):

insert $+A4+B\$3$ into cell B4.

Next values of displacement and velocity:

insert $+I3$ into cell D4; insert $+G3$ into cell C4.

Next values of a , dv , $v + dv$, ds and $s + ds$:

copy cells E3 to I3 into cells E4 to I4 respectively. Use the 'copy down' facility for this.

Next values:

copy down the row of cells A4 to I4 as far as required.

EXAMPLE 1

Calculate the displacement every second from O, of an object moving with a constant velocity of 5 m s^{-1} . The object is initially at O.

The displacement s at time t is given by $s = vt$, where $v = 5 \text{ m s}^{-1}$.

Step 1 Insert the velocity value in cell A1 and the time interval (dt) in cell B1.

Step 2 Label cell A2 't' and cell B2 's'; use " before t or s to insert the letters.

Step 3 Insert 0 into cells A3 and B3 for the initial values of the time and displacement.

Step 4 Insert the formula $+A3+\$B\$1$ into cell A4 to display the time at the end of the first time interval. The \$ sign is necessary to give an absolute reference to cell B1.

Step 5 Insert the formula $+\$A\$1*A4$ into cell B4 to calculate and display the displacement.

Step 6 Copy cells A4 and B4 down as far as desired. The spreadsheet should then show the time and displacement on each row. Note that in each cell below A4 or B4, the formulae retain addresses labelled with the \$ sign (absolute addresses) and index addresses without the \$ sign.

EXAMPLE 2**An object released from rest**

Acceleration, $a = -9.8 \text{ m s}^{-2}$, hence $a = -9.8$, $b = c = 0$. Also, the initial values of displacement, velocity and time are zero. In the spreadsheet below, $dt = 0.200 \text{ s}$.

-9.800	0.000	0.000						
t	$t + dt$	v	s	a	$dv = a*dt$	$v + dv$	ds	$s + ds$
0.000	0.200	0.000	0.000	-9.800	-1.960	-1.960	-0.196	-0.196
0.200	0.400	-1.960	-0.196	-9.800	-1.960	-3.920	-0.588	-0.784
0.400	0.600	-3.920	-0.784	-9.800	-1.960	-5.880	-0.980	-1.764
0.600	0.800	-5.880	-1.764	-9.800	-1.960	-7.840	-1.372	-3.136
0.800	1.000	-7.840	-3.136	-9.800	-1.960	-9.800	-1.764	-4.900
1.000	1.200	-9.800	-4.900	-9.800	-1.960	-11.760	-2.156	-7.056
1.200	1.400	-11.760	-7.056	-9.800	-1.960	-13.720	-2.548	-9.604
1.400	1.600	-13.720	-9.604	-9.800	-1.960	-15.680	-2.940	-12.544
1.600	1.800	-15.680	-12.544	-9.800	-1.960	-17.640	-3.332	-15.876
1.800	2.000	-17.640	-15.876	-9.800	-1.960	-19.600	-3.724	-19.600
2.000	2.200	-19.600	-19.600	-9.800	-1.960	-21.560	-4.116	-23.716
2.200	2.400	-21.560	-23.716	-9.800	-1.960	-23.520	-4.508	-28.224

Note

Presentation of the spreadsheet may be improved by reducing the column widths to 8 characters per column, setting the number of decimal places to 3 and centring all the entries.

EXAMPLE 3**An object released from rest in a fluid**

Acceleration, $a = -9.8 \text{ m s}^{-2}$, hence $a = -9.8$, $b = -0.10$, $c = 0$. Also, the initial values of displacement, velocity and time are zero. In the spreadsheet below, $dt = 0.500 \text{ s}$.

-9.800	-0.100	0.000						
t	t + dt	v	s	a	dv = a*dt	v + dv	ds	s + ds
0.000	0.200	0.000	0.000	-9.800	-1.960	-1.960	-0.196	-0.196
0.200	0.400	-1.960	-0.196	-9.604	-1.921	-3.881	-0.584	-0.780
0.400	0.600	-3.881	-0.780	-9.412	-1.882	-5.763	-0.964	-1.744
0.600	0.800	-5.763	-1.744	-9.224	-1.845	-7.608	-1.337	-3.082
0.800	1.000	-7.608	-3.082	-9.039	-1.808	-9.416	-1.702	-4.784
1.000	1.200	-9.416	-4.784	-8.858	-1.772	-11.187	-2.060	-6.844
1.200	1.400	-11.187	-6.844	-8.681	-1.736	-12.924	-2.411	-9.255
1.400	1.600	-12.924	-9.255	-8.508	-1.702	-14.625	-2.755	-12.010
1.600	1.800	-14.625	-12.010	-8.337	-1.667	-16.293	-3.092	-15.102
1.800	2.000	-16.293	-15.102	-8.171	-1.634	-17.927	-3.422	-18.524
2.000	2.200	-17.927	-18.524	-8.007	-1.601	-19.528	-3.746	-22.270
2.200	2.400	-19.528	-22.270	-7.847	-1.569	-21.098	-4.063	-26.332

List of Experiments in the Book

Key

- D** short experiments that demonstrate a specific point
- E** standard experiments that can be completed in a single session
- I** short investigations or tests that develop or reinforce a topic
- P** longer experiments that could be developed into projects
- *** Website only (see page reference and www.palgrave.com/foundations/breithaupt)

Safety note

A risk assessment must be carried out before any practical work (including demonstrations) is carried out. All necessary safety information should be supplied to students.

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- I** Uncertainty estimates **7**
- D** The parallelogram of forces **12** Fig. 1.3
- P** Investigating friction **19–20** Fig. 1.24
- E*** Investigating motion **31** and website
- E** Measurement of g by the falling ball method **36** Fig. 2.16
- I** Investigating force and acceleration **42** Fig. 3.3
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- D*** Testing microwaves **127** and website
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See www.palgrave.com/foundations/breithaupt for exercises on these mathematical skills.

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Summary of Equations

General

Density

$$\rho = \frac{m}{V} \text{ where } m = \text{mass, } V = \text{volume}$$

$$\text{Weight} = mg$$

Circle of radius r

$$\text{area, } A = r^2$$

$$\text{circumference, } C = 2r$$

$$\text{arc length, } s = r\theta, \text{ where } \theta = \text{angle subtended by the arc in radians}$$

Sphere of radius r

$$\text{surface area} = 4\pi r^2$$

$$\text{volume, } V = \frac{4}{3}\pi r^3$$

Cylinder of radius r and length L

$$\text{volume} = \pi r^2 L$$

Part 1 Mechanics

Component of a force F along a line at angle θ

$$= F \cos \theta$$

$$\text{Moment of a force} = Fd$$

$$\text{Coefficient of friction } \mu = \frac{F}{N}$$

Equations for uniform acceleration

$$v = u + at;$$

$$s = \frac{1}{2}(u + v)t;$$

$$s = ut + \frac{1}{2}at^2; v^2 = u^2 + 2as$$

$$\text{Momentum} = mv$$

Newton's second law

$$F = \frac{d}{dt}(mv)$$

$$\text{becomes } F = ma \text{ for constant mass } m$$

$$\text{and } F = v \frac{dm}{dt} \text{ for constant rate of change of mass}$$

$$\text{Work} = \text{force} \times \text{distance}$$

$$\text{Power} = \frac{\text{energy transferred}}{\text{time taken}}$$

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$\text{Change of potential energy} = mgh$$

Part 2 Properties of Materials

$$\text{Thermal expansion } \Delta L = \alpha L \Delta T$$

$$\text{Energy transferred} = mc(T_2 - T_1) \\ \text{for change of temperature } (T_2 - T_1) \text{ of mass } m$$

$$\text{Energy transferred} = ml \\ \text{for change of state of mass } m$$

Power radiated

$$W = e\sigma AT^4 \text{ from a surface at temperature } T$$

$$\text{Heat transfer } \frac{q}{t} = \frac{kA(T_2 - T_1)}{L}$$

$$\text{through a thermal conductor of thermal conductivity } k$$

Hooke's law

$$T = ke \text{ where } T = \text{tension and } k = \text{spring constant}$$

Young modulus of elasticity

$$\frac{\text{stress}}{\text{strain}} = \frac{(T \div A)}{(e \div L)} = \frac{TL}{Ae}$$

Energy stored in a stretched spring

$$= \frac{1}{2}ke^2 \left[= \frac{1}{2}(AE/L)e^2 \text{ for a stretched wire} \right]$$

Pressure

$$p = \frac{F}{A} \text{ where } F = \text{force applied at right angles to a surface of area } A$$

$$\text{Pressure of a liquid column of depth } H \quad p = H\rho g$$

Part 3 Waves

Frequency

$$f = \frac{1}{T} \text{ where } T = \text{time period}$$

Wave speed

$$v = f\lambda \text{ where } f = \text{frequency and } \lambda = \text{wavelength}$$

Phase difference

$$\Delta\phi = \frac{2\pi x}{\lambda}$$

Stationary waves on a string of length L

$$2L = m\lambda$$

Stationary waves in a pipe of length L open at one end

$$L + e = \frac{(2m+1)\lambda}{4}$$

Stationary waves in a pipe of length L open at both ends

$$L + 2e = \frac{m\lambda}{2}$$

Young's slits

$$\lambda = \frac{yd}{X}, \text{ where } y = \text{fringe spacing, } d = \text{slit spacing,} \\ X = \text{slit-screen distance}$$

Snell's law

$$\frac{\sin i}{\sin r} = \frac{\lambda_i}{\lambda_r} = \frac{v_i}{v_e} = \text{refractive index, } n$$

$$\text{Lens formula } \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Diffraction grating equation

$$d \sin \theta_m = m\lambda, \text{ where } d = \text{slit spacing}$$

Part 4 Electricity**Charge**

$$Q = It \text{ for constant current and}$$

$$I = \frac{dQ}{dt} \text{ for changing current}$$

Potential difference

$$V = \frac{E}{Q} \text{ where } E = \text{electrical energy transferred}$$

$$\text{Resistance } R = \frac{V}{I}$$

Resistor combination rules

$$\text{in series, } R = R_1 + R_2;$$

$$\text{in parallel, } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Potential divider equation

$$\text{p.d. across } R_1 = \frac{R_1}{R_1 + R_2} V$$

$$\text{Potentiometer equation } \frac{E_1}{E_2} = \frac{I_1}{I_2}$$

$$\text{Wheatstone bridge equation } \frac{P}{R} = \frac{Q}{S}$$

$$\text{Resistivity } \rho = \frac{RA}{L}$$

$$\text{Capacitance } C = \frac{Q}{V}$$

Capacitor combination rules

$$\text{in series, } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{in parallel, } C = C_1 + C_2$$

$$\text{Energy stored in a capacitor} = \frac{1}{2} CV^2$$

$$\text{Capacitor discharge } Q = Q_0 e^{t/RC}$$

$$\text{Time constant} = CR$$

$$\text{Capacitance of parallel plates } C = A\epsilon_0\epsilon_r$$

Voltage gain

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_F}{R_1} \text{ for an inverting amplifier}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_F + R_1}{R_1} \text{ for a non-inverting amplifier}$$

Part 5 Fields

$$\text{Coulomb's law } F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

Electric field strength due to two parallel plates

$$E = \frac{V}{d} = \frac{Q}{A\epsilon_0}$$

Electric field near a point charge Q

$$\text{Electric field strength, } E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\text{Electric potential, } V = \frac{Q}{4\pi\epsilon_0 r}$$

Force on a current-carrying conductor

$$F = BIL \sin \theta$$

$$\text{Force on a moving charge } F = Bqv \sin \theta$$

$$\text{Hall voltage } V_H = Bvd$$

Magnetic flux density formulae

$$\text{at distance } r \text{ from a long straight wire } B = \frac{\mu_0 I}{2\pi r}$$

$$\text{inside a long solenoid, } B = \frac{\mu_0 NI}{l}$$

$$\text{Magnetic flux linkage } \Phi = BAN$$

Faraday's law of electromagnetic induction

$$\text{induced e.m.f., } V = -\frac{d\Phi}{dt}$$

Induced e.m.f. formulae

$$\text{for a moving rod: } V = BLv$$

$$\text{for a generator coil: } V = 2\pi fBAN \sin(2\pi ft)$$

Transformer equation = mgd

Self-inductance $L = \frac{\Phi}{I}$

Energy stored in a self-inductance = $\frac{1}{2}LI^2$

r.m.s. value

r.m.s. value of an alternating current = $\frac{\text{peak value}}{\sqrt{2}}$

Reactance of a capacitor $\frac{V_0}{I_0} = \frac{1}{2\pi fC}$

Reactance of an inductor $\frac{V_0}{I_0} = 2\pi fL$

Impedance of a series LCR circuit

$$Z = \left[R^2 + \left(2\pi fL - \frac{1}{2\pi fC} \right)^2 \right]$$

Resonant frequency = $\frac{1}{2\pi(LC)^{1/2}}$

Part 6 Atomic and Nuclear Physics

Electron gun equation $\frac{1}{2}mv^2 = eV_A$

Force on an electron between oppositely charged parallel plates

$$F = \frac{eV}{d}$$

Force on an electron moving across a magnetic field

$$F = Bev$$

Millikan's experiment $\frac{QV}{d} = mg$

Stokes' law $F = 6\pi\eta rV$

Photon energy $E = hf$

Maximum k.e. of emitted photoelectron = $hf - \phi$

Maximum frequency of X-rays from an X-ray tube

$$f_{\max} = \frac{eV_A}{h}$$

Radioactive decay equations

$$\text{activity, } A = \frac{dN}{dt} = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$

$$\text{decay constant, } \lambda = \frac{\log_e 2}{T_{1/2}}$$

Energy $E = mc^2$

de Broglie's equation $\lambda = \frac{h}{mv}$

Part 7 Further Physics

Continuity equation

$$\text{mass flow per second, } \frac{m}{t} = \rho vA$$

Poiseuille's equation $\frac{V}{t} = \frac{\pi r^4 \Delta p}{8\eta L}$

Stokes' Law $F = 6\pi\eta rV$

Bernoulli's equation $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

Pitot-static tube $V = \sqrt{\frac{2\Delta p}{\rho}}$

Boyle's law $pV = \text{constant}$

Charles' law $\frac{V}{T} = \text{constant}$

Pressure law $\frac{p}{T} = \text{constant}$

Ideal gas equation $pV = nRT$

Kinetic theory equation

$$pV = \frac{1}{3}Nm v_{\text{RMS}}^2 \text{ or } p = \frac{1}{3}\rho v_{\text{RMS}}^2$$

Mean kinetic energy of a gas molecule = $\frac{3}{2}kT$

Absolute temperature

$$T = \frac{pV}{(pV)_{\text{Tr}}} \times 273.16$$

Celsius temperature

$$t = \text{absolute temperature } T - 273.15$$

First Law of Thermodynamics $\Delta Q = \Delta U + \Delta W$

Work done by a gas = $p\Delta V$

Molar heat capacities of a gas

$$C_p - C_v = R$$

monoatomic gas: $C_p = \frac{5}{2}RT$, $C_v = \frac{3}{2}RT$, $\gamma = \frac{5}{3}$

diatomic gas: $C_p = \frac{7}{2}RT$, $C_v = \frac{5}{2}RT$, $\gamma = \frac{7}{5}$

polyatomic gas: $C_p = \frac{9}{2}RT$, $C_v = \frac{6}{2}RT$, $\gamma = \frac{4}{3}$

Equation for an adiabatic change of a gas

$$pV^\gamma = \text{constant}$$

Efficiency of a heat engine = $\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

Maximum efficiency of a heat engine = $\frac{T_1 - T_2}{T_1}$

Angular speed $\omega = \frac{2\pi}{T} = 2\pi f$

Centripetal acceleration $a = \frac{-v^2}{r} = -\omega^2 r$

Newton's law of gravitation $F = \frac{Gm_1m_2}{r^2}$

Gravitational field strength $g = \frac{F}{m}$

Potential energy of two masses $= -\frac{Gm_1m_2}{r}$

At distance r from the centre of a spherical planet of mass M

$$g = \frac{GM}{r^2}$$

Escape speed $= \sqrt{(2gr)}$

Satellite equation $r^3 = \frac{GM}{4\pi^2} T^2$

Simple harmonic motion equations

acceleration, $a = -\omega^2 s$ where s = displacement

maximum speed, $v_{\max} = \omega r$ where r = amplitude

time period, $T = 2\pi \left(\frac{m}{k}\right)^{1/2}$ for mass m on a spring

time period, $T = 2\pi \left(\frac{L}{g}\right)^{1/2}$ for a simple pendulum of length L

Angular acceleration $\alpha = \frac{\omega_2 - \omega_1}{t}$

Angular dynamics equations

$$\omega = \omega_0 + \alpha t$$

$$\theta = \frac{1}{2}(\omega + \omega_0)t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Resultant torque $T = I\alpha$

Work done by a torque $W = T\theta$

Power delivered by a torque $P = T\omega$

Angular momentum $= I\omega$

Useful Data

Table of physical constants

The Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
1 atomic mass unit	$= 1.666 \times 10^{-27} \text{ kg}$
The Stefan constant	$a = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Absolute permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
Absolute permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Speed of light in a vacuum	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
The Planck constant	$h = 6.63 \times 10^{-34} \text{ J s}$
Electronic charge	$e = 1.60 \times 10^{-19} \text{ C}$
Specific charge of the electron	$e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$
Mass of the electron	$= 9.11 \times 10^{-31} \text{ kg}$
Mass of the proton	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Molar gas constant	$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
The Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

Useful numerical data

The weight of 1 kg at the surface of the Earth	$= 9.8 \text{ N}$
2π radians	$= 360^\circ$
1 electron volt	$= 1.60 \times 10^{-19} \text{ J}$
1 u	$= 931 \text{ MeV}$

Answers to In-text Questions and Revision Questions

The numerical answers for each question are usually given to the same number of significant figures as the data in the question.

UNITS AND MEASUREMENTS

In-text questions

U1

- 1 (a) (i) 0.500 m (ii) 320 cm (iii) 95.60 m.
 (b) (i) 450 g (ii) 1.997 kg (iii) 5.4×10^7 g.
 2 (a) 1.50×10^{11} m (b) 3.15×10^7 s (c) 6.30×10^{-7} m.
 (d) 2.58×10^{-8} kg (e) 1.50×10^6 m (f) 1.25×10^{-6} m.

U2

- 1 (a) 1.90 kg (b) 2.40×10^{-4} m³ (c) 7920 kg m⁻³.
 2 (a) 0.048 kg (b) 1.77×10^{-5} m³ (c) 2710 kg m⁻³.
 3 (a) (i) 0.960 kg (ii) 9.60×10^{-4} m³ (iii) 0.101 m.
 (b) (i) 0.105 kg (ii) 4.0×10^{-5} m³ (iii) 2630 kg m⁻³.

Revision questions

- 1 (a) C (b) E (c) B (d) E.

- 2 (a) (i) 54.0 g (ii) 0.1 g (b) (i) 103.4 g (ii) 0.1 g (c) (i) 107 cm³.
 (ii) 2 cm³.

(d) (i) $\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{103.4 \times 10^{-3} \text{ kg}}{107 \times 10^{-6} \text{ m}^3} = 966 \text{ kg m}^{-3}$.

(ii) $\text{maximum density} = \frac{\text{maximum mass}}{\text{minimum volume}} = \frac{103.5 \times 10^{-3} \text{ kg}}{105 \times 10^{-6} \text{ m}^3} = 986 \text{ kg m}^{-3}$.

Hence the uncertainty in the density = 20 kg m⁻³. The density is therefore between 946 and 986 kg m⁻³, i.e. density = $966 \pm 20 \text{ kg m}^{-3}$. The percentage uncertainty in the density is therefore $\frac{20}{966} \times 100\% = 2\%$.

- 3 (a) 0.330 m (b) $6.5 \times 10^{-3} \text{ m}^2$.

(c) $1.63 \times 10^{-5} \text{ m}^3$ ($= 0.100 \text{ m} \times 0.065 \text{ m} \times 0.0025 \text{ m}$)

(d) 7380 kg m^{-3} ($= 0.120 \text{ kg} + 1.63 \times 10^{-5} \text{ m}^3$).

4 (a) volume = $\pi r^2 L$, where the radius, $r = 26 \text{ mm} = 0.026 \text{ m}$ and the length, $L = 1.2 \text{ m}$. $\therefore$ volume = $\pi \times (0.026)^2 \times 1.2 = 2.55 \times 10^{-3} \text{ m}^3$.

(b) mass = volume $\times$ density = $2.55 \times 10^{-3} \times 2700 = 6.88 \text{ kg}$.

5 (a) Volume $V = \frac{4}{3}\pi r^3$, where the radius, $r = 12 \text{ mm} = 0.012 \text{ m}$. Hence the volume = $\frac{4}{3}\pi \times (0.012)^3 = 7.24 \times 10^{-6} \text{ m}^3$.

(b) Rearranging the volume equation gives

$$r^3 = \frac{3V}{4\pi} = \frac{3 \times 0.60 \times 10^{-6}}{4\pi} = 1.43 \times 10^{-7} \text{ m}^3$$

Hence $r = (1.43 \times 10^{-7})^{1/3} = 5.2 \times 10^{-3} \text{ m} = 5.2 \text{ mm}$. $\therefore$ diameter = 10.4 mm

UNIT 1

In-text questions

1.1a

- 1 (a) 17.0 N due East (b) 7.0 N due West (c) 13.0 N at an angle of 67.4° North of due East (d) 15.1 N at 43.3° North of due East (e) 10.4 N at 84.5° North of due West. See Fig. A1.1

- 2 (a) 10.0 N (b) 13.0 N. See Fig. A1.2.

1.1b

- 1 (a) $A_x = +3 \cos 45^\circ$, $A_y = +3 \sin 45^\circ$, $B_x = +2.0 \text{ N}$, $B_y = 0$
 $\therefore R_x = 3 \cos 45^\circ + 2 = 4.1 \text{ N}$, $R_y = 2.0 \text{ N}$, so $R = (4.1^2 + 2^2)^{1/2} = 4.6 \text{ N}$ at $\theta = 27^\circ$ above the x-axis (since $\tan \theta = 2.0/4.1$).

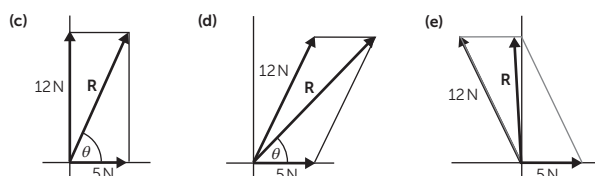


Fig. A1.1

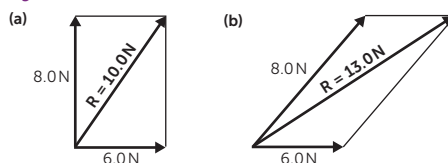


Fig. A1.2

- (b) $A_x = +4 \cos 30^\circ$, $A_y = +4 \sin 30^\circ$; $B_x = -2.5 \text{ N}$, $B_y = 0$
 $\therefore R_x = 4 \cos 30^\circ - 2.5 = +1.0 \text{ N}$, $R_y = +2.0 \text{ N}$, so $R = (1.0^2 + 2.0^2)^{1/2} = 2.2 \text{ N}$ at $\theta = 26^\circ$ above the x-axis (since $\tan \theta = 1.0/2.0$).

- 2 In each case, C is equal and opposite to R in question 1.

1.2

- 1 Use $W_1 d_1 + W_2 d_2 = W_3 d_3$. (a) 2.0 m (b) 1.5 m (c) 0.5 m.

- (d) 2.0 N.

- 2 (a) $W \times 0.50 = 2.0 \times 0.15$. $\therefore W = 2.0 \times 0.15/0.50 = 0.6 \text{ N}$.

- (b) $(W_x + 0.60) \times 0.50 = 2.0 \times 0.36$. $\therefore W_x + 0.60 = 2.0 \times 0.36/0.50 = 1.44$
 $\therefore W_x = 0.84 \text{ N}$.

1.3

- 1 Let d represent the distance from the centre of gravity of the whole placed to the free end of the pole. Consider the pole horizontal, as in Fig. A1.3. The moment of the pole about its free end = $30 \text{ N} \times 2.0 \text{ m} = 60 \text{ N m}$ anticlockwise. The moment of the square board about the free end of the pole = $20 \text{ N} \times 4.0 \text{ m} = 80 \text{ N m}$ anticlockwise.

$\therefore$ total moment = $60 + 80 = 140 \text{ N m}$ anticlockwise.

The total weight of 50 N acting at distance d from the free end of the pole gives the same moment. Hence $50d = 140$

$\therefore d = 140/50 = 2.8 \text{ m}$.

- 2 (a) Area of rectangle = 0.4 m^2 ; area of square = 0.16 m^2 . $\therefore$ weight of rectangle = $15g \times 0.40/0.56 = 105 \text{ N}$.

- (b) Weight of square = $15g - 105 = 42 \text{ N}$.

- (c) Consider the rectangular section horizontal as in Fig. A1.4. The moment of the rectangular section about the centre of the square = $105 \text{ N} \times 1.2 \text{ m} = 126 \text{ N m}$. This is the same as

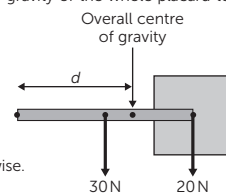


Fig. A1.3

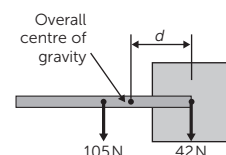


Fig. A1.4

the moment of the whole board about the centre of the square, i.e. $15gd$, where d is the distance from the centre of the square to the centre of gravity of the whole board.

$$\therefore 15gd = 126 \quad \therefore d = 126/15g = 0.86 \text{ m}$$

$\therefore$ the height of the centre of gravity
 $= 2.20 - 0.86 = 1.34 \text{ m}$.

1.4

- 1 (a) $F = W \sin \theta = 12 \sin 58 = 10.2 \text{ N}$. (b) $N = W \cos \theta = 12 \cos 58 = 6.4 \text{ N}$.
 (c) $\mu = \tan 58^\circ = 1.6$.

2 Fig. A1.5 shows a free-body diagram of the forces on the ladder. Let θ represent the angle between the ladder and the floor at the point of slipping. The force on the ladder due to the floor has been resolved into a normal component, N_1 , and a friction component, F . Considering all the forces acting on the ladder:

- The upward vertical forces = downward vertical components $\therefore N_1 = W$.
- The horizontal components acting leftwards = the horizontal components acting rightwards $\therefore N_2 = F$.

Taking moments about the lower end of the ladder at the point of contact with the floor:

- Clockwise moment = $N_2 h = F \times 8.0 \sin \theta$, since the height of the point of contact of the ladder on the wall, $h = 8.0 \sin \theta$, and $N_2 = F$.
- Anticlockwise moment = $Wd/2 = N_1 \times 8.0 \cos \theta/2$ since d , the distance from the wall of the point of contact of the ladder on the floor = $8.0 \cos \theta$ and $N_1 = W$.
- In equilibrium, $F \times 8.0 \sin \theta = N_1 \times 4.0 \cos \theta$.

Since $F = \mu N_1 = 0.4N_1$, then $0.4N_1 \times 8.0 \sin \theta = N_1 \times 4.0 \cos \theta$. Simplifying this expression gives $0.8 \sin \theta = \cos \theta$.

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{0.8} = 1.25 \quad \therefore \theta = 51^\circ$$

1.5

1 Consider the free-body force diagram for the shelf, as in Fig. 1.6. Let X and Y represent the two support forces as in the diagram.

$$\therefore X + Y = 25 + 40 = 65 \text{ N}$$

Take moments about the support at X :

- Clockwise moment
 $= (40 \times 0.5) + (25 \times 1.0) = 45 \text{ N m}$.
- Anticlockwise moment = $2.0Y$.
- For equilibrium, $2.0Y = 45 \quad \therefore Y = 22.5 \text{ N}$.
 Hence $X = 65 - 22.5 = 42.5 \text{ N}$.

2 Consider the free-body force diagram for the caravan, as shown in Fig. A1.7.

(i) Vertically, $R + T \sin \theta = 3200$, where T is the towing force and θ is the angle of the towing force above the horizontal.

(ii) Horizontally, $D + F = T \cos \theta$, where $D = 0.5 \times 1086 = 543 \text{ N}$ and $F = \mu R = 0.4R$. Hence $0.4R + 543 = T \cos \theta$.

(iii) Apply the principle of moments about the point where the tow bar is attached to the caravan, which is 0.5 m above the road and 1.5 m in front of the front wheels, as in Fig. 1.37.

- Total clockwise moment = $3.0R + 0.5F = 3.2R$, since $F = 0.4R$.
- Total anticlockwise moment = $2.0W + (1.3 - 0.5)D$
 $= 6400 + (0.8 \times 1086) = 7270 \text{ N m}$.

$$\text{Hence } 3.2R = 7270 \quad \therefore R = 7270/3.2 = 2270 \text{ N}$$

$$\therefore T \sin \theta = 3200 - R = 3200 - 2270 = 930 \text{ N}$$

$$\text{Also, } T \cos \theta = 0.4R + 543 = (0.4 \times 2270) + 543 = 1450 \text{ N}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{930}{1450} = 0.64\theta = 33^\circ \quad T = 930/\sin \theta = 930/\sin 33 = 1720 \text{ N}$$

Revision questions

1.1

(a) (i) mass, m = volume $\times$ density = $0.050 \times 0.500 \times 0.800 \times 2700 = 54 \text{ kg}$.

(ii) weight = $mg = 54 \times 9.8 = 530 \text{ N}$.

(b) (i) See Fig. A1.8. (ii) There are no other vertical components and therefore the weight is equal and opposite to the normal reaction at the floor.

(iii) $F = 0.4 \times 530 = 212 \text{ N}$. (iv) Take moments about the point of contact at the floor: $W \times 0.5b = R \times h$, where R is the reaction at the wall, b and h are as in

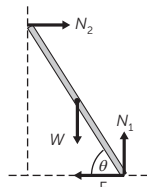


Fig. A1.5

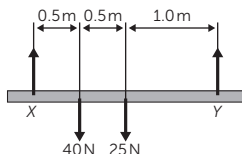


Fig. A1.6

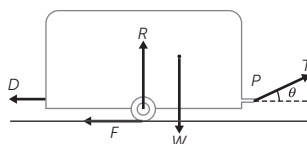


Fig. A1.7

Fig. A1.8. $\therefore b/h = 2R/W$. Hence $\tan \theta = b/h = 2R/W$. Since reaction R is equal and opposite to the frictional force, $R = 212 \text{ N}$. $\tan \theta = 2 \times 212/530 = 0.8 \quad \therefore \theta = 39^\circ$.

1.2

(a) Initially, the front wheels bear less weight than the rear wheels. As the skip is raised and moved onto the lorry, an increasing fraction of the weight is carried by the front wheels.

(b) Let X = reaction on the front wheels, Y = reaction at the rear wheels. $\therefore X + Y = 130 \text{ kN}$. Take moments about the rear wheels to give $4.5X + 10 \times 2.5 = 120 \times 3.2$.

$$4.5X = 384 - 25 = 359 \quad \therefore X = 359/4.5 = 80 \text{ kN} \quad \therefore Y = 130 - 80 = 50 \text{ kN}$$

1.3

(a) (i) Take moments about the pivot: $W \times (250 - 10) = 6 \times (50 - 10)$

$\therefore W \times 240 = 6 \times 40 = 240$. Hence $W = 240/240 = 1.0 \text{ N}$. (ii) Force on the rule due to the pivot = $6 - 1 = 5 \text{ N}$ downwards.

(b) Take moments about the pivot:

$$S \times (50 - 10) = 2 \times (410 - 10) + 1.0 \times (250 - 10) \quad \therefore 40S = 800 + 240 = 1040$$

$$\text{Hence } S = 1040/40 = 26 \text{ N}$$

1.4

(a) Downward pull = $80 + 50g = 570 \text{ N}$. Let X and Y represent the support forces, so $X + Y = 2400 + 570 = 2970 \text{ N}$.

Take moments about Y : $8.0X = 2400 \times 4.0 + 570 \times 6.0 = 9600 + 3420 = 13020$

$$\therefore X = 13020/8 = 1630 \text{ N at the end near the hose. Hence } Y = 2970 - 1630 = 1340 \text{ N}$$

1.5

(a) Take moments about the front wheels: $F \times 1.5 = 1500 \times (0.40 - 0.25)$

$$\therefore F \times 1.5 = 1500 \times 0.15 \quad \text{Hence } F = 1500 \times 0.15/1.5 = 150 \text{ N}$$

(b) Take moments about the front wheels: $300h = 1500 \times (0.40 - 0.25)$

$$\therefore h = 1500 \times 0.15/300 = 0.75 \text{ m}$$

UNIT 2

In-text questions

2.1

1 110 km.

$$2 \text{ Speed} = \frac{\text{distance}}{\text{time}} = \frac{20 \text{ km}}{0.5 \text{ h}} = 40 \text{ km h}^{-1}$$

3 (a) $50 \text{ km} (= 180 - 110 - 20 \text{ km})$.

(b) 30 minutes.

$$(c) \text{ Average speed for final section} = \frac{\text{distance}}{\text{time}} = \frac{50 \text{ km}}{0.5 \text{ h}} = 100 \text{ km h}^{-1}$$

4 (a) See Fig. A2.1.

(b) (i) Average speed for the whole journey

$$= \frac{180 \text{ km}}{2 \text{ h}} = 90 \text{ km h}^{-1}$$

$$(ii) 90 \text{ km h}^{-1} = \frac{90 \times 1000 \text{ m}}{3600 \text{ s}} = 25 \text{ m s}^{-1}$$

2.2

$$1 \text{ (a) (i) Average speed} = \frac{\text{distance}}{\text{time}} = \frac{84 \text{ km}}{(70/60) \text{ h}} = 72 \text{ km h}^{-1}$$

$$(ii) 72 \text{ km h}^{-1} = \frac{72 \times 1000 \text{ m}}{3600 \text{ s}} = 20 \text{ m s}^{-1}$$

(b) Direct distance from junction 22 to junction 5 = $(48^2 + 30^2)^{1/2} = 57 \text{ km}$. Junction 5 is at angle θ East of due South from Junction 22, where

$$\tan \theta = \frac{30}{48} = 0.625 \quad \therefore \theta = 32^\circ$$

$\therefore$ displacement from junction 22 to junction 5 is 57 km at 32° East of due South.

(c) (i) due East (ii) due South.

2.3

1 (a) The vehicle slows down from an initial speed, u , of 30 m s^{-1} to rest. (i.e. $v = 0$) in 20 s

$$\therefore \text{acceleration, } a = \frac{(v - u)}{t} = \frac{(0 - 30)}{20} = -1.5 \text{ m s}^{-2}$$

(b) Distance travelled

$$= \text{area under the line} = \frac{1}{2}(v + u)t = 0.5 \times 30 \times 20 = 300 \text{ m}$$

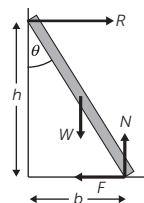


Fig. A1.8

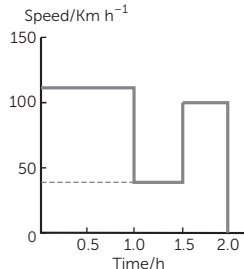


Fig. A2.1

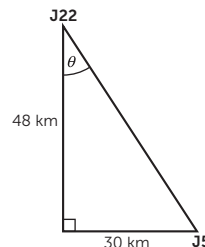


Fig. A2.2

$$2 \text{ (a) Acceleration} = \frac{(v-u)}{t} = \frac{(12-0)}{30} = 0.4 \text{ m s}^{-2}.$$

$$\text{Distance travelled} = \frac{1}{2}(v+u)t = 0.5 \times 12 \times 30 = 180 \text{ m}.$$

(b) (i) The speed is constant for this period.

$$(ii) \text{ Distance travelled} = \text{speed} \times \text{time} = 12 \times 100 = 1200 \text{ m}.$$

$$(c) (i) \text{ Acceleration} = \frac{(v-u)}{t} = \frac{(0-12)}{20} = -0.6 \text{ m s}^{-2}.$$

$$(ii) \text{ Distance travelled} = \frac{1}{2}(u+v)t = 0.5 \times 12 \times 20 = 120 \text{ m}.$$

$$(d) (i) \text{ Total distance travelled} = 180 + 1200 + 120 \text{ m} = 1500 \text{ m}.$$

$$(ii) \text{ Average speed} = \frac{1500 \text{ m}}{150 \text{ s}} = 10 \text{ m s}^{-1}.$$

2.4

$$1 \text{ } u = 0, s = 210 \text{ m}, v = 85 \text{ m s}^{-1}.$$

$$(a) \text{ To calculate } t, \text{ use } s = \frac{(u+v)}{2}t \text{ which becomes } s = \frac{vt}{2}$$

since $u = 0$. Rearranging this equation gives

$$t = \frac{2s}{v} = \frac{2 \times 210}{85} = 4.9 \text{ s}.$$

$$(b) \text{ To calculate } a, \text{ use } a = \frac{(v-u)}{t} = \frac{(85-0)}{4.9} = 17 \text{ m s}^{-2}.$$

$$2 \text{ } u = 115 \text{ m s}^{-1}, v = 0, s = 0.045 \text{ m}.$$

$$(a) \text{ To calculate } t, \text{ rearrange } s = \frac{(u+v)t}{2} \text{ with } v = 0 \text{ to give}$$

$$t = \frac{2s}{v} = \frac{2 \times 0.045}{115} = 0.00078 \text{ s}.$$

$$(b) \text{ To calculate } a, \text{ use } a = \frac{(v-u)}{t} = \frac{(0-115)}{0.00078}$$

$$= -147000 \text{ m s}^{-2} = -1.47 \times 10^5 \text{ m s}^{-2}.$$

$$3 \text{ } u = 28 \text{ m s}^{-1}, v = 0, s = 1.2 \text{ m}. \text{ To calculate } a, \text{ rearrange } v^2 = u^2 + 2as \text{ with } v = 0 \text{ to give}$$

$$a = \frac{-v^2}{2s} = \frac{-28 \times 28}{2 \times 1.2} = -330 \text{ m s}^{-2}.$$

2.5a

$$1 \text{ } u = 0, t = 1.8 \text{ s}, a = g.$$

(a) To calculate the depth of the well, use

$$s = ut + \frac{1}{2}at^2 = (0 \times 1.8) + (\frac{1}{2} \times 9.8 \times 1.8^2) = 16 \text{ m}.$$

(b) To calculate the speed just before impact, use

$$v = u + at = 0 + (9.8 \times 1.8) = 18 \text{ m s}^{-1}.$$

$$2 \text{ } u = 4.0 \text{ m s}^{-1} \text{ downwards}, s = 36 \text{ m}, a = g.$$

(a) To calculate the speed of the package just before impact, use

$$v^2 = u^2 + 2as = 4.0^2 + (2 \times 9.8 \times 36) = 722 \text{ m}^2 \text{ s}^{-2} \therefore v = \sqrt{722} = 27 \text{ m s}^{-1}.$$

(b) To calculate the time taken, rearrange $v = u + at$ to give

$$t = \frac{(v-u)}{a} = \frac{(27-4)}{9.8} = 2.3 \text{ s}.$$

2.5b

1 (a) The ball takes 2.2 s ($= 0.5 \times 4.4$ s) to reach maximum height. (b) Consider the ball from the point of projection to maximum height: $t = 2.2$ s, $v = 0$, $a = -9.8 \text{ m s}^{-2}$.

To calculate u , rearrange $v = u + at$ with $v = 0$ to give

$$u = -at = -(-9.8 \times 2.2) = 22 \text{ m s}^{-1}.$$

(c) To calculate the maximum height, use

$$s = ut + \frac{1}{2}at^2 = (22 \times 2.2) + (0.5 \times -9.8 \times 2.2^2) = 24 \text{ m}.$$

(d) Its speed on returning to the same height is the same as its speed of projection $= 0.22 \text{ m s}^{-1}$.

$$2 \text{ } u = +15 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2}, s = -90 \text{ m}.$$

$$(a) \text{ To calculate } v, \text{ use } v^2 = u^2 + 2as = 15^2 + (2 \times -9.8 \times -90) = 1989 \text{ m}^2 \text{ s}^{-2}.$$

$$\therefore v = \sqrt{1989} = 45 \text{ m s}^{-1}$$

$$(b) \text{ To calculate the time taken, rearrange } v = u + at \text{ to give } t = \frac{(v-u)}{a}.$$

Note that $v = -45 \text{ m s}^{-1}$ since the package is moving downwards before impact.

$$\text{Hence } t = \frac{(-45-15)}{-9.8} = 6.1 \text{ s}.$$

2.6

$$1 \text{ } t/\text{s} \quad s/\text{m} \quad v/\text{m s}^{-1} \quad t/\text{s} \quad s/\text{m} \quad v/\text{m s}^{-1}$$

$$0.000 \quad 0.000 \quad 24.500 \quad 3.000 \quad 29.400 \quad -4.900$$

$$0.500 \quad 11.020 \quad 19.600 \quad 3.500 \quad 25.730 \quad -9.800$$

$$1.000 \quad 19.600 \quad 14.700 \quad 4.000 \quad 19.600 \quad -14.700$$

$$1.500 \quad 25.730 \quad 9.800 \quad 4.500 \quad 11.020 \quad -19.600$$

$$2.000 \quad 29.400 \quad 4.900 \quad 5.000 \quad 0.000 \quad -24.500$$

$$2.500 \quad 30.630 \quad 0.000$$

Revision questions

2.1

$$(a) a = 1.5 \text{ m s}^{-2}, u = 0, t = 20 \text{ s}.$$

To calculate v , use

$$v = u + at = 0 + 1.5 \times 20 = 30 \text{ m s}^{-1}.$$

(b) See Fig. A2.3.

$$(c) t = 40 \text{ s}, u = 30 \text{ m s}^{-1}, a = 0.$$

To calculate a , use $v = u + at$:

$$0 = 30 + 40a. \text{ Hence } 40a = -30.$$

$$\therefore a = -30/40 = -0.75 \text{ m s}^{-2}.$$

$$(d) (i) 0 \rightarrow 20 \text{ s: } s = \frac{1}{2}(u+v)t = \frac{1}{2}(0+30) \times 20 = 300 \text{ m}.$$

$$\text{Next } 90 \text{ s: } s = \frac{1}{2}(u+v)t = 30 \times 90 = 2700 \text{ m (average speed} = \frac{1}{2}(u+v).$$

$$\text{Final } 40 \text{ s: } s = \frac{1}{2}(u+v)t = \frac{1}{2}(30+0) \times 40 = 600 \text{ m}.$$

$$(ii) \text{ Average speed} = \frac{\text{total distance}}{\text{time taken}} = \frac{(300 + 2700 + 600) \text{ m}}{150 \text{ s}} = 24 \text{ m s}^{-1}.$$

2.2

$$(a) u = 0, v = 6.5 \text{ m s}^{-1}, t = 30 \text{ s}. \text{ To calculate } a, \text{ use } v = u + at \text{ to give}$$

$$6.5 = 0 + 30a \text{ so } 30a = 6.5. \therefore a = 6.5/30 = 0.22 \text{ m s}^{-2}.$$

$$\text{To calculate } s, \text{ use } s = \frac{1}{2}(u+v)t = \frac{1}{2}(0+6.5) \times 30 = 97.5 \text{ m}.$$

$$(b) u = 6.5 \text{ m s}^{-1}, u = 0, s = 80 \text{ m}. \text{ To calculate } a, \text{ use } v^2 = u^2 + 2as \text{ to give}$$

$$0 = u^2 + 2as \text{ or } 2as = -u^2. \text{ Hence } a = \frac{-u^2}{2s} = \frac{-6.5^2}{(2 \times 80)} = -0.26 \text{ m s}^{-2}.$$

$$\text{To calculate } t, \text{ use } s = \frac{1}{2}(u+v)t \text{ to give } s = ut/2 \text{ or } t = 2s/u$$

$$\therefore t = 2 \times 80/6.5 = 25 \text{ s}.$$

(c) (i) See Fig. A2.4.

$$(ii) \text{ Average speed} = \frac{\text{total distance}}{\text{time taken}}$$

$$= \frac{97.5 + 80}{(30 + 25)} = \frac{177.5}{55} = 3.25 \text{ m s}^{-1}.$$

2.3

$$(a) u = +15 \text{ m s}^{-1}, v = 0 \text{ at max height},$$

$$a = -9.8 \text{ m s}^{-2}.$$

To calculate s (its maximum height), use

$$v^2 = u^2 + 2as \text{ to give } 0 = u^2 + 2as \text{ or } 2as = -u^2.$$

$$\text{Hence } s = \frac{-u^2}{2a} = \frac{-15^2}{2 \times -9.8} = 11.5 \text{ m}.$$

To calculate t , use $v = u + at$ to give

$$0 = u + at \text{ or } at = -u. \text{ Hence}$$

$$t = -u/a = -15/-9.8 = 1.5 \text{ s}.$$

$$(b) (i) u = +15 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2},$$

$$s = +5.0 \text{ m}.$$

$$\text{To calculate } u, \text{ use } v^2 = u^2 + 2as. \text{ Hence}$$

$$v^2 = 15^2 + 2 \times -9.8 \times 5.0 = 225 - 98 = 127$$

$$\therefore u = \sqrt{127} = -11.3 \text{ m s}^{-1},$$

(- for downwards).

(ii) To calculate t , use $v = u + at$ to give

$$at = v - u \text{ or } t = (v - u)/a$$

$$\text{Hence } t = \frac{(-11.3 - 15)}{-9.8} = \frac{-26.3}{-9.8} = 2.7 \text{ s}.$$

2.4

$$u = -3.2 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2},$$

$$s = -100 \text{ m}.$$

$$(a) \text{ To calculate } u, \text{ use } v^2 = u^2 + 2as. \text{ Hence } u^2 = 3.2^2 + 2 \times -9.8 \times -100 = 1970$$

$$\therefore u = \sqrt{1970} = 44 \text{ m s}^{-1}.$$

$$(b) \text{ To calculate } t, \text{ rearrange } v = u + at \text{ and substitute } v = -44 \text{ m s}^{-1}.$$

$$\text{Hence } t = \frac{(v-u)}{a} = \frac{(-44-3.2)}{-9.8} = 4.8 \text{ s}.$$

(c) In 4.8 s, the parachutist descends a distance of $3.2 \times 4.8 = 15 \text{ m}$ ($= \text{speed} \times \text{time}$). She will therefore be 85 m above the ground when the object hits the ground.

2.5

$$(a) (i) u = 0, a = 8.0 \text{ m s}^{-2}, t = 40 \text{ s}. \text{ To calculate } u, \text{ use } v = u + at = 0 + 8.0 \times 40 = 320 \text{ m s}^{-1}. \text{ To calculate its height at cut-off, use } s = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 8.0 \times 40^2 = 6400 \text{ m}.$$

(ii) To calculate its maximum height, calculate the distance moved upwards after

cut-off, i.e. $u = +320 \text{ m s}^{-1}, a = -9.8 \text{ m s}^{-2}, u = 0$. To calculate s , use

$$v^2 = u^2 + 2as \text{ to give } 0 = u^2 + 2as \text{ or } 2as = -u^2.$$

$$\text{Hence } s = \frac{-u^2}{2a} = \frac{-320^2}{(2 \times -9.8)} = 5200 \text{ m}$$

$$\therefore \text{maximum height} = 6400 + 5200 = 11600 \text{ m}.$$

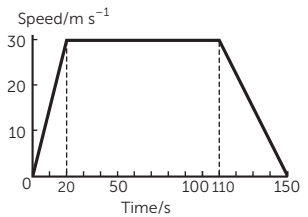


Fig. A2.3

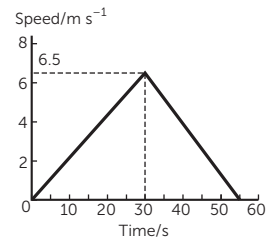


Fig. A2.4

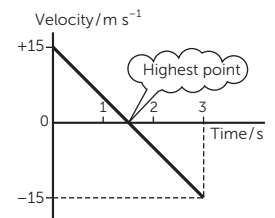


Fig. A2.5

(iii) To calculate its velocity, v , just before impact, consider its motion under gravity from cut-off to impact: $a = -9.8 \text{ m s}^{-2}$, $u = +320 \text{ m s}^{-1}$, $s = -6400 \text{ m}$.

Use $v^2 = u^2 + 2as = 320^2 + (2 \times -9.8 \times -6400)$

$= 2.28 \times 10^5 \text{ m}^2 \text{ s}^{-2}$ $\therefore v = -480 \text{ m s}^{-1}$ (– for downwards).

(iv) To calculate the time, t , taken from cut-off to impact, use $v = u + at$ or $at = v - u$.

Hence $t = \frac{(v - u)(-480 - 320)}{a} = \frac{-800}{-9.8} = 82 \text{ s}$.

$\therefore$ total time taken $= 82 + 40 = 122 \text{ s}$.

(b) See Fig. A2.6.

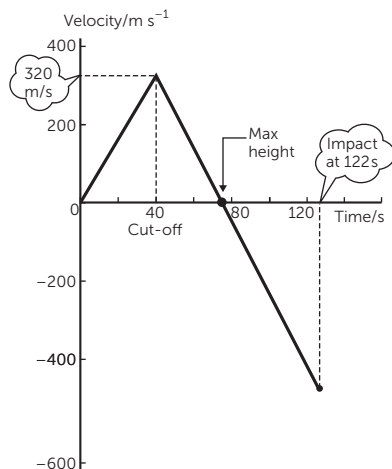


Fig. A2.6

UNIT 3

In-text questions

3.1

1 (a) $u = 25 \text{ m s}^{-1}$, $v = 0$, $t = 8.0 \text{ s}$. To calculate a , rearrange $v = u + at$ to give

$$a = \frac{(v - u)}{t} = \frac{(0 - 25)}{8.0} = -3.1 \text{ m s}^{-2}$$

(b) Force needed = mass $\times$ acceleration $= 1200 \times 3.1 = 3700 \text{ N}$.

2 (a) Weight $= mg = 8000 \times 9.8 = 78400 \text{ N}$. (b) $T - mg = ma$, where T is the thrust and a is the acceleration $\therefore T = mg + ma = 78400 + (8000 \times 6.0) = 126400 \text{ N}$.

3 (a) $T = mg = 1500 \times 9.8 = 14700 \text{ N}$.

(b) $a = +1.2 \text{ m s}^{-2}$. Using $T - mg = ma$ gives

$$T = mg + ma = 14700 + (1500 \times 1.2) = 16500 \text{ N}$$

(c) $a = +1.2 \text{ m s}^{-2}$ since the deceleration direction is upwards, in the opposite direction to the velocity. Using $T - mg = ma$ gives $T = mg + ma = 14700 + (1500 \times 1.2) = 16500 \text{ N}$.

3.2

1 (a) Change of momentum = final momentum – initial momentum $= (-0.12 \times 25) - (0.12 \times 25) = -6.0 \text{ kg m s}^{-1}$.

(b) Impact force $= \frac{\text{change of momentum}}{\text{time taken}} = \frac{-6.0}{0.004} = -1500 \text{ N}$.

2 (a) Weight $= mg = 2000 \times 9.8 = 19600 \text{ kg}$.

(b) $T - mg = ma$, where T is the thrust $\therefore T = mg + ma = 19600 + (2000 \times 6.5) = 32600 \text{ N}$.

3.3

1 (a) Loss of momentum of first vehicle $= (800 \times 25) - (800 \times 5) = 16000 \text{ kg m s}^{-1}$.

(b) Gain of momentum of second vehicle $= 1600v$, where v is its speed after the impact $\therefore 1600v = 16000$, which gives $v = 10 \text{ m s}^{-1}$.

(c) Impact force $= \frac{\text{change of momentum of either vehicle}}{\text{time taken}} = \frac{16000}{0.2} = 80000 \text{ N}$.

2 Let v = recoil speed of gun $\therefore 500v = 4.0 \times 115$, to give $v = 0.86 \text{ m s}^{-1}$.

Revision questions

3.1

(a) (i) $a = \frac{(v - u)}{t} = \frac{12 - 0}{8.0} = 1.2 \text{ m s}^{-2}$.

(ii) $F = ma = 1500 \times 1.2 = 1800 \text{ N}$.

(b) (i) $a = 0$ $\therefore 4T = mg = 850 \times 9.8 = 8300 \text{ N}$. $T = 2075 \text{ N}$.

(ii) $a = +0.5 \text{ m s}^{-2}$, $4T - mg = ma$ $\therefore 4T = mg + ma = 8300 + (850 \times 0.5) = 8700 \text{ N}$ (to 2 sig. figs) $\therefore T = 2175 \text{ N}$.

(c) (i) Fig. A3.1 shows the forces acting on the block. Consider the forces and their components parallel and perpendicular to the surface:

- Perpendicular: $N = mg = 12 \times 9.8 = 118 \text{ N}$.
- Parallel: $F - F_0 = ma$, where $F_0 = \mu N = \mu mg = 0.4 \times 12 \times 9.8 = 47 \text{ N}$. $\therefore 12a = 50 - 47 = 3$. Hence $a = 3/12 = 0.25 \text{ m s}^{-2}$.

(ii) Fig. A3.2 shows the forces acting on the block. Consider the forces and their components parallel and perpendicular to the surface:

- Perpendicular: $N = mg \cos 30 = 12 \times 9.8 \times \cos 30 = 102 \text{ N}$.
- Parallel: $F + mg \sin 30 - F_0 = ma$, where $F_0 = \mu N = 0.4 \times 102 = 41 \text{ N}$. $\therefore 12a = 50 + (12 \times 9.8 \times \sin 30) - 41 = 68$ $\therefore a = 68/12 = 5.7 \text{ m s}^{-2}$.

3.2

(a) Loss of momentum $= 0.001 \times 100 = 0.10 \text{ kg m s}^{-1}$.

(b) Rearrange $s = \frac{1}{2}(u + v)t$ with $u = 0$ to give

$$t = \frac{2s}{(u + v)} = \frac{2 \times 0.050}{100} = 0.0010 \text{ s}$$

(c) Force $= \frac{\text{change of momentum}}{\text{time taken}} = \frac{0.10}{0.0010} = 100 \text{ N}$

3.3

(a) Weight $= mg = 11800 \text{ N}$; acceleration $= \frac{\text{resultant force}}{\text{mass}} = \frac{16000 - 11800}{1200} = 3.5 \text{ m s}^{-2}$.

(b) Thrust force, $T = v \frac{dm}{dt}$ $\therefore \frac{dm}{dt} = \frac{T}{v} = \frac{16000}{1200} = 13.3 \text{ kg s}^{-1}$.

(c) Time taken to use 800 kg of fuel at $13.3 \text{ kg s}^{-1} = \frac{800}{13.3} = 60 \text{ s}$.

3.4

(a) Total initial momentum $= 3.0 \times 1000 = 3000 \text{ kg m s}^{-1}$. Total final momentum $= (1000 + 2000)V$, where V is the final speed of the two wagons $\therefore 3000V = 3000$. Hence $V = 1 \text{ m s}^{-1}$.

(b) Total initial momentum $= (3.0 \times 1000) + (2.0 \times 2000) = 7000 \text{ kg m s}^{-1}$. Total final momentum $= (1000 + 2000)V$, where V is the final speed of the two wagons $\therefore 3000V = 7000$. Hence $V = 7/3 = 2.3 \text{ m s}^{-1}$.

(c) Total initial momentum $= (3.0 \times 1000) - (2.0 \times 2000) = -1000 \text{ kg m s}^{-1}$ where the minus sign indicates the direction of the 2000 kg wagon. Total final momentum $= (1000 + 2000)V$, where V is the final speed of the two wagons $\therefore 3000V = -1000$. Hence $V = -1/3 = -0.3 \text{ m s}^{-1}$.

3.5

Let m represent the mass of the unknown object

$$\therefore (m + 0.5) \times 0.22 = 0.60 \times 0.35$$

$$m + 0.5 = \frac{0.60 \times 0.35}{0.22} = 0.95 \text{ kg} \therefore m = 0.95 - 0.50 = 0.45 \text{ kg}$$

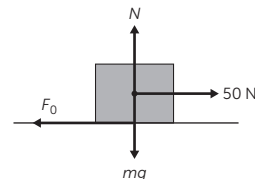


Fig. A3.1

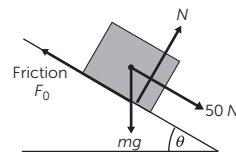


Fig. A3.2

UNIT 4

In-text questions

4.1

1 (a) Initial k.e. $= \frac{1}{2}mu^2 = 0.5 \times 0.12 \times 22^2 = 29 \text{ J}$.

(b) At maximum height, the ball is stationary for an instant and has zero k.e.

(c) Gain of p.e. = loss of k.e. $= 29 \text{ J}$.

(d) $mgh = 29$; $h = 29/0.12g = 25 \text{ m}$.

- 2 (a) Loss of p.e. = $mgh = 150 \times 9.8 \times 200 = 290 \text{ kJ}$.
 (b) Its gain of k.e. = $\frac{1}{2}mv^2 = 0.5 \times 150 \times 55^2 = 230 \text{ kJ}$.
 (c) Work done against friction, etc. = $290 - 230 = 60 \text{ kJ}$.
 (d) Friction force, $F \times \text{distance} = 60 \text{ kJ}$ $\therefore F = 60\,000/1500 = 40 \text{ N}$.

4.2

- 1 (a) (i) k.e. = $\frac{1}{2}mv^2 = 0.5 \times 70 \times 10^2 = 3500 \text{ J}$.
 (ii) power output = $\frac{\text{k.e. gain}}{\text{time taken}} = \frac{3500 \text{ J}}{3.5 \text{ s}} = 1000 \text{ W}$.

- (b) (i) p.e. gain = $mgh = 55 \times 9.8 \times 0.40 = 220 \text{ J}$.

- (ii) Time taken for 1 step = 3.0 s

$$\therefore \text{power output} = \frac{\text{p.e. gain}}{\text{time taken}} = \frac{220 \text{ J}}{3.0 \text{ s}} = 73 \text{ W}$$

- 2 (a) Use $P = Fv$ to give $F = \frac{P}{v} = \frac{175\,000}{55} = 3200 \text{ N}$.

- (b) At constant speed, resistive force = output force = 3200 N .

- (c) Method 1: rearrange $v^2 = u^2 + 2as$ with $u = 0$, $v = 55 \text{ m s}^{-1}$, $s = 1000 \text{ m}$, to obtain

$$a = \frac{(v^2 - u^2)}{2s} = \frac{3025}{(2 \times 1000)} = 1.5 \text{ m s}^{-2}. \text{ Hence } F = ma = 60\,000 \times 1.5 = 90\,000 \text{ N}.$$

Method 2: Calculate the loss of k.e. and use work done, Fs = loss of k.e. to calculate F .

4.3

- 1 (a) Electrical energy supplied to motor = power $\times$ time = $300 \times 25 = 7500 \text{ J}$.

- (b) Useful energy from motor = $mgh = 320 \times 9.5 = 3000 \text{ J}$.

- (c) Energy wasted = $7500 - 3000 = 4500 \text{ J}$.

- (d) Efficiency = $\frac{3000}{7500} = 0.40$.

- 2 (a) p.e. gain = $150 \text{ N} \times 2.5 \text{ m} = 375 \text{ J}$.

- (b) Work done by effort = effort $\times$ distance moved = $40 \text{ N} \times (4 \times 2.5 \text{ m}) = 400 \text{ J}$.

- (c) Efficiency = $\frac{375}{400} = 0.94$.

4.4

- 1 (a) 10 m^2 (= $5000 \text{ W}/500 \text{ W}$).

- (b) Energy needed to heat 0.012 kg from 15°C to 30°C = $0.012 \times 4200 \times (30 - 15) = 760 \text{ J}$ $\therefore$ power = 760 W .

- 2 (a) $P = \frac{1}{2} \times 1.2 \times \pi \times 15^2 \times 5^3 = 50\,000 \text{ W}$.

- (b) $P = \frac{1}{2} \times 1.2 \times \pi \times 15^2 \times 15^3 = 1.4 \times 10^6 \text{ W}$.

Revision questions

4.1

- (a) (i) $h = 40 \sin 5^\circ = 3.5 \text{ m}$. (ii) p.e. gain = $mgh = 120 \times 9.8 \times 3.5 = 4120 \text{ J}$.

- (b) (i) $F = \frac{\text{p.e. gain}}{\text{distance}} = \frac{4120}{40} = 103 \text{ N}$.

- (ii) Friction between the wheels and the wheel bearings must be overcome.

- (c) Work done/sec = $\frac{\text{p.e. gain}}{\text{time taken}} = \frac{4120}{45} = 91 \text{ W}$.

4.2

- (a) (i) $h = \frac{s}{20} = \frac{21}{20} = 1.05 \text{ m}$, since $s = 21 \text{ m}$ in 1 second.

- (ii) p.e. gain = $mgh = 600 \times 9.8 \times 1.05 = 6.2 \text{ kJ}$.

- (b) (i) Energy delivered by engine in $1 \text{ s} = 25 \text{ kJ}$. Work done against resistive forces = energy delivered - p.e. gain = $25 - 6.2 = 18.8 \text{ kJ}$.

- (ii) Resistive force = $\frac{\text{work done against friction}}{\text{distance}} = \frac{18\,800}{21} = 895 \text{ N}$.

4.3

- (a) k.e. gain of pile driver = loss of p.e. $\frac{1}{2}mv^2 = mgh$

$$\therefore v = \sqrt{2gh} = \sqrt{(2 \times 9.8 \times 3.0)} = 7.7 \text{ m s}^{-1}.$$

- (b) $(4000 + 6000)V = 4000 \times 7.7$, where V is the speed of the pile driver and the girder immediately after impact.

$$\text{Thus } V = \frac{4000 \times 7.7}{10\,000} = 3.1 \text{ m s}^{-1}.$$

- (c) k.e. of girder and block after impact = $\frac{1}{2}(4000 + 6000) \times 3.1^2 = 48\,050 \text{ J}$

$$\text{Average resistive force} = \frac{\text{loss of k.e. of girder and block}}{\text{distance moved}} = \frac{48\,050}{1.5} = 32\,000 \text{ N}.$$

4.4

- (a) (i) p.e. gain = $m_xgh_0 = 0.25 \times 9.8 \times 0.10 = 0.245 \text{ J}$.

- (ii) k.e. gain = loss of p.e. on downswing $\therefore \frac{1}{2}m_xu_0^2 = mgh$ hence $u_0 = \sqrt{(2gh_0)} = \sqrt{(2 \times 9.8 \times 0.10)} = 1.4 \text{ m s}^{-1}$.

- (b) (i) p.e. gain = $m_ygh_2 = 0.15 \times 9.8 \times 0.05 = 0.27 \text{ J}$.

- (ii) k.e. loss = p.e. gain on upswing $\therefore \frac{1}{2}m_yv_2^2 = mgh$ hence $v_2 = \sqrt{(2gh_2)} = \sqrt{(2 \times 9.8 \times 0.05)} = 0.99 \text{ m s}^{-1}$.

- (c) (i) $m_xu_0 = m_xv_1 + m_yv_2$. Hence $0.25 \times 1.4 = 0.25v_1 + 0.15 \times 0.99$

$$\therefore 0.25v_1 = 0.20. \text{ Hence } v_1 = 0.20/0.25 = 0.80 \text{ m s}^{-1}.$$

- (ii) Using p.e. gain = k.e. loss on the upswing, $mgh_1 = \frac{1}{2}m_xv_1^2$

$$\therefore h_1 = \frac{v_1^2}{2g} = \frac{0.8^2}{2 \times 9.8} = 0.032 \text{ m}.$$

4.5

- (a) Using conservation of momentum, $(1200 + 800)V = 1200 \times 30$, where V is the speed after the impact

$$\therefore V = \frac{1200 \times 30}{2000} = 18 \text{ m s}^{-1}.$$

- (b) Loss of k.e. = initial k.e. - final k.e. = $(\frac{1}{2} \times 1200 \times 30^2) - (\frac{1}{2} \times 2000 \times 18^2) = 220\,000 \text{ J}$.

- (c) Force = $\frac{\text{change of momentum}}{\text{time taken}} = \frac{(1200 \times 18) - (1200 \times 30)}{0.12} = 120\,000 \text{ N}$.

UNIT 5

In-text questions

5.1

- 1 (a) Solid: it keeps its own shape after removal from its dish. (b) Liquid: the spray consists of tiny droplets which gradually fall unless they adhere to a surface.

- 2 (a) Ice is a solid which turns to water in the liquid state at 0°C , which then boils and turns to steam at 100°C .

- (b) Water droplets in the fabric evaporate and the vapour is removed by the wind.

5.2a

- 1 (a) $1p + 2n$ (b) $8p + 8n$ (c) $11p + 12n$ (d) $82p + 124n$. (e) $92p + 146n$.

- 2 (a) ${}^4_2\text{He}$ (b) ${}^{12}_6\text{C}$.

5.2b

- 1 (a) 18 (b) 28 (c) 16 (d) 17 (e) 80.

- 2 Radius of oil droplet, $r = 0.40 \text{ mm}$; diameter of oil patch, $D = 350 \text{ mm}$

$$\therefore \text{volume of oil patch, } \frac{\pi D^2 t}{4} = \text{volume of oil droplet, } \frac{4\pi r^3}{3}$$

where t = patch thickness $\therefore$ estimate of length of oil

$$\text{molecule} = t = \frac{4\pi(0.40 \times 10^{-3})^3 \times 4}{3\pi(0.35)^2} = 2.8 \times 10^{-9} \text{ m}.$$

5.3

- 1 (a) (i) 12 g of carbon contains 6.02×10^{23} atoms $\therefore 1 \text{ kg}$ of carbon contains 5.0×10^{25} atoms ($6.02 \times 10^{23}/0.012$).

- (ii) 235 g of Uranium contains 6.02×10^{23} atoms $\therefore 1 \text{ kg}$ of uranium contains 2.5×10^{24} atoms ($6.02 \times 10^{23}/0.235$).

- (b) (i) 44 g of CO_2 contains 6.02×10^{23} molecules $\therefore 1 \text{ kg}$ of carbon dioxide contains 1.4×10^{25} molecules.

- (ii) 16 g of methane contains 6.02×10^{23} molecules $\therefore 1 \text{ kg}$ of methane contains 3.8×10^{25} molecules.

- 2 (a) 27 g of aluminium contains 6.02×10^{23} atoms

$$\therefore \text{the mass of 1 atom of aluminium} = 27 \text{ g}/6.02 \times 10^{23} = 4.5 \times 10^{-23} \text{ grams}.$$

- (b) The volume of 1 aluminium atom = mass of

$$1 \text{ atom}/\text{density} = 1.67 \times 10^{-29} \text{ m}^3.$$

$$\therefore \text{diameter} = (1.67 \times 10^{-29})^{1/3} = 2.6 \times 10^{-10} \text{ m}.$$

5.4

- 1 (a) (i) covalent (ii) metallic (iii) ionic (iv) molecular (v) none.

- (b) (i) 2, 4 (ii) See Fig. 5.14.

- 2 (a) no, yes (b) yes, yes (c) yes, no (d) yes, no.

Revision questions

5.1

- (a) The atoms and molecules are locked together. (b) The molecules of a liquid move about at random and therefore the liquid flows to take the shape of its container. (c) Carbon dioxide molecules are heavier than air molecules so they do not move away as quickly as the molecules of a lighter gas would.

5.2

- (a) Water droplets freeze. (b) Water vapour condenses to form a film of water droplets on the windscreen. (c) Solid ice turns directly to vapour.

5.3

- (a) $17p + 20n$, $17p + 18n$. (b) 35.5 g (= $(3 \times 35 + 1 \times 37) \div 4$).

5.4

- (a) (i) 2, 4 (ii) 1.

- (b) (i) 16 g (= $12 + 4$) (ii) $2.7 \times 10^{-26} \text{ kg}$ (= $0.016 \text{ kg} \div N_A$)

(iii) See Fig. A5.1.

5.5

(a) $92p + 143n$ (b) 2.6×10^{21} ($= N_A \div 235$).

5.6

(a) 3.0×10^{-26} kg ($= 0.018 \div N_A$). (b) 3.1×10^{-10} m ($= (3.0 \times 10^{-26} \div 1000)^{1/3}$).

5.7

(a) 3.4×10^{-25} kg ($= 0.207 \div N_A$) (b) 3.1×10^{-10} m ($= (3.4 \times 10^{-25} \div 11340)^{1/3}$).

5.8

(a) $0.80 \times 28 \text{ g} + 0.20 \times 32 \text{ g} = 28.8 \text{ g}$. (b) (i) 3.4 nm ($= [0.029 \div (N_A \times 1.2)]^{1/3}$)

(ii) The average spacing is about $10 \times$ the diameter of an air molecule.

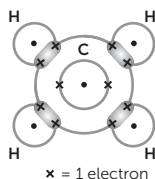


Fig. A5.1

2 (a) The mass of water boiled away in 1 second $= \frac{0.10 \text{ kg}}{120 \text{ s}} = 8.3 \times 10^{-4} \text{ kg s}^{-1}$
 $\therefore$ energy needed per second $= ml = 8.3 \times 10^{-4} \times 2.3 \times 10^6 = 1.9 \times 10^3 \text{ J s}^{-1}$.

(b) Base area, $A = \frac{\pi D^2}{4} = \frac{\pi (0.12)^2}{4} = 1.13 \times 10^{-2} \text{ m}^2$.

Rearranging $\frac{Q}{t} = kA \frac{(T_1 - T_2)}{L}$ gives $(T_1 - T_2) = \frac{LQ/t}{kA}$

$\therefore (T_1 - T_2) = \frac{0.120 \times 1.9 \times 10^3}{210 \times 1.13 \times 10^{-2}} = 96 \text{ K} = 96^\circ \text{C}$.

Since the top side of the base is at 100°C , the underside must therefore be at a temperature of 196°C .

3 (a) Total window area $= 4 \times 1 \times 1.5 = 6 \text{ m}^2$.

(b) (i) Total heat loss per second $= 6 \times 4.3 \times 10 = 258 \text{ W}$

(ii) Total heat loss per second $= 6 \times 3.2 \times 10 = 192 \text{ W}$.

(c) For (b)(i), heat loss per day $= 258 \times 24 \times 3600 = 2.2 \times 10^7 \text{ J}$.

$\therefore$ cost per day $= 22 \text{ p}$.

For (b)(ii), heat loss per day $= 192 \times 24 \times 3600 = 1.7 \times 10^7 \text{ J}$.

$\therefore$ cost per day $= 17 \text{ p}$.

UNIT 6

In-text questions

6.1

1 See Fig. A6.1.

2 (a) $\Delta L = \alpha L \Delta T = 1.9 \times 10^{-5} \times 0.80 \times 25 = 7.6 \times 10^{-5} \text{ m}$.

(b) $\Delta L = \alpha L \Delta T = 2.6 \times 10^{-5} \times 1.80 \times 25 = 9.8 \times 10^{-4} \text{ m}$.

6.2

1 (a) Energy needed

$= 5.0 \times 2100 \times (50 - 10) = 420\,000 \text{ J}$.

(b) Energy needed $= 1500 \times 850 \times (35 - 5) = 3.8 \times 10^7 \text{ J} = 38 \text{ MJ}$.

(c) Energy needed

$= (15 \times 900 \times 60) + (95 \times 4200 \times 60)$

$= 0.8 \times 10^6 + 23.9 \times 10^6 = 24.7 \text{ MJ}$.

2 Energy needed to heat the calorimeter

$= 0.055 \times 390 \times (62 - 15) = 1010 \text{ J}$

$\therefore$ energy available to heat the liquid $= 9340 - 1010 = 8330 \text{ J}$.

Mass of liquid $= 143 - 55 = 88 \text{ g} = 0.088 \text{ kg}$; temperature rise of liquid $= 47^\circ \text{C}$.

Since energy needed to heat the liquid $=$ mass of liquid $\times$ specific heat capacity $\times$ temperature rise, then $0.088 \times c \times 47 = 8330$.

$\therefore c = \frac{8330}{0.088 \times 47} = 2010 \text{ J kg}^{-1} \text{ K}^{-1}$

6.3

1 Energy to heat 0.20 kg of ice from -10°C to $0^\circ \text{C} = 0.20 \times 2100 \times 10 = 4200 \text{ J}$.

Energy needed to melt 0.20 kg of ice $= 0.20 \times 340\,000 = 68\,000 \text{ J}$.

Energy needed to heat 0.20 kg of water from 0°C to 100°C

$= 0.20 \times 4200 \times (100 - 0) = 84\,000 \text{ J}$. Energy needed to boil away 0.20 kg of

water $= 0.20 \times 2.3 \text{ MJ} = 460\,000 \text{ J}$. Total energy needed

$= 4.2 \text{ kJ} + 68 \text{ kJ} + 84 \text{ kJ} + 460 \text{ kJ} = 616.2 \text{ kJ}$.

2 (a) Energy needed $= 0.12 \times 4200 \times (52 - 20) = 2.6 \times 10^4 \text{ J}$.

(b) Energy released by mass, m , of steam when it condenses $= ml$, where

$l = 2.3 \times 10^6 \text{ J kg}^{-1}$. Energy released when mass, m , of water cools from 100°C to

$52^\circ \text{C} = m \times 4200 \times (100 - 52) = 2.18 \times 10^5 \text{ m}$. $\therefore$ total energy released by

mass $m = 2.18 \times 10^5 \text{ m} + 2.3 \times 10^6 \text{ m} = 2.32 \times 10^6 \text{ m}$.

Assuming energy released by the steam $=$ energy needed to heat the water,

$2.32 \times 10^6 \text{ m} = 2.6 \times 10^4$

$\therefore m = \frac{2.6 \times 10^4}{2.32 \times 10^6} = 1.13 \times 10^{-2} \text{ kg} = 11.3 \text{ g}$.

6.4

1 (a) To minimise heat loss due to thermal radiation.

(b) To reduce heat loss due to thermal conduction.

(c) To prevent heat loss due to convection and evaporation.

2 Cork, polystyrene or any other material which is a good thermal insulator.

2 The surface area of the hot plate $= \frac{\pi d^2}{4} = \frac{\pi (0.10)^2}{4} = 7.9 \times 10^{-3} \text{ m}^2$.

Using Stefan's Law, energy per second radiated from the surface

$= e\sigma AT^4 = 1 \times 7.9 \times 10^{-3} \times 5.67 \times 10^{-8} \times (1200)^4 = 920 \text{ W}$.

6.5

1 Using $\frac{Q}{t} = kA \frac{(T_1 - T_2)}{L}$ with $A = 1.0 \text{ m}^2$, $(T_1 - T_2) = 15^\circ \text{C}$

and $L = 0.120 \text{ m}$ $\therefore \frac{Q}{t} = 0.40 \times 1 \times \frac{15}{0.12} = 50 \text{ W}$.

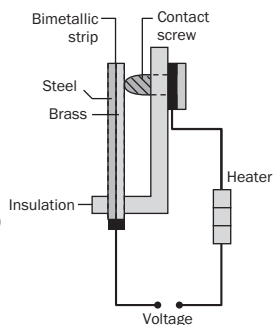


Fig. A6.1

Revision questions

6.1

(a) (i) 373 K (ii) 243 K (b) (i) -173 K (ii) 727 K .

6.2

(a) (i) To allow for thermal expansion in summer. Without the gaps, the road spans would press against each other and crack.

(ii) The tyre expands when heated. It can then be fitted on the wheel and when it cools it tightens on the wheel because it contracts.

(b) Change of length $= \alpha L \Delta T = 1.1 \times 10^{-5} \times 120 \times (35 - -5) = 5.3 \times 10^{-2} \text{ m}$.

6.3

(a) (i) Energy to heat water $= 0.185 \times 4200 \times (27 - 15) = 9320 \text{ J}$.

(ii) Energy needed to heat the calorimeter $= 0.065 \times 390 \times (27 - 15) = 304 \text{ J}$.

(b) Total energy released by metal $= 9320 + 304 = 9624 \text{ J}$

$\therefore 0.235 \times c \times (100 - 27) = 9624$.

Hence $c = \frac{9624}{0.235 \times 73} = 560 \text{ J kg}^{-1} \text{ K}^{-1}$

6.4

(a) (i) Energy removed $= (0.080 \times 390 \times 16) + (0.120 \times 4200 \times 16) = 8560 \text{ J}$.

(ii) Energy removed on freezing $= 0.120 \times 340\,000 = 40\,800 \text{ J}$.

(b) Total energy removed in 35 minutes $= 40\,800 + 8560 = 49\,360 \text{ J}$

$\therefore$ energy removed per second $= \frac{49\,360}{35 \times 60} = 24 \text{ J s}^{-1}$

6.5

(a) Net rate of heat transfer

$= [0.6 \times 5.67 \times 10^{-8} \times 1.6 \times (273 + 45)^4] - [0.6 \times 5.67 \times 10^{-8} \times 1.6 \times (273 + 25)^4]$

$= 557 - 429 = 128 \text{ W}$.

(b) For $A = 1 \text{ m}^2$, net rate of heat loss $= [1 \times 5.67 \times 10^{-8} \times 1 \times (273 + 10)^4] -$

$[1 \times 5.67 \times 10^{-8} \times 1 \times (273 + 0)^4] = 364 - 315 = 49 \text{ W}$.

6.6

(a) (i) Temperature gradient $= \frac{(54 - 26)}{15 \times 10^{-3}} = 1870 \text{ K m}^{-1}$

(ii) Heat conducted per second $= 0.037 \times 0.95 \times 1870 = 66 \text{ J s}^{-1}$.

(b) Heat conducted per second $= 0.2 \times 0.12 \times \frac{(22 - 18)}{10 \times 10^{-3}} = 9.6 \text{ W}$.

6.7

(a) Heat loss/second $= 2.0 \times 80 \times 20 = 3200 \text{ W}$

(b) Heat loss/second $= 0.5 \times 80 \times 20 = 800 \text{ W}$.

6.8

(a) (i) Area of sides - windows area $= (24 \times 2.5) - 4 = 56 \text{ m}^2$.

(ii) Floor area $= 9 \times 3 = 27 \text{ m}^2$ (iii) roof area $= 9 \times 3 = 27 \text{ m}^2$.

(b) (i) Heat loss/second $= (2.5 \times 56 \times 5)$ for the sides $+ (2.5 \times 27 \times 5)$ for the roof $+ (2.0 \times 27 \times 5)$ for the floor $+ (4.0 \times 4 \times 5)$ for the windows $= 1390 \text{ W}$.

(ii) Heat loss in 1 week $= 1390 \times 3600 \times 24 \times 7 = 8.4 \times 10^8 \text{ J}$.

Cost $= 2p \times 8.4 \times 10^8 / 1 \times 10^6 = \text{£}16.78$.

UNIT 7

In-text questions

7.1

1 (a) (i) Weight $= 0.40 \times 9.8 = 3.9 \text{ N}$ (ii) Extension $= 120 \text{ mm}$.

(iii) $k = 3.9 \text{ N} / 0.120 \text{ m} = 32.5 \text{ N m}^{-1}$.

(b) (i) Extension, $e = 390 - 300 = 90$ mm

$\therefore$ tension $= ke = 32.5 \times 0.090 = 2.93$ N $\therefore$ weight $= 2.93$ N.

(ii) Mass $= \text{weight}/g = 0.30$ kg.

2 (a) Tension/N 0 2.0 4.0 6.0 8.0
Extension/m 0 0.052 0.100 0.152 0.198

(b) (i) See Fig. A7.1.

(ii) Spring constant = gradient of line $= 40$ N m $^{-1}$.

7.2

1 (a) Brittle, not too stiff or strong.

(b) Tough, strong, not too stiff, lightweight. (c) Tough, strong, stiff, lightweight.

2 No, the extension is not proportional to the length so equal weights would not extend the rubber band by equal increases of length. Also, the unloading curve differs to the loading curve so the extension depends on whether the rubber band is being loaded or unloaded.

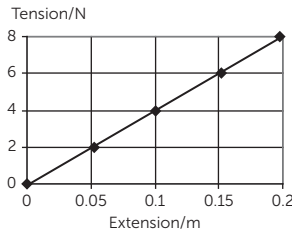


Fig. A7.1

7.3

1 (a) (i) Area of cross-section, $A = \pi(0.38 \times 10^{-3})^2/4 = 1.13 \times 10^{-7}$ m ^{2}
 $\therefore$ stress $= \frac{\text{tension}}{\text{area of cross-section}} = 1.0 \times 10^9$ Pa

(ii) Strain $= \frac{\text{stress}}{E} = \frac{1.0 \times 10^9}{2.0 \times 10^{11}} = 5.0 \times 10^{-3}$

$\therefore$ extension $= \text{strain} \times \text{length} = 5.0 \times 10^{-3} \times 2.37 = 1.2 \times 10^{-2}$ m.

(b) Maximum weight = breaking stress $\times$ area of cross-section
 $= 1.1 \times 10^9 \times 1.13 \times 10^{-7} = 120$ N.

2 (a) See Fig. A7.2.

(b) (i) Approx. 2.5 mm.

(ii) Gradient of straight section of line

$$= \frac{10}{1.6 \times 10^{-3}} = 6.25 \times 10^3 \text{ N m}^{-1}$$

$$\therefore \text{Young modulus} = \frac{\text{gradient} \times \text{length}}{\text{area of cross-section}} = \frac{6.25 \times 10^3 \times 1.28}{\pi(0.28 \times 10^{-3})^2/4} = 1.3 \times 10^{11} \text{ Pa}.$$

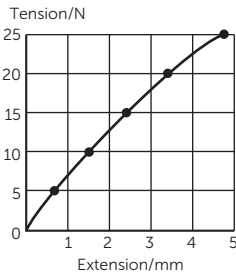


Fig. A7.2

7.4

1 (a) (i) See Fig. 7.14. (ii) The force between two atoms is proportional to the change of separation, up to a limit.

(b) See p. 102.

2 (a) B is stronger because its UTS is greater.

(b) A is stiffer because its Young modulus (= initial gradient) is greater.

7.5

1 (a) (i) 0.195 m (= 515 - 320 mm) (ii) $k = \frac{T}{e} = \frac{5.0}{0.195} = 25.6$ N m $^{-1}$.

(b) Energy stored $= \frac{1}{2}Te = \frac{1}{2} \times 5.0 \times 0.195 = 0.49$ J.

2 (a) (i) 0.035 m (= 785 - 750 mm)

(ii) Area of cross-section $A = \pi d^2/4 = \frac{1}{4}\pi(5.0 \times 10^{-4})^2 = 1.96 \times 10^{-7}$ m ^{2}

Rearranging $E = \frac{TL}{Ae}$ gives

$$T = \frac{AEe}{L} = \frac{1.96 \times 10^{-7} \times 3.0 \times 10^9 \times 0.035}{0.750} = 27.5 \text{ N}.$$

(b) Volume $= AL = 1.96 \times 10^{-7} \times 0.75 = 1.47 \times 10^{-7}$ m ^{3} .

Energy stored $= \frac{1}{2}Te = \frac{1}{2} \times 27.5 \times 0.035 = 0.48$ J.

$$\therefore \text{energy stored per unit volume} = \frac{0.48}{1.47 \times 10^{-7}} = 3.3 \times 10^6 \text{ J m}^{-3}$$

Revision questions

7.1

(a) (i) Extension $= 345$ mm $= 0.345$ m

$\therefore$ spring constant, $k = T/e = 6.0$ N/0.345 m $= 17.4$ N m $^{-1}$.

(ii) Energy stored $= \frac{1}{2}Te = 0.5 \times 6.0 \times 0.345 = 1.04$ J.

(b) (i) Using $T = ke$, weight = tension $= 17.4 \times (0.728 - 0.500) = 4.0$ N.

(ii) Mass $= \text{weight}/g = 4.0/9.8 = 0.40$ kg.

7.2

(a) (i) B (ii) B.

(b) (i) See Fig. 7.5. (ii) Elasticity is the ability to regain shape after being distorted. The elastic band regains its original length when it is unloaded. The polythene strip does not regain its original length and it therefore undergoes plastic behaviour.

7.3

(a) Extension $= 1.414 - 1.393 = 0.021$ m due to 60 N of extra tension.

(b) Young modulus $= \frac{TL}{eA} = \frac{60 \times 1.393}{0.021 \times \pi(0.20 \times 10^{-3})^2/4} = 1.3 \times 10^{11}$ Pa.

(c) Energy stored $= \frac{1}{2}Te = 0.5 \times 60 \times 0.021 = 0.63$ J.

7.4

(a) Maximum weight = maximum allowed stress $\times$ cross-sectional area of the cable $= 2.0 \times 10^8 \times \pi(0.030)^2/4 = 1.4 \times 10^5$ N.

(b) (i) $E = \frac{TL}{Ae} \therefore e = \frac{TL}{AE} = \frac{1.4 \times 10^5 \times 55}{\pi(0.03)^2/4 \times 2.0 \times 10^{11}} = 5.5 \times 10^{-2}$ m.

(ii) Energy stored $= \frac{1}{2}Te = 0.5 \times 1.4 \times 10^5 \times 5.5 \times 10^{-2} = 3.9 \times 10^3$ J.

Volume of wire $= LA = 55 \times \pi(0.030)^2/4 = 3.9 \times 10^3$ m ^{3} .

$$\therefore \text{energy stored per unit volume} = \frac{3.9 \times 10^3 \text{ J}}{3.9 \times 10^3 \text{ m}^3} = 1.0 \times 10^5 \text{ J m}^{-3}.$$

7.5

(a) 0.25 mm.

(b) Rearranging $E = \frac{TL}{Ae}$ gives

$$T = \frac{EAe}{L} = \frac{1.3 \times 10^{11} \times \pi(0.010)^2 \times 0.25 \times 10^{-3}}{60 \times 10^{-3} \times 4} = 4.3 \times 10^4 \text{ N}.$$

7.6

(a) Area of cross-section of each cable $= \pi(0.005)^2/4 = 2.0 \times 10^{-5}$ m ^{2}

$\therefore$ maximum safe tension in each cable = maximum allowed stress $\times$ area of cross-section $= 2.0 \times 10^3$ N

$\therefore$ maximum allowed weight $= 2.0 \times 10^3 \times 8 = 1.6 \times 10^4$ N. Maximum passenger weight $= 16.0$ kN $- 1.80$ kN $= 14.2$ kN.

(b) Rearrange $E = \frac{TL}{Ae}$ to give $e = \frac{TL}{AE}$

$$\therefore \text{extension for tension of } 2.0 \times 10^3 \text{ N} = \frac{2.0 \times 10^3 \times 65}{2.0 \times 10^{-5} \times 2.0 \times 10^{11}} = 3.2 \times 10^{-2} \text{ m}.$$

UNIT 8

In-text questions

8.1

1 (a) The sharper the needle, the less the area of contact between its tip and the cloth when it is pushed into the cloth. The pressure of a sharp needle on the cloth is therefore greater than that of a blunt needle for the same force.

(b) The contact area between the tyre and the earth is larger for a wide tyre in comparison with a narrow tyre. Hence the pressure of the tractor on the earth is less and it is less likely to sink into the earth.

2 (a) $p = \frac{F}{A} = \frac{750}{8.5 \times 10^{-3}} = 8.8 \times 10^4$ Pa.

(b) Total area, $A = \frac{F}{p} = 12\,000\,250,000 = 4.8 \times 10^{-2}$ m ^{2}

$\therefore$ area of each tyre in contact $= 1.2 \times 10^{-2}$ m ^{2} .

8.2

1 The effort is applied to one end of the lever which causes a greater force to be applied to the piston in the narrow cylinder. The force on the piston in the master cylinder is greater than the force on the piston in the narrow cylinder. The force of the master cylinder on the load acts via another lever system which enables the piston in the master cylinder to exert a larger force on the load.

2 Maximum force = maximum pressure $\times$ total area of all four pistons $= 500 \times 10^3 \times 4 \times 0.012 = 2.4 \times 10^4$ N $\therefore$ maximum additional weight $= 24\,000 - 2500$ N $= 21\,500$ N.

8.3

1 $p = H\rho g = 20.0 \times 1030 \times 9.8 = 2.0 \times 10^5$ Pa.

2 Rearrange $p = H\rho g$ to give $H = \frac{p}{\rho g} = \frac{101 \times 10^3}{1030 \times 9.8} = 10.1$ m.

8.4

1 (a) (i) $p = H\rho g = 0.25 \times 1000 \times 9.8 = 2.5$ kPa. (ii) $2.4\% \left(= \frac{2.5 \text{ kPa}}{101 \text{ kPa}} \times 100\% \right)$.

(b) (i) It would be insufficient to drive gas through the pipes. (ii) It would cause gas to leak from the pipes.

2 (a) The pressure in the space at the top is negligible. The column of mercury is supported by atmospheric pressure acting on the open surface of mercury in the reservoir.

(b) (i) For $H = 80$ mm, $p = H\rho g = 0.080 \times 13\,600 \times 9.8 = 1.1 \times 10^4$ kPa $= 11$ kPa. For $H = 120$ mm, $p = H\rho g = 0.120 \times 13\,600 \times 9.8 = 1.6 \times 10^4$ Pa $= 16$ kPa.

(ii) To produce the same pressure with water rather than mercury, the manometer tube would need to be about 13 times longer, which is unrealistic.

8.5

1 (a) $p = \frac{F}{A} = \frac{700}{2.0 \times 0.060} = 580 \text{ Pa}$.

(b) Rearrange $p = H\rho g$ to give

$$H = \frac{p}{\rho g} = \frac{580}{1000 \times 9.8} = 0.060 \text{ m}.$$

2 (a) Weight of water displaced when it carries maximum load = $0.05 \times A\rho g$, where A is the area of the base of the hull. This is equal to the extra upthrust to keep it floating and the extra upthrust is equal to the extra weight, i.e. $0.050A\rho g = 40\,000$. When empty, the upthrust = weight of water displaced = $1.20A\rho g$. This is equal to the weight of the empty boat.

$$\therefore \text{weight of empty boat} = 1.20A\rho g = \frac{1.20 \times 40\,000}{0.050} = 960\,000 \text{ N}.$$

(b) The total weight when fully loaded = $1000\,000 \text{ N}$. $\therefore$ the weight of water displaced = upthrust = $1000\,000 \text{ N}$. Hence $H'\rho g = 1000\,000$ where H' = depth of the base of the hull below the water line, and $\rho = 1000 \text{ kg m}^{-3}$.

$$\text{Since } 0.050A\rho g = 40\,000, \text{ then } \frac{H'}{0.050} = 25 \times \frac{1050}{1000} = 26.25$$

$$\therefore H' = 26.3 \times 0.050 = 1.31 \text{ m}.$$

Revision questions

8.1

(a) Weight = mass $\times g$ = volume $\times$ density $\times g$
 $= (0.450 \times 0.300 \times 0.025) \times 2600 \times 9.8 = 86 \text{ N}$.

(b) (i) Face area = $0.45 \times 0.30 = 0.135 \text{ m}^2$

$$\therefore \text{pressure} = \frac{\text{weight}}{\text{area}} = \frac{86}{0.135} = 640 \text{ Pa}.$$

(ii) Edge area = $0.30 \times 0.025 = 0.0075 \text{ m}^2$

$$\therefore \text{pressure} = \frac{\text{weight}}{\text{area}} = \frac{86}{0.0075} = 11500 \text{ Pa}.$$

8.2

(a) Area of piston = $\pi(0.45)^2/4 = 0.16 \text{ m}^2$

$$\therefore p = \frac{F}{A} = \frac{120\,000}{0.16} = 755\,000 \text{ Pa}.$$

(b) Effort force = pressure $\times$ area of narrow piston
 $= 755\,000 \times \pi(0.025)^2/4 = 370 \text{ N}$.

3 (a) Rearrange $p = H\rho g$ to give $H = \frac{p}{\rho g} = \frac{101 \times 10^3}{1.2 \times 9.8} = 8600 \text{ m}$.

(b) The density of the atmosphere decreases with increased height.

8.4

(a) $p = H\rho g = 2.0 \times 1000 \times 9.8 = 2.0 \times 10^4 \text{ Pa}$.

(b) The maximum lung pressure of a human is no more than about 20 kPa. At a depth of more than 2.0 m, the pressure due to the water would be greater than maximum lung pressure so it would not be possible to inflate the lungs naturally at this depth or below.

8.5

(a) See p. 111.

(b) If the arm is raised, the blood pressure in it is lowered so the reading would be less.

8.6

(a) The submarine weighs less when the water is forced from its ballast tanks. The upthrust is unchanged because the volume of the boat and hence the weight of water displaced is the same. Therefore the submarine rises because the upthrust on it exceeds its weight.

(b) Weight of test tube = upthrust for flotation. Upthrust = weight of water displaced = $HA\rho g$, where A is the area of cross-section of the test tube and H is its depth in the water

$\therefore HA\rho g$ is the same in both liquids (= weight of test tube)

$$\therefore H_1\rho_1 = H_2\rho_2 \text{ which gives } \rho_2 = \frac{H_1\rho_1}{H_2} = \frac{0.060 \times 1000}{0.068} = 880 \text{ kg m}^{-3}.$$

UNIT 9

In-text questions

9.2

1 (a) 12 mm (b) 50 mm.

2 (a) 0.2 s (b) 0.25 m s⁻¹.

3 (a) 3400 Hz (b) 0.11 m.

4 (a) 200 kHz (b) 250 m.

5 (a) (i) 180° (ii) 180° (iii) 360° (iv) 270°.

(b) (i) π (ii) π (iii) 2π (iv) $\frac{3}{2}\pi$.

9.3

1 See Fig. A9.1.

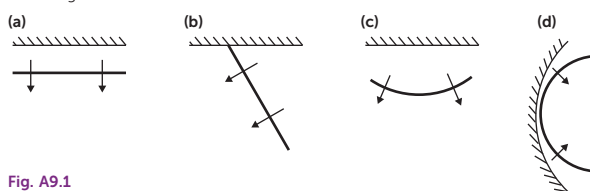


Fig. A9.1

2 (a) refraction (b) reflection (c) diffraction.

9.4

1 (a) Move one end of the coil from side to side rapidly.

(b) (i) longitudinal (ii) and (iii) transverse.

2 (a) The pressure of the sound waves forces the ear drum to and fro repeatedly.

(b) The plane of the aerial is at right angles to the plane of polarisation of the radio waves so the radio waves cannot be detected.

Revision questions

9.1

(a) gamma, X, UV, visible, IR, microwave, radio.

(b) (i) radio (ii) visible (iii) microwave (iv) gamma.

9.2

(a) Transverse: vibration at 90° to direction of propagation;

longitudinal: vibrations parallel to direction of propagation.

(b) transverse: microwaves, light; longitudinal: sound waves, ultrasonics, primary seismic waves.

9.3

(a) (i) 15 mm (ii) 30 mm. (b) (i) π (ii) $\frac{3}{2}\pi$ (iii) $\frac{1}{2}\pi$.

(c) (i) -15 mm (ii) 0 (iii) 0 (iv) +15 mm.

9.4

(a) (i) $6.0 \times 10^{14} \text{ Hz}$ (ii) 600 nm. (b) (i) 6800 Hz (ii) 28 mm.

9.5

(a) See Fig. A9.2.

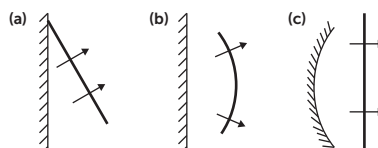


Fig A9.2

(b) (i) To focus the microwaves to a point, (ii) The detector is placed at the focal point of the dish.

(c) The waves slow down and lose energy as the water becomes shallower.

9.6

(a) (i) See Fig. A9.3.

(ii) The wavelength is less in the shallow water than in the deep water. The direction of the waves is closer to the normal (i.e. closer to the line perpendicular to the boundary) in the shallow water.

(iii) Waves at an angle to the beach turn towards the beach because the part of each wavefront nearer the beach slows down, so the rest of the wavefront catches up and the wavefront turns towards the beach.

9.7

(a) See Fig. A9.4.

(b) Diffraction is less for a larger dish so the focus is more difficult to find.

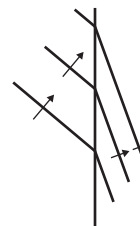


Fig. A9.3

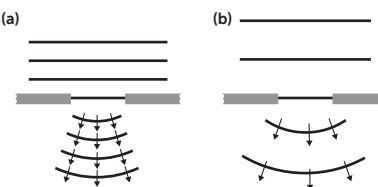


Fig. A9.4

9.8

- (a) See p. 128.
 (b) (i) Polarised light from the filter nearest the light source cannot pass through the other filter when the two filters are 'crossed'.
 (ii) The intensity of light changes from minimum to maximum at 90° then minimum at 180° then maximum at 270° and back to minimum at 360° .
 (iii) The same as in (i).

UNIT 10

In-text questions

10.1

- 1 (a) 50 Hz (b) (i) and (ii) See Fig. A10.1(i) and (ii).

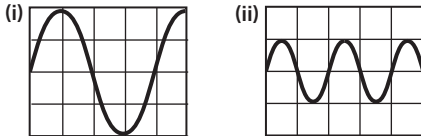


Fig. A10.1

- 2 (a) 0.05 s (b) 75 m.

10.2

- 1 (a) To amplify the force of the vibrations, to filter out unwanted sounds, to protect the inner ear from excessive loudness.
 (b) A recorder does not record higher frequencies as effectively as lower frequencies. Therefore, a recording sounds deeper than your direct speech. Also, the speed of sound through bone depends on frequency, so direct sounds are distorted.
 (c) (i) 30 dB (ii) 1000 times as much ($= 10 \times 10 \times 10$).
 2 (a) About 3000 Hz, (b) about 25 dB, (c) Hearing loss at the upper end of the frequency range is greater than at the other end. The upper frequency limit is reduced to about 15 kHz.

10.3

- 1 (a) If the pulses last too long, reflected pulses might return before the end of a transmitted pulse and would therefore not be detected.
 (b) Time for one cycle, $T = \frac{1}{f} = \frac{1}{(1.5 \times 10^6)} = 0.67 \times 10^{-6}$ s
 Hence the time for 10 pulses = 6.7 μ s.
 (c) Ratio = $\frac{6.7 \mu\text{s}}{1 \text{ ms}} = \frac{6.7 \times 10^{-6}}{1 \times 10^{-3}} = 6.7 \times 10^{-3}$.
 2 (a) $\lambda = \frac{v}{f} = \frac{1500 \text{ m s}^{-1}}{40 \text{ kHz}} = \frac{1500}{40000} = 3.75 \times 10^{-2}$ m.
 (b) (i) The wavelength is greater than the width of the source of waves so diffraction is significant. (ii) This is desirable so that waves spread out in all directions from the transducer.

10.4.

- 1 (a) See Fig. A10.2.



Fig. A10.2

- (b) The fundamental frequency $f_0 = \frac{v}{2L}$

Hence the frequency is inversely proportional to the length.

- (i) Doubling the length halves the frequency, hence the frequency becomes 60 Hz.
 (ii) Halving the length doubles the frequency, hence the frequency becomes 240 Hz.

- 2 (a) See Fig. A10.3.



Fig. A10.3

- (b) (i) The amplitude is zero at the ends and the middle, and greatest at one quarter and three quarters of the way along the wire.
 (ii) The phase difference is zero for any two points on either side of the centre and 180° for any two points separated by the node in the middle.

10.5

- 1 (a) $f_1 = 3f_0 = 312$ Hz.
 (b) (i) $\lambda = \frac{v}{f_0} = \frac{340}{104} = 3.27$ m. (ii) $e = \frac{1}{4}\lambda_0 - L = 0.817 - 0.815 = 0.002$ m.
 2 (a) (i) $\lambda_0 = 2L = 2 \times 0.60 = 1.20$ m. (ii) $f = \frac{v}{\lambda_0} = \frac{340}{1.2} = 283$ Hz.

Revision questions

10.1

- (a) (i) 0.025 s (ii) 40 Hz. (b) 8.5 m
 ($= 340 \text{ m s}^{-1} \div 40 \text{ Hz}$). (c) See Fig. A10.4.

10.2

- (a) (i) D (ii) B.
 (b) (i) 204 m ($= 340 \text{ m s}^{-1} \times 1.2 \text{ s} \div 2$), (ii) Time a further short blast. If the timing is less than the first timing, the ship has moved closer to the cliffs.

10.3

- (a) (i) 18 kHz, (ii) Worn bones in the middle ear, inner ear nerve damage.
 (b) (i) $\times 10$ (ii) $\times 10$ (iii) $\times 10\,000$. (c) 100.

10.4

- (a) (i) Each reflected pulse is due to an internal boundary. Each boundary both transmits and reflects some ultrasonic energy. Transmitted ultrasonics from one boundary will partially reflect at the next boundary.
 (ii) Ultrasonic energy is partly absorbed by the medium it passes through. The amount of absorption depends on the distance travelled, which depends on how far the tissue boundary is. Also, the strength of a reflection depends on the type of boundary.
 (b) (i) The slice will not vibrate at resonance unless the frequency matches.
 (ii) The wavelength is small enough at frequencies of the order of MHz for diffraction to be insignificant. If diffraction was significant, the ultrasonic pulses would spread out and weaken.

10.5

- (a) (i) increased (ii) increased.
 (b) (i) These two frequencies are the fundamental and the second overtone.
 (ii) Both the frequencies in (i) have an antinode at the centre of the wire. The wire was plucked at the middle so a node is not likely there. Hence the first overtone at 512 Hz is missing.

10.6

- (a) Since the frequency is inversely proportional to the length

(i.e. $f = \frac{\text{constant}}{L}$), then fL is constant.

- (i) $f \times 0.4 = 384 \times 0.80$ gives $f = 384 \times 2 = 768$ Hz.
 (b) $f \times 1.2 = 384 \times 0.80$ gives $f = 384 \times 0.80 \div 1.20 = 768 \div 3 = 256$ Hz.
 (b) 5% of 384 Hz = 19 Hz. To raise the frequency to 403 Hz, the length must be reduced to 0.76 m (from $384 \times 0.8 = 403 \times L$). To reduce the frequency to 365 Hz, the length must be raised to 0.84 m ($= 384 \times 0.8 = 365 \times L$).

10.7.

- (a) (i) $\lambda = \frac{340}{150} = 2.27$ m.
 (ii) See Fig. A10.5.

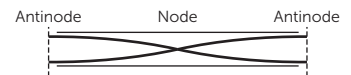


Fig. A10.5

- (iii) One half wavelength at 150 Hz = $1.13(3)$ m. Since $L + 2e = \frac{1}{2}\lambda$, where the pipe length, $L = 1.10$ m and e is the end correction, then $2e = 1.133 \text{ m} - 1.10 \text{ m} = 0.033 \text{ m}$. Hence $e = 0.017$ m.
 (b) The first overtone frequency = $2 \times 150 = 300$ Hz.
 (c) For a closed pipe vibrating at its fundamental frequency,
 $\frac{\lambda}{4} = L + e$. Hence $\lambda = 4 \times 1.117 = 4.47 \text{ m}$. $\therefore f = \frac{340}{4.47} = 76$ Hz.

10.8

- (a) The fundamental wavelength = $2 \times$ the pipe length, ignoring the end corrections. Hence the wavelengths range from 80 mm to 8 m. The fundamental frequency range is therefore

from 42.5 Hz ($= \frac{340}{8}$) to 4250 Hz ($= \frac{340}{0.080}$).

- (b) 8.5 kHz, 12.75 kHz, 17.0 kHz.

UNIT 11

In-text questions

11.1

- 1 (a) (i) $d = 0.50$ mm, $y = 0.90$ mm, $X = 0.80$ m
 $\lambda = \frac{yd}{X} = \frac{0.90 \times 10^{-3} \times 0.50 \times 10^{-3}}{0.80} = 563 \times 10^{-9} \text{ m} = 563 \text{ nm}$.
 (ii) green.
 2 (a) The fringe pattern would disappear.

(b) (i) The fringes would be closer together. This is because the fringe spacing $\propto$ the slit spacing is constant, so if the slit spacing is made larger, the fringe spacing is decreased. (ii) The fringe spacing would be unchanged but there would be fewer fringes visible because there is less overlap. This is because diffraction is less from a wider slit.

11.2

1 (a) See Fig. 11.6 (b) 3.0 m.

2 (a) See Fig. 11.11.

(b) $\sin c = \frac{\text{refractive index of cladding}}{\text{refractive index of the core}} = \frac{1.2}{1.5} = 0.80$

$\therefore c = 53^\circ$.

(c) The lower the critical angle, the more the fibre can be bent without light leaving the core. A higher refractive index for the cladding would mean a larger critical angle at the core-cladding boundary.

11.3

1 (a) See Figs 11.17–11.20.

(b) (i) $\frac{1}{0.5} + \frac{1}{v} = \frac{1}{0.2} \therefore \frac{1}{v} = \frac{1}{0.2} - \frac{1}{0.5} = 5.0 - 2.0 = 3.0$

$\therefore v = 0.33$ m, so $m = \frac{v}{u} = \frac{0.333}{0.50} = 0.67$.

(ii) $\frac{1}{0.25} = \frac{1}{v} = \frac{1}{0.2} \therefore \frac{1}{v} = \frac{1}{0.2} - \frac{1}{0.15} = 5.0 - 4.0 = 1.0$

$\therefore v = 1.00$ m, so $m = \frac{v}{u} = \frac{1.00}{0.25} = 4.0$.

(iii) $\frac{1}{0.15} + \frac{1}{v} = \frac{1}{0.2} \therefore \frac{1}{v} = \frac{1}{0.2} - \frac{1}{0.15} = 5.0 - 6.7 = -1.7$

$\therefore v = -0.60$ m, so $m = \frac{v}{u} = \frac{-0.60}{0.15} = -4.0$.

2 (a) See Fig. 11.20. (b) See Fig. 11.22.

Revision questions

11.1

(a) See Fig. 11.2 and pp. 145–7.

(b) (i) $\lambda = 590$ nm, $X = 0.90$ m, $y = 0.50$ mm. Rearranging

$\frac{\lambda}{d} = \frac{y}{X}$ gives $d = \frac{\lambda X}{y} = \frac{590 \times 10^{-9} \times 0.90}{0.5 \times 10^{-3}} = 1.1 \times 10^{-3}$ m.

(ii) $X = 0.90$ m, $y = 0.50$ mm, $d = 0.80$ mm. Rearranging

$\frac{\lambda}{d} = \frac{y}{X}$ gives $d = \frac{yX}{\lambda} = \frac{0.5 \times 10^{-3} \times 0.8 \times 10^{-3}}{0.90} = 440 \times 10^{-9}$ m.

11.2

(a) (i) 650 nm (ii) 450 nm.

(b) (i) refractive index, $n = \frac{\text{wavelength in air}}{\text{wavelength in glass}}$

$\therefore \text{wavelength in glass} = \frac{\text{wavelength in air}}{n}$

$= \frac{590 \text{ nm}}{1.55} = 381 \text{ nm}$.

(ii) The speed is slower in glass.

11.3

(a) Towards.

(b) For light passing from air into glass, $n = \frac{\sin i}{\sin r}$

$\therefore \sin r = \frac{\sin i}{n}$

(i) For $i = 25^\circ$ and $n = 1.50$, $\sin r = \frac{\sin 25}{1.50} = 0.28 \therefore r = 16^\circ$.

(ii) For $i = 50^\circ$ and $n = 1.50$, $\sin r = \frac{\sin 50}{1.50} = 0.51 \therefore r = 31^\circ$.

(c) (i) For light passing from glass to air, $\frac{\sin i}{\sin 3} = \frac{1}{n}$
 $\therefore \sin r = n \sin i = 1.5 \sin 40 = 0.96 \therefore r = 75^\circ$.

(ii) $\sin c = \frac{1}{n} = \frac{1}{1.50} = 0.67 \therefore c = 42^\circ$.

(iii) The light ray undergoes total internal reflection.

See Fig. A11.1.

11.4

(a) (i) The image is real, inverted, 0.18 m from the lens on the other side, and $0.18 \times$ the size of the object.

(ii) The image is real, inverted, 0.60 m from the lens on the other side, and $3.0 \times$ the size of the object.

(iii) The image is virtual, upright, 0.3 m from the lens on the same side, and $3.0 \times$ the size of the object.

(b) Rearranging $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$.

(i) $u = 1.00$ m, $f = 0.15$ m

$\therefore \frac{1}{v} = \frac{1}{0.15} - \frac{1}{1.00} = 6.7 - 1.0 = 5.7 \therefore v = 0.18$ m

$\therefore \text{Linear magnification} = \frac{v}{u} = \frac{0.18}{1.00} = 0.18$.

(ii) $u = 0.20$ m, $f = 0.15$ m

$\therefore \frac{1}{v} = \frac{1}{0.15} - \frac{1}{0.20} = 6.7 - 5.0 = 1.7 \therefore v = 0.60$ m

Linear magnification $= \frac{v}{u} = \frac{0.60}{0.20} = 3$.

(iii) $u = 0.10$ m, $f = 0.15$ m

$\therefore \frac{1}{v} = \frac{1}{0.15} - \frac{1}{0.10} = 6.7 - 10 = -3.3 \therefore v = -0.30$ m

$\therefore \text{Linear magnification} = \frac{v}{u} = \frac{-0.30}{0.10} = -3.0$.

(c) $u = +0.20$ m, $f = -0.30$ m

$\therefore \frac{1}{v} = \frac{1}{-0.30} + \frac{1}{0.20} = -8.33 \therefore v = -0.12$ m.

The image is upright and at a distance of 0.12 m from the lens on the same side of the lens as the object.

The magnification $= \frac{v}{u} = \frac{-0.12}{0.20} = -0.60$.

11.5

(a) See Fig. 11.26 and p. 155.

(b) The lens should be moved away from the film to focus the image on the film because the object distance is less than before. The aperture should be made wider if the light level indoors is poor.

11.6

(a) See Fig. 11.27 and p. 156.

(b) Diffraction is reduced using blue light because it has a smaller wavelength than all the other components of white light. Consequently, points closer together on the object can be seen separately.

(c) Spherical aberration of one or both lenses is the likely cause. See p. 156.

11.7

(a) See Fig. 3.28 and p. 156.

(b) A wide objective would collect more light and make the images of point objects brighter. Also, there would be less diffraction and more detail could be seen in extended images.

UNIT 12

In-text questions

12.1

1 (a) See Fig. 12.1.

(b) Only the blue part of the spectrum would be seen. All the other colours would not be reflected from the screen.

2 (a) Red + cyan; green + magenta; blue + yellow. (b) (i) red (ii) black.

12.2

1 (a) $m = 1$, $d = \frac{1}{600 \text{ mm}} = 1.67 \times 10^{-6}$ metres,

$\theta_1 = 122^\circ 06' - 101^\circ 22' = 20^\circ 44' = 20.73^\circ$

Using $m\lambda = d \sin \theta_m$ gives $\lambda = d \sin \theta_1 = 1.67 \times 10^{-6} \times \sin 20.73$
 $= 5.91 \times 10^{-7}$ metres.

(b) For $m = 2$, $\sin \theta_2 = \frac{2\lambda}{d} = 2 \sin 20.73 = 0.708 \therefore \theta = 45.07^\circ$;

for $m = 3$, $\sin \theta_3 = \frac{3\lambda}{d} = 3 \sin 20.73 = 1.062$; no third order is possible.

2 (a) Order number $m = 1$, $\lambda = 629$ nm, $\theta_1 = 10^\circ 52' = 10.87^\circ$.

Rearranging $m\lambda = d \sin \theta_m$ for $m = 1$ gives

$d = \frac{\lambda}{\sin \theta_1} = \frac{629 \times 10^{-9}}{\sin 10.87} = 3.33 \times 10^{-6}$ metres.

No. of lines per mm $= \frac{1}{3.33 \times 10^{-3}} = 300$.

(b) Maximum order number $= \frac{d}{\lambda}$ rounded down

$= \frac{3.33 \times 10^{-6}}{629 \times 10^{-9}} = 5.29$ rounded down = 5.

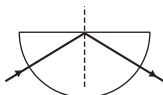


Fig. A11.1

12.3a

- 1 (a) The sea is usually colder than the land so emits less infra-red radiation.
 (b) Urban areas are usually warmer than rural areas so emit more infra-red radiation.
 2 (a) More infra-red radiation is received than is emitted due to the presence of the hot plate. (b) Less infra-red radiation is received than is emitted due to the presence of the beaker of ice.

12.3b

- (a) Microwaves penetrate food and heat water molecules inside the food.
 (b) Microwaves have a very high frequency so a beam can carry many more digital signals than radio waves. Also, they are not absorbed by the atmosphere at this frequency.
 2 (a) To ensure there are no cold spots in the food due to microwave nodes.
 (b) Microwaves at such high power are harmful to humans and animals.

12.3c

- 1 (a) (i) $c = 300\,000\text{ km s}^{-1}$, $\lambda = 1500\text{ m}$, $f = \frac{c}{\lambda} = \frac{300\,000 \times 10^3}{1500} = 200\text{ kHz}$.
 (ii) These radio waves travel round the Earth following the Earth's curvature to reach distant countries.
 (b) Absorption by the atmosphere makes the signal too weak beyond a certain range. Radio waves above 30 MHz pass through the ionosphere into space.
 2 (a) Polarisation. (b) The microwave beam from the satellite has further to travel through the atmosphere to reach N. Europe and therefore it becomes weaker and a wider dish is needed to compensate. A wider dish produces less diffraction so it is more difficult to focus the microwaves onto the aerial.

12.4a.

1. Produce a visible spectrum in a dark room as in Fig. 12.1. Mark some white paper using a fluorescent marker pen. Place the marks on the paper beyond the violet part of the visible spectrum. The marks should glow if ultraviolet radiation is present.
 2 (a) The fabric of the white shirt contains chemicals from washing powder that glow in ultraviolet radiation. (b) The skin cream absorbs the ultraviolet radiation and prevents it from reaching the skin.

12.4b

- 1 (a) The image on the film would be blurred otherwise as the shadow would be ill-defined.
 (b) Light would blacken the film and the X-rays would make no difference if the film was not in a light-proof wrapper.
 2 (a) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{1 \times 10^{-14}} = 3.0 \times 10^{22}\text{ Hz}$.
 (b) (i) It kills living cells (ii) It kills living cells.

Revision questions**12.1**

- (a) See Fig. 12.1. (b) Direct a narrow beam of white light into a semi-circular glass block, as in Fig. 1.10. The beam is split into colours where it emerges from the block. Adjust the incident angle so the red component emerges along the boundary. The blue component is totally internally reflected at this position because the critical angle for blue light is less than for red light.

12.2

- (a) Black, since it absorbs blue light. (b) (i) Blue print on a black background, as the background absorbs blue light. (ii) Black print on a red background, since the print absorbs yellow light and the background absorbs the green component of yellow light.

12.3

- (a) (i) $\tan \theta_1 = \frac{0.29}{0.15} = 0.193$ $\therefore \theta_1 = 10.9^\circ$.
 (ii) $d = \frac{1\text{ mm}}{300\text{ mm}} = 3.33 \times 10^{-6}\text{ m}$. Using $d \sin \theta_m = m\lambda$ gives
 $\lambda = d \sin \theta_1 = 3.33 \times 10^{-6} \times \sin 10.9 = 6.3 \times 10^{-7}\text{ m} = 630\text{ nm}$.
 (b) $\frac{d}{\lambda} = \frac{3.33 \times 10^{-6}}{6.3 \times 10^{-7}} = 5.3$ $\therefore$ five orders are produced.

12.4

- (a) (i) Blue: $\theta_1 = (125^\circ 33' - 95^\circ 39') \div 2 = 14^\circ 57'$;
 Yellow: $\theta_1 = (131^\circ 20' - 89^\circ 52') \div 2 = 20^\circ 44'$.
 (ii) For the first order yellow line, $\lambda = 590\text{ nm}$, $\theta_1 = 20^\circ 44'$.
 Using $d \sin \theta_m = m\lambda$ gives $d = \frac{\lambda}{\sin \theta_1} = \frac{590 \times 10^{-9}}{\sin 20^\circ 44'} = 1.67 \times 10^{-6}\text{ m}$
 $\therefore$ the number of lines per mm = $\frac{1}{1.67 \times 10^{-3}} = 600$.

- (iii) For the first order blue line, $\theta_1 = 14^\circ 57'$
 $\therefore \lambda = d \sin \theta_1 = 1.67 \times 10^{-6} \times \sin 14^\circ 57' = 4.3 \times 10^{-7}\text{ metres} = 430\text{ nm}$.
 (b) (i) Blue: $\sin \theta_2 = \frac{2m\lambda}{d} = \frac{2 \times 4.3 \times 10^{-7}}{1.67 \times 10^{-6}} = 2 \times 0.257 = 0.514$ $\therefore \theta_2 = 31.0^\circ$;
 $\sin \theta_3 = \frac{3m\lambda}{d} = \frac{3 \times 4.3 \times 10^{-7}}{1.67 \times 10^{-6}} = 3 \times 0.257 = 0.77$
 $\therefore \theta_3 = 50.4^\circ$; no fourth order is formed.
 Yellow: $\sin \theta_2 = \frac{2m\lambda}{d} = \frac{2 \times 5.9 \times 10^{-7}}{1.67 \times 10^{-6}} = 2 \times 0.257 = 0.514$
 $\therefore \theta_2 = 31.0^\circ$; no third order is formed.

12.5

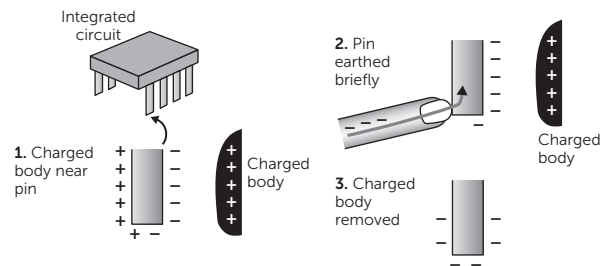
- (a) (i) radio (ii) infra-red (iii) ultraviolet (b) (i) 300 kHz (ii) 10 GHz (iii) $5 \times 10^{14}\text{ Hz}$.

12.6

- (a) Humans and animals are hotter and therefore emit more infra-red radiation than their surroundings. (b) Invisible ink glows in ultraviolet radiation.
 (c) Microwaves are not absorbed by the atmosphere and are at a high enough frequency to carry several TV channels. (d) X-rays pass through soft tissue and are absorbed by bone. (e) Gamma radiation kills living cells due to its ionising properties. (f) Line of sight communication; high carrier frequency allows many channels.

UNIT 13**In-text questions****13.1**

- 1 (a) Electrons from the cloth transfer to the rod. (b) Negative. (c) Charge the electroscope negatively by direct contact with the charged polythene rod. Charge the unknown rod and hold it directly above the cap of the electroscope. If the leaf rises further, the unknown rod is negative. If the leaf falls, the rod could be positive or uncharged. To decide which, earth the electroscope and repeat the test with the electroscope charged positively from the charged perspex rod.
 2 (a) See Fig. A13.1.

**Fig. A13.1**

- (b) Friction between the flowing oil and the pipe generates static electricity which would build up on the pipe if the pipe was not earthed. A spark due to static electricity could cause an explosion as oil is flammable.

13.2

- 1 (a) air, glass, germanium (b) 5 A, 0.3 C, 3 s, 2.5 A.
 2 (a) 2.0 A (b) 0.35 A (c) 4.2 A leaving the junction.

13.3

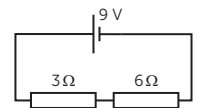
- 1 (a) 3 A (= 36 W/12 V). (b) $Q = It = 3 \times 10 \times 60 = 1800\text{ C}$.
 (c) Energy supplied = $36\text{ W} \times 600\text{ s} = 21\,600\text{ J}$.
 2 (a) 2.8 kW h = 2800 W for 3600 s = 10 MJ.
 (b) Rearrange $V = \frac{E}{Q}$ to give $Q = \frac{E}{V} = \frac{10\text{ MJ}}{230\text{ V}} = 44\text{ kC}$.
 (c) Current, $I = \frac{Q}{t} = \frac{44\text{ kC}}{30 \times 60\text{ s}} = 24\text{ A}$.

13.4a

- 1 7.5 Ω , 200 V, 0.68 mA, 2.4 k Ω , 0.50 mA. 2 (a) 3900 Ω (b) 1.5 mA.

13.4b

- 1 The circuit diagram is shown in Fig. A13.2.
 (a) (i) 9 Ω . (ii) $I = V/R = 9\text{ V}/9\text{ Ω } = 1.0\text{ A}$.
 (iii) 3 Ω resistor: $V = IR = 1 \times 3 = 3\text{ V}$,
 $P = IV = 1 \times 3 = 3\text{ W}$. 6 Ω resistor:
 $V = IR = 1 \times 6 = 6\text{ V}$, $P = IV = 1 \times 6 = 6\text{ W}$.

**Fig. A13.2**

(b) The circuit diagram is shown in Fig. A13.3.

(i) $\frac{1}{R} = \frac{1}{3} + \frac{1}{6}$. $\therefore R = 2\Omega$. (ii) $I = \frac{V}{R} = \frac{9\text{ V}}{2\Omega} = 4.5\text{ A}$.

(iii) 3Ω resistor: $I = \frac{V}{R} = \frac{9\text{ V}}{3\Omega} = 3\text{ A}$.

$P = IV = 3 \times 9 = 27\text{ W}$.

6Ω resistor: $I = \frac{V}{R} = \frac{9\text{ V}}{6\Omega} = 1.5\text{ A}$.

$P = IV = 1.5 \times 9 = 13.5\text{ W}$.

2 (a) See Fig. A13.4 for the eight possible combinations. (b) All in series 11Ω ; all in parallel 1Ω ; two in series in parallel with the third $30/11\Omega$, $24/11\Omega$, $18/11\Omega$; two in parallel in series with the third 7.2Ω , 4.5Ω , 4.0Ω .

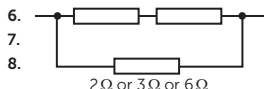
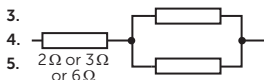
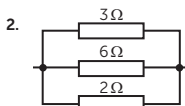
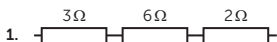


Fig. A13.4

Revision questions

13.1

- (a) Static electric charge is created on the balloon when it is rubbed. This charge attracts opposite charges in the ceiling which holds the balloon on the ceiling.
 (b) The plastic ruler becomes positively charged, the same as the perspex rod, when it is rubbed. The charge on the ruler forces the positive charge on the cap of the electroscope onto the leaf and the stem. Hence the leaf rises more.
 (c) Friction between clothing and the seat creates static electricity. The person becomes charged and discharges on touching the door handle.

13.2

- (a) More conduction electrons are created when the temperature is increased. The material therefore conducts more easily.
 (b) The atoms in the metal vibrate more when the temperature of the metal is raised. The vibrating atoms impede the flow of electrons through the metal. The material therefore conducts less easily.

13.3

- (a) (i) $Q = It = 0.25\text{ A} \times 600\text{ s} = 150\text{ C}$. (ii) $P = IV = 0.25\text{ A} \times 1.5\text{ V} = 0.375\text{ W}$.
 (iii) $E = Pt = 0.375 \times 600 = 225\text{ J}$.
 (b) (i) $Q = It = 0.25\text{ A} \times 3600\text{ s} = 900\text{ C}$. (ii) $E = QV = 900\text{ C} \times 1.5\text{ V} = 1350\text{ J}$.

13.4

- (a) (i) $I = 0.1 + 0.1 = 0.2\text{ A}$.
 (ii) $V_{\text{batt}} = V_{\text{bulb}} + V_{\text{res}} \therefore V_{\text{res}} = 4.5 - 3.0 = 1.5\text{ V}$.
 (iii) $P_x = P_y = IV = 0.1\text{ A} \times 3.0\text{ V} = 0.30\text{ W}$.
 (iv) $P_{\text{batt}} = I_{\text{batt}} V_{\text{batt}} = 0.2\text{ A} \times 4.5\text{ V} = 0.90\text{ W}$.
 (b) The power dissipated in the variable resistor $= 0.2\text{ A} \times 1.5\text{ V} = 0.3\text{ W}$. This is the difference between the power supplied by the battery and the power delivered to each torch bulb.

13.5

- (a) (i) $I = \frac{P}{V} = \frac{750\text{ W}}{230\text{ V}} = 3.3\text{ A}$.
 (ii) Number of units used $= 0.75\text{ kW} \times 0.5\text{ h} = 0.375\text{ kWh}$.
 (b) 5 A .

13.6

- (a) See Fig. A13.5.
 (b) (i) $R = \frac{V}{I} = \frac{5\text{ V}}{0.44\text{ A}} = 11.4\Omega$.
 (ii) $R = \frac{10\text{ V}}{0.66\text{ A}} = 15.1\Omega$.
 (c) The element becomes hotter as the current increases.

13.7

- (a) See Fig. A13.6.
 (b) (i) $V = 9.0 - 0.6 = 8.4\text{ V}$. (ii) $R = \frac{V_R}{I} = \frac{8.4\text{ V}}{2.0\text{ A}} = 4.2\Omega$.

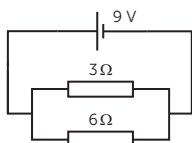


Fig. A13.3

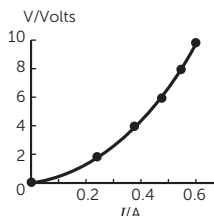


Fig. A13.5

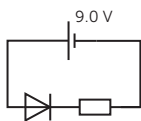


Fig. A13.6

13.8

- (a) See Fig. A13.7.
 (b) (i) $R = 4 + 6 = 10\Omega$.
 (ii) $I_{\text{batt}} = \frac{V_{\text{batt}}}{R} = \frac{6.0\text{ V}}{10\Omega} = 0.60\text{ A}$.
 (iii) 4Ω : $V_4 = IR_4 = 0.60 \times 4.0 = 2.4\text{ V}$.
 6Ω : $V_6 = IR_6 = 0.60 \times 6.0 = 3.6\text{ V}$.
 (iv) 4Ω : $P = IV_4 = 0.60 \times 2.4 = 1.44\text{ W}$.
 6Ω : $P = IV_6 = 0.60 \times 3.6 = 2.16\text{ W}$.

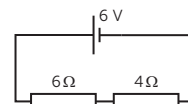


Fig. A13.7

13.9

- (a) See Fig. A13.8.
 (b) (i) $\frac{1}{R} = \frac{1}{6} + \frac{1}{12} = \frac{12+6}{12 \times 6} \therefore R = \frac{72}{18} = 4\Omega$.
 (ii) $I = \frac{V_{\text{batt}}}{R} = \frac{12}{4} = 3\text{ A}$.
 (iii) $I_6 = \frac{V_{\text{batt}}}{6} = \frac{12}{6} = 2\text{ A}$. $I_{12} = \frac{V_{\text{batt}}}{12} = \frac{12}{12} = 1\text{ A}$.
 (iv) $P_6 = I_6 V_{\text{batt}} = 2 \times 12 = 24\text{ W}$;
 $P_{12} = I_{12} V_{\text{batt}} = 1 \times 12 = 12\text{ W}$.

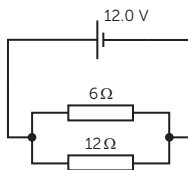


Fig. A13.8

13.10

- (a) See Fig. A13.9.
 (b) (i) $R = 4 + \left(\frac{1}{\frac{1}{3} + \frac{1}{6}}\right)^{-1} = 4 + 2 = 6\Omega$.
 (ii) $I_{\text{batt}} = \frac{V_{\text{batt}}}{R} = \frac{6}{6} = 1.0\text{ A}$.
 (iii) $V_4 = I_{\text{batt}} \times 4 = 1.0 \times 4 = 4.0\text{ V}$;
 $V_3 = V_6 = V_{\text{batt}} - V_4 = 6 - 4 = 2.0\text{ V}$.
 (iv) $I_4 = I_{\text{batt}} = 1.0\text{ A}$;
 $I_3 = \frac{V_3}{R_3} = \frac{2.0}{3} = 0.67\text{ A}$; $I_6 = \frac{V_6}{R_6} = \frac{2.0}{6} = 0.33\text{ A}$.
 (v) $P_4 = I_4 V_4 = 1.0 \times 4.0 = 4.0\text{ W}$; $P_6 = I_6 V_6 = 0.33 \times 2.0 = 0.7\text{ W}$;
 $P_3 = I_3 V_3 = 0.67 \times 2.0 = 1.3\text{ W}$.

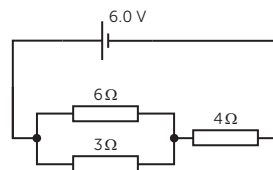


Fig. A13.9

UNIT 14

In-text questions

14.1

- 1 (a) See Fig. A14.1.
 (b) Use $E = V + Ir$ to find V , the p.d. across the torch bulb ($= IR$). Hence $2.0 = V + (0.25 \times 0.5)$
 $\therefore V = 2.0 - (0.25 \times 0.5) = 1.875\text{ V}$. Power delivered to torch bulb $= IV = 0.25 \times 1.875 = 0.47\text{ W}$.
 (c) Power supplied by cell $= IE = 0.25 \times 2.0 = 0.50\text{ W}$.
 (d) Power dissipated in the cell due to its internal resistance $= I^2 r = 0.25^2 \times 0.5 = 0.3\text{ W}$. The cell supplies 5.0 W of electrical power, the bulb uses 0.47 W of electrical power, the internal resistance of the cell causes 0.3 W of electrical power to be dissipated.

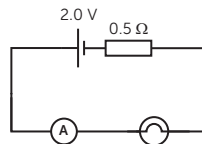


Fig. A14.1

2 (a) See Fig. A14.2.

The voltmeter reading decreases because the reduced resistance allows more current so there is a bigger lost voltage. Hence the cell p.d. (= e.m.f. - lost voltage) falls.

- (b) Use $E = V + Ir$ where $V = IR$, for each pair of values of I and R to give

$$E = 1.20 + I_1 r, \text{ where } I_1 = \frac{V}{R} = \frac{1.2}{16.0} = 0.075\text{ A}.$$

$$\text{Hence } E = 1.20 + 0.075r$$

$$\text{Also, } E = 1.00 + I_2 r \text{ where } I_2 = \frac{V}{R} = \frac{1.00}{8.0} = 0.125\text{ A}.$$

$$\text{Hence } E = 1.00 + 0.125r$$

$$\therefore E = 1.20 + 0.075r = 1.00 + 0.125r \text{ which gives } 0.125r - 0.075r = 1.20 - 1.00 \therefore 0.050r = 0.20 \text{ to give}$$

$$r = \frac{0.20}{0.050} = 4.0\Omega \text{ and } E = 1.00 + (0.125 \times 4.0) = 1.50\text{ V}.$$

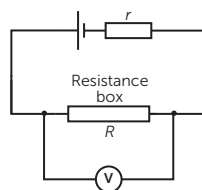


Fig. A14.2

14.2

- 1 Using $V_y = \frac{R_y}{(R_x + R_y)} V_{\text{batt}}$ gives (a) 1.0 mA , 1.0 V
 (b) 0.25 mA , 0.25 V (c) 0.11 mA , 3.45 V (d) 0.030 A , 6.0 V .

2 (a) A current passes through cell X and the 5 ohm resistor at balance. The cell p.d. is therefore less than the cell's e.m.f. because there is a 'lost voltage' across the internal resistance of the cell when current passes through the cell. Note that no current passes through the centre-reading meter because the sliding contact is at the balance position.

(b) (i) Using $E_x = \frac{I_x}{L_x} E_s = \frac{0.775}{0.558} \times 1.08 = 1.50 \text{ V}$.

(ii) Cell p.d. at balance with the 5 ohm resistor in circuit,

$$V = E_s L'_x = \frac{1.08}{0.558} \times 0.692 = 1.34 \text{ V}$$

$$\therefore \text{current } I = \frac{V}{R} = \frac{1.34}{5.0} = 0.27 \text{ A}$$

$$\text{Hence } r = \frac{(E - V)}{I} = \frac{(1.50 - 1.34)}{0.27} = 0.6 \Omega$$

14.3

1 (a) $\frac{X}{50} = \frac{0.452}{0.548} = 0.82 \therefore X = 0.82 \times 50 = 41 \Omega$.

(b) Let L_w = the length of wire from the balance point to the end of the wire at X. The effective resistance of two 50Ω resistors in parallel = 25Ω .

$$\therefore \frac{X}{25} = \frac{L}{(L_w - L)}, \text{ where } L_w, \text{ the length of the wire} = 1.00 \text{ m.}$$

$$\text{Hence } \frac{L}{(L_w - L)} = \frac{41}{25} = 1.64$$

$$\therefore L = 1.64(L_w - L) \text{ to give } 2.64L = 1.64L_w.$$

$$\text{Hence } L = \frac{1.64L_w}{2.64} = 0.621 \text{ m since } L_w = 1.00 \text{ m.}$$

2 (a) The LDR's resistance is greater in darkness than in daylight. The potential difference across the LDR therefore increases, thus reducing the p.d. across the variable resistor. Consequently, the potential difference from X to Z (which is unchanged) exceeds the potential difference from Y to Z. Therefore X becomes positive relative to Y.

(b) The variable resistance needs to be increased until it equals that of the LDR in darkness. The p.d. between Y and Z will then equal the p.d. between X and Z.

14.4

1 (a) $\rho = \frac{RA}{L} = \frac{58 \times \pi(0.24 \times 10^{-3})^2/4}{5.5} = 4.8 \times 10^{-7} \Omega \text{ m.}$

(b) $R = \frac{\rho L}{A} = \frac{4.8 \times 10^{-7} \times 1.0}{\pi(0.35 \times 10^{-3})^2/4} = 5.0 \Omega$.

2 Area of cross-section, A = width w $\times$ thickness t

$$\therefore \text{resistivity } \rho = \frac{RA}{L} = \frac{Rwt}{L}$$

$$\text{Hence } t = \frac{\rho L}{Rw} = \frac{3.0 \times 10^{-5} \times 10 \times 10^{-3}}{15 \times 1.0 \times 10^{-3}} = 2.0 \times 10^{-5} \text{ m.}$$

14.5

1 (a) $V_{\max} = 1.0 \text{ mA} \times 100 \Omega = 0.10 \text{ V}$.

(i) Current through shunt resistor, $i = 99 \text{ mA}$

$$\therefore \text{shunt resistance} = \frac{0.10 \text{ V}}{0.099 \text{ A}} = 1.01 \Omega$$

(ii) Current through shunt resistor, $i = 9.999 \text{ A}$

$$\therefore \text{shunt resistance} = \frac{0.10 \text{ V}}{9.999 \text{ A}} = 0.010 \Omega$$

(b) Current through series resistor = 1.0 mA .

(i) Voltage across series resistor = $10.0 - 0.10 = 9.9 \text{ V}$

$$\therefore \text{series resistance} = \frac{9.9 \text{ V}}{0.001 \text{ A}} = 9900 \Omega$$

(ii) Voltage across series resistor = $30.0 - 0.10 = 29.9 \text{ V}$

$$\therefore \text{series resistance} = \frac{29.9 \text{ V}}{0.001 \text{ A}} = 29900 \Omega$$

2 Maximum meter current = $\frac{200 \text{ mV}}{5.0 \text{ M}\Omega} = 40.0 \text{ nA}$.

(a) Maximum current through series resistor = 40.0 nA .

(i) Maximum voltage across series resistor = $2.0 - 0.2 = 1.8 \text{ V}$

$$\therefore \text{series resistance} = \frac{1.8 \text{ V}}{40.0 \text{ nA}} = 45 \text{ M}\Omega$$

(ii) Maximum voltage across series resistor = $100.0 - 0.2 = 99.8 \text{ V}$

$$\therefore \text{series resistance} = \frac{99.8 \text{ V}}{40.0 \text{ nA}} = 2495 \text{ M}\Omega$$

(b) Maximum voltage across shunt resistor = 200 mV .

(i) Maximum current through shunt resistor = 200 mA (-40 nA negligible).

$$\therefore \text{shunt resistance} = \frac{200 \text{ mV}}{200 \text{ mA}} = 1.0 \Omega$$

(ii) Maximum current through shunt resistor = 10 mA (-40 nA negligible).

$$\therefore \text{shunt resistance} = \frac{200 \text{ mV}}{10 \text{ A}} = 20.0 \text{ m}\Omega$$

Revision questions

14.1

(a) (i) See Fig. A14.3.

(ii) $E = I(R + r) \therefore I = \frac{2.0}{(4.2 + 0.8)} = 0.40 \text{ A}$

$$V = IR = 0.40 \times 4.2 = 1.68 \text{ V}$$

(b) (i) The lost voltage across the internal resistance of the cell = $Ir = 0.40 \times 0.80 = 0.32 \text{ V}$, which is the difference between the cell's e.m.f. and the p.d. across R .

14.2

(a) See Fig. A14.4.

(b) (i) $P = IV = 0.25 \times 2.5 = 0.625 \text{ W}$.

(ii) Electric energy/sec produced = $IE = 0.25 \times 3.0 = 0.75 \text{ W}$.

(c) Difference = $0.75 - 0.625 = 0.175 \text{ W}$; power dissipated in the cells due to internal resistance = $I \times \text{lost voltage } (Ir) = I^2 r = 0.25^2 \times 1.0 = 0.0625 \text{ W}$; power dissipated in variable resistor accounts for $(0.175 - 0.0625) \text{ W} = 0.1125 \text{ W}$.

14.3

(a) $E = IR + Ir \therefore E = (0.1 \times 15) + 0.1r = 1.5 + 0.1r$;

Also, $E = (0.2 \times 5.0) + 0.2r = 1.0 + 0.2r$

$$\therefore 1.0 + 0.2r = 1.5 + 0.1r \text{ so } 0.1r = 0.5$$

$$\therefore r = 5.0 \Omega; E = 1.0 + (0.2 \times 5.0) = 2.0 \text{ V}$$

(b) $I = \frac{E}{R + r} = \frac{2.0}{(5.0 + 3.0)} = 0.25 \text{ A}$.

14.4

(a) Thermistor resistance = $20 \text{ k}\Omega$ at 20°C

$$\therefore V_R = \frac{10}{(20 + 10)} \times 12.0 \text{ V} = 4.0 \text{ V}$$

(b) $V_R = \frac{10}{(10 + R)} \times 12.0 = 5.0$

$$\therefore 5(R + 10) = 12 \times 10 \text{ so } 5R = 120 - 50 = 70 \therefore R = 14 \text{ k}\Omega$$

$$\therefore \text{temperature} = 35^\circ\text{C}$$

14.5

(a) See Fig. A14.5.

(b) (i) 1.0 V . (ii) Total resistance = $1.5 \text{ k}\Omega$ since voltmeter and $1 \text{ k}\Omega$ resistor in parallel are equal to $0.5 \text{ k}\Omega$; hence cell current

$$= \frac{2.0 \text{ V}}{1.5 \text{ k}\Omega} = 1.33 \text{ mA} \text{ so voltmeter reading}$$

$$= \text{p.d. across } 0.5 \text{ k}\Omega \text{ combination}$$

$$= 0.5 \text{ k}\Omega \times 1.33 \text{ mA} = 0.67 \text{ V}$$

14.6

(a) $E_x = \frac{L_x}{L_s} E_s = \frac{0.651}{0.885} \times 1.5 = 1.1 \text{ V}$.

(b) (i) $V = \frac{L'_x}{L_s} E_s = \frac{0.542}{0.885} \times 1.5 = 0.92 \text{ V}$.

(ii) $I = \frac{V}{R} = \frac{0.92}{2.0} = 0.46 \text{ A}$

$$\therefore r = \frac{(E - V)}{I} = \frac{(1.1 - 0.92)}{0.46} = 0.4 \Omega$$

14.7

Use $\frac{P}{Q} = \frac{R}{S}$ (a) $R = \frac{PS}{Q} = \frac{5.0 \times 50.0}{12.0} = 20.8 \Omega$.

(b) $Q = \frac{PS}{R} = \frac{5.0 \times 50.0}{20.0} = 12.5 \Omega$. (c) $P = \frac{QR}{S} = \frac{12.0 \times 20.0}{50.0} = 4.8 \Omega$.

14.8

(a) (i) $\frac{R}{S} = \frac{(1000 - 438)}{438} \therefore R = 1.28S = 15.4 \Omega$.

(ii) For $L = 38 + 2 = 440 \text{ mm}$, $R = \frac{(1000 - 440)S}{440} = 15.3 \Omega$

$$\therefore \text{probable error in } R = 0.1\Omega$$

(b) $\frac{1}{S'} = \frac{1}{6} + \frac{1}{12} \therefore S' = 4.0 \Omega \therefore \frac{(1000 - L)}{L} = \frac{15.4}{4.0} \therefore 4000 - 4.0L = 15.4L$;

$$\text{hence } 19.4L = 4000 \therefore \frac{4000}{19.4} = 206 \text{ mm}$$

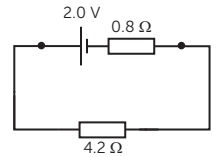


Fig. A14.3

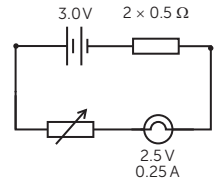


Fig. A14.4

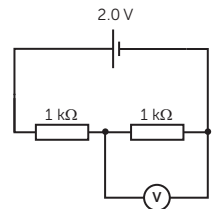


Fig. A14.5

14.9

$$(a) R = \frac{\rho L}{A} = \frac{4.8 \times 10^{-7} \times 1.50}{\pi(0.36 \times 10^{-3})^2/4} = 7.1 \Omega.$$

$$(b) A = \frac{\rho L}{R} = \frac{4.8 \times 10^{-7} \times 10}{105} = 4.57 \times 10^{-8} \text{ m}^2 \therefore \frac{\pi d^2}{4} = 4.57 \times 10^{-8};$$

$$\text{hence } d^2 = 4.57 \times 10^{-8} \times 4/\pi = 5.82 \times 10^{-8} \text{ m}^2 \therefore d = 2.4 \times 10^{-4} \text{ m}.$$

$$(c) L = \frac{RA}{\rho} = \frac{0.040 \times \pi(2.0 \times 10^{-3})^2/4}{1.7 \times 10^{-8}} = 7.4 \text{ m}.$$

14.10

$$(a) \text{ Fit a shunt resistor, } S, \text{ in parallel with the meter, where } S = \frac{ir}{(I-i)}.$$

$$(i) S = \frac{0.10 \text{ mA} \times 500 \Omega}{(100 - 0.1) \text{ mA}} = 0.50 \Omega. \quad (ii) S = \frac{0.10 \text{ mA} \times 500 \Omega}{5.0 - 0.0001 \text{ A}} = 10 \text{ m}\Omega.$$

$$(b) \text{ Fit a multiplier resistor, } R, \text{ in series with the meter where } R = \frac{(V-v)r}{v}$$

where $v = ir$.

$$(i) R = \frac{(1.0 - 0.05) \times 500}{0.05} = 9500 \Omega. \quad (ii) R = \frac{(15.0 - 0.05) \times 500}{0.05} = 149500 \Omega.$$

UNIT 15

In-text questions

15.1

$$1 \text{ (i) } Q = CV = 10 \mu\text{F} \times 6.0 \text{ V} = 60.0 \mu\text{C}. \quad (ii) V = Q/C = 0.33 \mu\text{C}/0.22 \mu\text{F} = 1.5 \text{ V}.$$

$$(iii) C = Q/V = 9.90 \times 10^4 \mu\text{C}/4.50 \text{ V} = 22000 \mu\text{F}.$$

$$(iv) V = Q/C = 5.00 \times 10^{-2} \mu\text{C}/1.00 \times 10^{-3} \mu\text{F} = 50 \text{ V}.$$

$$2 \text{ (a) } Q = CV = 2200 \mu\text{F} \times 9.0 \text{ V} = 19800 \mu\text{C}.$$

$$(b) \text{ Rearrange } Q = It \text{ to give } t = Q/I = 19800 \mu\text{C}/0.25 \text{ mA} = 79.2 \text{ s}.$$

15.2

$$1 \text{ (a) (i) } \frac{1}{C} = \frac{1}{3} + \frac{1}{6} = \frac{6+3}{3 \times 6} = \frac{9}{18} = \frac{1}{2} \therefore C = 2 \mu\text{F}. \quad (ii) C = 3 + 6 = 9 \mu\text{F}.$$

$$(b) \text{ CIRCUIT A: (i) The combined capacitance, } C_1, \text{ of the } 6 \mu\text{F} \text{ capacitor and the } 12 \mu\text{F} \text{ capacitor in series is given by}$$

$$\frac{1}{C_1} = \frac{1}{6} + \frac{1}{12} = \frac{12+6}{6 \times 12} = \frac{18}{72} \therefore C_1 = \frac{72}{18} = 4 \mu\text{F}$$

Hence the total capacitance, $C = 4 + 2 = 6 \mu\text{F}$.

$$(ii) \text{ Total charge stored, } Q = CV_{\text{batt}} = 6 \mu\text{F} \times 12 \text{ V} = 72 \mu\text{C}.$$

$$(iii) 2 \mu\text{F}: V = V_{\text{batt}} = 12 \text{ V} \therefore Q = CV = 2 \mu\text{F} \times 12 \text{ V} = 24 \mu\text{C};$$

$$6 \mu\text{F}: Q = 72 - 24 = 48 \mu\text{C} \therefore V = \frac{Q}{C} = \frac{48 \mu\text{C}}{6 \mu\text{F}} = 8 \text{ V};$$

$$12 \mu\text{F}: Q = 48 \mu\text{C}, V = 12 - 8 = 4 \text{ V}.$$

$$(c) \text{ CIRCUIT B: (i) The combined capacitance of } 1 \mu\text{F} \text{ and } 2 \mu\text{F} \text{ in parallel} = 3 \mu\text{F}.$$

Hence the combined capacitance of $3 \mu\text{F}$ and $6 \mu\text{F}$ in series $= 2 \mu\text{F}$.

$$(ii) Q = CV = 2 \mu\text{F} \times 5.0 \text{ V} = 10 \mu\text{C}.$$

$$(iii) 6 \mu\text{F}: Q = 10 \mu\text{C} \text{ (same as the total charge stored)}$$

$$\therefore V = \frac{Q}{C} = \frac{10 \mu\text{C}}{6 \mu\text{F}} = 1.67 \text{ V}; 1 \mu\text{F}: V = 5.0 - 1.67 = 3.33 \text{ V}$$

$$\therefore Q = CV = 1 \mu\text{F} \times 3.33 \text{ V} = 3.33 \mu\text{C}; 2 \mu\text{F}: V = 3.33 \text{ V}$$

$$\therefore Q = CV = 2 \mu\text{F} \times 3.33(3) \text{ V} = 6.67 \mu\text{C}.$$

$$2 \text{ (a) Total charge stored, } Q = CV = 4.7 \mu\text{F} \times 6.0 \text{ V} = 28 \mu\text{C}.$$

$$(b) \text{ Combined capacitance} = 10 + 4.7 = 14.7 \mu\text{F}.$$

$$\therefore \text{ final p.d.} = \frac{\text{total charge stored, } Q}{\text{combined capacitance}} = \frac{28 \mu\text{C}}{14.7 \mu\text{F}} = 1.90 \text{ V}.$$

$$(c) 4.7 \mu\text{F}: Q = CV = 4.7 \mu\text{F} \times 1.90 \text{ V} = 9 \mu\text{C};$$

$$10 \mu\text{F}: Q = CV = 10 \mu\text{F} \times 1.90 \text{ V} = 19 \mu\text{C}.$$

15.3

$$1 \text{ (a) (i) } E = \frac{1}{2} CV^2 = \frac{1}{2} \times 5.0 \times 10^{-6} \times 12^2 = 360 \times 10^{-6} \text{ J} = 360 \mu\text{J}.$$

$$(ii) V = \frac{Q}{C} = \frac{1.8 \text{ mC}}{100 \mu\text{F}} = 18 \text{ V} \therefore E = \frac{1}{2} CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 18^2 = 1.62 \times 10^{-2} \text{ J}.$$

$$(iii) E = \frac{1}{2} QV = 0.5 \times 30 \times 10^{-3} \times 6.0 = 9.0 \times 10^{-2} \text{ J}.$$

$$(b) (i) \text{ Rearrange } E = \frac{1}{2} CV^2 \text{ to give } V^2 = 2E/C = 2 \times 250 \times 10^{-6}/20 \times 10^{-6} = 25$$

$$\therefore V = 5.0 \text{ V}.$$

$$(ii) E = \frac{1}{2} CV^2 = \frac{1}{2} \times 50000 \times 10^{-6} \times 6.0^2 = 0.90 \text{ J}$$

$$\therefore \text{ time} = \frac{\text{energy}}{\text{power}} = \frac{0.90 \text{ J}}{0.2 \text{ W}} = 4.5 \text{ s}.$$

$$2 \text{ (a) (i) } Q = CV = 100 \mu\text{F} \times 6.0 \text{ V} = 600 \mu\text{C};$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 6.0^2 = 1.8 \times 10^{-3} \text{ J}.$$

$$(ii) \text{ Combined capacitance} = 100 + 50 = 150 \mu\text{F}.$$

$$\text{final p.d.} = \frac{\text{initial charge}}{\text{combined capacitance}} = \frac{600 \mu\text{C}}{150 \mu\text{F}} = 4.0 \text{ V}.$$

$$(iii) 100 \mu\text{F}: Q = CV = 100 \mu\text{F} \times 4.0 \text{ V} = 400 \mu\text{C};$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 4.0^2 = 8.0 \times 10^{-4} \text{ J};$$

$$50 \mu\text{F}: Q = CV = 50 \mu\text{F} \times 4.0 \text{ V} = 200 \mu\text{C};$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 50 \times 10^{-6} \times 4.0^2 = 4.0 \times 10^{-4} \text{ J}.$$

$$(b) \text{ Energy dissipated} = 1.8 \text{ mJ} - (0.8 + 0.4) \text{ mJ} = 0.6 \text{ mJ}.$$

15.4

$$1 \text{ (a) Using } C = \frac{A\epsilon_0\epsilon_r}{d} \text{ gives } C = \frac{1.5 \times 0.04 \times 8.85 \times 10^{-12} \times 2.5}{0.010 \times 10^{-3}} = 1.3 \times 10^{-7} \text{ F}.$$

$$(b) (i) \text{ For } 0.10 \text{ mm thickness, } V = 700 \text{ kV mm}^{-1} \times 0.01 \text{ mm} = 7 \text{ kV}.$$

$$(ii) E = \frac{1}{2} CV^2 = 0.5 \times 1.3 \times 10^{-7} \times 7000^2 = 3.2 \text{ J}.$$

$$2 \text{ (a) Using } C = \frac{A\epsilon_0\epsilon_r}{d} \text{ gives } C = \frac{6.0 \times 8.85 \times 10^{-12}}{0.10} = 5.3 \times 10^{-10} \text{ F}.$$

$$(b) Q = CV = 5.3 \times 10^{-10} \times 1000 = 5.3 \times 10^{-7} \text{ C};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 5.3 \times 10^{-10} \times 1000^2 = 2.7 \times 10^{-4} \text{ J}.$$

15.5

$$1 \text{ (a) } Q_0 = CV_0 = 5.0 \mu\text{F} \times 12.0 \text{ V} = 60 \mu\text{C};$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 5.0 \times 10^{-6} \times 12.0^2 = 360 \mu\text{J}.$$

$$(b) \frac{t}{CR} = \frac{5.0}{5.0 \times 10^{-6} \times 0.5 \times 10^6} = 2.0$$

$$\therefore Q = Q_0 e^{-t/CR} = 60 e^{-2} = 60 \times 0.135 = 8.1 \mu\text{C} \therefore V = \frac{Q}{C} = \frac{8.1 \times 10^{-6}}{5.0 \times 10^{-6}} = 1.6 \text{ V}.$$

$$2 \text{ (a) } Q_0 = CV_0 = 2200 \times 10^{-6} \times 6.0 = 0.013 \text{ C}.$$

$$(b) CR = 2200 \times 10^{-6} \times 100 \times 10^3 = 220 \text{ s}.$$

$$(c) \text{ At } t = CR, Q = Q_0 e^{-1} = 0.37 Q_0 = 0.0048 \text{ C}; V = \frac{Q}{C} = \frac{0.0048}{2200 \times 10^{-6}} = 2.2 \text{ V}.$$

$$(d) I = \frac{V}{R} = \frac{2.2}{100 \times 10^2} = 2.2 \times 10^{-5} \text{ A}.$$

Revision questions

15.1

$$(a) (i) \text{ The charging current is sufficient to light the LED. (ii) The charging current decreases and becomes too small to light the LED.}$$

$$(b) (i) Q = CV = 5000 \times 10^{-6} \text{ F} \times 3.0 \text{ V} = 1.5 \times 10^{-2} \text{ C}.$$

$$(ii) t = \frac{Q}{I} = \frac{1.5 \times 10^{-2}}{0.15 \times 10^{-3}} = 100 \text{ s}.$$

15.2

$$(a) (i) C = 3 + 6 = 9 \mu\text{F}.$$

$$(ii) \frac{1}{C} = \frac{1}{3} + \frac{1}{6} = \frac{6+3}{3 \times 6} = \frac{9}{18} \therefore C = \frac{18}{9} = 2 \mu\text{F}.$$

$$(b) (i) \text{ The combined capacitance of the } 3 \mu\text{F} \text{ and the } 6 \mu\text{F} \text{ capacitor in parallel is } 9 \mu\text{F}; \text{ therefore the total capacitance is given by}$$

$$\frac{1}{C} = \frac{1}{9} + \frac{1}{11} = \frac{11+9}{9 \times 11} = \frac{20}{99} \therefore C = \frac{99}{20} = 4.95 \mu\text{F}.$$

$$(ii) Q = CV = 4.95 \times 10^{-6} \times 6.0 = 2.97 \times 10^{-5} \text{ C}.$$

$$(iii) 11 \mu\text{F}: Q = 2.97 \times 10^{-5} \text{ C} \therefore V = \frac{Q}{C} = \frac{2.97 \times 10^{-5}}{11 \times 10^{-6}} = 2.7 \text{ V};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 11 \times 10^{-6} \times 2.7^2 = 4.0 \times 10^{-5} \text{ J};$$

$$3 \mu\text{F}: V = 6.0 - 2.7 = 3.3 \text{ V} \therefore Q = CV = 3 \times 10^{-6} \times 3.3 = 9.9 \times 10^{-6} \text{ C};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 3 \times 10^{-6} \times 3.3^2 = 1.6 \times 10^{-5} \text{ J};$$

$$6 \mu\text{F}: V = 6.0 - 2.7 = 3.3 \text{ V} \therefore Q = CV = 6 \times 10^{-6} \times 3.3 = 19.8 \times 10^{-6} \text{ C};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 6 \times 10^{-6} \times 3.3^2 = 3.2 \times 10^{-5} \text{ J}.$$

15.3

$$(a) C = \frac{A\epsilon_0\epsilon_r}{d} = \frac{0.30 \times 0.25 \times 8.85 \times 10^{-12} \times 3.5}{1.5 \times 10^{-3}} = 1.55 \times 10^{-9} \text{ F}.$$

$$(b) Q = CV = 1.55 \times 10^{-9} \times 50 = 7.7 \times 10^{-8} \text{ C};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 1.55 \times 10^{-9} \times 50^2 = 1.9 \times 10^{-6} \text{ J}.$$

15.4

$$(a) (i) Q_0 = CV = 4.7 \times 10^{-6} \times 9.0 = 4.2 \times 10^{-5} \text{ C},$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 4.7 \times 10^{-6} \times 9.0^2 = 1.9 \times 10^{-4} \text{ J}.$$

$$(ii) \text{ Time constant, } CR = 4.7 \times 10^{-6} \times 10 \times 10^6 = 47 \text{ s. (iii) At } t = 100 \text{ s,}$$

$$t/CR = 100/47 = 2.12 \therefore Q = Q_0 e^{-t/CR} = 4.2 \times 10^{-5} \times e^{-2.12} = 5.0 \times 10^{-6} \text{ C};$$

$$V = \frac{Q}{C} = \frac{5.0 \times 10^{-6}}{4.7 \times 10^{-6}} = 1.06 \text{ V};$$

$$E = \frac{1}{2} CV^2 = 0.5 \times 4.7 \times 10^{-6} \times 1.06^2 = 2.6 \times 10^{-6} \text{ J}.$$

$$(b) (i) \text{ Initial charge} = 4.2 \times 10^{-5} \text{ C; total capacitance} = 4.7 + 2.2 = 6.9 \mu\text{F}$$

$$\therefore V = \frac{Q}{C} = \frac{4.2 \times 10^{-5}}{6.9 \times 10^{-6}} = 6.1 \text{ V}.$$

$$(ii) 4.7 \mu\text{F}: E = \frac{1}{2} CV^2 = 0.5 \times 4.7 \times 10^{-6} \times 6.1^2 = 8.7 \times 10^{-5} \text{ J};$$

$$2.2 \mu\text{F}: E = \frac{1}{2} CV^2 = 0.5 \times 2.2 \times 10^{-6} \times 6.1^2 = 4.1 \times 10^{-5} \text{ J}.$$

$$(iii) \text{ Loss of stored energy} = 1.9 \times 10^{-4} \text{ J} - (8.7 \times 10^{-5} \text{ J} + 4.1 \times 10^{-5} \text{ J})$$

$$= 7.2 \times 10^{-5} \text{ J}.$$

15.5

$$(a) \text{ Input resistance} = \frac{6.0 \text{ V}}{0.5 \text{ mA}} = 12\,000 \, \Omega$$

$$(b) (i) CR = 50\,000 \times 10^{-6} \times 12\,000 = 600 \text{ s.}$$

$$\frac{t}{CR} = \frac{100}{600} = 0.167 \therefore Q = Q_0 e^{-t/CR} = Q_0 e^{-0.167} = 0.85Q_0$$

$$\therefore V = \frac{Q}{C} = 0.85 \frac{Q_0}{C} = 0.85V_0 = 5.1 \text{ V.}$$

$$(iii) Q = Q_0 e^{-t/CR} \therefore V = \frac{Q}{C} = \frac{Q_0}{C} e^{-t/CR} = V_0 e^{-t/CR} \therefore 5.0 = 6.0 e^{-t/CR}$$

$$\therefore e^{-t/CR} = \frac{5.0}{6.0} = 0.83 \therefore \frac{-t}{CR} = \ln 0.83 = -0.186,$$

$$t = 0.186CR = 0.186 \times 600 = 112 \text{ s.}$$

UNIT 16

In-text questions

16.1

- 1 Digital (a) and (b); analogue (c) and (d).

2 See Fig. A16.1.

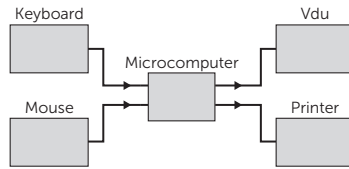


Fig. A16.1

16.2

1

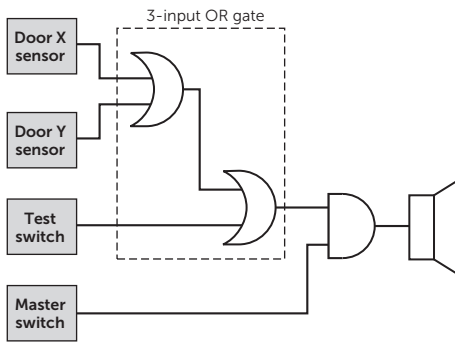


Fig. A16.2

A	B	Output for (a)	Output for (b)	Output for (c)
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	1	1	0

2 (a) ALARM = 1 if master switch = 1 AND door X sensor = 1 OR door Y sensor = 1 OR test switch = 1.

(b) ALARM = 1 if master switch = 1 AND smoke sensor OR panic button OR test switch = 1.

16.3

- 1 (a) The test switch supplies logic 1 to the OR gate, which therefore switches the transistor and relay on.

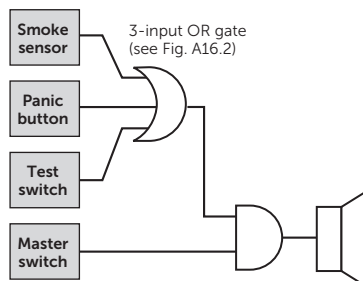


Fig. A16.3

- (b) (i) The output voltage from the temperature sensor rises as the thermistor warms up. At a certain temperature, the output voltage is high enough to count as logic 1 to the OR gate input. Hence the OR gate switches the fan on. (ii) The temperature at which the thermistor is switched on is raised. Reducing this resistance lowers the voltage from the potential divider output, so the thermistor must warm up more to raise the voltage to the level necessary to switch the output state of the OR gate.

2 See Fig. A16.4.

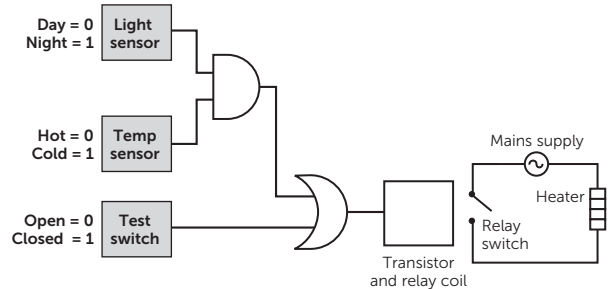


Fig. A16.4

16.4

1 Circuit A: (a) Voltage gain = $\frac{-R_F}{R_1} = \frac{-15 \text{ M}\Omega}{0.5 \text{ M}\Omega} = -30$.

(b) $V_{IN} = \frac{-V_{OUT}}{-30} = \frac{15}{30} = 0.5 \text{ V.}$

Circuit B: (a) Voltage gain = $\frac{R_F + R_1}{R_1} = \frac{(10 + 1)}{1} = 11$.

(b) $V_{IN} = \frac{V_{OUT}}{11} = \frac{15}{11} = 1.4 \text{ V.}$

- 2 (a) The voltage to input Q is constant. The voltage to input P increases as the variable resistor is increased. When the voltage at P becomes greater than the voltage at Q, the op-amp switches polarity. (b) The thermistor resistance increases as it becomes cooler. The voltage to input P therefore falls as the thermistor becomes cooler. When the voltage at P becomes less than the fixed voltage at Q, the op-amp output switches polarity to +15 V.

16.5

1 (a) The time delay would be lengthened because the time constant, CR , would be increased.

(b) The time delay would be reduced because the time constant, CR , would be reduced.

2 (a) (i) $T = CR = 1 \times 10^{-6} \text{ F} \times 1000 \, \Omega = 1 \times 10^{-3} \text{ s.}$ (ii) Each pulse would be $2 \times 10^{-3} \text{ ms}$ in duration so the frequency of the pulses would be 500 Hz ($= \frac{1}{2 \times 10^{-3}} \text{ s}$).

(b) One state would last $4.7 \times$ longer and the other state would last $10 \times$ longer. The pulse would therefore be $14.7 \times$ longer and the pulse frequency would be reduced to about 34 Hz ($= 500 \text{ Hz}/14.7$).

Revision questions

16.1

A	B	Output for (a)	Output for (b)	Output for (c)
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	0

16.2

(a) (i) To test the system (ii) Increase the variable resistance. (iii) OR.

(b) High voltage at the output of the logic circuit makes a current enter the base of the transistor, so allowing a current to pass through the relay coil and the transistor via its collector. The relay coil is therefore energised and the fan motor is switched on.

16.3

(a) See Fig. A16.5 overleaf.

(b) The time delay is increased because the capacitor takes longer to charge up, so the voltage at the input of the first NOT gate takes longer to reach logic level 1.

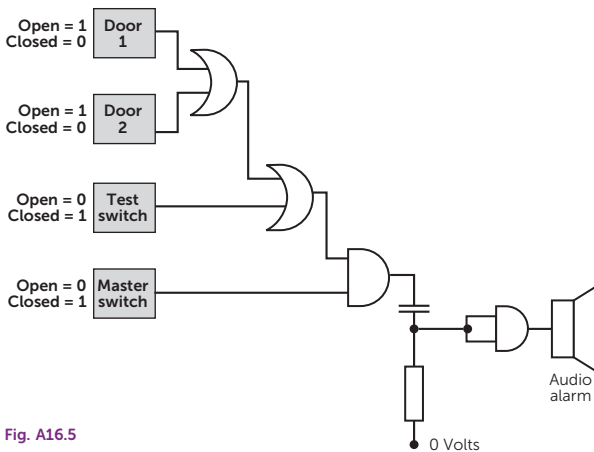


Fig. A16.5

16.4

(a) (i) See Fig. A16.6.

(ii) The output waveform is at positive saturation when the input voltage at Q is more than the voltage at P, and at negative saturation when the input voltage at P is more than the voltage at Q. Since the voltage at Q is +0.5 V and the input voltage is an alternating voltage of peak voltage 1.0 V, the output is at positive saturation more than it is at negative saturation.

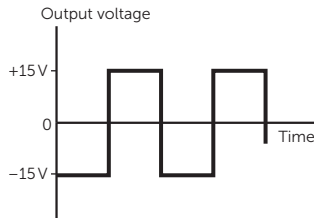


Fig. A16.6

 (b) (i) A non-inverting amplifier. (ii) $6 \left[\frac{(2.0 + 0.4)}{0.4} \right]$. (iii) 2.5 V (= 15 V/6).

16.5

 (a) (i) $-2.0 \text{ V} \left(= \frac{-R_F}{R_1} V_1 = \frac{-1.0}{1.0} \times 2.0 \right)$.

 (ii) $-3.0 \text{ V} \left(= \frac{-R_F}{R_2} V_2 = -1.005 \times 1.5 \right)$. (iii) $-5.0 \text{ V} (= -2.0 \text{ V} - 3.0 \text{ V})$.

(b) See Table A16.1 and Fig. A16.7.

Table A16.1

Binary number				Output voltage/volts
V_C	V_B	V_A		
0	0	1		1
0	1	0		2
0	1	1		3
1	0	0		4
1	0	1		5
1	1	0		6
1	1	1		7

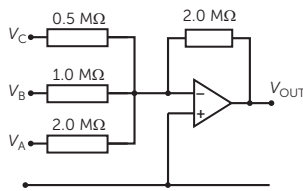


Fig. A16.7

UNIT 17
In-text questions
17.1

 1 $F = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2} = \frac{+1.6 \times 10^{-19} \times -1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (0.10 \times 10^{-9})^2} = -2.3 \times 10^{-8} \text{ N}$.

 2 The force between the +2.5 nC charge and the +1.5 μC charge, $F_1 = \frac{+2.5 \times 10^{-9} \times +1.5 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.10)^2} = 3.4 \text{ mN}$ (repulsion).

The force between the +2.5 nC charge and the -3.5 μC charge,

 $F_1 = \frac{+2.5 \times 10^{-9} \times -3.5 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (0.10)^2} = -7.9 \text{ mN}$ (attraction).

The +2.5 nC charge therefore experiences a force 3.4 mN away from the +1.5 μC charge and a force of 7.9 mN towards the 3.5 μC charge. These two forces on the +2.5 nC charge are in the same direction. Therefore, the total force on the +2.5 nC charge is 11.3 mN (= 7.9 mN + 3.4 mN).

17.2

 1 (a) Change of p.e., $\Delta PE = q(V_2 - V_1)$.

 (i) $\Delta PE = +2.5 \text{ nC} \times (6.0 - 4.0) = 5.0 \text{ nJ}$. (ii) $\Delta PE = 0$. (iii) $\Delta PE = -5.0 \text{ nJ}$.

 (b) $Fd = q(V_2 - V_1) \therefore F = \frac{5.0 \times 10^{-9} \text{ J}}{5.0 \times 10^{-3} \text{ m}} = 1.0 \times 10^{-6} \text{ N}$.

 2 (a) (i) Negative. (ii) Rearranging $\frac{qV}{d} = mg$ gives

 $q = \frac{mgd}{V} = \frac{2.5 \times 10^{-15} \times 9.8 \times 12.0 \times 10^{-3}}{600} = 4.9 \times 10^{-19} \text{ C}$.

(b) See Fig. A17.1.

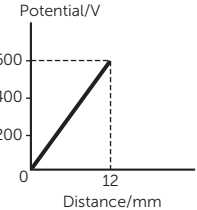


Fig. A17.1

 (c) (i) $V = 100 \text{ V}$ at 2.0 mm above the earthed plate $\therefore \text{p.e.} = qV = +4.9 \times 10^{-17} \text{ J}$.

 (ii) $V = 300 \text{ V}$ at 6.0 mm above the earthed plate $\therefore \text{p.e.} = qV = +1.47 \times 10^{-16} \text{ J}$.

 (iii) $V = 500 \text{ V}$ at 10.0 mm above the earthed plate $\therefore \text{p.e.} = qV = +2.45 \times 10^{-16} \text{ J}$.

17.3

 1 (a) $F = qE = 3.2 \times 10^{-19} \times 120\,000 = 3.8 \times 10^{-14} \text{ N}$.

 (b) $\Delta E_P = q\Delta V = qE\Delta x = 3.8 \times 10^{-14} \times 5.0 \times 10^{-6} = 1.9 \times 10^{-19} \text{ J}$.

 2 (a) $E = \frac{V}{d} = \frac{300}{0.010} = 30\,000 \text{ V m}^{-1}$.

 (b) $Q/A = \epsilon_0 E = 8.85 \times 10^{-12} \times 30\,000 = 2.7 \times 10^{-7} \text{ C m}^{-2}$.

 (c) No. of electrons per $\text{m}^2 = \frac{2.7 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.7 \times 10^{12}$
 $\therefore$ No. of electrons per $\text{mm}^2 = 1.7 \times 10^{12} \times 10^{-6} = 1.7 \times 10^6$.

17.4

 1 (a) $E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{+8.5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (5.0 \times 10^{-3})^2} = 3.1 \times 10^3 \text{ V m}^{-1}$.

 $V = \frac{Q}{4\pi\epsilon_0 r} = \frac{+8.5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (5.0 \times 10^{-3})} = 15.3 \text{ V}$.

 (b) $E = \frac{V_P}{d} = \frac{200}{40 \times 10^{-3}} = 5000 \text{ V m}^{-1}$. At the mid-point, $V = 100 \text{ V}$.

 2 (a) Rearrange $V_S = \frac{Q}{4\pi\epsilon_0 R_S}$ to give

 $Q = 4\pi\epsilon_0 R_S V_S = 4\pi \times (8.85 \times 10^{-12}) \times 0.20 \times 100\,000 = 2.2 \times 10^{-6} \text{ C}$.

 (b) $E = \frac{Q}{4\pi\epsilon_0 R_S^2} = \frac{4\pi\epsilon_0 R_S V_S}{4\pi\epsilon_0 R_S^2} = \frac{100\,000}{0.20} = 500\,000 \text{ V m}^{-1}$.

 (c) At $r = 2.0 \text{ m}$, $E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{+2.2 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (2.0)^2} = 5000 \text{ V m}^{-1}$.

 $V = \frac{Q}{4\pi\epsilon_0 r} = \frac{+2.2 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times (2.0)} = 10\,000 \text{ V}$.

Revision questions
17.1

(a) See Fig. A17.2.

 (b) $F_1 = \frac{+2.5 \times 10^{-12} \times -7.2 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (0.020)^2}$
 $= 4.0 \times 10^{-10} \text{ N}$ (attraction).

 (c) Let Q_1 represent the positive charge and Q_2 the negative charge. The electric field strength due to Q_1 ,

 $E_1 = \frac{Q_1}{4\pi\epsilon_0 r^2} = \frac{+2.5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (0.010)^2}$
 $= 225 \text{ V m}^{-1}$ away from Q_1 .

 The electric field strength due to Q_2 ,

 $E_2 = \frac{Q_2}{4\pi\epsilon_0 r^2} = \frac{-7.2 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (0.010)^2} = -648 \text{ V m}^{-1}$ towards Q_2 .

 The total electric field, $E = E_1 + E_2$ because the electric field contributions are in the same direction at the midpoint. Hence $E = 225 + 648 = 873 \text{ V m}^{-1}$.

 The electric potential, V_1 , at the midpoint, due to Q_1
 $= \frac{Q_1}{4\pi\epsilon_0 r} = \frac{+2.5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (0.010)} = 2.25 \text{ V}$.

 The electric potential, V_2 , at the midpoint, due to Q_2
 $= \frac{Q_2}{4\pi\epsilon_0 r} = \frac{-7.2 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times (0.010)} = -6.48 \text{ V}$.

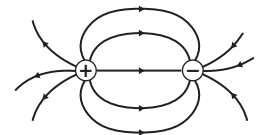
 The total potential, $V = V_1 + V_2 = (2.25) + (-6.48) = -4.23 \text{ V}$.


Fig. A17.2

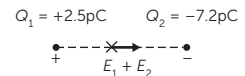


Fig. A17.3

17.2

$$(a) (i) E = \frac{V}{d} = \frac{430}{5.0 \times 10^{-3}} = 8.6 \times 10^4 \text{ V m}^{-1}.$$

(ii) Rearrange $\frac{qV}{d} = mg$ to obtain

$$q = \frac{mgd}{V} = \frac{5.6 \times 10^{-15} \times 9.8 \times 5.0 \times 10^{-3}}{430} = 6.4 \times 10^{-19} \text{ C}.$$

(b) For a balanced droplet with constant charge, its mass, m , is proportional to the holding voltage, V . $\therefore$ the new mass of the droplet

$$m' = \frac{mV'}{V} = \frac{5.6 \times 10^{-15} \times 620}{430} = 8.1 \times 10^{-15} \text{ kg}$$

$\therefore$ the mass of the uncharged droplet = $m' - m = 2.5 \times 10^{-15} \text{ kg}$.

17.3

$$(a) F = \frac{-e^2}{4\pi\epsilon_0 r^2} = \frac{-(1.6 \times 10^{-19})^2}{4\pi \times 8.85 \times 10^{-12} \times (1.0 \times 10^{-10})^2} = 2.3 \times 10^{-8} \text{ N}.$$

$$(b) \text{ Potential due to } +e \text{ at distance } r = \frac{e}{4\pi\epsilon_0 r}$$

$$= \frac{1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (1.0 \times 10^{-10})} = 14.4 \text{ V. } \therefore \text{ potential energy of an electron at } 14.4 \text{ V} = -14.4e = -14.4 \times 1.6 \times 10^{-19} = -2.3 \times 10^{-18} \text{ J}$$

$\therefore$ work done to separate electron and proton to infinity = $2.3 \times 10^{-18} \text{ J}$.

17.4

(a) A small test charge, q , placed at the mid point would experience a force towards one of the two charges and an equal and opposite force towards the other charge. Hence the resultant force on the test charge is zero.

$$(b) (i) \text{ Potential at } 20 \text{ mm from } 1.6 \text{ nC charge} \\ = \frac{Q}{4\pi\epsilon_0 r} = \frac{+1.6 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12} \times (0.020)} = +720 \text{ V}.$$

Potential due to the other charge at the same distance is also +720 V $\therefore$ total potential = $720 + 720 = 1440 \text{ V}$.

(ii) See Fig. A17.4.

$$(c) W = qV = 1.5 \times 10^{-17} \times 1440 = 2.2 \times 10^{-14} \text{ J}.$$

17.5

$$(a) E_S = \frac{Q_S}{4\pi\epsilon_0 R_S^2} = \frac{V_S}{R_S} \text{ since } V_S = \frac{Q}{4\pi\epsilon_0 R_S}.$$

$$(i) E_S = \frac{10\,000}{100} = 100 \text{ V m}^{-1}.$$

$$(ii) E_S = \frac{10\,000}{1.0 \times 10^{-3}} = 1.0 \times 10^7 \text{ V m}^{-1}.$$

(b) Charge gathers at sharp points. Because the radius of curvature is very small at a sharp point, the electric field is very strong.

$$(c) (i) \text{ Rearrange } V_S = \frac{Q}{4\pi\epsilon_0 R_S} \text{ to give}$$

$$Q = 4\pi\epsilon_0 R_S V = 4\pi \times 8.85 \times 10^{-12} \times (0.100) \times 5000 = 5.6 \times 10^{-8} \text{ C}.$$

(ii) The potential equalises when contact is made. Hence the charge is shared in proportion to the radius since $Q = 4\pi\epsilon_0 R_S V$. Therefore the larger sphere has twice as much charge as the smaller sphere since its radius is twice as large. The charge on the larger sphere therefore becomes $3.7 \times 10^{-8} \text{ C}$ ($= \frac{2}{3} \times 5.6 \times 10^{-8} \text{ C}$) and the charge on the smaller sphere becomes $1.9 \times 10^{-8} \text{ C}$ ($= \frac{1}{3} \times 5.6 \times 10^{-8} \text{ C}$).

$$(iii) V_S = \frac{Q}{4\pi\epsilon_0 R_S} = \frac{3.7 \times 10^{-8}}{4\pi \times (8.85 \times 10^{-12}) \times 0.100} = 3330 \text{ V for both spheres}.$$

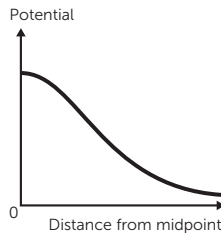


Fig. A17.4

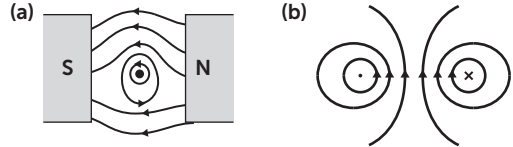


Fig. A18.1

(b) The resistivity of copper is very low, so the resistance of the windings is very small.

(c) It does not oxidise and it has a high melting point, so it withstands sparks.

2 (a) See p. 266.

(b) A fuse melts and needs to be replaced. A circuit breaker does not need to be replaced and is reset manually.

18.3

1 (a) Anticlockwise; see Fig. 18.14 if necessary.

(b) The split-ring commutator reverses the current direction around the coil every half-turn. This is necessary to ensure that when each side is adjacent to a certain pole, the current is always in the same direction, so the force is always in the same direction.

2 (a) X deflects less because the forces on the coil due to the magnet are less.

(b) X deflects more than Y because the spiral spring can be tightened more before it stops the coil turning.

18.4

$$1 (a) (i) F = BIL = 0.080 \times 5.2 \times 10^{-2} \times 3.2 = 13.3 \text{ mN due North}.$$

$$(ii) \text{ Rearrange } F = BIL \text{ to give } I = \frac{F}{BL} = \frac{0.015}{0.25 \times 0.040} = 1.5 \text{ A in a direction from north to south}.$$

(b) (i) The force on each long edge = $BILn$, where n is the number of turns. Hence $F = 0.065 \times 7.2 \times 40 \times 10^{-3} \times 80 = 1.5 \text{ N}$.

(ii) Couple = force $\times$ perpendicular distance between lines of action of the forces = $1.5 \text{ N} \times 0.030 \text{ m} = 0.045 \text{ N m}$.

$$2 (a) (i) \text{ Change of reading} = 105.38 - 104.92 = 0.46 \text{ g} = 4.6 \times 10^{-4} \text{ kg}$$

$$\therefore \text{ force} = 4.6 \times 10^{-4} \times 9.8 = 4.5 \times 10^{-3} \text{ N}.$$

$$(ii) B = \frac{F}{IL} = \frac{4.5 \times 10^{-3}}{6.5 \times 0.035} = 2.0 \times 10^{-2} \text{ T}.$$

(b) The angle between the lines of force and the wire is 60°

$$\therefore F = BIL \sin 60 = 2.0 \times 10^{-2} \times 6.5 \times 0.035 \times \sin 60 = 3.9 \times 10^{-3} \text{ N}$$

$$\therefore \text{ new reading} = 105.38 - \left(\frac{3.9 \times 10^{-3}}{g} \times 1000 \right) = 104.98 \text{ g}.$$

18.5

$$1 (a) \text{ Rearrange } B = \mu_0 nI \text{ to give } n = \frac{B}{\mu_0 I} = \frac{25 \times 10^{-3}}{4\pi \times 10^{-7} \times 8.0} = 2490$$

$$\therefore N = nL = 2490 \times 0.800 = 1990 \text{ turns}.$$

(b) (i) $B = 50 + 25 = 75 \text{ mT}$. (ii) For zero field, the solenoid's own field must be 50 mT in the opposite direction to the external field. The solenoid current must therefore be 16.0 A , since a current of 8.0 A is needed to produce a 25 mT field. If the external field is left to right, the solenoid field must be right to left. The solenoid current must therefore be anticlockwise around the solenoid, viewing the left-hand end of the solenoid.

2 (a) (i) For $r = 12.5 \text{ mm}$ ($= 25 \text{ mm}/2$),

$$B = \frac{\mu_0 I}{2\pi r} = \frac{2.0 \times 10^{-7} \times 1000}{12.5 \times 10^{-3}} = 1.6 \times 10^{-2} \text{ T}.$$

$$(ii) \text{ For } r = 10 \text{ m}, B = \frac{\mu_0 I}{2r} = \frac{2.0 \times 10^{-7} \times 1000}{10} = 2.0 \times 10^{-5} \text{ T}.$$

$$(b) \text{ The force per unit length on the second cable} = BI_2 = 2.0 \times 10^{-5} \times 1000 = 0.020 \text{ N m}^{-1}.$$

18.6

1 (a) The magnetic field of the solenoid aligns all the domains in the ferromagnet to magnetise the ferromagnet fully.

(b) The magnetic field of the solenoid alternates, making the domains repeatedly change direction. As the ferromagnet is withdrawn, the solenoid field has less effect on the ferromagnet and the domains gradually lose their alignment, which becomes disordered.

2 (a) (i) Steel remains more strongly magnetised than iron in a direct current solenoid after the current is switched off. (ii) Steel has a larger loop area as it is harder to magnetise and demagnetise.

$$(b) \text{ Number of turns per unit length, } n = \frac{200}{\pi \times 0.040}$$

$$\therefore B = \mu_r \mu_0 nI = 2000 \times 4\pi \times 10^{-7} \times \frac{200}{\pi \times 0.040} \times 0.05 = 0.20 \text{ T}.$$

UNIT 18

In-text questions

18.1

1 (a) Steel does not lose its magnetism whereas iron does.

(b) (i) Position the two magnets so they are equidistant from the plotting compass with the N-pole of each magnet pointing towards the plotting compass. The plotting compass needle is repelled more by the stronger N-pole.

(ii) With the magnets as in Fig. 18.6 and the N-pole of each magnet pointing towards the plotting compass, adjust the distances so the plotting compass needle is at 45° to the axis of each magnet. Turn one of the magnets around. If the plotting compass needle turns through more than 90° , the S-pole of that magnet is stronger than its N-pole.

2 See Fig. A18.1 opposite.

18.2

1 (a) Iron magnetises and demagnetises easily.

Revision questions

18.1

(a) See Fig. A18.2.

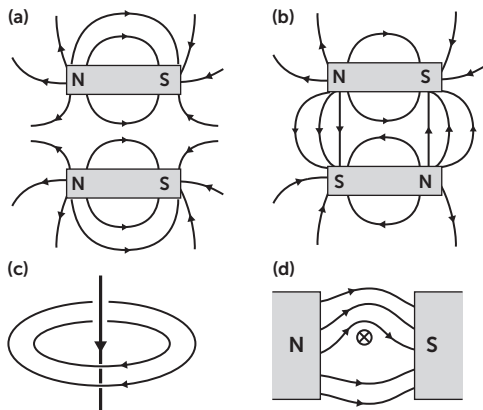


Fig. A18.2

- (b) (i) The bar magnet makes the domains align along the lines of force of the bar magnet's field. After the bar magnet is removed, the domains remain partly aligned.
 (ii) The bar magnet attracts the opposite pole of each domain so it leaves the end of the nail with the opposite polarity.

18.2

(a) See p. 267.

- (b) (i) $F = BILN = 0.120 \times 5.0 \times 0.040 \times 200 = 4.8 \text{ N}$. (ii) The force on each long side is unchanged (the perpendicular distance between the two forces is zero so the forces have no turning effect at this position).

18.3

(a) (i) Into the plane of the diagram.

(ii) Rearrange $\frac{qV}{d} = Bqv$ to give $B = \frac{V}{vd}$.(iii) $B = \frac{V}{vd} = \frac{3500}{2.8 \times 10^7 \times 50 \times 10^{-3}} = 2.5 \times 10^{-3} \text{ T}$.(b) V needs to be doubled if B is doubled. Hence $V = 7000 \text{ V}$.

18.4

(a) (i) $B = \mu_0 nI = 4\pi \times 10^{-7} \times \frac{500}{0.250} \times 6.5 = 1.63 \times 10^{-2} \text{ T}$.(ii) At the end, $B = 0.5\mu_0 nI = 8.2 \times 10^{-4} \text{ T}$.(b) Rearrange $B = \mu_0 nI$ to give

$$I = \frac{B}{\mu_0 n} = \frac{60 \times 10^{-6}}{4\pi \times 10^{-7} \times (500/0.250)} = 2.4 \times 10^{-2} \text{ T}.$$

18.5

(a) At the nearer edge, the magnetic flux density due to the wire,

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{2.0 \times 10^{-7} \times 8}{0.040} = 4.25 \times 10^{-5} \text{ T}$$

$\therefore$ force on the nearer edge $= B_1 I_{\text{coil}} L = 4.25 \times 10^{-5} \times 2.0 \times 0.060 = 5.1 \times 10^{-6} \text{ N}$.
 At the further edge, which is twice as far from the wire, $B = 0.5B_1$ so the force is half the force on the nearer edge and therefore equals $2.55 \times 10^{-6} \text{ N}$.

(b) The nearer edge is attracted to the wire and the further edge is repelled.

Therefore the resultant force is $(5.1 - 2.55) \times 10^{-6} \text{ N} = 2.55 \times 10^{-6} \text{ N}$ towards the wire.

UNIT 19

In-text questions

19.1

1 See Fig. A19.1 opposite.

2 The induced e.m.f. is due to the wingspan cutting the vertical component, B_v , of the Earth's field. Since $B_v = 60 \sin 70 = 56 \mu\text{T}$, then the induced voltage, $V = B_v L v = 56 \times 10^{-6} \times 22 \times 180 = 0.22 \text{ V}$.

19.2

1 (a) $v = u + at = 0 + (9.8 \times 0.5) = 4.9 \text{ ms}^{-1}$.

(b) The rod moves vertically and therefore cuts the horizontal component, B_H , of the Earth's field. $B_H = 60 \cos 70 = 20.5 \mu\text{T}$

$\therefore V = B_H L v = 20.5 \times 10^{-6} \times 0.750 \times 4.9 = 7.5 \times 10^{-5} \text{ V}$.

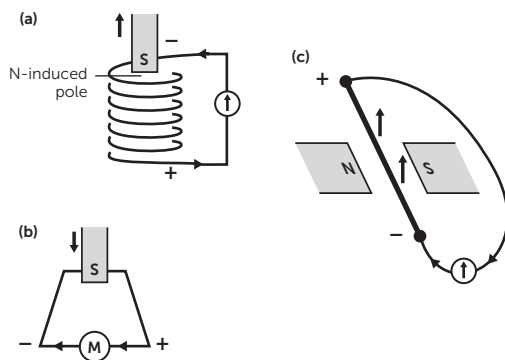


Fig. A19.1

2 (a) Total magnetic flux $= BAN = 90 \times 10^{-3} \times (0.040 \times 0.040) \times 50 = 7.2 \times 10^{-3} \text{ Wb}$.

(b) (i) Time taken, $t = \frac{\text{distance}}{\text{speed}} = \frac{0.040}{0.16} = 0.25 \text{ s}$.

(ii) $V = \frac{\text{change of total flux}}{\text{time taken}} = \frac{7.2 \times 10^{-3}}{0.25} = 2.9 \times 10^{-2} \text{ V}$.

19.3

1 (a) See Fig. 19.15b.

(b) (i) any zero (ii) any peak.

(c) The peak voltage is halved and the time for each cycle is doubled, so the waveform is half the height and twice as long as in (a).

(a) $V_{\text{MAX}} = 2\pi fBAN = 2\pi \times 50 \times 0.150 \times (0.500 \times 0.300) \times 1500 = 10\,600 \text{ V}$.

(b) See Fig. 19.15b.

(c) The force due to the magnetic field on each side $= BILN = 0.150 \times 10.0 \times 0.50 \times 1500 = 1125 \text{ N}$. The couple at peak current $= \text{force} \times \text{width of the coil} = 1125 \times 0.30 = 340 \text{ Nm}$.

19.4

1 (a) Rearrange $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ to give $V_p = \frac{N_p \times V_s}{N_s} = \frac{60 \times 230}{1200} = 11.5 \text{ V}$.

(b) (i) Power delivered by the secondary coil $I_s V_s = 100 \text{ W}$,
 hence $I_s = \frac{100 \text{ W}}{230 \text{ V}} = 0.43 \text{ A}$.

(ii) Assuming 100% efficiency, $I_p V_p = I_s V_s$. Rearranging this equation gives
 $I_p = \frac{I_s V_s}{V_p} = \frac{0.43 \times 230}{11.5} = 8.7 \text{ A}$.

(a) Turns ratio $= \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{230}{110} = 2.1$.

(b) Maximum generator current $= \frac{20 \text{ kW}}{110 \text{ V}} = 182 \text{ A}$.

(c) Maximum current from the cables $= I_s = \frac{I_p V_p}{V_s} = \frac{182 \times 110}{230} = 87 \text{ A}$.

19.5

1 (a) (i) The growth of current through the coil causes an induced e.m.f. which opposes the battery and slows the growth of current.

(ii) Estimated rate of growth of current $= 0.25 \text{ A/s} = 0.25 \text{ A s}^{-1}$ approximately. Hence $L = 3 \text{ V} / 0.25 \text{ A s}^{-1} = 12 \text{ H}$ approximately for the two coils.

(b) The magnetic fields of the two coils were in opposite directions as a result of the reversal of the connections to one coil. Hence the magnetic flux in the core is zero, regardless of the current in the coils. Hence no induced e.m.f. can be produced.

(a) Initial rate of change of current, $\frac{di}{dt} = \frac{V_{\text{batt}}}{L} = \frac{12}{25} = 0.48 \text{ A s}^{-1}$.

(b) (i) Total resistance, $R = 2.0 + 1.0 = 3.0 \Omega$. $\therefore I_0 = \frac{V_{\text{batt}}}{R} = \frac{12}{3.0} = 4.0 \text{ A}$.

(ii) Energy stored $= \frac{1}{2} L I^2 = 0.5 \times 25 \times 4.0^2 = 200 \text{ J}$.

Revision questions

19.1

(a) (i) North (ii) left to right.

(b) (i) Smaller and in the same direction. (ii) Larger and in the opposite direction.

19.2

(a) (i) When the coil is shorted, the induced e.m.f. forces current around the circuit. The magnetic field generated by this current acts against the rotating magnet, exerting a braking effect.

(ii) The loss of k.e. is converted into electrical energy, which is converted into heat due to the resistance of the windings.

(b) (i) The increased load slows the motor down, which reduces the back e.m.f. enabling the voltage supply unit to push more current through the motor.

(ii) Connecting an extra load across the secondary coil causes the secondary current to increase. The magnetic flux of the secondary coil is in the opposite direction to the primary flux, so the total flux decreases and the back e.m.f. across the primary coil becomes smaller. This allows the primary voltage supply to push more current through the primary coil, increasing the flux in the core back to its initial level.

19.3

(a) $V = BLv = 80 \times 10^{-6} \times 35 \times 550 = 1.54 \text{ V}$.

(b) (i) Each block represents a change of flux of $1.0 \times 10^{-4} \text{ Wb}$ ($= 5 \text{ mV} \times 0.02 \text{ s}$). There are 36 blocks under the curve. Hence the change of flux linkage is $3.6 \times 10^{-3} \text{ Wb}$ ($= 36 \times 1.0 \times 10^{-4} \text{ Wb}$).

(ii) $BAN = 3.6 \times 10^{-4} \text{ Wb} \therefore B = \frac{3.6 \times 10^{-3}}{\pi(12.5 \times 10^{-3})^2 \times 120} = 6.1 \times 10^{-2} \text{ T}$.

19.4

(a) (i) The sudden growth of current through the primary coil causes a sudden growth of magnetic flux in the core which induces a large e.m.f. in the secondary coil. This e.m.f. is large enough to cause a spark across the gap between the rods.

(ii) The magnetic flux in the core suddenly collapses, generating a large reverse e.m.f. in the secondary coil which also creates a spark.

(iii) No change in the primary current occurs so there is no change in the magnetic flux in the core and hence no induced e.m.f.

(b) (i) Rearrange $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ to give $N_s = \frac{N_p \times V_s}{V_p} = \frac{1200 \times 9}{230} = 47$

(ii) $I_s = \frac{36 \text{ W}}{9.0 \text{ V}} = 4.0 \text{ A}$. (iii) $I_p = \frac{36 \text{ W}}{230 \text{ V}} = 0.16 \text{ A}$.

19.5

(a) (i) $B = 2000 \times 4\pi \times 10^{-7} \times \frac{80}{\pi \times 0.035} \times 0.06 = 0.11 \text{ T}$.

(ii) $\Phi = BAN = 0.11 \times 6.0 \times 10^{-5} \times 80 = 5.3 \times 10^{-4} \text{ Wb}$

$\therefore L = \frac{\Phi}{I} = \frac{5.3 \times 10^{-4}}{0.06} = 8.8 \times 10^{-3} \text{ H}$.

(b) (i) Initial rate of growth of current $= \frac{V_{\text{batt}}}{L} = 170 \text{ A s}^{-1}$.

(ii) $E = \frac{1}{2}LI^2 = 0.5 \times 8.8 \times 10^{-3} \times (0.06)^2 = 1.6 \times 10^{-5} \text{ J}$.

UNIT 20

In-text questions

20.1

1 The trace measures 58 mm from the bottom to the top. Also, one complete cycle covers a horizontal distance of 32 mm.

(a) $2V_0 = 5.8 \text{ cm} \times 0.2 \text{ V cm}^{-1} = 1.16 \text{ V} \therefore V = 0.58 \text{ V}$;

$T = 3.2 \text{ cm} \times 10 \mu\text{s cm}^{-1} = 32 \mu\text{s}$

$\therefore f = \frac{1}{32 \times 10^{-6} \text{ s}} = 3.13 \times 10^4 \text{ Hz}$.

(b) $2V_0 = 5.8 \text{ cm} \times 5.0 \text{ V cm}^{-1} = 29 \text{ V}$

$\therefore V_0 = 14.5 \text{ V}$;

$T = 3.2 \text{ cm} \times 5 \text{ ms cm}^{-1} = 16 \text{ ms}$

$\therefore f = \frac{1}{16 \times 10^{-3} \text{ s}} = 62.5 \text{ Hz}$.

2 (a) See Fig. A20.1.

(b) (i) $V = V_0 \sin(2\pi ft)$
 $= 4.0 \sin(2\pi \times 200 \times 1.0 \times 10^{-3} \text{ radians})$
 $= 3.8 \text{ V}$.

(ii) $V = V_0 \sin(2\pi ft)$
 $= 4.0 \sin(2\pi \times 200 \times 3.0 \times 10^{-3} \text{ radians}) = -2.4 \text{ V}$.

20.2

1 (a) (i) Right to left (ii) A and C.

(b) A half-wave rectified waveform is produced. See Fig. 20.7.

2 (a) Right to left. (b) The peak current is greater in alternate half-cycles when diode A conducts, because current passes through the lesser resistance when A conducts and through the higher resistance when B conducts. See Fig. A20.2.

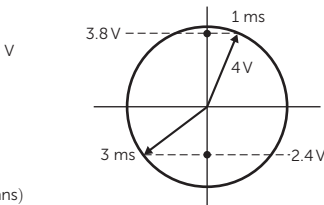


Fig. A20.1

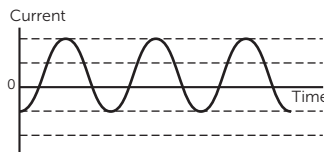


Fig. A20.2

20.3

1 (a) (i) $V_{pp} = 7.2 \text{ cm} \times 0.2 \text{ V cm}^{-1} = 1.44 \text{ V}$. (ii) $V_0 = 0.5V_{pp} = 0.72 \text{ V}$.

(iii) $V_{rms} = \frac{1V_0}{\sqrt{2}} = 0.51 \text{ V}$.

(b) (i) $R = \frac{V_{rms}}{I_{rms}} = \frac{0.51}{0.12} = 4.2 \Omega$.

(ii) Mean power $= I_{rms}V_{rms} = 0.120 \times 0.51 = 0.061 \text{ W}$.

2 (a) $V_0 = \sqrt{2}V_{rms} = \sqrt{2} \times 230 \text{ V} = 325 \text{ V}$.

(b) (i) Rearrange mean power $= I_{rms}V_{rms}$ to give

$I_{rms} = \frac{\text{mean power}}{V_{rms}} = \frac{1000 \text{ W}}{230 \text{ V}} = 4.3 \text{ A}$.

(ii) $I_0 = \sqrt{2}I_{rms} = \sqrt{2} \times 4.3 = 6.1 \text{ A}$. (iii) $R = \frac{V_{rms}}{I_{rms}} = 53 \Omega$.

20.4

1 (a) See Fig. A20.3.

(b) Reactance $=$ gradient of graph

$= \frac{V_{rms}}{I_{rms}} = \frac{10.0}{60.0 \times 10^{-3}} = 167 \Omega$.

(c) Rearrange $X_C = \frac{1}{2\pi fC}$ to give

$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi \times 2000 \times 167} = 4.8 \times 10^{-7} \text{ F}$.

2 (a) (i) Trace height (top to bottom) $= 60 \text{ mm}$

$\therefore$ peak-to-peak voltage $= 2.0 \times 6.0$

$= 12.0 \text{ V} \therefore V_0 = 6.0 \text{ V}$.

(ii) $V_{rms} = \frac{V_0}{\sqrt{2}} = 4.24 \text{ V}$.

(b) One full cycle on the screen $= 40 \text{ mm}$ horizontally
 $\therefore$ time period $= 0.5 \times 4.0 \text{ ms} = 2.0 \text{ ms}$

$\therefore$ frequency, $f = \frac{1}{T} = \frac{1}{2.0 \times 10^{-3}} = 500 \text{ Hz}$.

(c) (i) Reactance $= \frac{V_{rms}}{I_{rms}} = \frac{4.24}{0.48 \times 10^{-3}} = 8830 \Omega$.

(ii) Rearrange $X_C = \frac{1}{2\pi fC}$ to give

$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi \times 500 \times 8830} = 3.6 \times 10^{-8} \text{ F}$.

20.5

1 (a) (i) $X = \frac{V_{rms}}{I_{rms}} = \frac{12.0}{0.110} = 109 \Omega$.

(ii) $X_L = 2\pi fL \therefore L = \frac{X_L}{2\pi f} = \frac{109}{2\pi \times 1500} = 1.16 \times 10^{-2} \text{ H}$.

(b) $X_L = 2\pi fL = 2\pi \times 20000 \times 1.16 \times 10^{-2} = 1460 \Omega$

$\therefore I_{rms} = \frac{V_{rms}}{X_L} = \frac{12.0}{1460} = 8.2 \times 10^{-3} \text{ A}$.

2 (a) (i) $X_L = \frac{V_{rms}}{I_{rms}} = \frac{6.0}{0.028} = 214 \Omega$.

(ii) $X_L = 2\pi fL \therefore f = \frac{X_L}{2\pi L} = \frac{214}{2\pi \times 0.045} = 757 \text{ Hz}$.

(b) $X_C = X_L$ for the same current

$\therefore \frac{1}{2\pi fC} = X_L$, hence $C = \frac{1}{2\pi \times 757 \times 214} = 9.8 \times 10^{-7} \text{ F}$.

20.6

1 (a) (i) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318 \Omega$.

(ii) $X_L = 2\pi fL = 2\pi \times 50 \times 0.48 = 151 \Omega$.

(iii) $Z = [R^2 + (X_C - X_L)^2]^{1/2} = [55^2 + (318 - 151)^2]^{1/2} = 176 \Omega$.

(iv) $I_{rms} = \frac{V_{rms}}{Z} = \frac{6.0}{176} = 0.034 \text{ A}$.

(b) (i) r.m.s. value of $V_R = I_{rms}R = 0.034 \times 55 = 1.9 \text{ V}$.

(ii) r.m.s. value of $V_L = I_{rms}X_L = 0.034 \times 151 = 5.1 \text{ V}$.

(iii) r.m.s. value of $V_C = I_{rms}X_C = 0.034 \times 318 = 10.8 \text{ V}$.

(c) (i) See Fig. A20.4.

(a) Phasor diagram

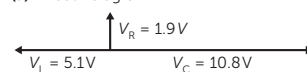
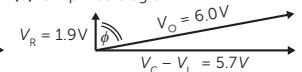


Fig. A20.4

(ii) $\tan \phi = \frac{V_C - V_L}{V_R} = \frac{10.8 - 5.1}{1.9} = 3.0 \therefore \phi = 72^\circ$.

(b) Simplified diagram



2 (a) (i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.15 \times 10^{-3}) \times (0.47 \times 10^{-6})}} = 1.9 \times 10^4 \text{ Hz}$.

(ii) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 1.9 \times 10^4 \times 0.47 \times 10^{-6}} = 17.8 \Omega$.

(b) (i) $R = \frac{V_{rms}}{I_{rms}} = \frac{5.0}{0.350} = 14.3 \Omega$.

(ii) r.m.s. voltage across the capacitor
 $= I_{rms} X_C = 0.350 \times 17.8 = 6.2 \text{ V}$.

(iii) The capacitor voltage and the inductor voltage are equal and opposite. See Fig. A20.5 for the phasor diagram.

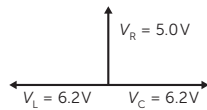


Fig. A20.5

Revision questions

20.1

(a) and (b) See p. 303.

20.2

(a) (i) $I_0 = \sqrt{2} I_{rms} = 1.06 \text{ A}$.

(ii) $V_0 = I_0 R = 1.06 \times 47 = 50 \text{ V}$.

(iii) Peak power $= I_0 V_0 = 1.06 \times 50 = 53 \text{ W}$.

(iv) Mean power $= 0.5 \times \text{peak power} = 26 \text{ W}$.

(b) See Fig. A20.6 (ii) 5.1 V (iii) 11.2 V.

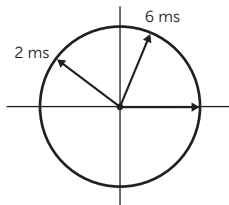


Fig. A20.6

20.3

(a) (i) Time period, $T = 4.0 \text{ cm} \times 5 \text{ ms cm}^{-1} = 20 \text{ ms}$

$\therefore f = \frac{1}{0.020 \text{ s}} = 50 \text{ Hz}$.

(ii) $V_0 = 3.0 \text{ cm} \times 2.0 \text{ V cm}^{-1} = 6.0 \text{ V}$.

(iii) $V_{rms} = \frac{V_0}{\sqrt{2}} = 4.2 \text{ V}$.

(b) (i) $X_C = \frac{V_{rms}}{I_{rms}} = \frac{4.2 \text{ V}}{2.8 \times 10^{-3} \text{ A}} = 1.5 \times 10^3 \Omega$.

(ii) Rearrange $X_C = \frac{1}{2\pi fC}$ to give

$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 1500} = 2.1 \times 10^{-6} \text{ F}$.

20.4

(a) (i) $X_L = \frac{V_{rms}}{I_{rms}} = \frac{6.0 \text{ V}}{0.35 \text{ A}} = 17.1 \Omega$.

(ii) Rearrange $X_L = 2\pi fL$ to give $L = \frac{X_L}{2\pi f} = \frac{17.1}{2\pi \times 50} = 5.4 \times 10^{-2} \text{ H}$.

(b) $X_L = X_C$ and $X_C = \frac{1}{2\pi fC}$ $\therefore \frac{1}{2\pi fC} = 17.1 \Omega$.

$\therefore C = \frac{1}{2\pi \times 50 \times 17.1} = 1.9 \times 10^{-4} \text{ F}$.

(c) (i) X_L would double and therefore the r.m.s. current would halve to 0.175 A.

(ii) X_C would halve and therefore the r.m.s. current would double to 0.70 A.

20.5

(a) The circuit is in resonance at 750 Hz. The capacitor reactance is exactly equal to the inductor reactance at this frequency so the circuit impedance is a minimum at 750 Hz. Hence the current is at a maximum.

(b) (i) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 750 \times 2.2 \times 10^{-6}} = 97 \Omega$.

(ii) $X_L = X_C$ and $X_L = 2\pi fL$ $\therefore 2\pi fL = X_C$

$\therefore L = \frac{X_C}{2\pi f} = \frac{96.5}{2\pi \times 750} = 2.1 \times 10^{-2} \text{ H}$.

(iii) $R = \frac{\text{r.m.s. supply voltage}}{I_{rms}} = \frac{6.0 \text{ V}}{65 \times 10^{-3} \text{ A}} = 92 \Omega$.

(c) (i) r.m.s. voltage across $R = 6.0 \text{ V}$.

(ii) r.m.s. voltage across

$C = I_{rms} X_C = 65 \times 10^{-3} \times 97 = 6.3 \text{ V}$.

r.m.s. voltage across $L = 6.3 \text{ V}$.

(d) See Fig. A 20.7: at resonance, $V_C - V_L = 0$ so the supply voltage is equal to V_R which is in phase with I .

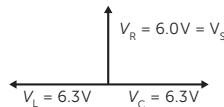


Fig. A20.7

(b) $v^2 = \frac{2eV_A}{m} = 2 \times 1.76 \times 10^{11} \times 4000 = 1.41 \times 10^{15}$

$\therefore v = 3.8 \times 10^7 \text{ m s}^{-1}$.

2 (a) Rearrange $\frac{1}{2}mv^2 = eV_A$ to calculate v .

Hence $v^2 = \frac{2eV_A}{m} = 2 \times 1.76 \times 10^{11} \times 3200 = 1.1 \times 10^{15}$.

(b) Rearrange $Bev = \frac{(eV_p)}{d}$ to give

$B = \frac{V_p}{vd} = \frac{4200}{3.4 \times 10^7 \times 40 \times 10^{-3}} = 3.1 \text{ mT}$.

21.2

1 (a) Positive, as the top plate is negative.

(b) Rearranging $\frac{QV_p}{d} = mg$ gives

$Q = \frac{mgd}{V_p} = \frac{3.8 \times 10^{-15} \times 9.8 \times 5.0 \times 10^{-3}}{595} = 3.2 \times 10^{-19} \text{ C}$.

2 (a) (i) $v = \frac{s}{t} = \frac{1.0 \times 10^{-3}}{16.5} = 6.1 \times 10^{-5} \text{ m s}^{-1}$.

(ii) $r^2 = \frac{9\eta v}{2\rho g} = \frac{9 \times 1.8 \times 10^{-5} \times 6.1 \times 10^{-5}}{2 \times 960 \times 9.8} = 5.2 \times 10^{-13} \text{ m}^2$.

$\therefore r = 7.2 \times 10^{-7} \text{ m}$.

(iii) $m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi (7.2 \times 10^{-7})^3 \times 960 = 1.5 \times 10^{-15} \text{ kg}$.

(iv) $Q = \frac{mgd}{V_p} = \frac{1.5 \times 10^{-15} \times 9.8 \times 4.0 \times 10^{-3}}{375} = 1.6 \times 10^{-19} \text{ C}$.

(b) (i) The drag force increases with speed as the droplet falls. Hence the resultant force (= weight - drag force) and therefore the acceleration decreases to zero. At the terminal speed, the drag force is equal and opposite to the weight.

(ii) The electrostatic force on the droplet would reverse direction and the droplet would be attracted to the lower plate. It would therefore accelerate towards the lower plate, reaching a constant speed when the drag force became equal and opposite to the electrostatic force + the weight.

21.3

1 (a) (i) $E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{600 \times 10^{-9}} = 3.3 \times 10^{-19} \text{ J}$.

(ii) $E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{100 \times 10^{-9}} = 2.0 \times 10^{-18} \text{ J}$.

(b) (i) $c = 0.64 \text{ eV} = 0.64 \times 1.6 \times 10^{-19} \text{ J} = 1.0 \times 10^{-19} \text{ J}$

$\therefore \text{max. k.e.} = hf - \phi = 3.3 \times 10^{-19} - 1.0 \times 10^{-19} = 2.3 \times 10^{-19} \text{ J}$.

(ii) $\text{Max. k.e.} = hf - \phi = 2.0 \times 10^{-18} - 1.0 \times 10^{-19} = 1.9 \times 10^{-18} \text{ J}$.

2 (a) (i) $E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{550 \times 10^{-9}} = 3.6 \times 10^{-19} \text{ J}$.

(ii) Work done $= eV = 1.6 \times 10^{-19} \times 0.58 = 9.3 \times 10^{-20} \text{ J}$.

(iii) Rearrange $eV_s = hf - \phi$ to give

$\phi = hf - eV_s = 3.6 \times 10^{-19} - 9.3 \times 10^{-20} = 2.7 \times 10^{-19} \text{ J}$.

(b) The threshold frequency, $f_0 = \frac{\phi}{h} = \frac{2.7 \times 10^{-19}}{6.6 \times 10^{-34}} = 4.1 \times 10^{14} \text{ Hz}$

$\therefore \text{minimum wavelength} = \frac{c}{f_0} = \frac{3.0 \times 10^8}{4.1 \times 10^{14}} = 7.3 \times 10^{-7} \text{ m}$.

21.4

1 (a) Photon energy $= E_1 - E_2 = 7.2 - 0 = 7.2 \text{ eV} = 7.2 \times 1.6 \times 10^{-19} \text{ J}$

$= 11.5 \times 10^{-19} \text{ J}$. Rearrange photon energy, $E = hf = \frac{hc}{\lambda}$ to give $\lambda = \frac{hc}{E}$

$= \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{11.5 \times 10^{-19}} = 1.7 \times 10^{-7} \text{ m}$.

(b) (i) Photon energy $= E_1 - E_2 = 8.9 - 7.6 = 1.3 \text{ eV} = 1.3 \times 1.6 \times 10^{-19} \text{ J}$

$= 2.1 \times 10^{-19} \text{ J}$. Rearrange photon energy,

$E = hf = \frac{hc}{\lambda}$ to give $\lambda = \frac{hc}{E}$

$= \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{2.1 \times 10^{-19}} = 9.4 \times 10^{-7} \text{ m}$.

(ii) Photon energy $= E_1 - E_2 = 8.9 - 4.9 = 4.0 \text{ eV} = 4.0 \times 1.6 \times 10^{-19} \text{ J}$

$= 6.4 \times 10^{-19} \text{ J}$. Rearrange photon energy,

$E = hf = \frac{hc}{\lambda}$ to give $\lambda = \frac{hc}{E}$

$= \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{6.4 \times 10^{-19}} = 3.1 \times 10^{-7} \text{ m}$.

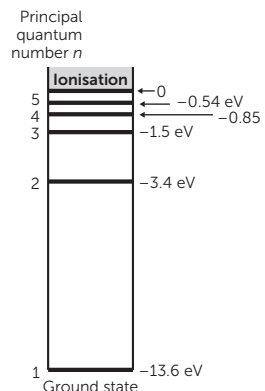


Fig. A21.1

UNIT 21

In-text questions

21.1

1 Rearranging $\frac{1}{2}mv^2 = eV_A$ gives $v^2 = \frac{2eV_A}{m}$.

(a) $v^2 = \frac{2eV_A}{m} = 2 \times 1.76 \times 10^{11} \times 100 = 3.52 \times 10^{13}$

$\therefore v = 5.9 \times 10^6 \text{ m s}^{-1}$.

$$(a) E_2 = \frac{-13.6}{4} = -3.4 \text{ eV}; E_3 = \frac{-13.6}{9}$$

$$= -1.5 \text{ eV}; E_4 = \frac{-13.6}{16} = -0.85 \text{ eV};$$

$$E_5 = \frac{-13.6}{25} = -0.54 \text{ eV}.$$

(b) (i) See Fig. A21.1.

1 (a) Photon energy $= E = (-0.54) - (-0.85) = 0.31 \text{ eV} = 0.31 \times 1.6 \times 10^{-19} \text{ J} = 5.0 \times 10^{-20} \text{ J}.$

$$\text{Rearrange photon energy, } E = hf = \frac{hc}{\lambda} \text{ to give } \lambda = \frac{hc}{E} \\ = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{5.0 \times 10^{-20}} = 4.0 \times 10^{-6} \text{ m}.$$

(iii) Infra-red.

21.5

$$1 (a) \lambda_{\min} = \frac{hc}{eV_A} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-19} \times 25000} = 5.0 \times 10^{-11} \text{ m}.$$

(b) Electrical power used by the tube $= IV_A = 30 \times 10^{-3} \times 25000 = 750 \text{ W}.$

(c) Heat produced per second $= 98\% \times 750 \text{ W} = 735 \text{ W}.$

2 See p. 340–1.

21.6

1 (a) Electrons in a beam directed at a thin metal sample are diffracted by the rows of atoms in the sample.

$$(b) v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34}}{1.7 \times 10^{-27} \times 1.5 \times 10^{-9}} = 260 \text{ m s}^{-1}$$

2 (a) In the STM, a metal probe with a very fine tip is placed with the tip very close to a flat surface as shown in Fig. 21.29. When a small p.d. is applied between the tip and the surface, the wave nature of the electrons allows them to cross the gap from the negative side to the positive side. Even though the work function of the material on the negative side is much greater than the kinetic energy of the electrons on the negative side, there is a finite probability that the electron waves can extend across the gap if the gap is very narrow. In operation, as the probe scans the surface, the electron current increases or decreases according to whether the gap becomes narrower or wider. The varying electron current is used to provide an image of the surface.

$$(b) \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 9.2 \times 10^6} = 7.9 \times 10^{-11} \text{ m}.$$

Revision questions

21.1

(a) (i) k.e. $= eV_A = 1.6 \times 10^{-19} \times 4200 = 6.7 \times 10^{-16} \text{ J}.$

$$(ii) \frac{1}{2}mv^2 = 6.7 \times 10^{-16} \therefore v^2 = \frac{2 \times 6.7 \times 10^{-16}}{0.1 \times 10^{-31}} = 1.48 \times 10^{15}$$

$$\therefore v = 3.8 \times 10^7 \text{ m s}^{-1}.$$

$$(b) (i) F = \frac{eV_p}{d} = \frac{1.6 \times 10^{-19} \times 5000}{50 \times 10^{-3}} = 1.6 \times 10^{-14} \text{ N}.$$

(ii) Using $F = Bev$ gives $B = \frac{F}{ev} = \frac{1.6 \times 10^{-14}}{1.6 \times 10^{-19} \times 3.8 \times 10^7} = 2.6 \times 10^{-3} \text{ T}$ into the plane of the diagram.

21.2

(a) (i) Negative charge. (ii) The viscous drag increases as the speed increases until it becomes equal and opposite to the weight when the droplet is falling at its terminal speed.

$$(b) (i) v = \frac{s}{t} = \frac{1.2 \times 10^{-3}}{14.6} = 8.2 \times 10^{-5} \text{ m s}^{-1}.$$

$$(ii) r^2 = \frac{9\eta v}{2\rho g} = \frac{9 \times 1.8 \times 10^{-5} \times 8.2 \times 10^{-5}}{2 \times 960 \times 9.8} = 7.0 \times 10^{-13} \therefore r = 8.4 \times 10^{-7} \text{ m}.$$

$$(iii) m = \frac{4}{3}\pi r^2 \rho = \frac{4}{3}\pi (8.4 \times 10^{-7})^3 \times 960 = 2.4 \times 10^{-15} \text{ kg}.$$

$$(iv) Q = \frac{mgd}{V} = \frac{2.4 \times 10^{-15} \times 9.8 \times 4.0 \times 10^{-3}}{590} = 1.6 \times 10^{-19} \text{ C}.$$

21.3

$$(a) (i) f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{500 \times 10^{-9}} = 6.0 \times 10^{14} \text{ Hz};$$

$$E = hf = 6.6 \times 10^{-34} \times 6.0 \times 10^{14} = 4.0 \times 10^{-19} \text{ J}.$$

$$(ii) f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{50 \times 10^{-9}} = 6.0 \times 10^{15} \text{ Hz};$$

$$E = hf = 6.6 \times 10^{-34} \times 6.0 \times 10^{15} = 4.0 \times 10^{-18} \text{ J}.$$

(b) (i) Work done $= eV = 1.6 \times 10^{-19} \times 0.36 = 5.8 \times 10^{-20} \text{ J}.$

$$(ii) \text{Rearranging } eV_s = hf - \phi \text{ gives } \phi = hf - eV_s = 4.0 \times 10^{-19} - 5.8 \times 10^{-20} \\ = 3.4 \times 10^{-19} \text{ J} = \frac{3.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.1 \text{ eV}.$$

$$(iii) \text{Threshold frequency, } f_0 = \frac{\phi}{h} = 5.2 \times 10^{14} \text{ Hz}$$

$$\therefore \lambda = \frac{c}{f_0} = \frac{3.0 \times 10^8}{5.2 \times 10^{14}} = 5.8 \times 10^{-7} \text{ m}.$$

21.4

$$(a) (i) hf = E_4 - E_3 = \frac{(-13.6)}{16} - \frac{(-13.6)}{9} \text{ eV} = 0.66 \text{ eV} = 0.66 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore f = \frac{E_4 - E_3}{h} = \frac{0.66 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.6 \times 10^{14} \text{ Hz}.$$

$$\therefore \lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{1.6 \times 10^{14}} = 1.9 \times 10^{-6} \text{ m}.$$

$$(ii) hf = E_2 - E_1 = \frac{(-13.6)}{4} - \frac{(-13.6)}{1} \text{ eV} = 10.2 \text{ eV} = 10.2 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore f = \frac{E_2 - E_1}{h} = \frac{10.2 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 2.5 \times 10^{15} \text{ Hz}.$$

$$\therefore \lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{2.5 \times 10^{15}} = 1.2 \times 10^{-7} \text{ m}.$$

(b) 1 For $\lambda = 565 \text{ nm},$

$$E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{565 \times 10^{-9}} = 3.5 \times 10^{-19} \text{ J} = 2.2 \text{ eV}$$

which corresponds to a transition from the energy level at -9.8 eV to the ground state at $-12.0 \text{ eV}.$

2 For $\lambda = 430 \text{ nm},$

$$E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{430 \times 10^{-9}} = 4.6 \times 10^{-19} \text{ J} = 2.9 \text{ eV}$$

which corresponds to a transition from the energy level at -6.9 eV to the energy level at $-9.8 \text{ eV}.$

21.5.

(a) (i) See Fig. 21.22. (ii) The beam electrons knock electrons out of the shells of the target atoms. When a vacancy in the shell is refilled, a photon of a certain energy is emitted. Since the frequency of a photon is proportional to its energy, photons of certain frequencies only are produced by this mechanism. The intensity distribution is therefore much more intense at these frequencies.

(b) Combine $\lambda = \frac{c}{f}$ and $eV_A = hf$ to give $\lambda = \frac{hc}{eV_A}.$

$$(i) \lambda = \frac{hc}{eV_A} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-19} \times 25000} = 5.0 \times 10^{-11} \text{ m}.$$

$$(ii) \lambda = \frac{hc}{eV_A} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{1.6 \times 10^{-19} \times 100000} = 1.25 \times 10^{-11} \text{ m}.$$

21.6

(a) Multiplying both sides of the equation $\frac{1}{2}mv^2 = eV$ by $2m$ gives $m^2v^2 = 2meV,$ hence $mv = \sqrt{(2meV)}.$

$$\text{Therefore } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{(2meV)}}.$$

$$(b) (i) \lambda = \frac{h}{\sqrt{(2meV)}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10000}} \\ = 1.2 \times 10^{-11} \text{ m}.$$

(ii) Increasing the anode p.d. increases the speed of the electrons. Since the de Broglie wavelength is inversely proportional to the speed, the de Broglie wavelength decreases. As a result of the decrease of wavelength, the electrons that pass through the sample are diffracted less when they pass through the lenses. Consequently, adjacent features in the image that were indistinct can be seen more clearly as separate features.

UNIT 22

In-text questions

22.1

1 (a) 92 p + 143 n (b) 6 p + 6 n (c) 17 p + 20 n.

2 (a) 235 grams of U 235 contains 6.02×10^{23} atoms, hence 1.0 kg of U 235 contains 2.6×10^{24} atoms ($= 6.02 \times 10^{23} / 0.235$).

(b) 4.0 grams of He 4 contains 6.02×10^{23} atoms, hence 1.0 g of He 4 contains 1.5×10^{23} atoms ($= 6.02 \times 10^{23} / 4$).

22.2

1 (a) 2, 234 (b) 27, 0, 60.

2 (a) Similarity; both cause ionisation or both emitted by the nucleus.

(b) (i) The nucleus loses two protons and two neutrons. (ii) A neutron in the nucleus changes into a proton, creating and emitting a beta particle at the instant of change.

22.3

1 (a) (i) Beta radiation; the paper made no difference so there was no alpha radiation emitted; also, the 4 mm plate prevented the radiation from the source reaching the tube so no gamma radiation was present.

(ii) With no source present, the count rate should decrease to the background count rate if the tube is moved more than about 50 cm from the source.

(b) (i) Corrected count rate without the metal plate present = 602 counts per minute; corrected count rate with the metal plate present = 456 counts per minute.

(ii) Percentage transmitted = $\frac{456}{602} \times 100\% = 76\%$.

(iii) Corrected count rate with two metal plates present = $76\% \times 76\%$ of 602 counts per minute = 348 counts per minute. Hence the corrected count rate = 374 counts per minute (= 348 + 26 counts per minute).

2 (a) Average background count rate = $\frac{(143 + 124 + 136)}{3} \div 5$ minutes

= 27 counts per minute. Average count rate with source present

= $\frac{(725 + 746 + 738)}{3} \div 3$ minutes = 245 counts per minute. Corrected count rate

due to source = 245 - 27 = 218 counts per minute.

(b) The corrected count rate varies with distance from the source according to the inverse square law.

(i) At 0.20 m, the distance is halved so the corrected count rate increases by $\times 4$ to 872 counts per minute (= 4×218). Hence the count rate would be 899 counts per minute (= 872 + 27).

(ii) At 0.15 m, the distance is reduced by a factor of 0.375 (= $0.15/0.40$). Hence the corrected count rate would increase by a factor of $1/0.375^2 = \times 7.1$ to approximately 1550 counts per minute (= 7.1×218). Hence the count rate would be about 1580 counts per minute (= 1550 + 27 to the nearest 10).

22.4

1 (a) (i) 105 counts per minute (c.p.m.) (ii) 52 c.p.m.

(b) Approximately four half-lives (= 270 s to the nearest 10 s).

2 (a) (i) 80 kBq (ii) 40 kBq.

(b) 100 years is approx. four half-lives so the activity would be about 10 kBq.

22.5

1 (a) $\lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{140 \times 24 \times 3600 \text{ s}} = 5.7 \times 10^{-8} \text{ s}^{-1}$.

(b) 210 g of Po 210 contains 6.02×10^{23} atoms

$\therefore$ 1.0 mg contains 2.87×10^{18} atoms $\left(= 6.02 \times 10^{23} \times \frac{0.001}{210} \right)$.

(c) Activity = $\lambda N = 5.7 \times 10^{-8} \times 2.87 \times 10^{18} = 1.6 \times 10^{11}$ Bq.

(d) $A = A_0 e^{-\lambda t} = 1.6 \times 10^{11} \times \exp\{-(5.7 \times 10^{-8} \times 365 \frac{1}{4} \times 24 \times 3600)\}$
 $= 2.6 \times 10^{10}$ Bq. (Note: $\exp\{-(5.7 \times 10^{-8} \times 365 \frac{1}{4} \times 24 \times 3600)\} = 0.166$).

2 (a) $\lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{1940 \times 24 \times 3600} = 4.1 \times 10^{-9} \text{ s}^{-1}$.

(b) $N = \frac{A}{\lambda} = \frac{2.0 \times 10^6}{4.1 \times 10^{-9}} = 4.8 \times 10^{14}$.

(c) $A = A_0 e^{-\lambda t}$ gives $0.1 = 2.0 e^{-\lambda t}$ $\therefore e^{-\lambda t} = \frac{0.1}{2.0} = 5.0 \times 10^{-2}$.

Hence $-\lambda t = \log_e 5.0 \times 10^{-2} = -3.0$

$\therefore t = \frac{-3.0}{-\lambda} = \frac{-3.0}{-4.1 \times 10^{-9}} = 0.73 \times 10^9 \text{ s} = 23.2 \text{ years}$.

Revision questions.

22.1

(a) 28, 63, -1 (b) 214, 2, 82.

22.2

(a) Average background count rate = $\frac{(128 + 136 + 138)}{3 \times 5} = 26.8$ counts per minute.

(b) Beta radiation; alpha radiation would not reach the tube at 10 cm; gamma radiation would not be affected as much by a metal plate of thickness 0.8 mm.

(c) Corrected count rate without the absorber = $\frac{(565 + 572 + 552)}{3 \times 3} = 26.8$

= 35.8 c.p.m. Corrected count rate with the absorber present

= $\frac{(384 + 368 + 372)}{3 \times 3} = 26.8 = 14.9$ c.p.m.

Percentage transmitted = $\frac{14.9}{35.8} \times 100\% = 42\%$.

(d) Percentage transmitted by 2 plates = 42% of 42% = 17.6%. Corrected count rate = 17.6% of 35.8 c.p.m. = 6.3 c.p.m. Expected count rate = corrected count rate + background count rate = 6.3 + 26.8 = 33.1 c.p.m.

22.3

(a) Let d = distance of the source inside the container. Using the inverse square law in the form, corrected count rate $C = k/r^2$, then

$$C_1 = \frac{k}{(0.090 + d)^2} \text{ and } C_2 = \frac{k}{(0.190 + d)^2}.$$

$$\text{Hence } \frac{C_1}{C_2} = \frac{(0.190 + d)^2}{(0.090 + d)^2}. \text{ As } \frac{C_1}{C_2} = \frac{1284}{330} = 4 \text{ then } \frac{(0.190 + d)^2}{(0.090 + d)^2} = 4$$

$$\text{so } \frac{(0.190 + d)}{(0.090 + d)} = 2 \therefore 0.190 + d = 2(0.090 + d) = 0.180 + 2d$$

$$\therefore d = 0.190 - 0.180 = 0.010 \text{ m} = 10 \text{ mm}.$$

(b) Measure the count rate with paper and with metal plates of different thicknesses between the source and the tube. If the paper absorbs the radiation, it is alpha radiation. If the count rate is scarcely affected by several millimetres of aluminium, the radiation is gamma radiation. Otherwise, it is beta radiation.

22.4

$$(a) \lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{14.8 \text{ h}} = 0.047 \text{ h}^{-1} = 1.3 \times 10^{-5} \text{ s}^{-1}.$$

(b) 24 g of Na 24 contains 6.0×10^{23} atoms

$$\therefore 1.0 \text{ mg contains } 2.5 \times 10^{19} \text{ atoms } \left(= \frac{1.0 \times 10^{-3} \times 6.0 \times 10^{23}}{24} \right).$$

(c) Activity = $\lambda N = 3.3 \times 10^{14}$ Bq.

$$(d) A = A_0 e^{-\lambda t} = 3.3 \times 10^{14} \times \exp(-1.3 \times 10^{-5} \times 24 \times 3600) = 1.1 \times 10^{14} \text{ Bq}.$$

22.5

$$(a) (i) \lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{100 \times 365.25 \times 24 \times 3600 \text{ s}} = 2.2 \times 10^{-10} \text{ s}^{-1}.$$

(ii) 239 g contains 6.0×10^{23} atoms

$$\therefore 1.0 \text{ g contains } 2.5 \times 10^{21} \text{ atoms } \left(= \frac{6.0 \times 10^{23}}{239} \right).$$

(b) (i) Activity of 1.0 g = $\lambda N = 2.2 \times 10^{-10} \times 2.5 \times 10^{21} = 5.5 \times 10^{11}$ Bq

$$\therefore \text{mass of sample of activity } 1000 \text{ MBq} = \frac{10 \times 10^9}{5.5 \times 10^{11}} = 1.8 \times 10^{-3} \text{ g}.$$

$$(ii) A = A_0 e^{-\lambda t} = 1000 \times \exp(-2.2 \times 10^{-10} \times 10 \times 365.25 \times 24 \times 3600) = 933 \text{ MBq}.$$

UNIT 23

In-text questions

23.1

1 (a) Gravity. (b) (i) The strong nuclear force. (ii) The weak nuclear force.

(c) The electromagnetic force.

$$2 (a) (i) F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{2 \times 1.6 \times 10^{-19} \times 7 \times 1.6 \times 10^{-19}}{4\pi\epsilon_0 \times (10 \times 10^{-15})^2} = 32 \text{ N}.$$

$$(ii) F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{2 \times 1.6 \times 10^{-19} \times 7 \times 1.6 \times 10^{-19}}{4\pi\epsilon_0 \times (2 \times 10^{-15})^2} = 800 \text{ N}.$$

(b) Attraction due to the strong nuclear force exceeds the repulsion due to the electrostatic force only if the alpha particle has enough kinetic energy to overcome the electrostatic force and come within the range of the strong nuclear force.

23.2

1 (a) (i) Mass defect, $\Delta m = Zm_p + (A - Z)m_n - M$

$$= 8 \times 1.00728 + 8 \times 1.00867 - (15.99492 - 8 \times 0.00055) = 0.137 \text{ u}$$

$$\therefore \text{BE of O 16 nucleus} = 0.137 \text{ u} \times 931 \text{ MeV/u} = 128 \text{ MeV}.$$

$$\therefore \text{BE per nucleon} = \frac{\text{BE}}{A} = \frac{128}{16} = 8.0 \text{ MeV per nucleon}.$$

(ii) Mass defect, $\Delta m = Zm + (A - Z)m - M$

$$= 38 \times 1.00728 + 52 \times 1.00867 - (89.90730 - 90 \times 0.00055) = 0.870 \text{ u}$$

$$\therefore \text{BE of Sr 90 nucleus} = 0.870 \text{ u} \times 931 \text{ MeV/u} = 810 \text{ MeV}.$$

$$\therefore \text{BE per nucleon} = \frac{\text{BE}}{A} = \frac{810}{90} = 9.0 \text{ MeV per nucleon}.$$

2 (a) (i) ${}_{92}^{238}\text{U} \rightarrow {}_{90}^{234}\text{Th} + {}_2^4\text{He}$

(ii) Δm = atomic mass of U 238 - [atomic mass of Th 234 + atomic mass of He 4] = 238.05076 - [234.04357 + 4.00260] = 0.00459 u

$$\therefore Q = \Delta mc^2 = 0.00459 \text{ u} \times 931 \text{ MeV/u} = 4.3 \text{ MeV}.$$

(b) (i) ${}_{27}^{60}\text{Co} \rightarrow {}_{28}^{60}\text{Ni}$

(ii) Δm = atomic mass of Co 60 - atomic mass of Ni 60 = 59.93381 - 59.93078 = 0.00303 u $\therefore Q = \Delta mc^2 = 0.00303 \text{ u} \times 931 \text{ MeV/u} = 2.8 \text{ MeV}.$

23.3

1 (a) (i) Induced fission is the splitting of a heavy nucleus into two approximately equal fragments as a result of a neutron colliding with and being absorbed by the nucleus.

(ii) Two or three neutrons are released when a heavy nucleus is fissioned. These neutrons can then go on to produce further fission, leading to more released neutrons and more fission, etc.

(b) (i) Below a certain mass, too many neutrons would escape and not produce further fission. The probability of escape is proportional to the surface area and the probability of fission is proportional to the volume of the material. If the material is below a certain mass, its surface to volume ratio is too low for a chain reaction to occur.

(ii) For a steady rate of fission, exactly one neutron per fission must go on to produce further fission. Neutron-absorbing control rods are inserted into the core to keep the fission rate constant by absorbing surplus neutrons. Energy is released at a steady rate when the fission rate is constant.

(c) (i) The moderator reduces the kinetic energy of fission neutrons so they can produce further fission of U 235. The fission neutrons collide with the moderator atoms and transfer kinetic energy to the atoms.

(ii) For each successive collision, the k.e. of a neutron is reduced to 75%. (i.e. to $0.75 \times$ the k.e. before impact). Hence for n successive collisions, the k.e. is reduced to $(0.75)^n$. $\therefore 0.01 \text{ eV} = (0.75)^n \times 1 \text{ MeV}$,

$$\text{hence } (0.75)^n = \frac{0.01 \text{ eV}}{1 \text{ MeV}} = 10^{-8}.$$

Taking logs to the base 10 either side therefore gives $n \log 0.75 = -8$. $\therefore n = 64$ ($= -8 / \log 0.75$).

2 (a) Energy output in 1 day = $1000 \text{ MJ} \times 24 \text{ h} \times 3600 \text{ s} = 8.6 \times 10^{13} \text{ J}$.

$$\therefore \text{energy needed from fuel per day} = \frac{8.6 \times 10^{13}}{0.25} = 3.4 \times 10^{14} \text{ J}.$$

$$(i) \text{ Mass of oil needed per day} = \frac{3.4 \times 10^{14}}{30 \times 10^6} = 10 \text{ million kg approx.}$$

$$(ii) \text{ Mass of uranium needed per day} = \frac{10 \text{ million kg approx.}}{50\,000} = 200 \text{ kg approx.}$$

(b) (i) See p. 379. (ii) To recover uranium 238 and plutonium 239. (ii) The fuel is plutonium 239 which is fissioned by fast neutrons. Fission neutrons do not therefore need to be slowed down to produce further fission of plutonium.

23.4

1 (a) ${}^9_4\text{Be} + {}^4_2\alpha \rightarrow {}^{12}_6\text{C}$.

(b) (i) Charge of udd = $+\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e$; charge of udd = $+\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

(ii) Charge of uus = $+\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e$; strangeness = $0 + 0 + 1$.

2 (a) (i) The particle and antiparticle carry opposite charges.

(ii) The particle and antiparticle lose kinetic energy and slow down. Each is forced into a spiral as it slows down gradually.

(b) Total energy available = $2 + 2 + 1 + 1 = 6 \text{ GeV}$. $\therefore$ maximum gamma energy = 3 GeV .

Revision questions

23.1

$$(a) Q_1 = 2e, Q_2 = 4e$$

$$\therefore \text{assuming initial k.e.} = p.e \text{ at closest approach, } 5 \times 10^6 e = \frac{2e \times 4e}{4\pi\epsilon_0 r}$$

$$\therefore r = \frac{8e^2}{4\pi\epsilon_0 \times 5 \times 10^6 e} = \frac{8 \times 1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times 5 \times 10^6} = 2.3 \times 10^{-15} \text{ m}.$$

(b) The alpha particle will be attracted into the nucleus at a separation of 2.3 fm , since the strong nuclear force extends to this distance. A nuclear reaction is therefore likely.

23.2

$$(a) (i) \Delta m = (6 \times 1.00728 + 6 \times 1.00867) - (12.0000 - 6 \times 0.00055) = 0.092 \text{ u}$$

$$\therefore \text{BE} = 0.092 \times 931 \text{ MeV} = 86.0 \text{ MeV}; \text{BE/nucleon} = 86.0/12 = 7.2 \text{ MeV}.$$

$$(ii) \Delta m = (82 \times 1.00728 + 124 \times 1.00867) - (205.97446 - 82 \times 0.00055)$$

$$= 1.652 \text{ u} \therefore \text{BE} = 1.652 \times 931 \text{ MeV} = 1540 \text{ MeV}; \text{BE/nucleon}$$

$$= 1540/206 = 7.5 \text{ MeV}.$$

(b) (i) See Fig. 23.5.

(ii) When a heavy nucleus fissions, the binding energy per nucleon rises as the fission fragments are more stable. The energy released is equal to the rise of the total binding energy.

23.3

$$(a) (i) {}^{210}_{84}\text{Po} \rightarrow {}^4_2\alpha + {}^{206}_{82}\text{Pb}.$$

$$(ii) Q = 931 \times (209.98287 - 205.97446 - 4.00260) = 5.4 \text{ MeV}.$$

$$(b) (i) {}^{24}_{11}\text{Na} \rightarrow {}^0_1\beta + {}^{24}_{12}\text{Mg}.$$

$$(ii) Q = 931 \times (23.99097 - 23.98505) = 5.5 \text{ MeV}.$$

23.4

(a) (i) Fission neutrons go on to produce further fission which releases more neutrons which produce further fission, etc.

(ii) Control rods are inserted into the reactor core to absorb surplus neutrons so that exactly 1 neutron per fission goes on to produce further fission.

(b) (i) The moderator reduces the k.e. of the fission neutrons so they produce further fission of U 235.

(ii) Low neutron absorption, high melting point if solid, high specific heat capacity.

(c) (i) Spent fuel rods contain radioactive fission fragments, and plutonium 239 and uranium 238 which are highly radioactive.

$$(ii) \text{Decay constant, } \lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{100 \text{ yrs}};$$

$$\text{number of atoms in 1 kg, } N = \frac{6.02 \times 10^{23}}{0.235};$$

$$\text{activity} = \lambda N = \frac{0.693}{100 \times 365.25 \times 24 \times 3600 \text{ s}} = \frac{6.02 \times 10^{23}}{0.235} = 5.6 \times 10^{14} \text{ Bq}.$$

23.5

(a) (i) A ring of magnets forces the particles around a circular path. (ii) The speed increases until the particle radiates energy at the same rate as it gains it from the accelerator. The speed is then constant.

$$(b) (i) E = mc^2 \therefore m = \frac{E}{c^2} = \frac{20\,000 \text{ MeV}}{931} = 21.5 \text{ u}.$$

(ii) As it gains kinetic energy, its mass increases and its speed approaches closer to the speed of light. Its mass tends to infinity as it approaches the speed of light.

$$(c) (i) Q = +\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0; S = 0 + 0 - 1 = -1.$$

$$(ii) Q = +\frac{2}{3}e + \frac{1}{3}e = +e; S = 0 + 1.$$

(d) (i) $S = 0$ so there are no strange quarks present. Since $Q = +2e$, there must be 3 up quarks present ($Q = 3 \times +\frac{2}{3}e$).

(ii) $S = +1$, so the meson contains the strange antiquark. The charge of this antiquark is $+\frac{1}{3}e$ so the meson must also include a down quark (charge = $-\frac{1}{3}e$) so the total charge is 0.

UNIT 24

In-text questions

24.1

1. Streamlined flow is flow in which the path of any small amount of fluid does not move about and is stable. Fluid passing through any particular point always follows the same path which is referred to as a streamline.

2 (a) The flow from the tap into the bucket of water is steady and therefore streamlined. The flow of water in the bucket is turbulent as the water swirls round in the bucket.

(b) The oil flow from the bottle to the pan is stable and therefore streamlined flow. The pool of oil in the pan steadily becomes larger as oil flows into it. Any turbulence after the oil enters the pool decreases as the pool expands steadily or the oil level in the pan rises.

24.2

1. The volume of water per second flowing along the river is the same at any place along the river. At a given place along the river, the water flows through an area equal to the width times the depth of the river at that place. Where the area is smaller, the water must flow faster in order that the same volume of water flows past each second. Under the bridge compared with further along, the river is narrower so the area it flows through is smaller and therefore the flow is faster.

2 (a) The pipe radius $r = 5.5 \times 10^{-2} \text{ m}$

$$\text{area of cross-section} = \pi r^2 = \pi \times (5.5 \times 10^{-2})^2 = 9.5 \times 10^{-3} \text{ m}^2.$$

$$(b) \text{Mass flow per second} = \rho Av = 960 \times 9.5 \times 10^{-3} \times 0.65 = 5.9 \text{ kg s}^{-1}.$$

24.3

1 (a) A viscous fluid possess internal friction which acts when the velocity of the fluid changes with position in the fluid. This internal friction causes slower moving layers in the fluid to drag on faster-moving layers and to drag on any solid surface the fluid moves past.

(b) For a pipe of radius r , the volume per second flowing along a pipe is proportional to r^4 . Therefore

$$\frac{(\text{the effective radius})^4}{(\text{the actual radius})^4} = \frac{\text{the reduced volume per second}}{\text{the volume per second at first}}$$

$$\frac{(\text{the effective radius})^4}{(\text{the actual radius})^4} = \frac{4.0 \text{ cm}^3}{10 \text{ cm}^3} = 0.4.$$

$$\frac{(\text{the effective radius})}{(\text{the actual radius})} = 0.80 \text{ (to 2 significant figures)}$$

$$\text{the effective radius} = 0.80 \times 5.0 \text{ mm} = 4.0 \text{ mm}.$$

2 Using Poiseuille's equation

$$V/t = \frac{\pi r^2 \Delta p}{8\eta L} \text{ gives } V/t = \frac{\pi \times (6.0 \times 10^{-3})^4 \times 6.7 \times 10^3}{8 \times 7.5 \times 3.8} = 1.2 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}.$$

24.4

1 (a) The air flows faster over the top of the wing than it does under the wing. Therefore, the air pressure on top of the wing is less than it is above the wing. Therefore the downward force of the air pressure on the top surface of the wing is less than the upward force of the air pressure on the lower surface of the wing. There is therefore a net upward force on the wing.

(b) The upward force on the wings must be greater than the weight of the aircraft for it to take off. Since the upward force increases with speed, the aircraft must therefore attain a certain speed for it to take off.

2 (a) When the gas flows faster, its pressure is lower. Gas flows faster through the narrow section than it does through the wider sections so the pressure in the narrow section is less than in the wider sections. If the flow is fast enough, the pressure is reduced to less than the outside pressure so air is drawn through the small pipe into the flow.

(b) The dynamic pressure $\Delta p = \frac{1}{2} \rho v^2$. Rearranging this equation gives

$$v = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2 \times 210}{1000}} = 0.65 \text{ m s}^{-1}.$$

Revision questions

24.1

(a) Streamlined flow is steady flow such that the path of any the path of fluid passing a given point is always the same and the fluid does not become mixed up.

(b) The drag force on the vehicle is reduced by the wind deflector so the engine power needed to maintain a certain speed is reduced. Hence the engine uses less fuel per second.

24.2

(a) From the continuity equation, the volume of water flowing each second in each section = the water speed $\times$ the area of cross-section. As the volume of water per second flowing in each section is the same and the area of cross-section of each section is the same, the speed of the water in each section is the same.

(b) The viscosity of water means that the force of the pressure in the water does work to overcome internal friction in the water. The pressure in Z is less than in X because work is done by the water to overcome internal friction. Hence the water level in the vertical tube at Z is lower than the water level in vertical tube at X.

24.3

(a) See Figure 24.3A. The velocity gradient is greatest at the pipe surface.

(b) Using Poiseuille's equation

$$V/t = \frac{\pi r^2 \Delta p}{8\eta L} \text{ gives } V/t = \frac{\pi \times (8.0 \times 10^{-5})^4 \times \Delta p}{8 \times 2.4 \times 0.16} = 4.2 \times 10^{-9} \Delta p$$

$$\text{Hence } \Delta p = \frac{1.1 \times 10^{-5}}{4.2 \times 10^{-9}} = 2600 \text{ Pa (to 2 significant figures).}$$

24.4

(a) (i) Speed $v = \frac{\text{distance}}{\text{time}} = \frac{0.50}{30 \text{ s}} = 1.3 \times 10^{-2} \text{ m s}^{-1}$.

(ii) Volume of ball $= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (0.5 \times 5.4 \times 10^{-3})^3 = 8.2 \times 10^{-8} \text{ m}^3$
 mass of ball = volume $\times$ density $= 8.2 \times 10^{-8} \times 7800 = 6.4 \times 10^{-4} \text{ kg}$
 weight $= mg = 6.4 \times 10^{-4} \times 9.8 = 6.3 \times 10^{-3} \text{ N}$.

(b) Substituting known values into Stokes' Law gives
 $mg = 6\pi\eta rv = 6.3 \times 10^{-3} = 6\pi\eta \times (0.5 \times 5.4 \times 10^{-3}) \times 1.3 \times 10^{-2} = 6.6 \times 10^{-4} \eta$.

Rearranging this equation gives $\eta = \frac{6.3 \times 10^{-3}}{6.6 \times 10^{-4}} = 9.5 \text{ N s m}^{-2}$.

24.5

(a) Applying Bernoulli's equation to points X and Y along the streamline gives

$$p_X + \frac{1}{2} \rho v_X^2 + \rho gh_X = p_Y + \frac{1}{2} \rho v_Y^2 + \rho gh_Y$$

Since X and Y are open to the atmosphere, their pressures (p_X and p_Y) are equal. Also, assuming the flow speed at X, v_X , is negligible because the tank is much wider than the pipe, hence Bernoulli's equation becomes $\rho gh_X = \frac{1}{2} \rho v_Y^2 + \rho gh_Y$. Rearranging this equation gives $\frac{1}{2} \rho v_Y^2 = \rho gh_X - \rho gh_Y = \rho gh$ where $h = h_X - h_Y$. Hence $v_Y = (2gh)^{1/2}$.

(b) (i) $v_Y = (2gh)^{1/2} = (2 \times 9.8 \times 0.52)^{1/2} = 3.2 \text{ m s}^{-1}$.

(ii) Area of cross-section of the outlet,
 $A = \pi r^2 = \pi \times (6.0 \times 10^{-3})^2 = 1.1 \times 10^{-4} \text{ m}^2$

Volume/second flowing from the tap
 $= Av_Y = 1.1 \times 10^{-4} \times 3.2 = 3.6 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$.

Mass/second = density $\times$ volume per second $= 1000 \times 3.6 \times 10^{-4} = 0.36 \text{ kg s}^{-1}$.

UNIT 25

In-text questions

25.1a

1 (a) Initial volume of air = volume of pump + tyre = 1545 cm^3 . Final volume of air in tyre = 1500 cm^3

$$\therefore p \times 1500 = 110 \times 1545, \text{ so } p = \frac{110 \times 1545}{1500} = 113.3 \text{ kPa}.$$

(b) Pressure increase = 3.3 kPa $\therefore$ % increase of pressure = $\frac{3.3}{110} \times 100\% = 3\%$.

(c) Pressure after 20 strokes = $103\%^{20} \times 110 \text{ kPa} = 1.03^{20} \times 110 \text{ kPa} = 199 \text{ kPa}$.

2 (a) Initial volume of air = $V_f + V_p$ where V_f = volume of flask and V_p = volume of pump. Final volume of air = V_f so $100V_p = 18V_f$.

$$V_p = 0.18V_f = 0.18 \times 250 \text{ cm}^3 = 45 \text{ cm}^3.$$

(b) Initial volume of air = $295 \text{ cm}^3 - v$, where v is the powder volume.

$$\text{Final volume of air} = 250 \text{ cm}^3 - v \therefore 100(295 - v) = 140(250 - v)$$

$$\therefore 29500 - 100v = 35000 - 140v. \text{ Hence } 40v = 5500 \text{ so } v = \frac{5500}{40} = 138 \text{ cm}^3.$$

25.1b

1 Use $\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$ to give

(a) $V_2 = \frac{p_1 V_1 T_2}{p_2 T_1} = \frac{100 \times 0.05 \times 350}{110 \times 300} = 0.054 \text{ m}^3$;

(b) $T_2 = \frac{p_2 V_2 T_1}{p_1 V_1} = \frac{101 \times 0.12 \times 400}{105 \times 0.24} = 192 \text{ K}$.

(c) $p_2 = \frac{p_1 V_1 T_2}{V_2 T_1} = \frac{0.35 \times 0.85 \times 250}{0.58 \times 350} = 0.37 \text{ kPa}$.

(d) $T_1 = \frac{p_1 V_1 T_2}{p_2 V_2} = \frac{101 \times 0.42 \times 300}{101 \times 0.38} = 332 \text{ K}$.

(e) $V_1 = \frac{p_2 V_2 T_1}{p_1 T_2} = \frac{101 \times 0.161 \times 290}{110 \times 27} = 0.157 \text{ m}^3$.

2 (a) $p_1 = 105 \text{ kPa}$; $V_1 = 20 \text{ cm}^3 = 20 \times 10^{-6} \text{ m}^3$; $T_1 = 288 \text{ K}$; $p_2 = 101 \text{ kPa}$; V_2 = to be calculated; $T_2 = 273 \text{ K}$.

Using $\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$ gives $\frac{101 \times 10^3 \times V_2}{273} = \frac{105 \times 10^3 \times 2.0 \times 10^{-5}}{288}$

$$\text{Hence } V_2 = \frac{105 \times 10^3 \times 2.0 \times 10^{-5} \times 273}{288 \times 101 \times 10^3} = 1.97 \times 10^{-5} \text{ m}^3 = 19.7 \text{ cm}^3.$$

(b) $p_1 = 101 \text{ kPa}$; $V_1 = 30 \text{ cm}^3 = 30 \times 10^{-6} \text{ m}^3$; $T_1 = 20 + 273 = 293 \text{ K}$;

p_2 = to be calculated; $V_2 = 31 \text{ cm}^3$; $T_2 = 100 + 273 = 373 \text{ K}$.

Using $\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1}$ gives $\frac{p_2 \times 3.1 \times 10^{-5}}{273} = \frac{101 \times 10^3 \times 3.0 \times 10^{-5}}{293}$

$$\text{Hence } p_2 = \frac{101 \times 10^3 \times 3.0 \times 10^{-5} \times 373}{293 \times 3.1 \times 10^{-5}} = 124 \text{ kPa}.$$

25.2

1 (a) Rearrange $pV = nRT$ to give

$$n = \frac{pV}{RT} = \frac{120 \times 10^3 \times 200 \times 10^{-6}}{8.31 \times (273 + 15)} = 1.00 \times 10^{-2} \text{ mol}.$$

(b) V is constant $\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2}$ where $p_1 = 120 \text{ kPa}$, $p_2 = 150 \text{ kPa}$, $T_1 = 288 \text{ K}$,

$$T_2 = ? \text{ Rearranging this equation gives } T_2 = \frac{p_2 T_1}{p_1} = \frac{150 \times 10^3 \times 288}{120 \times 10^3} = 360 \text{ K}.$$

(c) (i) At 100°C , $T = 373 \text{ K}$, $V = 200 \text{ cm}^3 = 200 \times 10^{-6} \text{ m}^3$, $p = 150 \text{ kPa}$

$$\therefore n = \frac{pV}{RT} = \frac{150 \times 10^3 \times 200 \times 10^{-6}}{8.31 \times 373} = 9.7 \times 10^{-3} \text{ mol}$$

$$\text{Number of moles lost} = 1.00 \times 10^{-2} - 9.7 \times 10^{-3} = 3 \times 10^{-4} \text{ mol}.$$

(ii) Fraction of gas lost = $\frac{\text{number of moles lost}}{\text{initial number of moles}} = \frac{3 \times 10^{-4}}{1.00 \times 10^{-2}} = 0.03$.

2 (i) $V_m = \frac{RT}{p} = \frac{8.31 \times 273}{101 \times 10^3} = 0.0224 \text{ m}^3$;

density, $\rho = \frac{\text{mass}}{\text{volume}} = \frac{0.028}{0.0224} = 1.25 \text{ kg m}^{-3}$.

(ii) $V_m = \frac{RT}{p} = \frac{8.31 \times 373}{101 \times 10^3} = 0.0306 \text{ m}^3$;

density, $\rho = \frac{\text{mass}}{\text{volume}} = \frac{0.028}{0.0306} = 0.91 \text{ kg m}^{-3}$.

25.3

1. (a) (i) Use $pV_m = RT$ to calculate the volume of 1 mole at 0°C and 101 kPa pressure.

$$\text{Hence } V_m = \frac{RT}{p} = \frac{8.31 \times (273 + 100)}{150 \times 10^3} = 2.07 \times 10^{-2} \text{ m}^3.$$

$$\therefore \text{density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{molar mass}}{\text{molar volume}} = \frac{0.028 \text{ kg}}{2.07 \times 10^{-2} \text{ m}^3} = 1.36 \text{ kg m}^{-3}.$$

(ii) Rearrange $p = \frac{1}{3}\rho v_{\text{RMS}}^2$ to give

$$v_{\text{RMS}}^2 = \frac{3p}{\rho} = \frac{3 \times 150 \times 10^3}{1.36} = 3.31 \times 10^5 \text{ m}^2 \text{ s}^{-2} \therefore v_{\text{RMS}} = 575 \text{ m s}^{-1}.$$

(b) (i) Mean k.e. = $\frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \times (100 + 273) = 7.72 \times 10^{-21} \text{ J}$.

(ii) Since the mean k.e. is proportional to T , then the absolute temperature needs to be doubled to double the mean k.e., i.e. T needs to be raised to 746 K ($= 2 \times 373 \text{ K}$).

(2) (a) (i) At 0°C , mean k.e. = $\frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \times 273 = 5.65 \times 10^{-21} \text{ J}$.

(ii) For 1 mole, $1 N_A m v_{\text{RMS}}^2 = \frac{3}{2}RT$ and $N_A m$ = molar mass, M .

$$\therefore \frac{1}{2}M v_{\text{RMS}}^2 = \frac{3}{2}RT \text{ so } v_{\text{RMS}}^2 = \frac{3RT}{M} = \frac{3 \times 8.31 \times 273}{0.002} = 3.40 \times 10^6 \text{ m}^2 \text{ s}^{-2}.$$

$$v_{\text{RMS}} = 1.84 \times 10^3 \text{ m s}^{-1}.$$

(b) (i) At 100°C , mean k.e. = $3kT = 1.5 \times 1.38 \times 10^{-23} \times (100 + 273) = 7.72 \times 10^{-21} \text{ J}$.

$$(ii) v_{\text{RMS}}^2 = \frac{3RT}{M} = \frac{3 \times 8.31 \times 373}{0.002} = 4.65 \times 10^6 \text{ m}^2 \text{ s}^{-2}; v_{\text{RMS}} = 2.16 \times 10^3 \text{ m s}^{-1}.$$

Revision questions

25.1

(a) (i) Rearrange $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ with $p = 103 \text{ kPa}$, $V = 40 \text{ cm}^3 = 4.0 \times 10^{-5} \text{ m}^3$,

$T_1 = 273 + 25 = 298 \text{ K}$, $p_2 = 101 \text{ kPa}$, $T_2 = 273 \text{ K}$, V_2 to be calculated.

$$\therefore V_2 = \frac{p_1 V_1 T_2}{p_2 T_1} = \frac{103 \times 10^3 \times 4.0 \times 10^{-5} \times 273}{101 \times 10^3 \times 298} = 3.7 \times 10^{-5} \text{ m}^3.$$

(ii) Rearrange $pV = nRT$ to give

$$n = \frac{pV}{RT} = \frac{103 \times 10^3 \times 4.0 \times 10^{-5}}{8.31 \times 298} = 1.66 \times 10^{-3} \text{ moles}.$$

Mass = nM , where M = molar mass

$$\therefore \text{mass} = 1.66 \times 10^{-3} \times 0.032 = 5.3 \times 10^{-5} \text{ kg}.$$

(b) (i) Rearrange $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ with $p_1 = 101 \text{ kPa}$, $V_1 = 1.2 \text{ cm}^3 = 1.2 \times 10^{-6} \text{ m}^3$,

$T_1 = 273 + 10 = 283 \text{ K}$, $p_2 = 200 \text{ kPa}$, $T_2 = 273 + 20 = 293 \text{ K}$, V_2 to be calculated.

$$\therefore V_2 = \frac{p_1 V_1 T_2}{p_2 T_1} = \frac{101 \times 10^3 \times 1.2 \times 10^{-6} \times 293}{200 \times 10^3 \times 283} = 6.3 \times 10^{-7} \text{ m}^3.$$

(ii) Rearrange $pV = nRT$ to give $n = \frac{pV}{RT} = \frac{101 \times 10^3 \times 1.2 \times 10^{-6}}{8.31 \times 283} = 5.2 \times 10^{-5}$

Mass = nM , where M = molar mass

$$\therefore \text{mass} = 5.2 \times 10^{-5} \times 0.029 = 1.5 \times 10^{-6} \text{ kg}.$$

25.2

(a) (i) Rearrange $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ with $p_1 = 150 \text{ kPa}$, $V_1 = 1600 \text{ cm}^3$

$$= 1.60 \times 10^{-3} \text{ m}^3, T_1 = 273 + 25 = 298 \text{ K}, V_2 = 1500 \text{ cm}^3 = 1.5 \times 10^{-3} \text{ m}^3,$$

$T_2 = 273 \text{ K}$, p_2 to be calculated.

$$\therefore p_2 = \frac{p_1 V_1 T_2}{V_2 T_1} = \frac{150 \times 10^3 \times 1.6 \times 10^{-3} \times 273}{1.5 \times 10^{-3} \times 298} = 1.47 \times 10^5 \text{ Pa}.$$

(ii) Rearrange $pV = nRT$ to give

$$n = \frac{pV}{RT} = \frac{150 \times 10^3 \times 1.6 \times 10^{-3}}{8.31 \times 298} = 9.69 \times 10^{-2} \text{ moles}.$$

Mass = nM , where M = molar mass

$$\therefore \text{mass} = 9.69 \times 10^{-2} \times 0.029 = 2.8 \times 10^{-3} \text{ kg}.$$

(b) Sealed end lower than the upper end: the pressure in the trapped air, in millimetres of mercury, $p_1 = p_0 + L$, where L is the length of the mercury thread, and p_0 is the atmospheric pressure in millimetres of mercury. The volume of the trapped air column, V_1 = length of air column $\times$ area of cross-section, $A = 0.12A$.

Sealed end above the lower end: the pressure in the trapped air, in millimetres of mercury, $p_2 = p_0 - L$, where L is the length of the mercury thread, and p_0 is the atmospheric pressure in millimetres of mercury. The volume of the trapped air column, V_2 = length of air column $\times$ area of cross-section, $A = 0.18A$.

Using $p_1 V_1 = p_2 V_2$ gives $(p_0 + L) \times 0.12A = (p_0 - L) \times 0.18A$.

$$\therefore 0.18p_0 - 0.12p_0 = 0.18L + 0.12L, 0.06p_0 = 0.30L$$

$$\therefore p_0 = \frac{0.30}{0.06} L = 5L = 5 \times 0.150 \text{ m} = 0.750 \text{ m} = 750 \text{ mm of mercury}.$$

25.3

(a) and (b) See Fig. A25.1 opposite.

25.4

(a) (i) Rearrange $pV = nRT$ to give

$$n = \frac{pV}{RT} = \frac{140 \times 10^3 \times 500 \times 10^{-6}}{8.31 \times (20 + 273)} = 2.87 \times 10^{-2} \text{ moles} \therefore \text{mass} = nM,$$

where M = molar mass, hence mass = $2.87 \times 10^{-2} \times 0.028 \text{ kg} = 8.0 \times 10^{-4} \text{ kg}$.

(ii) Number of molecules = $n \times N_A = 2.87 \times 10^{-2} \times 6.02 \times 10^{23} = 1.73 \times 10^{22}$.

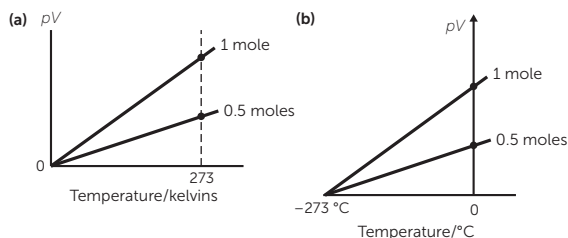


Fig. A25.1

(iii) Rearrange $pV = \frac{1}{3}Nm v_{\text{RMS}}^2$ to give

$$v^2 = \frac{3pV}{Nm}, \text{ where } Nm = \text{the total mass of gas}$$

$$\therefore v_{\text{RMS}}^2 = \frac{3 \times 140 \times 10^3 \times 500 \times 10^{-6}}{8.0 \times 10^{-4}} = 2.63 \times 10^5, \text{ hence } v_{\text{RMS}} = 512 \text{ m s}^{-1}.$$

(b) (i) Use $\frac{p_2}{T_2} = \frac{p_1}{T_1}$ where $p_1 = 140 \text{ kPa}$, $p_2 = 170 \text{ kPa}$, $T = (20 + 273) = 293 \text{ K}$

and T_2 is to be calculated.

$$T_2 = \frac{p_2 T_1}{p_1} = \frac{170 \times 293}{140} = 356 \text{ K} = 83^\circ\text{C}.$$

(ii) With the valve open, the pressure and volume remain constant and the number of moles falls.

Using $pV = nRT$, $nT = \frac{pV}{R}$ = constant, hence $n_2 T_2 = n_1 T_1$

$$\therefore n_2 = \frac{n_1 T_1}{T_2} = \frac{2.87 \times 10^{-2} \times 356}{(273 + 100)} = 2.74 \times 10^{-2} \text{ moles}$$

$$\therefore \text{number of moles lost} = 2.87 \times 10^{-2} - 2.74 \times 10^{-2} = 1.3 \times 10^{-3} \text{ moles}$$

$$\therefore \text{mass loss} = 1.3 \times 10^{-3} \times 0.028 \text{ kg} = 3.6 \times 10^{-5} \text{ kg}.$$

25.5

(a) (i) The molecules hit the sides less often because they have more distance to travel between successive impacts on average, so the rate of impacts is less and hence the pressure is less.

(ii) The molecules hit the sides less often because they are moving more slowly and the average impact force is less because the molecules move more slowly. Hence the pressure is less because the rate of impacts is smaller and the force of each impact is less.

(b) Rearrange $pV = \frac{1}{3}Nm v_{\text{RMS}}^2$ to give $v_{\text{RMS}}^2 = \frac{3pV}{Nm} = \frac{3nRT}{Nm} = \frac{3RT}{N_A m}$ where $M = N_A m$ is the molar mass.

$$(i) M = 0.002 \text{ kg} \therefore v_{\text{RMS}}^2 = \frac{3RT}{M} = \frac{3 \times 8.31 \times (273 + 20)}{0.002} = 3.65 \times 10^6 \text{ m}^2 \text{ s}^{-2}$$

$$\therefore v_{\text{RMS}} = 1910 \text{ m s}^{-1}.$$

$$(ii) M = 0.032 \text{ kg} \therefore v_{\text{RMS}}^2 = \frac{3RT}{M} = \frac{3 \times 8.31 \times (273 + 20)}{0.032} = 2.28 \times 10^5 \text{ m}^2 \text{ s}^{-2}$$

$$\therefore v_{\text{RMS}} = 478 \text{ m s}^{-1}.$$

(iii) The rms speed of hydrogen molecules is over $4 \times$ more than the rms speed of oxygen molecules at the same temperature. Hydrogen molecules are moving fast enough to escape from the Earth's gravity whereas oxygen molecules are not.

UNIT 26

In-text questions

26.1

$$1 \text{ (a) } t = \frac{h - h_0}{h_{100} - h_0} \times 100^\circ\text{C} = \frac{105 - 50}{220 - 50} \times 100^\circ\text{C} = \frac{155}{270} \times 100 = 57.4^\circ\text{C}.$$

(b) (i) Midway = 50 degrees.

$$(ii) t = \frac{h - h_0}{h_{100} - h_0} \times 100^\circ\text{C} = \frac{80 - 50}{220 - 50} \times 100^\circ\text{C} = \frac{130}{270} \times 100 = 48.1^\circ\text{C}.$$

$$2 \text{ } E = at - bt^2, \text{ where } a = 40.5 \mu\text{V } ^\circ\text{C}^{-1} \text{ and } b = 0.065 \mu\text{V } ^\circ\text{C}^{-2}.$$

$t/^\circ\text{C}$	0	20	40	60	80	100
$E/\mu\text{V}$	0	784	1516	2196	2824	3400

(b) At $E = 1700 \mu\text{V}$, $t = 45^\circ\text{C}$ (to within 0.5°C).

26.2

1 (a) (i) $\Delta Q = \Delta U + \Delta W = +1000 + 300 = +1300 \text{ J}$; i.e. 1300 J of heat transfer to the body.

(ii) $\Delta Q = \Delta U + \Delta W = +1000 - 300 = +700 \text{ J}$; i.e. 700 J of heat transfer to the body.

(b) (i) $\Delta Q = \Delta U + \Delta W = -500 + 200 = -300$ J; i.e. 300 J of heat transfer from the body.

(ii) $\Delta Q = \Delta U + \Delta W = -500 - 200 = -700$ J; i.e. 700 J of heat transfer from the body.

2 (a) Rearrange $\Delta Q = \Delta U + \Delta W$ to give $\Delta W = \Delta Q - \Delta U = 400 - 2000 = -1600$ J (i.e. 1600 J of work is done on the body).

(b) $\Delta Q = \Delta U + \Delta W = -2000 + 2400 = +400$ J (i.e. heat transfer = 400 J into the body).

(c) Rearrange $\Delta Q = \Delta U + \Delta W$ to give $\Delta U = \Delta Q - \Delta W = -800 - (-2400) = 1600$ J (i.e. heat transfer into body = 1600 J).

(d) Rearrange $\Delta Q = \Delta U + \Delta W$ to give $\Delta W = \Delta Q - \Delta U = -400 - (-1000) = 600$ J (i.e. 600 J of work is done by the body).

26.3

1 (a) (i) $\frac{V}{T} = \text{constant} \therefore T_2 = \frac{V_2}{V_1} T_1 = \frac{360}{240} \times (273 + 17) = 435$ K.

(ii) $\Delta W = p\Delta V = 80 \times 10^3 (360 \times 10^{-6} - 240 \times 10^{-6}) = 9.6$ J

$\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} p\Delta V = 1.5 \times 9.6 = 14.4$ J

$\Delta Q = \Delta U + \Delta W = 24.0$ J.

(iii) See Fig. A26.1.

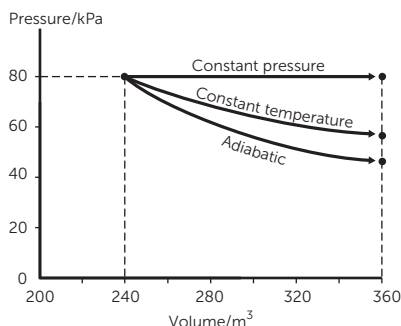


Fig. A26.1

(b) (i) $pV = \text{constant} \therefore p_2 = \frac{p_1 V_1}{V_2} = \frac{80 \times 10^3 \times 240 \times 10^{-6}}{360 \times 10^{-6}} = 53.3$ kPa.

(ii) See Fig. A26.1.

(iii) Estimate the area under the curve to give $\Delta W = 7.8$ J, $\Delta U = 0$ $\therefore \Delta Q = 7.8$ J.

(c) (i) Use $pV^\gamma = \text{constant}$, hence $p \times 360^{1.4} = 80 \times 240^{1.4}$

$\therefore p \times 3790 = 80 \times 2149$

$\therefore p = \frac{80 \times 2149}{3790} = 45$ kPa.

To find T , use $pV = nRT \therefore \frac{pV}{T} = \text{constant} \text{ so } \frac{45 \times 360}{T} = \frac{80 \times 240}{(273 + 17)}$

$\therefore T = \frac{45 \times 360 \times 290}{80 \times 240} = 245$ K = -28°C .

(ii) See Fig. A26.1.

(iii) $\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} \frac{p_1 V_1}{T_1} \Delta T = 1.5 \times \frac{80 \times 10^3 \times 240 \times 10^{-6}}{290} \times (245 - 290)$

$= -4.5$ J; $\Delta Q = 0$ so $\Delta W = -\Delta U = 4.5$ J.

2 (a) (i) Rearrange $pV = nRT$ to give

$n = \frac{pV}{RT} = \frac{110 \times 10^3 \times 1200 \times 10^{-6}}{8.31 \times (273 + 15)} = 0.055$ moles.

(ii) Mass of air = $n \times \text{molar mass} = 0.055 \times 0.029$ kg = 1.6×10^{-3} kg.

(iii) Internal energy, $U = \frac{5}{2} nRT = \frac{5}{2} pV = 2.5 \times 110 \times 10^3 \times 1200 \times 10^{-6} = 330$ J.

(b) (i) Use $pV^\gamma = \text{constant}$, hence $p \times 150^{1.4} = 110 \times 1200^{1.4}$

$\therefore p \times 1110 = 110 \times 20460 \therefore p = \frac{110 \times 20460}{1110} = 2030$ kPa.

To find the temperature, use $pV = nRT$

$\therefore \frac{pV}{T} = \text{constant}$, so $\frac{2030 \times 150}{T} = \frac{110 \times 1200}{(273 + 15)}$

$T = \frac{2030 \times 150 \times 288}{110 \times 1200} = 664$ K = 391°C

(ii) $\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} \times 0.055 \times 8.31 \times (664 - 288) = 258$ J.

(iii) $\Delta Q = 0$, so $\Delta W = -\Delta U = -258$ J.

26.4

1 (a) (i) 500 s ($= 15000$ m/15 m s $^{-1}$).

(ii) 20 MJ/500 s = 40 kW.

(b) Efficiency = $\frac{\text{output power}}{\text{energy supplied per second}} = \frac{6 \text{ kW}}{40 \text{ kW}} = 0.15$.

2 (a) $Q_1 = 160$ W + 40 W = 200 W.

(b) (i) Efficiency = $\frac{40 \text{ W}}{200 \text{ W}} = 0.20$

(ii) Maximum possible efficiency = $\frac{400 - 300}{400} = 0.25$.

Revision questions

26.1

(a) (i) $t = \frac{20 - (-6)}{29 - (-6)} \times 100 = 74^\circ\text{C}$. (ii) $T = t + 273 = 347$ K.

(b) (i) $V = 1.25 \times 50 \times (1 - 50 \times 20 \times 10^{-4}) = 56.3$ mV.

(ii) Centigrade temperature = 56.3 degrees.

26.2

(a) $\Delta Q = \Delta U + p\Delta V = 1000 + (100 \times 10^3 \times 2.0 \times 10^{-3}) = 1200$ J.

(b) $\Delta U = \Delta Q - p\Delta V = -600 - (150 \times 10^3 \times -2.0 \times 10^{-3}) = -600 + 300 = -300$ J.

(c) $p\Delta V = \Delta Q - \Delta U \therefore 100 \times 10^3 \Delta V = 200 - (-800) = 1000$ $\therefore \Delta V = 10 \times 10^{-3}$ m ^{3} .

(d) $p\Delta V = \Delta Q - \Delta U \therefore p \times -5.0 \times 10^{-3} = 400 - 1200 = -800$ $\therefore p = 160$ kPa.

26.3

(a) (i) Rearrange $pV = nRT$ to give

$n = \frac{pV}{RT} = \frac{10 \times 10^3 \times 60 \times 10^{-6}}{8.31 \times 290} = 2.5 \times 10^{-4}$.

(ii) $U = \frac{5}{2} nRT = 1.5 \times 2.5 \times 10^{-4} \times 8.31 \times 290 = 0.90$ J.

(b) (i) $pV = nRT$ for constant mass and volume becomes $\frac{p}{T} = \text{constant}$

$\therefore p_2 = \frac{T_2}{T_1} p_1 = \frac{380}{290} \times 10 \text{ kPa} = 13.1$ kPa.

(ii) $U_1 = 0.90$ J; $U_2 = \frac{5}{2} nRT = 1.5 \times 2.5 \times 10^{-4} \times 8.31 \times 380 = 1.18$ J. $\therefore \Delta U = 0.28$ J.

26.4

(a) (i) Rearrange $pV = nRT$ to give

$n = \frac{pV}{RT} = \frac{101 \times 10^3 \times 1.00 \times 10^{-4}}{8.31 \times (273 + 20)} = 4.1 \times 10^{-3}$ mol.

(ii) For a diatomic gas, $U = \frac{5}{2} nRT = 2.5 \times 4.1 \times 10^{-3} \times 8.31 \times 293 = 25.3$ J.

(b) (i) Use $pV^\gamma = \text{constant}$ where $\gamma = 1.4 \therefore p \times 20^{1.4} = 101 \times 100^{1.4}$.

$\therefore 66.3p = 101 \times 631$, so $p = \frac{101 \times 631}{66.3} = 961$ kPa.

To calculate T , use $pV = nRT$, hence

$T = \frac{pV}{nR} = \frac{961 \times 10^3 \times 20 \times 10^{-6}}{4.1 \times 10^{-3} \times 8.31} = 564$ K.

(ii) $U = \frac{5}{2} nRT = 2.5 \times 4.1 \times 10^{-3} \times 8.31 \times 564 = 48.7$ J.

(iii) $\Delta W = -\Delta U$ as $\Delta Q = 0 \therefore \Delta W = -(48.7 - 25.3) = -23.4$ J. $\therefore$ work done on air = 23.4 J.

26.5

(a) (i) $Q_2 = 2000 - 600 = 1400$ J in 1 s.

(ii) Efficiency = $\frac{600}{2000} = 0.30$.

(iii) Maximum possible efficiency = $\frac{450 - 300}{450} = 0.33$.

(b) (i) Consider A to B: $T_B = 1000$ K,

$p_B = p_A \times \frac{T_B}{T_A} = 100 \text{ kPa} \times \frac{1000}{300} = 333$ kPa.

Consider B to C: using $pV^\gamma = \text{constant}$

$\therefore 333 \times 0.0020^\gamma = 100 \times V_C^\gamma$, hence $V_C^{1.4} = 3.33 \times 0.0020^{1.4} = 5.54 \times 10^{-4}$

$\therefore 1.4 \ln V_C = \ln 5.54 \times 10^{-4} = -7.50$

$\therefore \ln V_C = -5.36 \therefore V_C = 4.7 \times 10^{-3}$ m ^{3} .

Using $pV = nRT$, $T_C = \frac{p_C V_C}{nR} = \frac{p_C V_C T_B}{p_B V_B} = \frac{100 \times 4.7 \times 10^{-3} \times 1000}{333 \times 0.0020} = 706$ K.

(ii) Maximum possible efficiency = $\frac{1000 - 300}{1000} = 0.70$.

(iii) Estimating the work done from the 'area' of the loop gives about 220 J.

(iv) Heat gain occurs between A and B; as no work is done, then

$\Delta Q_1 = \Delta U = \frac{5}{2} nR(T_B - T_A) = \frac{\frac{5}{2} p_A V_A (T_B - T_A)}{T_A}$
 $= \frac{2.5 \times 100 \times 10^3 \times 0.0020 \times 700}{300} = 1170$ J.

(v) Efficiency = $\frac{W}{Q_1} = \frac{220}{1170} = 0.19$.

UNIT 27

In-text questions

27.1

- 1 (a) $T = \frac{1}{f} = \frac{60}{800} = 0.075$ s.
 (b) $\omega = 2\pi f = 2\pi \times \frac{800}{60} = 83.8$ rad s⁻¹.
 (c) $v = \omega r = 83.8 \times 0.20 = 16.8$ m s⁻¹.
 2 (a) $f = \frac{3000}{60} = 50$ Hz, $T = \frac{1}{f} = 0.02$ s.
 (b) $\omega = 2\pi f = 2\pi \times 50 = 314$ rad s⁻¹.
 3 (a) (i) $C = 2\pi r = \pi \times 0.040 = 0.126$ m. (ii) $n = \frac{\text{tape length}}{C} = \frac{2.0}{0.126} = 15.9$.
 (b) (i) $v = 2\pi fr = 2\pi \times 2 \times 0.020 = 0.25$ m s⁻¹. (ii) $t = \frac{2.0 \text{ m}}{0.25 \text{ m s}^{-1}} = 8.0$ s
 (c) (i) $\theta = 0.9$ of a turn $\times 2\pi = 5.75$ rad. (ii) $\theta = 0.9 \times 360^\circ = 330^\circ$. $\therefore$ angle between ends $= 30^\circ$.

27.2

- 1 (a) $a = \frac{v^2}{r} = \frac{20 \times 20}{30} = 13.3$ m s⁻². (b) $F = ma = 800 \times 13.3 = 10\,700$ N.
 2 $F = ma = \frac{mv^2}{r}$. $\therefore v^2 = \frac{Fr}{m} = \frac{0.5mgr}{m} = 0.5gr = 0.5 \times 9.8 \times 30 = 147$.
 $\therefore v = 12.1$ m s⁻¹.
 3 (b) $v = \sqrt{gr} = \sqrt{(9.8 \times 6\,400\,000)} = 7920$ m s⁻¹.

27.3

- 1 (a) $F = mg - \frac{mv^2}{r} = 2000 \times 9.8 - \frac{2000 \times 15^2}{25} = 1600$ N.
 (b) $F = mg - \frac{mv^2}{r} = 0$ at the maximum speed.
 $v_{\text{MAX}} = \sqrt{gr} = \sqrt{9.8 \times 25} = 15.6$ m s⁻¹.
 2 Centripetal acceleration $= \frac{v^2}{r} = \frac{95^2}{450} = 20.0$ m s⁻²
 extra 'g force' $= (20/9.8)g = 2.0g$.

Revision questions

27.1

- (a) Time for one rotation, $T = 2 \times 60 \times 60 = 7200$ s
 $\therefore \omega = 2\pi/T = 2\pi/7200 = 8.7 \times 10^{-4}$ rad s⁻¹.
 (b) Speed, $v = \omega r = 8.7 \times 10^{-4} \times 8000 \times 10^3 = 7.0 \times 10^3$ m s⁻¹.
 (c) Centripetal acceleration, $a = \frac{v^2}{r} = \frac{(7.0 \times 10^3)^2}{8 \times 10^6} = 6.1$ m s⁻².

27.2

- (a) Angular speed, $\omega = 2\pi/T = 2\pi f = 2\pi \times 50 = 314$ rad s⁻¹.
 (b) Speed, $v = \omega r = 314 \times 0.04 = 12.6$ m s⁻¹.
 (c) Centripetal acceleration, $a = v^2/r = 12.6^2/0.04 = 3.9 \times 10^3$ m s⁻².

27.3

- (a) Centripetal acceleration, $a = v^2/r = 15^2/60 = 3.75$ m s⁻².
 (b) At its maximum speed, v_{MAX} , the centripetal force, $mv_{\text{MAX}}^2/r = 0.5mg$
 $\therefore v_{\text{MAX}}^2 = 0.5gr = 0.5 \times 9.8 \times 60 = \sqrt{294}$.
 Hence $v_{\text{MAX}} = \sqrt{294} = 17$ m s⁻¹.

27.4

- (a) (i) Gain of kinetic energy = loss of potential energy, hence the speed at the lowest point, v , is given by the equation $\frac{1}{2}mv^2 = mgh$, where h is the drop in height.
 $\therefore v = \sqrt{2gh} = \sqrt{(2 \times 9.8 \times 45)} = 30$ m s⁻¹.
 (ii) Frictional and drag forces are negligible.
 (b) (i) Centripetal acceleration, $a = \frac{v^2}{r} = \frac{(2 \times 9.8 \times 45)}{25} = 35$ m s⁻².
 (ii) Ratio $= (mv^2/r)/mg = v^2/gr = 35/9.8 = 3.6$.

27.5

- (a) The horizontal distance between the two ends of each cable $= 5 \sin 30$
 $\therefore$ radius of rotation, $r = 7.5 + 5 \sin 30 = 10.0$ m.
 (b) The vertical component of the tension, T , in each cable, $T \cos 30 = mg$, where m is the total mass of each aircraft and occupants.
 The horizontal component of the tension, $T \sin 30$ = centripetal force $= mv^2/r$, where v is the speed of each aircraft.
 Combining these two equations to eliminate T gives $mv^2/r = mg \tan 30$.
 $\therefore v^2 = gr \tan 30 = 9.8 \times 10 \times \tan 30 = 56$ m² s⁻². Hence $v = \sqrt{56} = 7.5$ m s⁻¹.
 (c) Time for one rotation = circumference/speed $= 2\pi \times 10/7.5 = 8.4$ s.

27.6

- (a) Let L = the lift force, θ = the angle of banking, r = radius of curvature of the flight path.
 The vertical component of the lift force, $L \cos \theta = mg$, the aircraft's weight.
 The horizontal component of the lift force, $L \sin \theta = mv^2/r$, the centripetal force.
 Combining these equations to eliminate L gives $\tan \theta = v^2/gr = 210^2/(9.8 \times 6400) = 0.703$. $\therefore \theta = 35^\circ$.
 (b) (i) The lift force is due to the shape of the wings and is always perpendicular to the plane of the wings, pushing on the 'underside' of the wing. When upside down, the 'underside' is uppermost so the lift force is downwards.
 (ii) At the highest point, the combined effect of the lift force, L , and the weight, mg , provides the centripetal force. Hence $L + mg = mv^2/r$. At the top of the loop, for $L = mg$, $mv^2/r = 2mg$. $\therefore v^2 = 2gr = 2 \times 9.8 \times 250 = 4900$ m² s⁻². $\therefore$ at the top, $v = \sqrt{4900} = 70$ m s⁻¹ for $L = mg$.

UNIT 28

In-text questions

28.1

- 1 (a) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 70 \times 5.98 \times 10^{24}}{(6380 \times 10^3)^2} = 686$ N.
 (b) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 5.98 \times 10^{24}}{(384\,000 \times 10^3)^2} = 1.99 \times 10^{20}$ N.
 (c) $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 70}{(384\,000 \times 10^3)^2} = 2.32 \times 10^{-3}$ N.
 2 (a) Force due to the Sun, F_2 at a distance of 150×10^6 km
 $= G \frac{m_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.0}{(1.50 \times 10^8 \times 10^3)^2} = 5.90 \times 10^{-3}$ N.

- (b) Let d = the distance from the centre of the Earth to the point at which the force on a mass, m , due to the Earth is equal and opposite to the force due to the Sun. See Fig. A28.1.

Force on m due to the Earth, $F_1 = \frac{GM_1m}{d^2}$
 where M_1 is the mass of the Earth.
 Force on m due to the Sun, $F_2 = \frac{GM_2m}{(D-d)^2}$

where M_2 is the mass of the Sun and D is the distance between the Earth and the Sun.

$$F_1 = F_2 \text{ gives } \frac{GM_1m}{d^2} = \frac{GM_2m}{(D-d)^2}$$

Cancelling G and m therefore gives

$$\frac{M_1}{d^2} = \frac{M_2}{(D-d)^2} \therefore \frac{(D-d)^2}{d^2} = \frac{M_2}{M_1} = \frac{1.99 \times 10^{30}}{5.98 \times 10^{24}} = 3.33 \times 10^5$$

Taking square roots of both sides gives

$$\frac{(D-d)}{d} = \sqrt{(3.33 \times 10^5)} = 577$$

$$\therefore D-d = 577d, \text{ which gives } 578d = D$$

$$\therefore d = \frac{D}{578} = \frac{150 \times 10^6}{578} \text{ km} = 260\,000 \text{ km}.$$

The distance from the Earth's surface is therefore $d - 6380$ km $= 254\,000$ km.

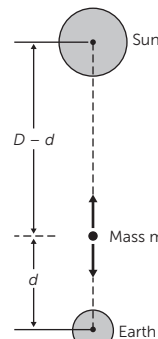


Fig. A28.1

28.2

- 1 (a) $g_s = \frac{GM}{r_s^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(696\,000 \times 10^3)^2} = 2.74 \times 10^2$ N kg⁻¹.
 (b) $g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(57.9 \times 10^6 \times 10^3)^2} = 0.0396$ N kg⁻¹.

- 2 Let d represent the distance from the Sun to the point at which the gravitational field strengths are equal and opposite.

Hence $\frac{GM_1}{d^2} = \frac{GM_2}{(D-d)^2}$ where D is the distance between the Sun and Jupiter,

M_1 is the mass of the Sun and M_2 is the mass of Jupiter.

$$\text{Hence } \frac{(D-d)^2}{d^2} = \frac{M_2}{M_1}$$

Since $M_1 = 1.99 \times 10^{30}$ kg and $M_2 = 1.90 \times 10^{27}$ kg,

$$\text{then } \frac{M_2}{M_1} = \frac{1.90 \times 10^{27}}{1.99 \times 10^{30}} = 9.55 \times 10^{-4} \therefore \frac{D-d}{d} = \sqrt{(9.55 \times 10^{-4})} = 3.09 \times 10^{-2}$$

$$\therefore D-d = 0.0309d \text{ which gives } 1.0309d = D, \text{ thus } d = 0.970D.$$

Since $D = 778$ million km, then $d = 755$ million km.

28.3

$$1 \text{ (a) Work done} = \frac{GMm}{r_s} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 1.0}{1740 \times 10^3} = 2.82 \text{ MJ.}$$

$$\text{(b) Work done} = \frac{GMm}{r} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.0}{150 \times 10^{11}} = 8.84 \text{ MJ.}$$

$$2 \text{ } R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(3.0 \times 10^8)^2} = 2.95 \times 10^3 \text{ m.}$$

28.4

$$1 \text{ (a) } v^2 = \frac{GM}{r} = \frac{g_s r_s^2}{r} = \frac{9.8 \times (6380 \times 10^3)^2}{7380 \times 10^3} = 5.41 \times 10^7 \text{ m}^2 \text{ s}^{-2}$$

$$\therefore v = 7.35 \text{ km s}^{-1}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(7380000)}{7350} = 6307 \text{ s} = 105 \text{ minutes.}$$

$$\text{(b) } r^3 = \frac{GMT^2}{4\pi^2} = \frac{g_s r_s^2 T^2}{4\pi^2} = \frac{9.8 \times (6380 \times 10^3)^2 \times (4 \times 3600)^2}{4\pi^2} = 2.10 \times 10^{21} \text{ m}^3.$$

$$r = 12.8 \times 10^6 \text{ m} \therefore \text{its height, } h = 12800 - 6380 = 6420 \text{ km.}$$

$$\text{Its speed, } v = \frac{2\pi r}{T} = \frac{2\pi(12800 \times 10^3)}{4 \times 3600} = 5590 \text{ m s}^{-1}.$$

$$2 \text{ (a) Rearrange } r^3 = \frac{GMT^2}{4\pi^2} \text{ to give}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2(1840 \times 10^3)^3}{6.67 \times 10^{-11} \times 7.35 \times 10^{22}} = 5.02 \times 10^7 \text{ s}^2$$

$$\therefore T = 7080 \text{ s} = 118 \text{ minutes.}$$

$$\text{(b) Rearrange } v^2 = \frac{GM}{r} \text{ to give } r = \frac{GM}{v^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{(1000)^2}$$

$$= 4.90 \times 10^6 \text{ m} \therefore h = 4900 - 1740 = 3160 \text{ km.}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(4.90 \times 10^6)}{1000} = 3.08 \times 10 \text{ s} = 513 \text{ minutes.}$$

Revision questions

28.1

$$\text{(a) (i) Force due to the Earth, } F_1$$

$$= \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 5.98 \times 10^{24}}{(384000 \times 10^3)^2} = 1.99 \times 10^{20} \text{ N.}$$

$$\text{(ii) Force due to the Sun, } F_2$$

$$= \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 1.99 \times 10^{30}}{(150 \times 10^9)^2} = 4.33 \times 10^{20} \text{ N.}$$

$$\text{(b) Overall force} = F_1 - F_2 = 2.34 \times 10^{20} \text{ N towards the Sun.}$$

28.2

$$\text{(a) } r = 5.20 - 1.00 = 4.20 \text{ A.U.}$$

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 318 \times 5.98 \times 10^{24}}{(4.20 \times 150 \times 10^9)^2} = 3.20 \times 10^{-7} \text{ N kg}^{-1}.$$

$$\text{(b) (i) } \frac{GM_1}{d^2} = \frac{GM_2}{(D-d)^2}$$

where M_1 = the mass of the Earth, M_2 = the mass of Jupiter = $318M_1$, d = distance to be calculated and D = distance from Earth to Jupiter at opposition = $5.20 - 1.00 = 4.20 \text{ A.U.}$

$$\text{Hence } (D-d)^2 = \frac{M_2 d^2}{M_1} = 318d^2 \therefore (D-d) = 17.8d \text{ so } 18.8d = D$$

$$\text{hence } d = \frac{4.20}{18.8} = 0.22 \text{ A.U.}$$

$$\text{(ii) } r = 1.00 + 0.22 = 1.22 \text{ A.U.} = 1.83 \times 10^{11} \text{ m.}$$

$$g_{\text{SUN}} = \frac{GM_{\text{SUN}}}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(1.83 \times 10^{11})^2} = 3.96 \times 10^{-3} \text{ N kg}^{-1}.$$

28.3

$$\text{(a) (i) } \frac{1}{2}mv^2 = 62.5 \text{ MJ} \therefore \text{for } m = 1 \text{ kg, } v^2 = 2 \times 62.5 \times 10^6,$$

$$\text{hence } v = 1.12 \times 10^4 \text{ m s}^{-1}.$$

$$\text{(ii) } \frac{1}{2}mv^2 = 2.8 \text{ MJ} \therefore \text{for } m = 1 \text{ kg, } v^2 = 2 \times 2.8 \times 10^6,$$

$$\text{hence } v = 2.37 \times 10^3 \text{ m s}^{-1}.$$

(b) (i) Its initial kinetic energy of 2.8 MJ kg^{-1} is used to enable it to escape from the Moon. Its potential energy at the Earth is -62500 MJ ($= -62.5 \text{ MJ kg}^{-1} \times 1000 \text{ kg}$). Its potential energy due to the Earth when it is on the Moon is negligible. Therefore its kinetic energy just before reaching the Earth = 62500 MJ .

$$\text{(ii) } \frac{1}{2}mv^2 = 62500 \text{ MJ} \therefore \text{for } m = 1000 \text{ kg, } v^2 = 2 \times 62.5 \times 10^6, \text{ hence } v = 1.12 \times 10^4 \text{ m s}^{-1}.$$

28.4

$$\text{(a) (i) } g_s = \frac{GM}{r_s^2} = \frac{6.67 \times 10^{-11} \times 0.108 \times 5.98 \times 10^{24}}{(3400 \times 10^3)^2} = 3.73 \text{ N kg}^{-1}.$$

$$\text{(ii) } v_{\text{esc}} = \sqrt{2g_s r_s} = 5.04 \times 10^3 \text{ m s}^{-1}.$$

$$\text{(b) (i) } r^3 = \frac{GMT^2}{4\pi^2} = \frac{g_s r_s^2 T^2}{4\pi^2} = \frac{3.73 \times (3400 \times 10^3)^2 \times (7.65 \times 3600)^2}{4\pi^2}$$

$$= 8.28 \times 10^{20} \text{ m}^3 \therefore r = 9.39 \times 10^6 \text{ m} = 9390 \text{ km}$$

$$\text{so } h = 9390 - 3400 = 5990 \text{ km.}$$

$$\text{(ii) } r = 20100 + 3400 = 23500 \text{ km} \therefore r^3 = \frac{GMT^2}{4\pi^2} \text{ to give}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 r^3}{g_s r_s^2} = \frac{4\pi^2 \times (23500 \times 10^3)^3}{3.73 \times (3400 \times 10^3)^2} = 1.19 \times 10^{10} \text{ s}^2$$

$$\therefore T = 1.09 \times 10^5 \text{ s} = 30.3 \text{ hours.}$$

28.5

$$\text{(a) (i) Rearrange } r^3 = \frac{GMT^2}{4\pi^2} \text{ to give}$$

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times (354000 \times 10^3)^3}{6.67 \times 10^{-11} \times (5.88 \times 24 \times 3600)^2} = 1.02 \times 10^{26} \text{ kg.}$$

$$\text{(ii) } r^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26} \times (359 \times 24 \times 3600)^2}{4\pi^2}$$

$$= 1.66 \times 10^{29} \text{ m}^3 \therefore r = 5.49 \times 10^9 \text{ m.}$$

$$\text{(b) (i) } r = 6380 + 1680 = 8060 \text{ km.}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 r^3}{g_s r_s^2} = \frac{4\pi^2 \times (8060 \times 10^3)^3}{9.80 \times (6380 \times 10^3)^2} = 5.14 \times 10^7 \text{ s}^2 \therefore T = 7170 \text{ s.}$$

$$\text{(ii) The Earth turns through } 7170 \div 24 \times 3600 \text{ of a full rotation in } 7170 \text{ s.}$$

Therefore successive transits of the equator are separated by distance,

$$d = \frac{7170}{24 \times 3600} \times \text{Earth's circumference } (2\pi \times 6380 \text{ km}) = 3330 \text{ km.}$$

UNIT 29

In-text questions

29.1

$$1 \text{ (a) (i) Northwards (ii) Southwards.}$$

(b) Its displacement is reversing from northwards to southwards, its velocity is southwards.

$$2 \text{ (a) (i) Upwards (ii) Zero (b) (i) Downwards (ii) Zero.}$$

29.2

$$1 \text{ (a) (i) } T = \frac{12}{10} = 1.2 \text{ s. (ii) } \omega = \frac{2\pi}{T} = 5.2 \text{ rad s}^{-1}.$$

$$\text{(b) (i) amplitude, } r = 0.20 \text{ m. (ii) } v_{\text{MAX}} = \omega r = 5.2 \times 0.20 = 1.0(4) \text{ m s}^{-1}.$$

$$\text{(iii) } a_{\text{MAX}} = (-)\omega^2 r = (-)5.2^2 \times 0.20 = 5.4 \text{ m s}^{-2}.$$

$$2 \text{ (b) (i) } T = \frac{1}{40} = 0.025 \text{ s. (ii) } \omega = \frac{2\pi}{T} = 250 \text{ rad s}^{-1}.$$

$$\text{(c) } v_{\text{MAX}} = \omega r = 250 \times 8.0 = 2000 \text{ mm s}^{-1}.$$

29.3

$$1 \text{ (a) } T = \frac{22.0}{20} = 1.1 \text{ s. (b) } \omega = \frac{2\pi}{T} = 5.7 \text{ rad s}^{-1}.$$

$$\text{(c) } v_{\text{MAX}} = \omega r = 5.7 \times 25 \times 10^{-3} = 0.14 \text{ m s}^{-1}.$$

(d) The maximum tension = weight of 0.20 kg + extra tension due to a displacement of 25 mm .

$$\text{Weight of } 0.20 \text{ kg} = mg = 0.20 \times 9.8 = 1.96 \text{ N.}$$

Extra tension = ks , where $s = 25 \text{ mm} = 0.025 \text{ m}$ and k is calculated from

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

$$\text{Hence } k = \frac{4\pi^2 m}{T^2} = 6.5 \text{ N m}^{-1}, \text{ hence the extra tension} = 6.5 \times 0.025 = 0.16 \text{ N}$$

$$\therefore \text{the maximum tension} = 1.96 + 0.16 = 2.1 \text{ N.}$$

$$2 \text{ (a) } T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.0}{9.8}} = 2.83 \text{ s.}$$

$$\text{(b) (i) For Fig. 29.11, } h = L - L \cos \theta = 2.0 - 2.0 \cos 5^\circ = 7.6 \times 10^{-3} \text{ m} = 7.6 \text{ mm.}$$

(ii) Maximum displacement from equilibrium, $s = L \sin \theta$, provided θ is less than about 10° . Hence amplitude, $r = s_{\text{MAX}} = 2.0 \sin 5^\circ = 0.17(4) \text{ m}$.

$$\therefore v_{\text{MAX}} = \omega r = \frac{2\pi r}{T} = \frac{2\pi}{2.83} \times 0.174 = 0.39 \text{ m s}^{-1}.$$

29.4

1 (a) Each panel of the machine is capable of vibrating at its own natural frequency of oscillation. If the motor frequency is equal to the natural frequency of a panel, that panel vibrates in resonance.

(b) Each panel needs to be fixed to the chassis at sufficient points to prevent resonance at any possible motor frequency.

$$2 \text{ (a) (i) Additional weight} = 300g/4 = 735 \text{ N.}$$

(ii) $k = \text{additional weight} + \text{change of length of spring} = 735 \text{ N}/0.05 \text{ m} = 1.47 \times 10^4 \text{ N m}^{-1}$.

(b) (i) Time period, $T = 2\pi\left(\frac{m}{k}\right)^{1/2} = 2\pi\left(\frac{375}{1.47 \times 10^4}\right)^{1/2} = 1.0 \text{ s}$.

(ii) At this speed, the car would hit a speed bump once every second, so it would resonate as its time period is 1.0 s.

Revision questions

29.1

(a) (i) $T = 15.0/20 = 0.75 \text{ s}$. (ii) $\omega = 2\pi/T = 2\pi/0.75 = 8.4 \text{ rad s}^{-1}$;

$v_{\text{MAX}} = \omega r = 8.4 \times 0.100 = 0.84 \text{ m s}^{-1}$.

(iii) $a_{\text{MAX}} = (-)\omega^2 r = 8.4^2 \times 0.100 = 7.1 \text{ m s}^{-2}$.

(b) (i) See Fig. A29.1. (ii) X is any point where the displacement is zero.

(iii) Y is any point where the displacement is a positive or a negative maximum.

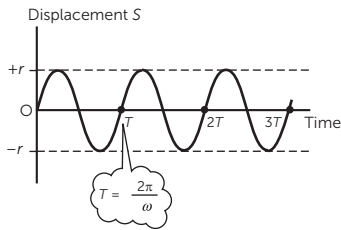


Fig. 29.1

29.2

(a) $k = 0.30 \times 9.8/0.090 = 33 \text{ N m}^{-1}$.

(b) (i) $T = 2\pi(m/k)^{1/2} = 0.60 \text{ s}$. (ii) $\omega = 2\pi/T = 2\pi/0.60 = 10.5 \text{ rad s}^{-1}$; amplitude, $r = 0.060 \text{ m}$. $\therefore v_{\text{MAX}} = \omega r = 10.5 \times 0.060 = 0.63 \text{ m s}^{-1}$.

29.3

(a) (i) $T = 3.0/2 = 1.50 \text{ s}$. (ii) Rearrange $T = 2\pi(m/k)^{1/2}$ to give

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 \times 900/4}{1.50^2} = 3950 \text{ N m}^{-1}.$$

(b) $m = 750/4 \text{ kg}$, $k = 3950 \text{ N m}^{-1}$.

$$\therefore T = 2\pi\left(\frac{m}{k}\right)^{1/2} = 2\pi\left(\frac{750/4}{3950}\right)^{1/2} = 1.37 \text{ s}.$$

29.4

(a) Length of simple pendulum $= 1.20 \text{ m}$, $T = 2\pi(L/g)^{1/2} = 2.2 \text{ s}$.

(b) (i) For angles of no more than 10° , $s = L \sin \theta = 1.20 \sin 10^\circ = 0.21 \text{ m}$.

(ii) $\omega = 2\pi/T = 2\pi/2.2 = 2.9 \text{ rad s}^{-1}$; amplitude, $r = s_{\text{MAX}} = 0.21 \text{ m}$
 $\therefore v_{\text{MAX}} = \omega r = 2.9 \times 0.21 = 0.60 \text{ m s}^{-1}$.

29.5

(a) (i) Weight of sand $= 120\text{g} = 1180 \text{ N}$. (ii) Additional weight on each spring $= 1180/4 = 294 \text{ N}$. $\therefore k = 294/0.020 = 1.47 \times 10^4 \text{ N m}^{-1}$.

(iii) $T = 2\pi\left(\frac{m}{k}\right)^{1/2} = 2\pi\left(\frac{1.30}{1.47 \times 10^4}\right)^{1/2} = 0.59 \text{ s}$.

(b) If the trailer is driven over speed bumps at a rate of 1 every 0.59 s, the trailer suspension system resonates.

UNIT 30

In-text questions

30.1

1 (a) 0.20 rad s^{-2} , (b) (i) 360 rad , (ii) 57 .

2 (a) $1.38 \times 10^5 \text{ rad}$, (b) $22\,000$.

30.2

1 0.38 N m .

2 (b) (i) 27 rad s^{-2} , (ii) 300 rad s^{-1} , (iii) 524 turns .

30.3

1 (a) 1.7 J , (b) (i) 0.028 N m , (ii) 60 rad

2 (a) 5.9 J , (b) (i) 0.41 m s^{-1} , (ii) 0.061 J ,

(c) (i) $N = 0.91/\pi \times 8.5 \times 10^{-3} = 34 \text{ turns}$,

(ii) k.e. gained by flywheel $(\frac{1}{2}I\omega^2) = \text{p.e. lost by } W_1 - \text{k.e. gained by } W_0 \text{ and } W_1 = 5.9 - 0.06 = 5.8(4) \text{ J}$,

(iii) $\frac{1}{2}I\omega^2 = 5.84 \text{ J}$; $I = 5.84/(\frac{1}{2} \times (96)^2) = 1.3 \times 10^{-3} \text{ kg m}^2$.

Revision questions

30.1

(a) 151 rad s^{-1} , (b) (i) 5.2 rad s^{-2} , (ii) 348 .

30.2

Y has a greater moment of inertia than X but a greater proportion of its mass is distributed near the perimeter of the disc and their radii are the same.

30.3

(b) (i) 13 kg m^2

30.4

As the ball rolls down the slope, it loses potential energy and gains kinetic energy due to its linear motion and due to its rotational motion. During the time it is moving its total energy decreases due to friction. After it moves onto the horizontal surface, the frictional forces causes its kinetic energy to decrease to zero.

30.5

$2.4 \times 10^{-2} \text{ kg m}^2$

Answers to Further Questions

Q1.1 (a) (i) 6.93 N due north, 4.0 N
(ii) 2.74 N due north, 7.52 N due west
(b) (i) 2 N due north **(ii)** 12.8 N at 39° east of due north **(iii)** 15.6 N at 26.3° east of due north
Q1.2 (a) 1.0 N upward **(b)** 7.81 N acting upwards at 40° to the vertical
Q1.3 (b) X: 48 N; Y: 32 N **(c)** 0.56 m
Q1.4 (a) 1.20 m **(b)** 70 N
Q2.1 (a) 15 m **(b)** 6.25 ms^{-2}
Q2.2 (b) (i) 0.2 ms^{-2} , 0, -0.25 ms^{-2}
(ii) 563 m, 3600 m, 450 m
(c) 4613 m, 12.3 ms^{-1}
Q2.3 (a) (i) 13.7 ms^{-1} **(ii)** 9.6 m
Q2.4 (a) (i) 1.6 s, **(ii)** 12.5 m **(iii)** 15.7 ms^{-1}
(iv) 15.7 ms^{-1}
Q3.1 (a) (i) -3.0 ms^{-2} **(ii)** 30 ms^{-1} **(iii)** 150 kg ms^{-1}
(b) (i) 3.6 kg ms^{-1} **(ii)** 144 N
Q3.2 (a) (i) 12.8 ms^{-1} **(ii)** -1960 ms^{-2}
(iii) 7840 N
(b) (i) 2.6 ms^{-2} **(ii)** 98.7 kN
Q3.3 (a) (i) 1.87 kg ms^{-1} **(ii)** 1.87 N
(b) (i) 2030 ms^{-2} **(ii)** 25.4 kN **(iii)** 2.7 ms^{-1}
Q3.4 (a) 1.03 ms^{-1} in the same direction as the first wagon **(b)** 0.04 ms^{-1} in the opposite direction to the first wagon
Q4.1 (a) (i) 3.53 J, 3.53 J **(ii)** 0, 7.06 J
(b) (i) 1.18 J, 5.42 ms^{-1} **(ii)** 0.94 J, 4.85 ms^{-1}
(iii) 0.24 J
Q4.2 (a) 1.35 MJ **(b)** 0.78 MJ **(c)** 0.57 MJ
Q4.3 (a) 13 MW **(b)** 63 kW
Q4.4 (a) (i) 11 ms^{-1} **(ii)** 99 kJ **(b)** 80 kJ m^{-1} , 80 kN
Q5.1 (a) (i) 0.018 kg **(ii)** 0.044 kg
(b) (i) $3.0 \times 10^{-26} \text{ kg}$ **(ii)** 0.31 nm
Q5.3 (b) (i) ${}^{20}_{10}\text{Ne}$, ${}^{22}_{10}\text{Ne}$
(ii) 90% Ne 20, 10% Ne 10
Q5.4 (a) (i) $1.0 \times 10^{-5} \text{ m}^3$ **(ii)** 6.0×10^{22}
(b) 0.26 nm
Q6.1 (a) 1.42 mm **(b)** 44 mm
Q6.2 (a) (i) 7.2 MJ **(ii)** 48°C **(b)** 383 s
Q6.3 (b) 128 W
Q6.4 (a) 5800 K **(b) (i)** $4.0 \times 10^{26} \text{ W}$ **(ii)** 1.0
Q7.1 (b) (i) 325 mm **(ii)** 4.4 N
Q7.2 (b) (i) $1.0 \times 10^9 \text{ Pa}$ **(ii)** 102 N
Q7.3 (a) $3.2 \times 10^8 \text{ Pa}$ **(b)** 7.3 mm
Q7.4 (a) $1.26 \times 10^{11} \text{ Pa}$ **(b)** 0.6 J
Q8.1 (a) 40 kPa **(b)** 2.5 kPa
Q8.2 (a) 51.5 kPa
Q8.3 (b) (i) 500 kPa **(ii)** 3000 N
Q8.4 (a) 490 Pa **(b)** 36.7 m^2 **(c)** 72 kN
Q9.1 (b) (i) $5.17 \times 10^{14} \text{ Hz}$ **(ii)** 3.18 m
Q9.2 (b) (i) 3.0 m **(ii)** $2\pi/3$ radians ($= 120^\circ$)
Q10.1 (b) 86 dB
Q10.2 (b) (i) 0.40 ms **(ii)** 0.60 ms
Q10.3 (b) (i) 3.6 m, 216 ms^{-1} **(ii)** 180 Hz
Q10.4 (b) (i) 340 ms^{-1} **(ii)** 360 Hz

Q11.1 (a) (i) 13° **(ii)** 31° **(b) (i)** 42°
Q11.2 (a) (i) 1.5 mm **(ii)** 0.65 mm **(b)** 8.0 mm
Q11.3 (a) (i) $v = -300 \text{ mm}$: virtual, upright, $\times 3$ magnification
(ii) $v = +300 \text{ mm}$: real, inverted, $\times 1$ magnification
(b) $v = -0.10 \text{ m}$
Q11.4 (b) 4.0° **(c)** 0.075 m
Q12.3 (a) 7.9° ($33.4 - 25.5^\circ$) **(b)** 59.3° for 430 nm
Q13.2 (a) (i) 9.0Ω **(ii)** 2.0Ω **(b)** 4.0Ω : 1.0 A, 4.0 V, 4.0 W; 3.0 Ω : 0.67 A, 2.0 V, 1.33 W; 6.0 Ω : 0.33 A, 2.0 V, 0.67 W
Q13.3 (a) (i) 10.4 A **(ii)** 1870 C **(iii)** 450 J
Q13.4 (b) (iii) diode: 0.36 mW; resistor: 0.54 mW; cell: 0.90 mW
Q14.1 (a) 3.0 A **(b)** 7.5 V, 1.5 A
(c) (i) 36 W, 11.25 W **(ii)** 13.5 W dissipated due to internal resistance in the battery
Q14.2 (a) 6.1 Ω **(b)** 291 mm from the end at S
Q14.3 (a) (iii) 240 k Ω
Q14.4 (a) $4.81 \times 10^{-7} \Omega \text{ m}$ **(b)** 592 mm
Q15.1 (a) 72 mC **(b)** 22 mF
Q15.2 (a) (i) 18 μF **(ii)** 4 μF
(b) (i) 6 μF : 24 μC , 48 μJ ; 12 μF : 24 μC , 24 μJ
(ii) 24 μC , 72 μJ
Q15.3 (a) (i) 300 pF **(ii)** 3.6 nC, 21.6 nJ
(b) (i) 600 pF **(ii)** 6.0 V
Q15.4 (a) 42 μC , 190 μJ **(b)** 6.0 μA , 21 μC , 46 μJ
Q16.2 (b) 3.0 V
Q16.3 (b) (ii) 2 k Ω
Q17.1 (a) $9.6 \times 10^{10} \text{ V m}^{-1}$, 2.88 V **(b)** $4.6 \times 10^{-18} \text{ J}$
Q17.2 (a) $1.94 \times 10^{-5} \text{ J}$ **(b)** 7.56 kV m^{-1} , 918 V
Q17.3 (a) (i) 125 kV m^{-1} **(b)** $1.35 \times 10^{-11} \text{ J}$
Q17.4 (b) (i) 76.6 nC **(ii)** 600 V
Q18.1 (b) (i) 15.2 A **(ii)** 3.8 mT
Q18.2 (a) 55 mT **(b)** 0.96 mN
Q18.3 (a) 18 μT
Q19.1 (b) 0.19 V
Q19.2 (b) (i) 61 mWb **(ii)** 7.7 V
Q19.3 (b) (i) 11.4 V **(ii)** 1.72 A
Q19.4 (a) 12 V **(b)** 5.0 A
Q20.1 (a) 1.60 A, 50 Hz
(b) (i) 1.13 A **(ii)** 1.60 A **(iii)** -0.94 A
Q20.2 (a) (i) 1.7 A **(ii)** 17.0 V
(b) (i) 1.2 A **(ii)** 7.2 W
Q20.3 (a) (i) 2.43 mA **(ii)** 4.86 V across R, 3.52 V across C **(iii)** 11.8 mW
(b) (i) 2.95 mA **(ii)** 5.90 V across R, 1.09 V across C **(iii)** 17.4 mW
Q20.4 (a) 848 Hz **(b) (i)** 54 n **(ii)** 0.11 A **(iii)** 8.3 V
Q21.1 (b) $7.2 \times 10^{-16} \text{ J}$, $4.0 \times 10^7 \text{ m s}^{-1}$
Q21.2 (a) (i) $2.2 \times 10^7 \text{ m s}^{-1}$, $2.2 \times 10^{-16} \text{ J}$
Q21.3 (a) 0.18 mm s^{-1}
(b) $1.3 \times 10^{-6} \text{ m}$, $8.2 \times 10^{-15} \text{ kg}$
(c) $3.2 \times 10^{-19} \text{ C}$

Q21.4 (a) $1.3 \times 10^{19} \text{ Hz}$
(b) $2.77 \times 10^{-19} \text{ J}$, $4.2 \times 10^{14} \text{ Hz}$
Q22.1 (b) (i) 0.16 MBq **(ii)** 270 Bq
Q22.2 (b) 1.37 mm
Q22.3 (b) 1770 years
Q22.4 (a) 360 kBq **(b)** 220 counts per second
Q23.1 (a) 2.0 fm
Q23.2 (b) (i) 7.1 MeV **(ii)** 7.4 MeV
Q23.3 (b) 2.82 MeV **(c)** $4.1 \times 10^{13} \text{ Bq}$
Q24.2 (a) $6.5 \text{ m}^3 \text{ s}^{-1}$
Q24.3 (b) $0.45 \text{ m}^3 \text{ s}^{-1}$
Q24.4 (b) (ii) 0.51 ms^{-1}
Q25.1 (a) (i) 4.9 **(ii)** 22.7 kPa **(b)** 18.4 cm^3
Q25.2 (a) (i) 85 kPa **(ii)** 16 kPa **(b)** $2.4 \times 10^{24} \text{ m}^{-3}$
Q25.3 (b) 12.4 moles **(ii)** 0.35 kg **(iii)** 508 ms^{-1}
(iv) $6.0 \times 10^{-21} \text{ J}$
Q25.4 (b) (i) 474 ms^{-1} **(ii)** 461 ms^{-1}
(iii) 1845 ms^{-1}
Q26.1 (b) (i) 39°C **(ii)** 312 K
Q26.2 (a) (i) 2.4 moles **(ii)** 8.8 kJ
(b) (i) 129 kPa **(ii)** 2.4 kJ
Q26.3 (a) (i) 0.100 m^3 **(ii)** 0.116 m^3 **(iii)** 1.63 kJ
(b) (ii) 3.29 kJ
Q26.4 (a) 1720 kPa, **(b)** 0.55
Q27.1 (a) 0.10 s **(b)** 11.0 ms^{-1} , 690 ms^{-2}
Q27.2 (a) 46 ms **(b)** 22 Hz **(c)** 4270 ms^{-2}
Q27.3 (a) 9.31 kN **(b)** 4.5 ms^{-2} **(c)** 5.03 kN
Q27.4 (a) 1.11 J **(b)** 4.31 ms^{-1} **(c)** 19.6 ms^{-2}
(d) 3.53 N
Q28.1 (a) 453 N
(b) 6.52 km s^{-1} , 2 hours 31 minutes
Q28.2 (a) 278 km
Q28.3 (b) (ii) 2.37 km s^{-1}
Q28.4 (a) $5.9 \times 10^{-3} \text{ N kg}^{-1}$
(b) (i) $3.7 \times 10^9 \text{ m}$ **(ii)** 0.025
Q29.1 (b) (i) 2.52 s **(ii)** 65 mm **(iii)** 1.62 ms^{-1}
Q29.2 (a) 1.94 s **(b) (i)** 81 mm **(ii)** 0.262 ms^{-1}
(iii) 0.850 ms^{-2}
Q29.3 (a) (i) 51 N m^{-1} **(ii)** 0.44 s **(b) (i)** 0.29 ms^{-1}
(ii) 1.43 N
Q30.1 (a) 107 rad s^{-1} **(b) (i)** 5.0 s **(ii)** 42.4
(c) 21 rad s^{-1}
Q30.2 The moment of inertia about axis Y is greater than about axis X. This is because the mass of the ball is distributed further from the axis of Y than from the axis of X.
Q30.3 (a) (i) 47 kg **(ii)** 0.57 kg m^2 **(b) (i)** 28 kJ, **(ii)** 2.0 kW
Q30.4 (a) (i) $(-0.21 \text{ rad s}^{-2})$ **(ii)** 11 N m **(b)** 9.3 W
Q30.5 (a) 50 kg m^2 **(b) (i)** 39 MJ **(ii)** 6.3 MJ
(iii) Work is done to overcome the frictional forces preventing the drill from turning. Also, work is done between the clutch and the flywheel when they first come into contact. For both these reasons, energy is transferred to the surroundings as heat.

Glossary

- absolute temperature** The scientific scale of temperature, measured in kelvins (K). To convert from °C to K, add 273 to the temperature in °C. Absolute temperature in K = temperature in °C + 273.15.
- absolute zero** The lowest possible temperature (-273.16°C or 0 K).
- acceleration** The change of velocity of an object per second. The unit of acceleration is the metre per second² (m s⁻²).
- activity** Number of radioactive nuclei that decay per second. The unit of activity is the becquerel (Bq).
- adiabatic change** A change in which no heat transfer occurs.
- alpha particle** A particle consisting of two protons and two neutrons emitted by a large unstable nucleus.
- alternating current** Current that repeatedly reverses its direction. A sinusoidal alternating current may be represented by the equation $I = I_0 \sin(2\pi ft)$.
- amorphous solid** A solid in which the atoms are arranged in random order.
- amplitude** The maximum displacement of a wave from equilibrium (e.g. the height of a transverse wave from the middle).
- amplitude modulation** Variation of the amplitude of a carrier wave to carry information.
- angular acceleration** Rate of change of angular velocity.
- angular frequency** 2π /time period.
- angular momentum** Moment of inertia $\times$ angular velocity.
- angular speed** Rate of rotation of an object.
- angular velocity** Angular speed in a given direction.
- anode** Positive terminal in an electrolytic cell or an electron tube.
- antimatter** Made of antiparticles.
- antinode** A point of maximum displacement in a stationary wave pattern.
- antiparticle** An antiparticle and its particle annihilate each other to produce radiation.
- atomic mass** The mass of an atom on a scale where one atom of carbon 12 has a mass of exactly 12 units. The mass of a proton or a neutron is 1 on this scale.
- atomic mass number (A)** The number of protons and neutrons in the nucleus of an atom.
- atomic number (Z)** The number of protons in the nucleus of an atom.
- avogadro constant, (N_A)** The number of atoms in exactly 12 grams of carbon 12.
- background radioactivity** Radioactivity due to materials in buildings or due to cosmic radiation.
- bandwidth** The frequency width of a communication channel (e.g. a channel from 100 to 104 kHz has a bandwidth of 4 kHz).
- barometer** Instrument used to measure atmospheric pressure.
- the Bernoulli effect** The increase (or decrease) of pressure in a fluid when it gains (or loses) speed or height.
- beta particle** An electron created and emitted from an unstable nucleus.
- binding energy** The work done to separate a nucleus into its constituent nucleons.

- bourdon gauge** A pressure gauge used to measure the pressure of a gas or a liquid in a pipe.
- Brownian motion** The unpredictable motion of microscopic particles due to random and uneven collisions with fast-moving molecules.
- byte** Series of bits (1's and 0's) that are coded to carry information.
- capacitor** A device that can store charge.
- cathode** Negative terminal in an electrolytic cell or an electron tube.
- cathode rays** A beam of electrons.
- centre of gravity** The point where the weight of an object as a single force is considered to act.
- centripetal acceleration** Acceleration of an object on a circular path due to its circular motion.
- centripetal force** The force on an object that makes it go around on a circular path; the direction of the centripetal force is always towards the centre of the circle.
- chain reaction** Series of reactions in which each reaction causes one or more further reactions; a steady chain reaction is where each reaction causes exactly one more reaction.
- change of state** Change of physical state (e.g. solid changes to liquid).
- chromatic aberration** Splitting of white light into colours by a lens or a prism.
- concave lens** A lens that makes a parallel beam of light diverge.
- condensation** This occurs when a vapour turns to liquid.
- conservation of angular momentum** If the resultant torque on a system is zero, the total angular momentum does not change (i.e. is conserved).
- conservation of energy** The total energy is unchanged when energy changes from one form into other forms.
- conservation of momentum** When two or more objects collide, the total momentum is unchanged, provided no external forces act.
- continuous spectrum** A spectrum in which the colours are continuous from one end to the other.
- convection** Fluid flow due to hot fluid rising and cold fluid falling.
- convex lens** A lens that focuses a parallel beam of incident light.
- covalent bond** A bond in which electrons are shared.
- critical angle** The angle of incidence of a light ray that refracts along the boundary between two transparent substances.
- damping** Forces that cause an oscillating system to lose energy.
- decay constant** $\ln 2$ /half-life.
- decibels** A measure of the loudness level of a sound where the loudness increases tenfold for every extra ten decibels.
- dielectric substance** A substance that increases the capacitance of a capacitor.
- diffraction** The spreading of waves after passing through a gap or around an obstacle; the narrower a gap is, the greater the amount of diffraction there is.
- diffraction grating** A plate ruled with many regularly spaced slits.
- diffusion** Spreading of a substance into a fluid without stirring (e.g. a coloured crystal dissolves in water and the water gradually becomes evenly coloured).
- digital circuit** An electric circuit in which the voltage at any point can only take two values (high '1' or low '0').
- diode** A device that lets current through in one direction only.
- dispersion** The splitting of white light into colours by a prism; this happens because the speed of light in glass depends on the colour of the light.
- displacement** Distance from a given point in a certain direction; displacement is a vector quantity.
- dynamic pressure** The pressure of a fluid due to its motion.
- dynamo effect** Generation of an e.m.f. due to changing magnetic flux.
- echo** Sound waves that reflect off a smooth surface.
- eddy currents** Unwanted currents in a metal caused by electromagnetic induction.

- efficiency** Fraction (or percentage) of the energy supplied to a machine that is used by the machine to do useful work.
- elastic energy** Energy stored in a body when its shape is changed.
- elastic limit** Limit beyond which a material is permanently extended when stretched.
- elasticity** Ability of a material to regain its shape after being distorted.
- electric field** Region where a force is exerted on an electric charge.
- electric field strength (E)** The force per unit charge on a test charge.
- electric potential (V)** The work done per unit charge to move a test charge from infinity.
- electromagnetic induction** A voltage induced in a coil as a result of a magnet moving near the coil or the magnetic field through a coil changing.
- electromagnetic spectrum** Radio waves, microwaves, infra-red radiation, light, ultraviolet radiation, X-rays, gamma radiation.
- electromotive force (e.m.f.)** The electrical energy produced per unit charge by a source of electrical energy; the unit of e.m.f. is the volt (V).
- electron** Negative particle in the atom.
- electron diffraction** The diffraction of electrons in a beam when the beam is directed at a thin metal foil.
- electron volt** The work done to move an electron through a p.d. of 1 volt.
- element** A substance that cannot be broken down by chemical methods into other substances.
- enthalpy** Heat transfer at constant pressure.
- equipotential** A surface of constant potential.
- escape speed** Minimum speed needed by an object to leave a planet.
- evaporation** Change of state from a liquid state to a gas.
- excitation** Process in which an electron in an atom moves to a higher energy level.
- expansion** Increase of volume of a substance when heated (e.g. increase in the length of a bar when it is heated).
- exponential decay** Process in which a quantity decreases by 50% every half-life.
- extension** Increase in length of a solid when it is stretched.
- Faraday's law of electromagnetic induction** The induced e.m.f. is proportional to the rate of change of magnetic flux linkage.
- feedback** Part of the output voltage from an amplifier is returned to the input.
- ferromagnet** A material that can be magnetised permanently (e.g. iron or steel).
- fission** When an unstable nucleus splits into approximately two equal fragments.
- flow line** The path of a small amount of fluid.
- fluid** Any liquid or gas.
- focal length of a convex lens** The distance from the lens to the point where parallel rays are brought to a focus.
- force** Anything that can change the state of motion of an object.
- fossil fuel** Material formed from the decay of long-dead organisms that releases energy when it undergoes chemical change.
- free-body force diagram** A diagram showing a body with the forces acting on it.
- frequency** The number of complete cycles per second made by a vibrating object. The unit of frequency is the hertz (Hz), equal to 1 cycle per second.
- frequency modulation** Variation of the frequency of a carrier wave to carry information.
- fundamental frequency** The lowest frequency of vibration of a string or pipe.
- fuse** A thin wire that melts when excess current passes through it.
- fusion** Thermal fusion is when a solid melts; nuclear fusion is when two nuclei are forced to join together.
- gradient** The gradient of a straight line on a graph is the increase of vertical distance/increase of horizontal distance.
- gravitational potential** The work done per unit mass to move a test mass from infinity.

- gravity** The force of attraction between any two objects due to their mass.
- half-life** The time taken for the number of atoms of a radioactive isotope to decrease to half.
- hardness** Resistance of a surface to scratches and dents.
- heat** Energy transfer due to temperature difference.
- heat exchanger** A device in which thermal energy is transferred from a hot fluid to a cooler fluid without the fluids mixing with each other.
- Hooke's law** The extension of a spring is proportional to the tension in the spring.
- hydrometer** An instrument used to measure the density of a liquid.
- ideal gas** A gas that obeys Boyle's law.
- incompressible fluid** A fluid that cannot be compressed and therefore has constant density.
- induced e.m.f.** A voltage induced as a result of changing magnetic flux through a circuit.
- inductance** Flux linkage per unit current.
- inertia** Resistance to change of motion.
- insulator** A thermal insulator is a substance that will not conduct heat; an electrical insulator is a substance that will not conduct electricity.
- interference** When two or more sets of waves meet, points of cancellation and reinforcement are formed. Reinforcement occurs where a crest meets a crest or a trough meets a trough; cancellation occurs where a crest meets a trough.
- internal energy** The total kinetic and potential energy of the molecules due to their individual movements and positions.
- internal resistance** The resistance inside a source of electrical energy.
- ion** An atom that has become charged due to loss or gain of electrons.
- ionic bond** A chemical bond between two particles in which one or more electrons are transferred between the particles.
- ionisation** Formation of an ion by removal of an electron from an atom.
- ionising radiation** Radiation that causes ions to form; alpha radiation, beta radiation, X-radiation and γ -radiation are all ionising.
- isothermal change** A change at constant temperature.
- isotopes** Atoms that have different numbers of neutrons but the same number of protons.
- kilowatt hours** The energy supplied to a 1 kW electrical appliance in 1 hour. One kilowatt hour (kW h) = 3.6 million joules (= $1000 \text{ W} \times 3600 \text{ s}$).
- kinetic energy** Energy of a moving object.
- LDR** A light-dependent resistor.
- LED** A light-emitting diode.
- laser** A light source that produces a coherent, monochromatic parallel beam of light.
- latent heat** Energy needed or released when a material changes its physical state.
- Lenz's law** The induced current always acts in a direction so as to oppose the change.
- line emission spectrum** The pattern of coloured lines seen when light from a slit illuminated by a suitable light source is passed through a prism.
- linear magnification** Image height $\div$ object height.
- logic gate** A digital circuit which gives an output voltage that is determined by the state of the inputs.
- longitudinal waves** Waves in which the vibrations are parallel to the direction of travel of the wave.
- loudness** A measure of the amplitude of a sound wave.
- magnetic field** A region in which the pole of a magnet experiences a force.
- magnetic field strength (B)** See magnetic flux density.
- magnetic flux** Magnetic flux density, B , $\times$ area, A .
- magnetic flux density (B)** The force per unit length per unit current on a current-carrying wire perpendicular to the lines of force of the field.
- magnifying glass** A convex lens used to make an object appear larger.
- manometer** An instrument used to measure the pressure of a gas or a liquid.
- mass number** The number of neutrons and protons in a nucleus.
- metallic bonding** A type of bonding which holds atoms together in a metal.
- micrometer** An instrument used to measure thicknesses to within 1/100th of a millimetre.

- microwaves** Electromagnetic waves of wavelength between about 0.1 mm and 10 mm.
- moderator** Used in a nuclear reactor to slow the neutrons down to enable further fission to occur.
- modulation** Where radio waves or light waves are used to carry information.
- mole** The amount of matter consisting of Avogadro's number of identical particles.
- molecular bond** Weak, attractive bond between two uncharged atoms or molecules.
- molecule** The smallest particle of a compound or element that can exist independently.
- moment of a force about a point** Force $\times$ perpendicular distance from the line of action of the force to the point.
- moment of inertia** The sum of the mass $\times$ the square of the perpendicular distance from the axis for all points in a body.
- momentum** Mass $\times$ velocity.
- motor effect** The force on a current-carrying wire in a magnetic field. The wire must not be parallel to the lines of force of the field.
- multimeter** A meter that may be used as an ammeter or a voltmeter.
- multivibrator** A circuit which switches between two output states.
- neutron** An uncharged particle inside the nucleus that has approximately the same mass as the proton.
- Newton's first law of motion** An object remains at rest or in uniform motion unless acted on by a force.
- Newton's second law of motion** The force on an object is proportional to the change of momentum per second of the object. The equation for Newton's second law for constant mass is 'force = mass $\times$ acceleration'.
- node** A point of no displacement in a stationary wave pattern.
- noise** Sound produced at random.
- non-newtonian fluid** A fluid with viscosity that changes due to stirring it.
- nuclear energy** Energy released when a heavy nucleus splits or light nuclei fuse together.
- nuclear fission** See fission.
- nucleon** A neutron or proton in the nucleus.
- nucleus** The nucleus of an atom is a tiny part of the atom where most of the atom's mass is located and which has a positive charge. It consists of neutrons and protons.
- Ohm's law** The current through a conductor that obeys Ohm's law is proportional to the potential difference across the conductor, provided the physical conditions do not change.
- operational amplifier** An amplifier designed to carry out mathematical operations.
- orbit** The path of a planet around the Sun or the path of a satellite around a planet.
- oscilloscope** An electronic instrument used to display and measure waveforms.
- parallel components** Components connected together in a circuit with the same potential difference across their terminals.
- parallelogram of forces** The rule used to add together two force vectors.
- phase difference** The fraction of a cycle between the motion of two objects vibrating at the same frequency.
- photon** Light consists of photons. Each photon is a wave packet of electromagnetic energy emitted by an atom; $E = hf$.
- pitch** The frequency of a note or sound.
- Pitot-static tube** A device used to measure the flow speed of a fluid.
- plane mirror** A flat mirror.
- plastic behaviour** Permanent change of shape of an object
- polarised waves** Transverse waves that vibrate in one plane only.
- polymer** A solid composed of long molecules, each made from smaller, identical molecules.
- positron** The antimatter particle of the electron. If a positron and an electron collide, they annihilate each other to produce radiation.
- potential difference (or voltage)** The work done per unit charge when charge flows between two points in a circuit. The unit of potential difference is the volt, equal to 1 joule per coulomb.

potential divider Two resistors in series connected to a fixed voltage supply. The potential difference of the supply is shared or 'divided' between the two resistors.

potential energy Energy due to position.

potentiometer A potential divider used to compare cell e.m.f.s.

power Energy transferred per second; the unit of power is the watt (W), equal to 1 joule per second. Note that 1 kilowatt = 1000 watts.

prefixes Roman or Greek letters used to denote powers of ten. Prefixes to remember include: milli- ($m = 10^{-3}$), micro- ($\mu = 10^{-6}$), kilo- ($k = 10^3$), and mega- ($M = 10^6$).

pressure Force per unit area applied at right angles to a surface. The unit of pressure is the pascal (Pa), equal to 1 newton per square metre.

primary colours of light Red, blue or green light.

principle of moments The sum of the clockwise moments about a fixed point is equal to the sum of the anticlockwise moments about the same point.

projectile An object in motion acted upon only by the force of gravity.

proton A positive particle inside the nucleus that has approximately the same mass as the neutron and a charge equal and opposite to that of the electron.

pulse code modulation Using a carrier wave to transmit pulses in a coded sequence.

quark Protons and neutrons are composed of quarks.

radio waves Electromagnetic waves of wavelengths longer than about a millimetre.

radioactive decay Disintegration of an unstable nucleus of an atom as a result of emitting an alpha or a beta particle or gamma radiation.

radioisotope An isotope that is radioactive.

random motion Unpredictable motion of a particle.

reactance $1/2\pi fC$ for a capacitor; $2\pi fL$ for an inductor.

real image An image that can be projected on a screen.

rectification Conversion of alternating current into direct current, achieved using one or more diodes.

reflection This occurs when a wave bounces off a smooth surface. For a light ray reflecting off a mirror, the angle of incidence is equal to the angle of reflection.

refraction This occurs when a wave changes direction on passing from one substance to another. A light ray passing from air to glass bends towards the normal.

refractive index of a substance The ratio of the wavelength in air to the wavelength in the substance.

relay An electromagnetic device in which a small current is used to switch a larger current on or off.

renewable energy Energy directly or indirectly from the Sun that does not need a chemical change to become available.

resistance The resistance of a resistor is defined as the potential difference $\div$ current. The unit of resistance is the ohm (Ω), equal to 1 volt per ampere.

resistivity Resistance $\times$ area of cross-section/length.

resistors in parallel The potential difference is the same; the current divides between the branches; the combined resistance of two resistors, R_1 and R_2 , in parallel $= (1/R_1 + 1/R_2)^{-1}$.

resistors in series The current is the same; the potential difference divides across the components; the combined resistance of two resistors, R_1 and R_2 , in series $= R_1 + R_2$.

resonance An oscillating system subjected to regular pushes at its natural frequency vibrates with a large amplitude.

root mean square current The value of direct current that gives the same power in a given resistor.

root mean square speed Square root of the mean of the squared speeds of the molecules in a gas.

scalar Any physical quantity that is not directional (e.g. speed, mass, energy).

scanning tunnelling microscope Device that uses the tunnelling effect of electrons between a fine tip and a surface to observe surface features such as rows of atoms.

semiconductors Substances that conduct better with increasing temperature.

- series components** Components in an electric circuit that are connected together so they pass the same current.
- sound** Vibrations in a substance that travel through the substance.
- specific heat capacity** The energy needed to raise the temperature of 1 kg of material by 1 K.
- specific latent heat** The energy lost or gained by 1 kg of material when it changes its physical state.
- spectrometer** An instrument used to measure accurately the wavelengths of a spectrum.
- spectrum** The colours of light seen when a parallel beam of white light is split into colours by a prism.
- speed** Distance –; time. The unit of speed is the metre per second.
- spherical aberration** A lens defect in which the outer rays of a parallel beam are brought to a different focus to that of the inner rays.
- sphygmomanometer** Used to measure blood pressure.
- standard pressure** The mean pressure of the atmosphere at sea level.
- stationary waves** Wave pattern formed in which there are fixed points of no displacement.
- stiffness** A measure of the force needed to change the shape of an object.
- strain** Change of length per unit length.
- streamline** A flow line that is stable and does not move about and become mixed up.
- strength** A measure of the force needed to break an object.
- stress** Force per unit area of cross-section acting perpendicular to an area.
- sublimation** Occurs when a solid changes directly to a vapour.
- temperature** A measure of the hotness of an object.
- Temperature** is measured in degrees Celsius ($^{\circ}\text{C}$) or in kelvins (K).
- tension** The force in an object that has been stretched.
- terminal velocity** The velocity that an object acted on by a constant ‘motive’ force (e.g. force of gravity for a falling object, engine force for a powered vehicle) reaches when moving through a fluid.
- thermal conduction** Heat transfer through a substance due to individual movement of particles in the substance.
- thermal expansion** The increase of the length of an object due to increase of temperature.
- thermal radiation** Electromagnetic radiation emitted from a body due to its temperature.
- thermionic emission** The emission of electrons from a hot metal surface.
- thermistor** A resistor with a resistance designed to change with temperature.
- thermocouple** A junction formed by two metal wires; the potential difference across the junction depends on the temperature of the junction. A thermocouple connected to a suitable meter may be used as a thermometer.
- total internal reflection** This is seen when a light ray in a substance strikes the boundary of the substance and reflects totally. It occurs when the angle of incidence exceeds the critical angle.
- toughness** Ability of a material to withstand impacts.
- transformer** A device that steps an alternating voltage up or down.
- transistor** An electronic device in which a small current controls a much larger current.
- transmission electron microscope** Device that uses an electron beam focused by magnetic lenses on to a fluorescent screen in order to observe features as small as 10^{-8} m.
- transverse waves** Waves in which the vibrations are at right angles to the direction of travel of the wave.
- turbulent flow** Flow that is stable and becomes mixed-up.
- tunnelling** An effect due to the wave nature of matter particles where particles can pass through a barrier due to their wave nature.
- ultraviolet radiation** Electromagnetic radiation between the violet end of the visible spectrum (wavelength about 400 nm) and X-rays (wavelengths less than about 1 nm).
- ultrasonics** Sound waves of frequency higher than 18 kHz which is the upper frequency range of the human ear.

upthrust The upward force on an object in a fluid due to the pressure of the fluid.

U-tube manometer A device used to measure the pressure of a gas or a liquid.

U-value The heat loss per second per square metre for a 1 K (or 1°C) temperature difference across a wall or window or roof or floor.

vaporisation The change of physical state when a liquid becomes a vapour.

vapour A gas that can be turned into a liquid by compression only.

vector Any physical quantity that has a direction as well as a magnitude (e.g. velocity, acceleration, force, weight, momentum).

velocity Speed in a given direction.

velocity gradient in a fluid Change of velocity per unit distance across the flow of the fluid.

vernier calipers An instrument used to measure distances up to 200 mm or more to within 0.1 mm.

virtual image An image that is formed where light rays appear to come from after reflection by a mirror or refraction at a boundary.

viscosity Internal friction in a fluid; the coefficient of viscosity of a fluid is the internal friction per unit area of layers of the fluid where the velocity gradient is 1 m s^{-1} per metre across the flow.

voltage Another word for potential difference.

voltage gain Output voltage $\div$ input voltage.

wave particle duality Matter particles and photons have a wave-like nature and a particle-like nature.

wavelength The distance between two adjacent wavecrests.

weight The force of gravity on an object. For a mass, m , its weight $= mg$.

Wheatstone bridge A circuit used to compare resistances.

work Energy transferred by a force.

work function The work necessary for an electron to leave an uncharged surface.

X-rays Electromagnetic waves of wavelength less than about 1 nm.

Young modulus of elasticity Stress $\div$ strain.

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